CMB Anomalies from Preinflationary Physics in Loop Quantum Cosmology

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Based on work with I. Agullo, A. Ashtekar, B. Gupt, D. Jeong and D. Kranas.

Phys. Rev. Lett. 125, 051302 (2020); Front.Astron.Space Sci. 8 (2021) 76

Gen.Rel.Grav. 53 (2021) 2, 17; Class.Quant.Grav. 38 (2021) 6, 065010; arXiv:2105.12993 [gr-qc].

Indian Institute of Technology Madras, Chennai

17 July, 2021

Introduction

Cosmic Microwave Background Temperature

CMB temperature $T(\hat{n})$ can be conveniently split into two parts [1]:

- Isotropic part :

$$\bar{T} = \frac{1}{4\,\pi} \int \mathrm{d}\Omega\, T(\hat{n})$$

- Anisotropic part :

$$\delta T(\hat{n}) \equiv \frac{T(\hat{n}) - \bar{T}}{\bar{T}} = \sum_{\ell\, m} a_{\ell\, m}\, Y_{\ell\, m}(\hat{n}),$$

[1] see, for instance, S. Weinberg, Cosmology (Oxford University Press, 2008); R. Durrer, The Cosmic Microwave Background (Cambridge University Press, Cambridge, 2008).

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COBE (Fixsen 2009)

$\bar{T} = 2.7260\, K$

At multipoles $\ell \lesssim 2500$:

Planck

CMB Temperature anisotropies

ΛCDM model predicts only the statistical properties of the temperature fluctuations. Hence, we are interested in moments of $\delta T(\hat{n})$ or in multipole space $a_{\ell\, m}$.

We will focus on two-point correlations of temperature fluctuations. For a statistically homogeneous and isotropic universe, $\langle a_{\ell\, m} a^*_{\ell'\, m'} \rangle$ must be diagonal in ℓ and m, and m-independent:

$$\langle a_{\ell m}\, a^*_{\ell' m'} \rangle \;=\; C_\ell\, \delta_{\ell\ell'}\, \delta_{mm'} \,.$$

Deviation from statistical isotropy or homogeneity would imply a non-diagonal covariance of $a_{\ell m}$.

Equivalently, in angular space, $\langle \delta T(\hat{n})\, \delta T(\hat{n}') \rangle$ depends only on the angle between the two directions.

$$C(\theta) \;\equiv\; \langle \delta T(\hat{n})\, \delta T(\hat{n}') \rangle \;=\; \frac{1}{4\pi} \sum_\ell (2\,\ell \,+\, 1)\, C_\ell\, P_\ell(\cos\theta) \,.$$

Cosmic Variance

Given a theoretical model, we can compute predictions for C_ℓ or $C(\theta)$ exactly. However, observations are constrained due to the fact that we only have one realisation of the Universe. Thus, a theoretical model will agree with the observations only within an error bar known as cosmic variance:

$$\begin{aligned} \sigma^2(C_\ell) &= \pm\sqrt{\frac{2}{2\,\ell\,+\,1}}\,C_\ell \\ \sigma^2(C(\theta)) &= \frac{1}{8\,\pi^2}\sum_\ell (2\,\ell\,+\,1)\,C_\ell^2\,P_\ell^2(\cos\theta) \end{aligned}$$

Anomalies in the Cosmic Microwave Background

Anomalies [2] refer to certain features in the CMB that deviate from the predictions of ΛCDM model together with the standard ansatz.

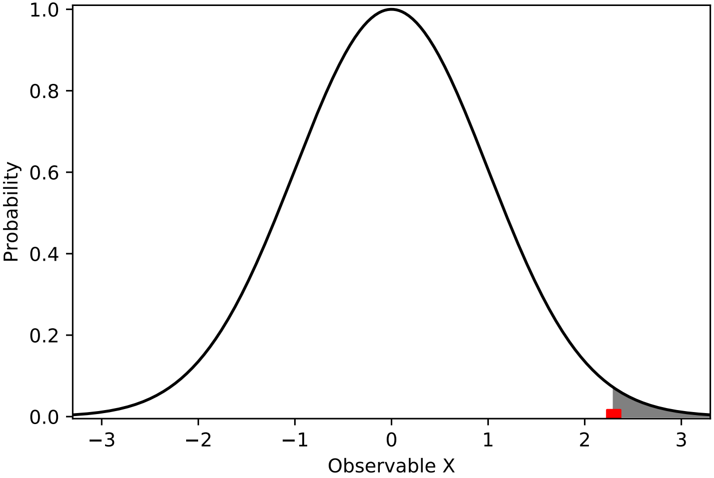
- Power suppression at large angular scales
- Dipolar modulation
- Lensing anomaly
- Preference for odd parity etc.

They are called anomalies because their departure from the predictions of standard model is only of the order of $2-3\,\sigma$. Hence, taken individually, the statistical significance of these anomalies is not enough to establish a violation of ΛCDM model. However, the presence of several of these anomalies, indicates either that we live in a very rare realisation of the universe predicted by ΛCDM model, or they may be signatures of new physics.

[2] Planck collaboration, A&A 571, A23 (2014); A&A 594, A16 (2016); A&A 641, A7 (2020).

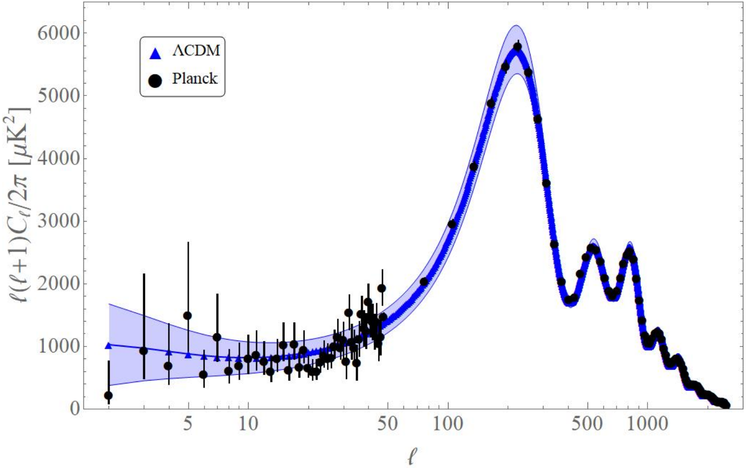
Statistical significance of CMB anomalies

ΛCDM model predicts only the statistical property of temperature fluctuations. Hence, in order to qualify an observation as a signal of departure from ΛCDM, one needs to quantify the amount by which an observation departs from theoretical predictions: p-value.



