

Towards a possible solution to the Hubble tension with Horndeski gravity

(Based on arXiv: 2301.09382)

Co-authors: Basundhara Ghosh & Rajeev Kumar Jain



Yashi Tiwari

Senior Research Fellow, Joint Astronomy Programme

Department of Physics, Indian Institute of Science, Bangalore

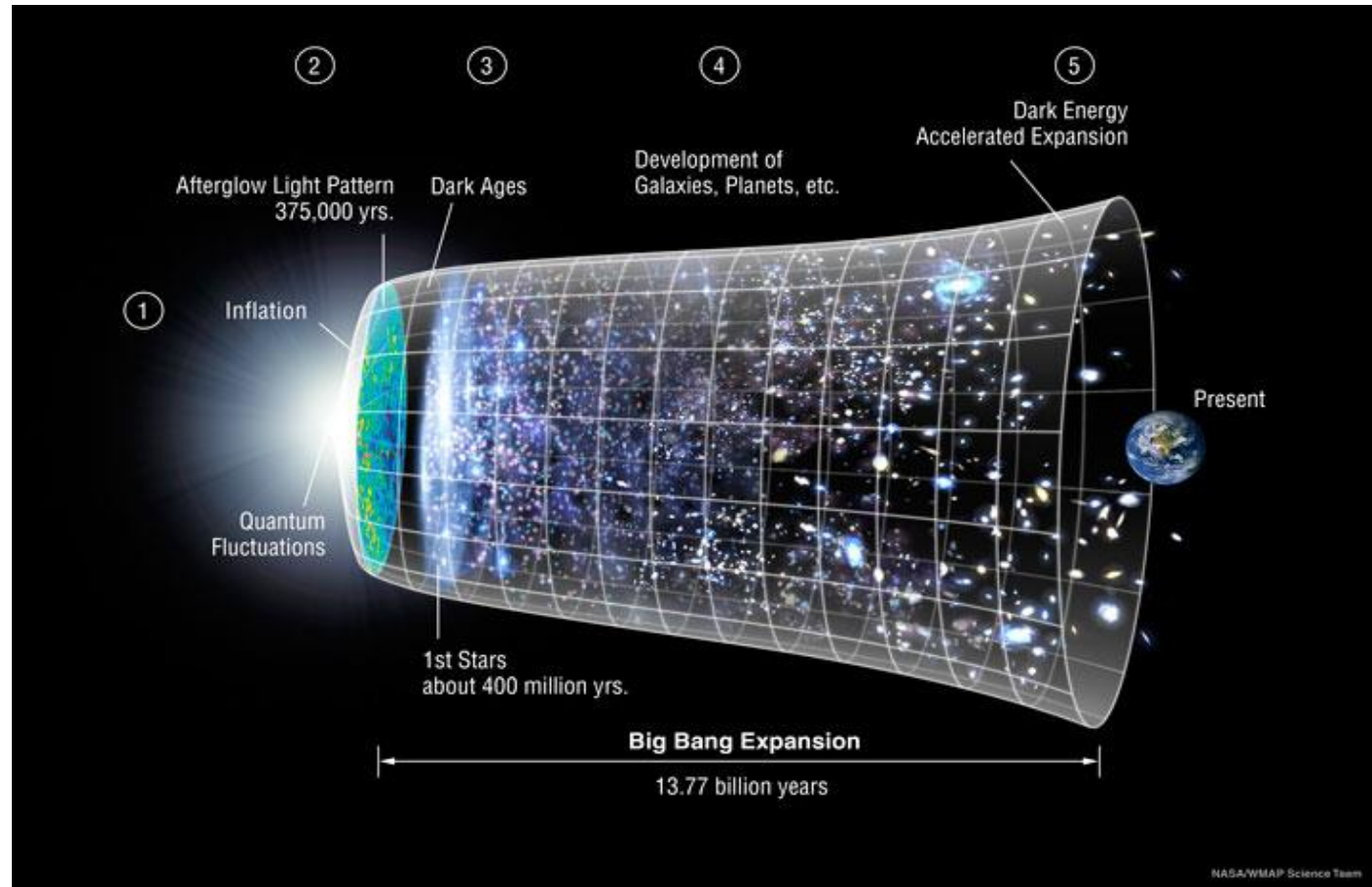
20th January, 2024

Weekly online meeting on Cosmology, IIT Madras

Outline

- Present Understanding of the Universe
- The Λ CDM Model
- Hubble Tension
- Understanding Tension
- Directions of resolution
- Hints of dynamical dark energy
- Horndeski gravity: a plausible approach
- Dark Energy in Horndeski gravity
- Constraints from Observations
- Conclusion and Future Prospects