Power Suppression

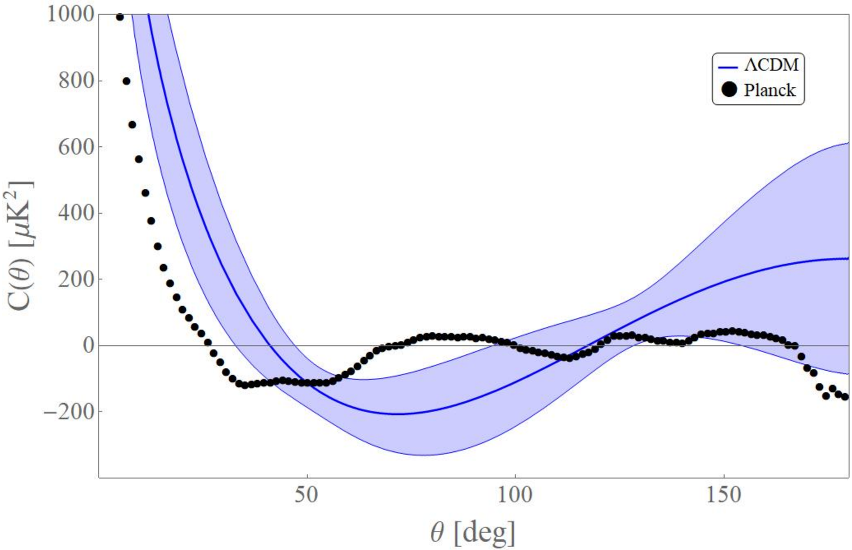
In multipole space:



ΛCDM refers to standard six parameter model together with the standard ansatz (SA) of nearly scale invariant spectrum.

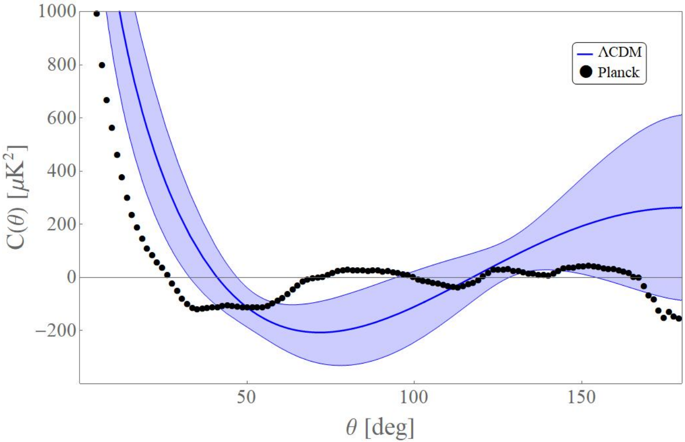
Power Suppression

In angular space:



Power Suppression[3]

In angular space:



Power at angular scales $\theta > 60^0$ is very small. It is quantified using

$$S_{1/2} = \int_{-1}^{1/2} C(\theta)^2 \, \mathrm{d}(\cos\theta)) \, .$$

Planck reports a value of $S_{1/2} = 1209$. ΛCDM leads to $S_{1/2} \approx 42000$.

[3] Planck collaboration: Y. Akrami *et al.*, A&A 641, A7 (2020).

Dipolar modulation anomaly

Planck has observed a dipolar modulation of the entire CMB which leads to a correlation between ℓ and $\ell + 1$ multipoles.

It was first modeled as[4]

$$T(\hat{n}) = T_0(\hat{n}) \left[1 + A_1\, \hat{n} \cdot \hat{d}\right] .$$

Planck has measured $A_1 \approx 0.07$ in the multipole range $[2 - 64]$ which departs from the ΛCDM by more than 3σ.

[4] C. Gordon, W. Hu, D. Huterer, and T. Crawford, Physical Review D 72 (2005).

Parity Anomaly

Observations from both WMAP and Planck have found a preference for odd parity two-point correlations at low multipoles. The asymmetry in the parity can be quantified using the estimator[5]

$$R^{TT}(\ell_{\rm max}) = \frac{D_{+}(\ell_{\rm max})}{D_{-}(\ell_{\rm max})},$$

where $D_{\pm}(\ell_{\rm max})$ are defined as

$$D_{\pm}(\ell_{\rm max}) = \frac{1}{\ell_{tot}^{\pm}} \sum_{2,\ell_{max}}^{\pm} \frac{\ell(\ell+1)}{2\,\pi} C_\ell$$

where the $+$ or $-$ signs on the right refer to the fact that we include only even or odd multipoles in the sum, respectively, and $\ell_{tot}^{\pm}$ refers to the total number of multipoles in the sum.

[5] Planck collaboration, A&A 571, A23 (2014); A&A 594, A16 (2016); A&A 641, A7 (2020).

Lensing Anomaly

- Along with measurements of temperature(T) and polarisation(E and B), Planck has also reconstructed lensing potential (ϕ).
- The effect of lensing is a smoothing of small angular scales in the CMB.
- The level of smoothing observed in the temperature and polarisation spectra should be consistent with the reconstructed power spectrum of lensing potential $C_\ell^{\phi\phi}$.
- Lensing parameter[6] A_L which scales $C_\ell^{\phi\phi}$ was introduced as an additional free parameter to check this consistency.
- If we perform a MCMC simulation by varying the standard six parameters along with A_L, if everything was consistent, $A_L = 1$.

Planck reports[7] $A_L = 1.072 \pm 0.041$ (when compared with TT+TE+EE+lowE+lensing data), which is greater than one by more than 1σ.

[6] E. Calabrese *et al*, Phys. Rev. D 77, 123531 (2008).
[7] Planck Collaboration : N. Aghanim *et al*, A & A 641, A6 (2020).

CMB Anomalies and Planck Scale Physics in LQC

Loop Quantum Cosmology

Effective equations describing background[8]:

$$H^2 = \frac{8\pi G}{3}\left(1 - \frac{\rho}{\rho_c}\right), \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\left(1 - \frac{4\rho}{\rho_c}\right) - 4\pi G P\,(1 - 2\frac{\rho}{\rho_c})$$

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0.$$

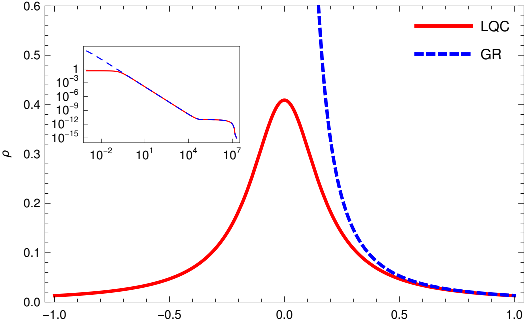
Use dressed metric approach to describe perturbations :

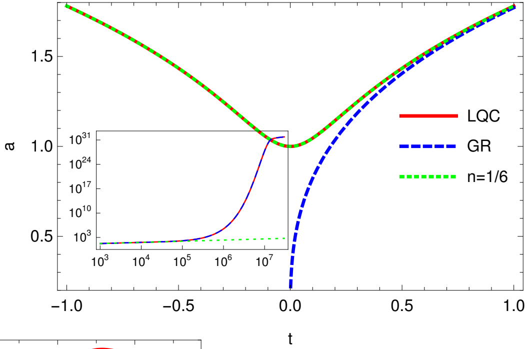
$$\left[\Box + \mathcal{U}/a^2\right] Q = 0.$$

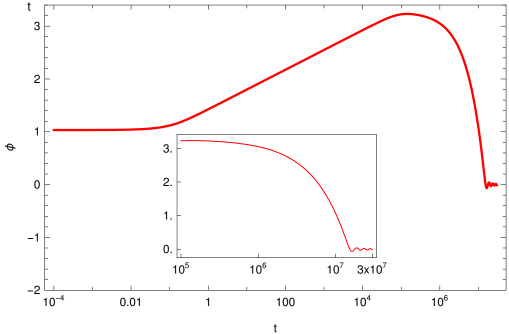
where $\Box$ and $\mathcal{U}$ are defined with respect to the dressed metric.

[8] See, for instance, A. Ashtekar and P. Singh, Class. Quant. Grav. 28, 213001.(2011); I. Agullo and P. Singh, World Scientific (2017).

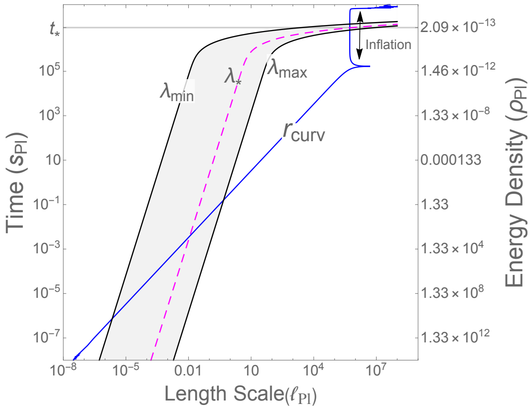
Background Dynamics

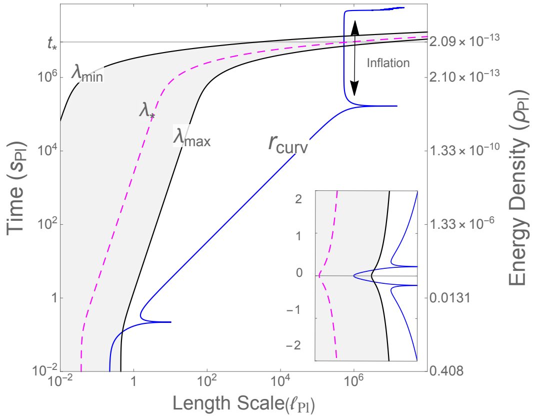






Loop Quantum Cosmology

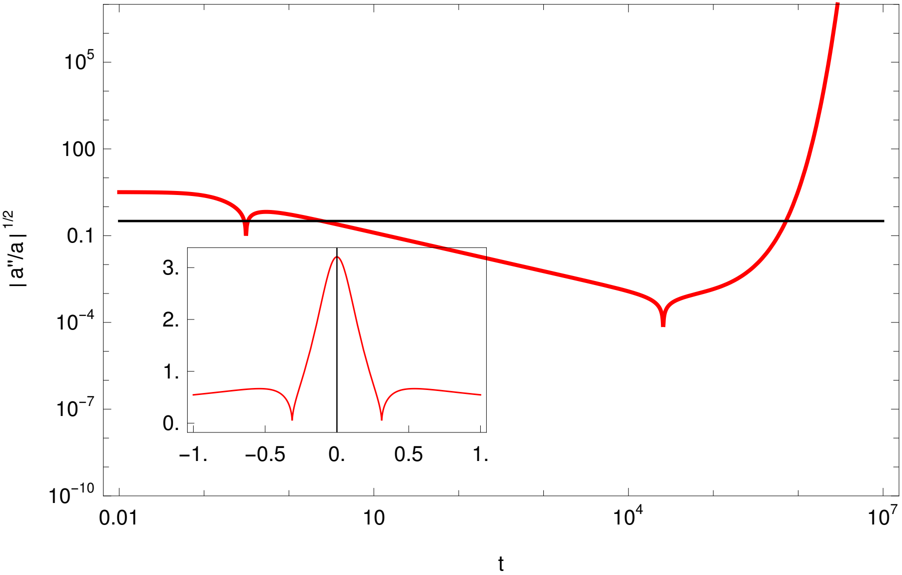




In LQC[9], there is an upper bound on the scalar curvature. This implies that only long wavelengths feel the effect of curvature at the bounce.

[9] See, for instance, A. Ashtekar and P. Singh, Class. Quant. Grav. 28, 213001.(2011); I. Agullo and P. Singh, World Scientific (2017).

Curvature at the bounce sets a new scale



Curvature at the bounce sets a new scale in the problem : k_{LQC}

Initial conditions

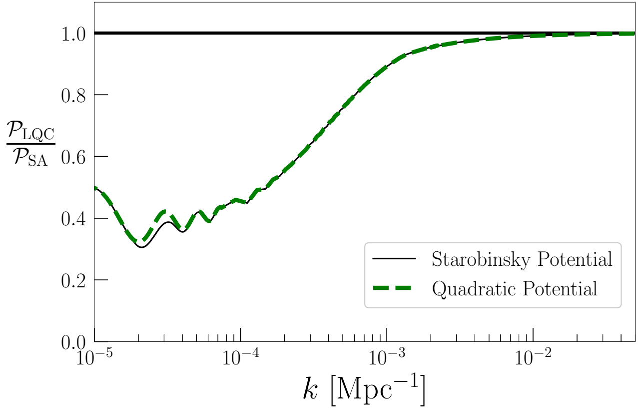
We work with some trial principles [10] to narrow down the initial state for both background and perturbations:

- The expansion of the universe is such that the area of the horizon at the bounce corresponds to the smallest area possible. This corresponds to an expansion of about 141 e-folds from the bounce till today.

- The initial state of perturbation is fixed using a quantum generalization of Penrose's Weyl curvature hypothesis, which requires the quantum state of perturbations to be as homogeneous and isotropic in the Planck regime as allowed by Heisenberg's uncertainty principle.