Present understanding of the universe



Cosmological Probes

- Cosmic microwave background
- Baryon acoustic oscillations
- Large scale structures
- Supernovae and Cepheids
Standard Candles
- Tip of Red Giant Branch
- Weak Lensing

Image Credit: NASA/
LAMBDA Archive / WMAP
Science Team

Present understanding of the universe: Λ CDM model

Cosmological Principle + General Relativity



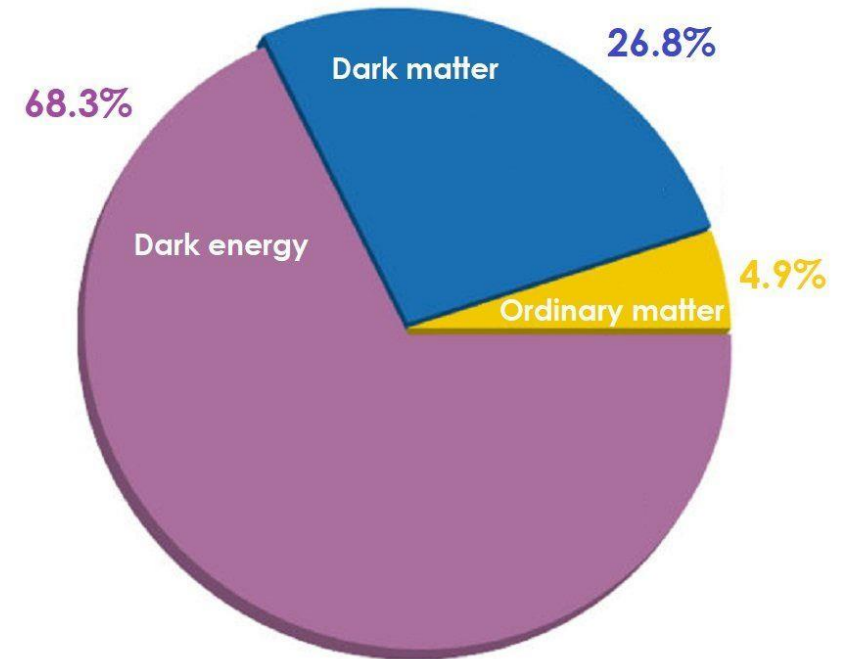
$$ds^2 = -dt^2 + a(t)^2 d\bar{x}^2, \quad R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$



Friedmann Equation $\longleftarrow H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \longrightarrow$ Total energy density

Lambda Cold Dark Matter (Λ CDM) Model : Simplest Scenario

$$H(z) = H_0 \sqrt{\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{\Lambda}}$$

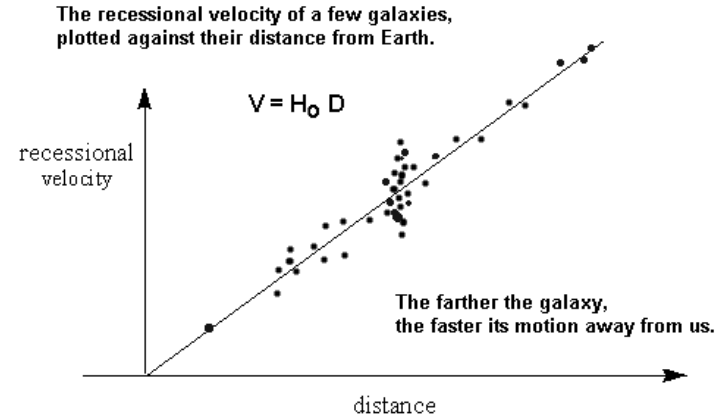


Here it is assumed that universe is spatially flat i.e. $\Omega_k=0$. Thanks to inflation!