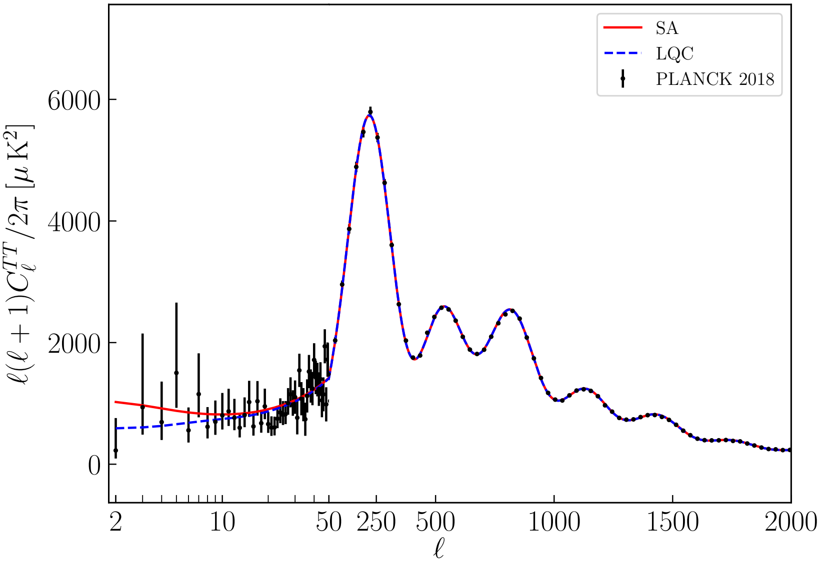
[10] A. Ashtekar and B. Gupt, Class. Quan. Grav. 34, 035004 (2017); Class. Quan. Grav. 34, 014002 (2017).

Primordial power spectrum



$$\mathcal{P}_{\rm LQC}(k) = f(k)\, A_s \left(\frac{k}{k_\star}\right)^{n_s-1}$$
$$= f(k)\, \mathcal{P}_{\rm SA}(k)$$

Anomalies Alleviated : Power suppression



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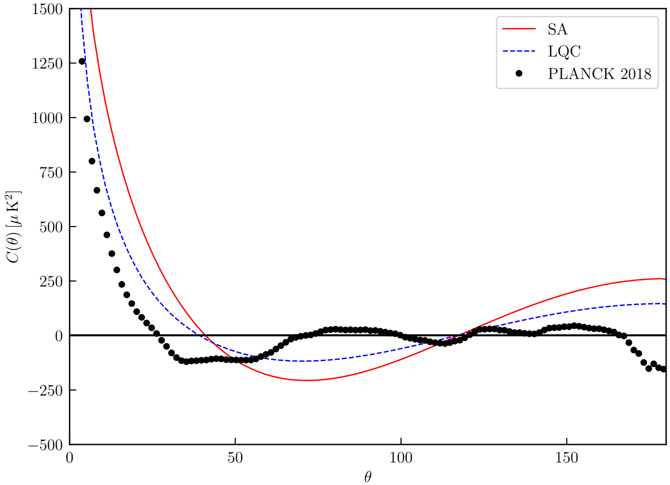
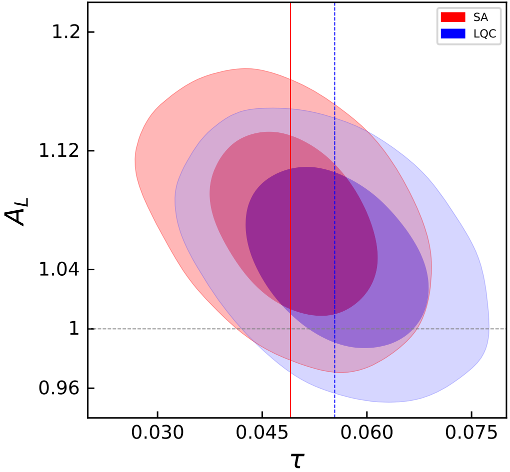


Model	$S_{1/2}$
SA	42496.5
LQC	14308.05

$S_{1/2}$ in LQC is only $1/3$ of that obtained in SA. These numbers should be compared with $S_{1/2} = 6771.7$ obtained from Planck C_ℓ^{TT} data.

Anomalies Alleviated : Lensing anomaly

Surprisingly, lower power at longer wavelengths also leads to a preference for lower values of A_L.



Model	A_L
SA	1.072 ± 0.041
LQC	1.049 ± 0.040

Lower power at longer wavelengths imply that value of A_s in LQC is larger than that in SA. This in turn leads to a larger value of the reionization depth. Since, τ and A_L are anti-correlated, LQC allows for a lower value of A_L.

Non-Gaussian Origins of CMB Anomalies in a Cosmic Bounce

Non-Gaussian modulation

- Temperature fluctuations $\delta T/T$ are seeded by primordial perturbations $\mathcal{R}$.

[11] L. Dai, D. Jeong, M. Kamionkowski, and J. Chluba, Phys. Rev. D 87, 123005 (2013); F. Schmidt and M. Kamionkowski, Phys. Rev. D82, 103002 (2010); F. Schmidt and L. Hui, Phys. Rev. Lett. 110, 011301 (2013); D. Jeong and M. Kamionkowski, Phys. Rev. Lett. 108, 251301 (2012).

Non-Gaussian modulation

- Temperature fluctuations $\delta T/T$ are seeded by primordial perturbations $\mathcal{R}$.
- Smallness of temperature fluctuations $\delta T/T \approx \mathcal{O}\left(10^{-5}\right)$ imply that perturbation theory is an appropriate tool.

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- If primordial perturbations generated in the early universe were linear, then only those perturbations with wavelength smaller than the horizon will affect the CMB.

- However, if non-Gaussianity is present, then it could introduce coupling between super-horizon modes and the observable modes in the CMB. Such a modulation of observable modes by long wavelength super horizon modes is known as non-Gaussian modulation [11].

[11] L. Dai, D. Jeong, M. Kamionkowski, and J. Chluba, Phys. Rev. D 87, 123005 (2013); F. Schmidt and M. Kamionkowski, Phys. Rev. D82, 103002 (2010); F. Schmidt and L. Hui, Phys. Rev. Lett. 110, 011301 (2013); D. Jeong and M. Kamionkowski, Phys. Rev. Lett. 108, 251301 (2012).

Effect of Non-Gaussian modulation

We will study the effects of non-Gaussian modulation in two steps:

1. Effect on primordial perturbations.
2. Effect on CMB covariance matrix.

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Curvature perturbations in the presence of non-Gaussianity

A convenient and general way to model the effects of non-Gaussian correlations, is to write the curvature perturbations in terms of a Gaussian field $\mathcal{R}^G$ as follows

$$\mathcal{R}_{\vec{k}}(t) = \mathcal{R}^G_{\vec{k}}(t) + \frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\, f_{\rm NL}(\vec{q}, \vec{k}-\vec{q})\, \mathcal{R}^G_{\vec{q}}(t)\, \mathcal{R}^G_{\vec{k}-\vec{q}}(t)\,.$$

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where $f_{\rm NL}$ is related to three-point function through the relations

$$\begin{aligned}\langle \mathcal{R}_{\vec{k}_1}\mathcal{R}_{\vec{k}_2}\mathcal{R}_{\vec{k}_3}\rangle &= (2\pi)^3\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\, B_{\mathcal{R}}(\vec{k}_1,\vec{k}_2,\vec{k}_3)\\ B_{\mathcal{R}}(\vec{k}_1,\vec{k}_2,\vec{k}_3) &= f_{\rm NL}(\vec{k}_1,\vec{k}_2)\,[P_{\mathcal{R}}(\vec{k}_1)P_{\mathcal{R}}(\vec{k}_2)+P_{\mathcal{R}}(\vec{k}_2)P_{\mathcal{R}}(\vec{k}_3)+P_{\mathcal{R}}(\vec{k}_3)P_{\mathcal{R}}(\vec{k}_1)].\end{aligned}$$

$P_{\mathcal{R}}(\vec{k})$ is the power spectrum of $\mathcal{R}^G$, defined as

$$\langle \mathcal{R}^G_{\vec{k}_1}\mathcal{R}^{G\star}_{\vec{k}_2}\rangle = (2\pi)^3\,\delta(\vec{k}_1-\vec{k}_2)\,P_{\mathcal{R}}(\vec{k}_1)\,.$$

The dimensionless power spectrum is defined as $\mathcal{P}_{\mathcal{R}}(\vec{k}) = k^3\,P_{\mathcal{R}}(\vec{k})/2\pi^2$.

Two-point function in the presence of spectator mode

The long wavelength mode is frozen and can be considered as a spectator field. In its presence, $\langle \mathcal{R}_{\vec{k}_1} \, \mathcal{R}_{\vec{k}_2} \rangle$ can be computed as follows

$$\begin{aligned}\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}^{\star}_{\vec{k}_2} \rangle|_{\mathcal{R}_{\vec{q}}} = \langle \mathcal{R}^G_{\vec{k}_1} \mathcal{R}^{G\star}_{\vec{k}_2} \rangle \quad &+ \quad \frac{1}{2} \int \frac{d^3 q'}{(2\pi)^3} \, f_{\rm NL}(\vec{q}', \vec{k}_1 - \vec{q}') \, \langle \mathcal{R}^G_{\vec{q}'} \, \mathcal{R}^G_{\vec{k}_1 - \vec{q}'} \, \mathcal{R}^{G\star}_{\vec{k}_2} \rangle \\ &+ \quad \frac{1}{2} \int \frac{d^3 q'}{(2\pi)^3} \, f_{\rm NL}(\vec{q}', \vec{k}_2 - \vec{q}') \, \langle \mathcal{R}^G_{\vec{k}_1} \, \mathcal{R}^{G\star}_{\vec{q}'} \, \mathcal{R}^{G\star}_{\vec{k}_2 - \vec{q}'} \rangle + \mathcal{O}(f^2_{\rm NL}).\end{aligned}$$

Taking the spectator mode out of the average, we obtain

$$\begin{aligned}\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}^{\star}_{\vec{k}_2} \rangle|_{\mathcal{R}_{\vec{q}}} \quad &= \quad (2\pi)^3 \, \delta(\vec{k}_1 - \vec{k}_2) \, P_{\mathcal{R}}(\vec{k}_1) \\ &+ \quad f_{\rm NL}(\vec{k}_1, -\vec{k}_2) \, \frac{1}{2} \left(P_{\mathcal{R}}(\vec{k}_1) + P_{\mathcal{R}}(\vec{k}_2)\right) \mathcal{R}_{\vec{q}} + \cdots .\end{aligned}$$

Modulated power spectrum

Modulated two-point function is

$$\begin{aligned}\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}^{\star}_{\vec{k}_2} \rangle|_{\mathcal{R}_{\vec{q}}} &= (2\pi)^3\, \delta(\vec{k}_1 - \vec{k}_2)\, P_{\mathcal{R}}(\vec{k}_1) \\ &+ f_{\rm NL}(\vec{k}_1, -\vec{k}_2)\, \frac{1}{2} \left(P_{\mathcal{R}}(\vec{k}_1) + P_{\mathcal{R}}(\vec{k}_2)\right) \mathcal{R}_{\vec{q}} + \cdots .\end{aligned}$$

- Non-Gaussianity leads to a modulation of power spectrum whose strength depends on the amplitude and shape of $f_{\rm NL}(\vec{k}_1, \vec{k}_2)$.
- Since q is non-zero, $k_1 \neq k_2$. Thus, modulation introduces correction to the non-diagonal terms. Such non-diagonal terms indicate a breaking of homogeneity in the local patch.
- When averaged over all realizations of $\mathcal{R}_{\vec{q}}$, modulated term vanishes (since, $\langle \mathcal{R}_{\vec{q}} \rangle = 0$) indicating underlying statistical homogeneity.

Modulated power spectrum

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Statistical homogeneity broken only in a particular realization due to modulation!

Modulation of CMB

The primordial perturbations $\mathcal{R}_{\vec{k}}$ are related to the CMB multipole coefficients $a_{\ell m}$ through the relation

$$a_{\ell m} = 4\pi \int \frac{d^3k}{(2\pi)^3} (-i)^\ell \, \Delta_\ell(k) \, Y^*_{\ell m}(\hat{k}) \, \mathcal{R}_{\vec{k}} \, .$$

Then, the covariance matrix is

$$\langle a_{\ell m} a^\star_{\ell' m'} \rangle = (4\pi)^2 \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} (-i)^{\ell-\ell'} \, \Delta_\ell(k_1) \, \Delta_{\ell'}(k_2) \, Y^*_{\ell m}(\hat{k}_1) \, Y_{\ell' m'}(\hat{k}_2) \, \langle \mathcal{R}_{\vec{k}_1} \mathcal{R}^\star_{\vec{k}_2} \rangle |_{\mathcal{R}_{\vec{q}}} \, ,$$

Upon expanding $f_{\rm NL}$ and $\mathcal{R}_{\vec{q}}$ in terms of Legendre polynomials and spherical harmonics respectively,

$$\begin{aligned} f_{\rm NL}(k_1, q, \mu) &= \sum_L G_L(k_1, q) \, \frac{2L+1}{2} \, P_L(\mu) \\ \mathcal{R}^G_{\vec{q}} &= \sum_{L'M'} \mathcal{R}^G_{L'M'}(q) \, Y_{L'M'}(\hat{q}), \end{aligned}$$

one can write

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \, \delta_{\ell\ell'} \, \delta_{mm'} + (-1)^{m'} \sum_{LM} A^{LM}_{\ell\ell'} \, C^{LM}_{\ell m \ell' - m'} \, .$$