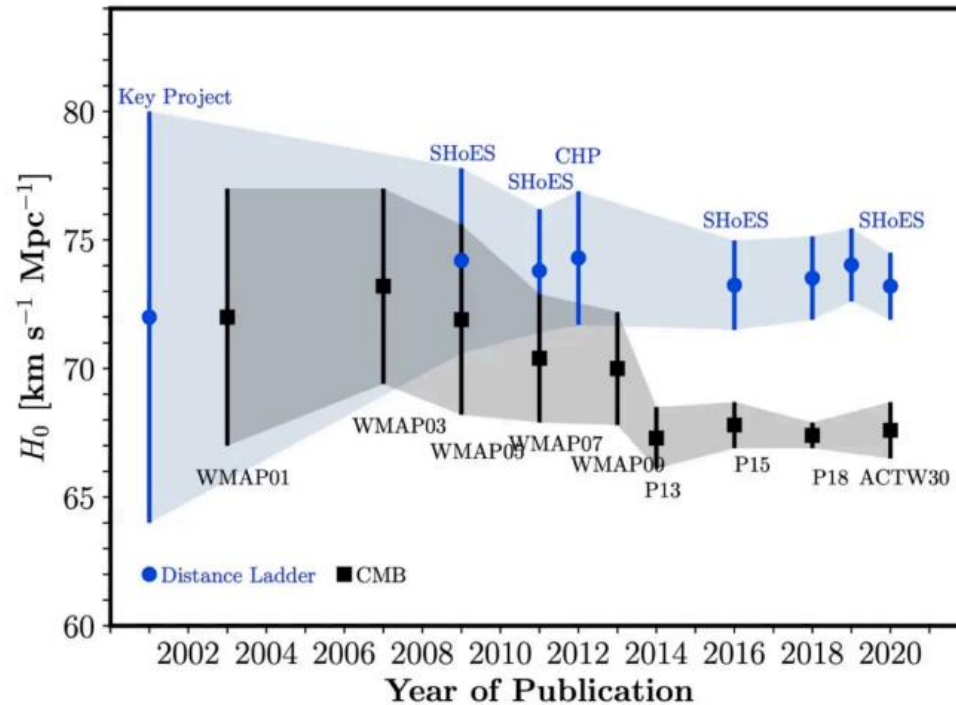
The Hubble Constant H_0



- The rate of expansion of universe at present.
- A crucial cosmological parameter.
- It has been a challenge to correctly measure the value of H_0 .