Modulated covariance

Modulated covariance matrix is

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In the above, $A^{LM}_{\ell\ell'}$ is the Bipolar Spherical Harmonic (BipoSH) coefficient and it is given by

$$\begin{aligned} A^{LM}_{\ell\ell'} &= \frac{4}{(2\pi)^3} \int dk_1 \, k_1^2 \, dq \, q^2 \, (-i)^{\ell-\ell'} \, \Delta_\ell(k_1) \, \Delta_{\ell'}(k_1) \, P_{\mathcal{R}}(k_1) \, G_L(k_1, q) \, \mathcal{R}^G_{LM}(q) \\ &\times \quad C^{L0}_{\ell 0 \ell' 0} \sqrt{\frac{(2\ell+1)(2\ell'+1)}{4\pi \, (2L+1)}} \, . \end{aligned}$$

Modulated covariance

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$$\times \quad C^{L0}_{\ell 0 \ell' 0} \sqrt{\frac{(2\ell+1)(2\ell'+1)}{4\pi \, (2L+1)}} \, .$$

Two Clebsch-Gordon coefficients impose the following properties on $A^{LM}_{\ell\ell'}$:

i. $L = 0$, then $\ell_1 = \ell_2$

ii. $L = 1$, then $|\ell_1 - \ell_2| = 1$

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Modulated covariance

Modulated covariance matrix is

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \, \delta_{\ell\ell'} \delta_{mm'} + (-1)^{m'} \sum_{LM} A^{LM}_{\ell\ell'} \, C^{LM}_{\ell m \ell' -m'} \, .$$

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$$\begin{aligned} A^{LM}_{\ell\ell'} &= \frac{4}{(2\pi)^3} \int dk_1 \, k_1^2 \, dq \, q^2 \, (-i)^{\ell-\ell'} \, \Delta_\ell(k_1) \, \Delta_{\ell'}(k_1) \, P_{\mathcal{R}}(k_1) \, G_L(k_1, q) \, \mathcal{R}^G_{LM}(q) \\ &\times \quad C^{L0}_{\ell 0 \ell' 0} \sqrt{\frac{(2\ell+1)(2\ell'+1)}{4\pi \, (2L+1)}} \, . \end{aligned}$$

Two Clebsch-Gordon coefficients impose the following properties on $A^{LM}_{\ell\ell'}$:

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Standard deviation of BipoSH coefficient

$A^{LM}_{\ell\ell'}$ depend on the mode $\mathcal{R}^G_{\vec{q}}$. Since, $\mathcal{R}^G_{\vec{q}}$ is a random variable, we cannot predict the exact value of $A^{LM}_{\ell\ell'}$. We can only compute the standard deviation of the BipoSH coefficients, *i.e*

$$\sqrt{\langle |A^{LM}_{\ell\ell'}|^2\rangle} = \left[\frac{1}{2\pi}\int dq\, q^2\, P_{\mathcal{R}}(q)\, |\mathcal{C}^L_{\ell\ell'}(q)|^2\right]^{1/2} \times C^{L0}_{\ell 0 \ell' 0}\sqrt{\frac{(2\ell+1)(2\ell'+1)}{4\pi\,(2L+1)}}\,,$$

where

$$\mathcal{C}^L_{\ell\ell'}(q) \equiv \frac{2}{\pi}\int dk_1\, k_1^2\, (i)^{\ell-\ell'}\, \Delta_\ell(k_1)\Delta_{\ell'}(k_1)\, P_{\mathcal{R}}(k_1)\, G_L(k_1,q)\,.$$

These are the typical values that the BipoSH coefficients are expected to take in the sky. If these values are large, the effects they entail should be expected in the CMB or, more precisely, they would have a large p-value and should not be considered anomalous.

The Model

Inspired from LQC, we consider a phenomenological model with a bounce preceding inflation. More accurately, we model the scale factor around the bounce as

$$a(t) = a_B(1 + b\,t^2)^n.$$

The Ricci curvature at the bounce is given by $R_B = 12\,n\,b$. So, bounce in this family of model can be described by two parameters R_B and n.

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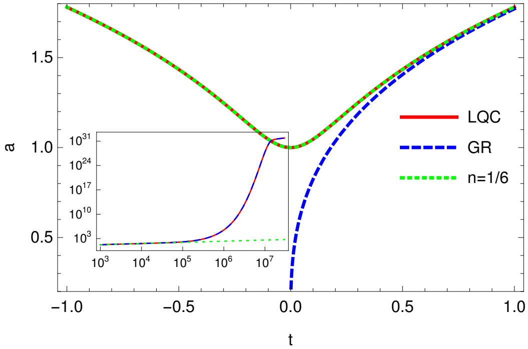
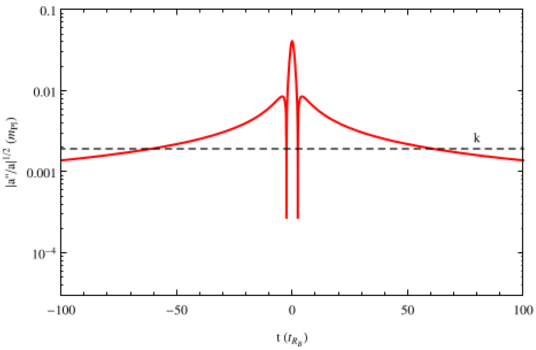
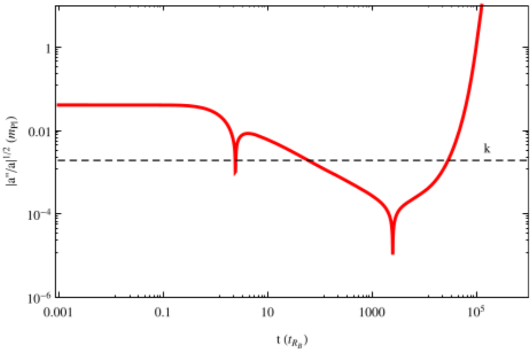


The scale factor near the bounce in LQC would correspond to $n \approx 1/6$.

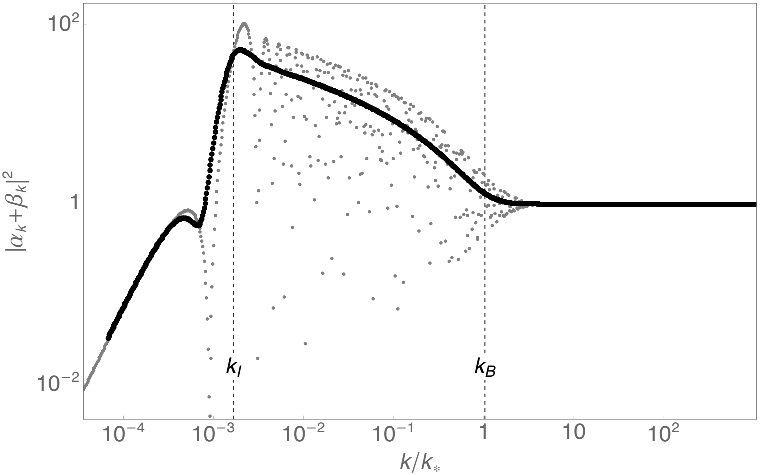
Evolution of perturbations





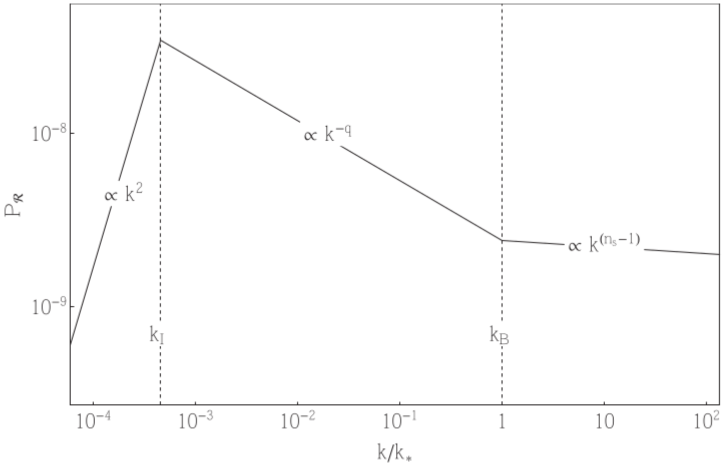
Power spectrum

An example power spectrum:



Plot of $\mathcal{P}(k)/\mathcal{P}_S A(k) = |\alpha_k + \beta_k|^2$.

Analytical ansatz for the power spectrum



Non-Gaussianity

We shall model[12] the non-Gaussianity generated in this model as

$$f_{\rm NL}(k_1,\, k_2,\, k_3) \;\simeq \mathfrak{f}_{\rm NL}\, e^{-\alpha\,(k_1\,+\,k_2\,+\,k_3)/k_{LQC}}\,,$$

where α depends on the curvature at the bounce. We will treat $\mathfrak{f}_{\rm NL}$ as a free parameter.

[12] I. Agullo I, B. Bolliet and V. Sreenath, Phys. Rev. D 97, 066021 (2018).

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Monopolar modulation ($L = 0$)

The modulated power spectrum is given by

$$C_\ell^{mod} = C_\ell \left(1 - \frac{(-1)^\ell}{C_\ell} \frac{A_{\ell\,\ell}^{00}}{\sqrt{2\,\ell + 1}} \right).$$

Note that $A_{\ell\,\ell}^{00}$ can be either positive or negative, leading to an enhancement or suppression of C_ℓ^{mod} with respect to C_ℓ. As explained before, we cannot predict the exact value of $A_{\ell\,\ell}^{00}$. The interesting quantity is rather the root-mean-square value of the modulation:

$$\sigma_0^2(\ell) = \frac{1}{C_\ell^2} \frac{\langle |A_{\ell\,\ell}^{00}|^2 \rangle}{2\,\ell + 1} = \frac{1}{C_\ell^2} \frac{1}{8\pi^2} \int dq\, q^2\, P_{\mathcal{R}}(q)\, |\mathcal{C}_{\ell\ell}^0(q)|^2 \,.$$

where

$$\mathcal{C}_{\ell\ell'}^L(q) \equiv \frac{2}{\pi} \int dk_1\, k_1^2\, (i)^{\ell-\ell'}\, \Delta_\ell(k_1) \Delta_{\ell'}(k_1)\, P_{\mathcal{R}}(k_1)\, G_L(k_1, q)\,.$$

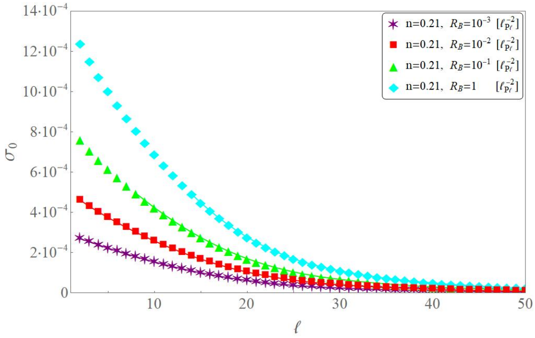
Power Suppression

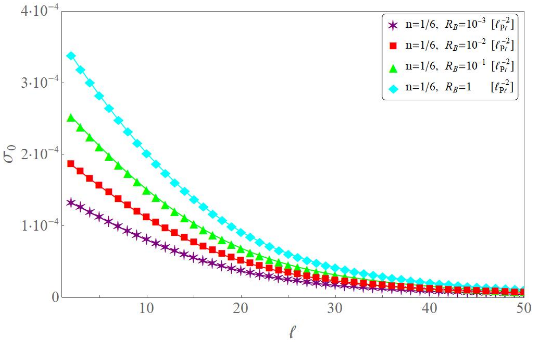
We fix $\mathfrak{f}_{\mathrm{NL}}$ by demanding that the probability of measuring $S_{1/2} \lesssim 1500$ is approximately 20%. This ensures that, observed value of $S_{1/2}$ is within $1\,\sigma$ away from the mean, making the observed value of $S_{1/2}$ less anomalous.

R_B \ n	$1/4$	0.21	$1/5$	$1/6$	$1/7$
$1\ell_{P\ell}^{-2}$	—	959	1334	3326	5031
$10^{-1}\ell_{P\ell}^{-2}$	—	1560	2065	4454	6298
$10^{-2}\ell_{P\ell}^{-2}$	—	2573	3238	6066	8024
$10^{-3}\ell_{P\ell}^{-2}$	—	4372	5234	8518	10530

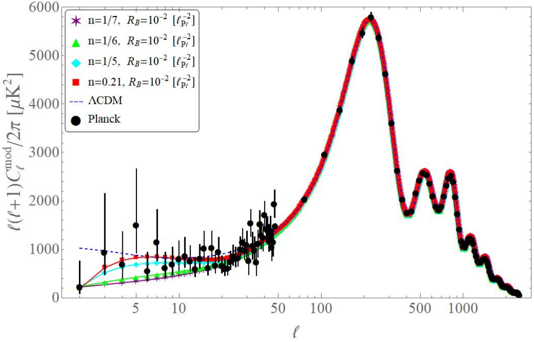
In the rest of this analysis, we work with the above values of $\mathfrak{f}_{\mathrm{NL}}$.

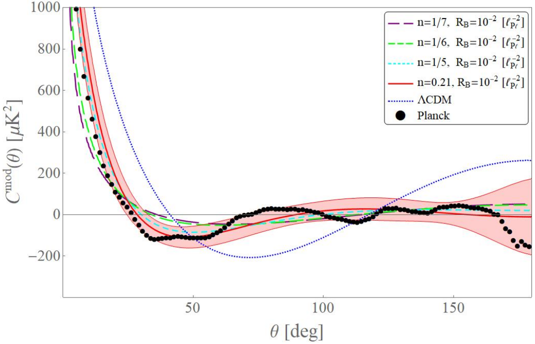
Scale dependent $\sigma_0(\ell)$





Modulated CMB power spectrum





Dipolar Modulation ($L = 1$)

The Planck team quantifies the dipolar modulation in terms of a scale-dependent amplitude $A_1(\ell)$[13]

$$A_1(\ell) \equiv \frac{3}{2}\sqrt{\frac{1}{3\pi}\left(|m_{1\,-1}|^2 + |m_{1\,0}|^2 + |m_{1\,1}|^2\right)},$$

where

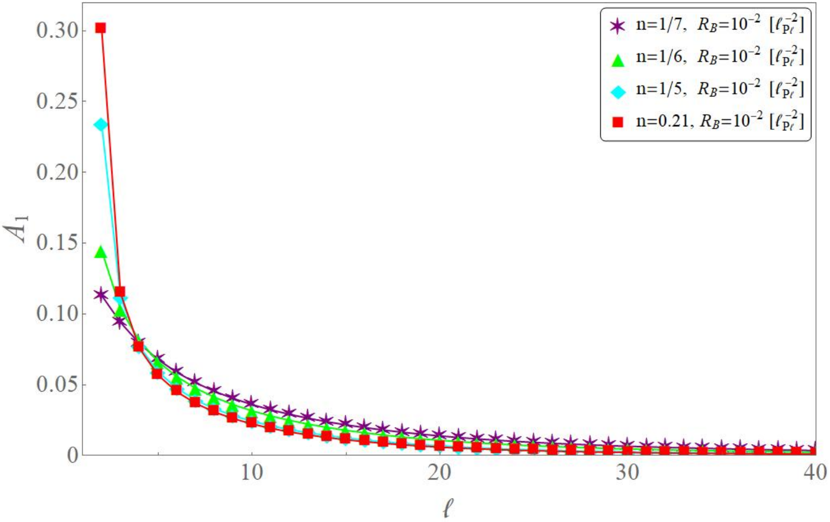
$$A^{1M}_{\ell\ell+1} \equiv m_{1M}\, G^1_{\ell\ell+1}, \qquad \text{with} \qquad G^1_{\ell\ell+1} \equiv (C_\ell + C_{\ell+1})\sqrt{\frac{(2\ell+1)(2\ell+3)}{4\pi\, 3}}\, C^{10}_{\ell,0,\ell+1,0}\,.$$

Comparing with root mean square of $A_1(\ell)$, we obtain

$$A_1(\ell) = \frac{3}{2}\frac{1}{\sqrt{\pi}}\frac{1}{C^{\mathrm{mod}}_\ell + C^{\mathrm{mod}}_{\ell+1}}\sqrt{\frac{1}{2\pi}\int dq\, q^2\, P_{\mathcal{R}}(q)\, |\mathcal{C}^1_{\ell\ell+1}(q)|^2}\,, \qquad (1)$$

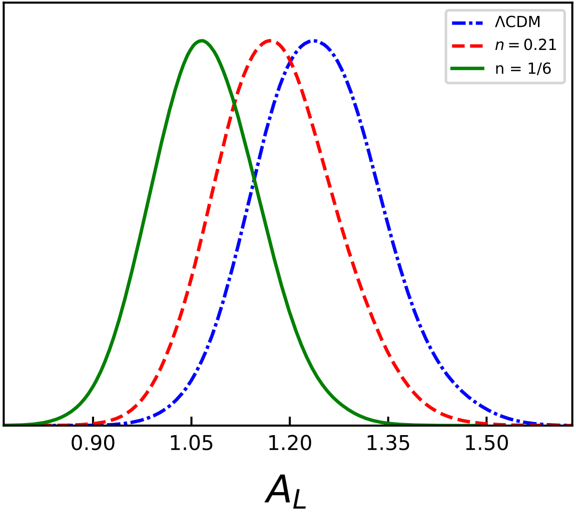
[13] P. A. R. Ade et al. (Planck), Astron. Astrophys. 594, A16 (2016)

Dipolar modulation from non-Gaussian modulation



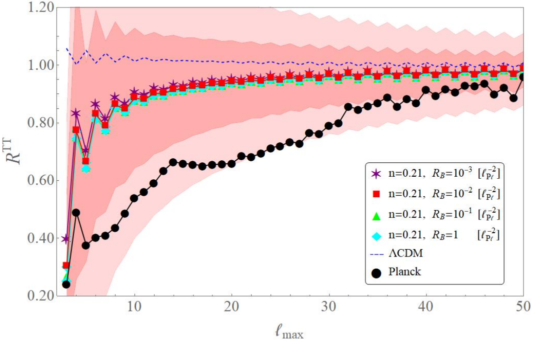
The amplitude and scale dependence of $A_1(\ell)$ is in consonance with observations. Recall that Planck observed a $A_1 \approx 0.07$ in the multipole bin $[2 - 64]$.

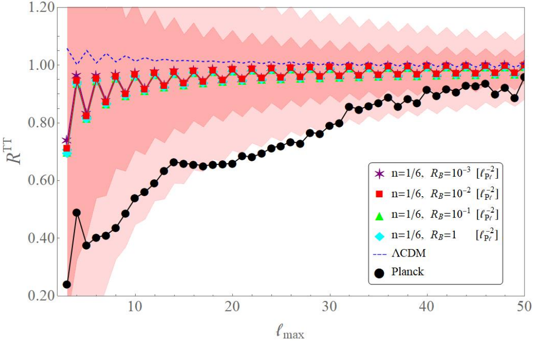
Lensing anomaly



For the modulation considered above the value of A_L becomes closer to one than in the standard model.

Preference for odd parity





Power suppression leads to a preference for odd parity.

Summary

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Motivated by this outlook, we considered two approaches:

- In one approach, we studied LQC in which we require the horizon size to be equal to the smallest possible area at the bounce and the initial conditions for quantum perturbations were chosen by Penrose's Weyl curvature hypothesis. We saw that power suppression and anomaly with lensing parameter gets alleviated.

- In the second approach, we considered phenomenological models motivated by LQC in which a bounce precedes inflation. We showed that non-Gaussianity generated in this model could lead to coupling between the long superhorizon scales and the long observable wavelengths. This coupling leads to a modulation of the CMB which makes multiple anomalies more likely to occur in the observed universe. In particular, this approach leads to both power suppression and dipolar modulation with amplitude and shape similar to the observations.

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Thank you!