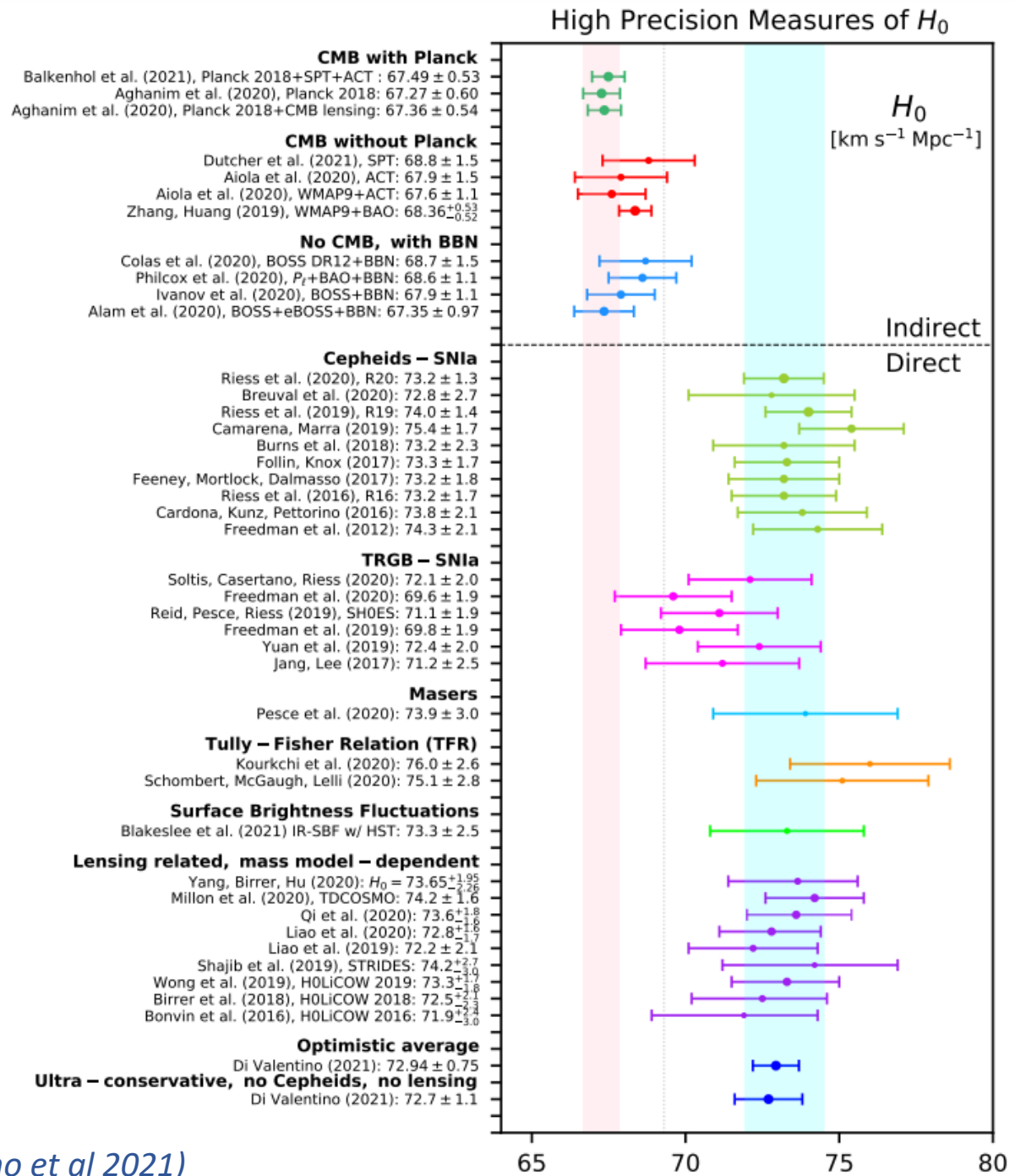


On this graph, the slope of the line is equal to Hubble's Constant (H_0)



The Hubble Tension

5 σ tension at present

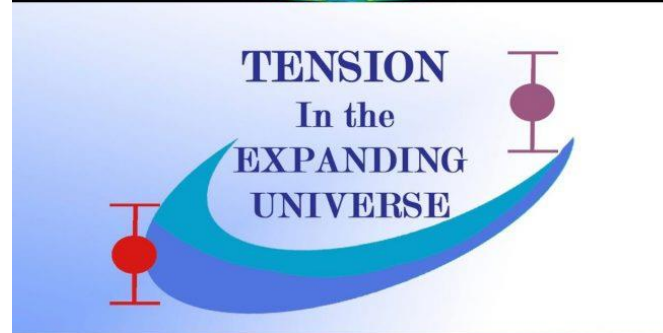
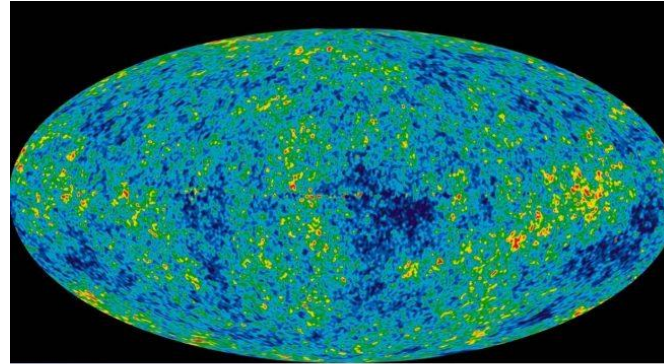


(Di Valentino et al 2021)

Understanding Hubble Tension

Early measurements

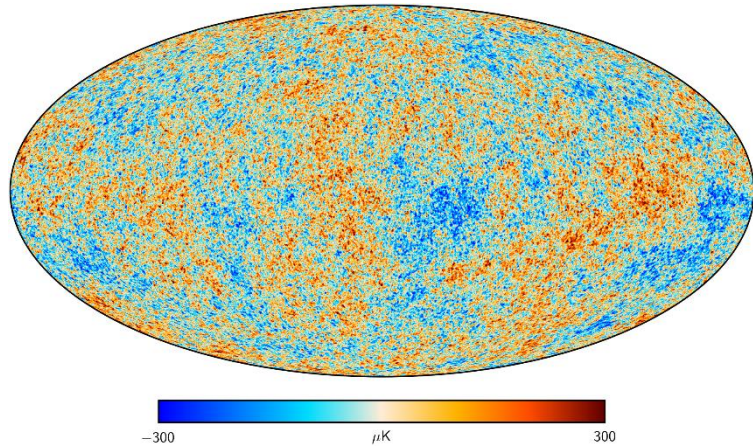
- Based on observations of cosmic microwave background coming from last scattering surface (redshift ~ 1100 , 13.76 Gyr back).
- Assumes Λ CDM model to calculate H_0 .
- Planck, WMAP



Late measurements

- Based on astrophysics of stars: observing standard candles in the nearby universe.
- Model independent measurement.
- SHOES, CHP

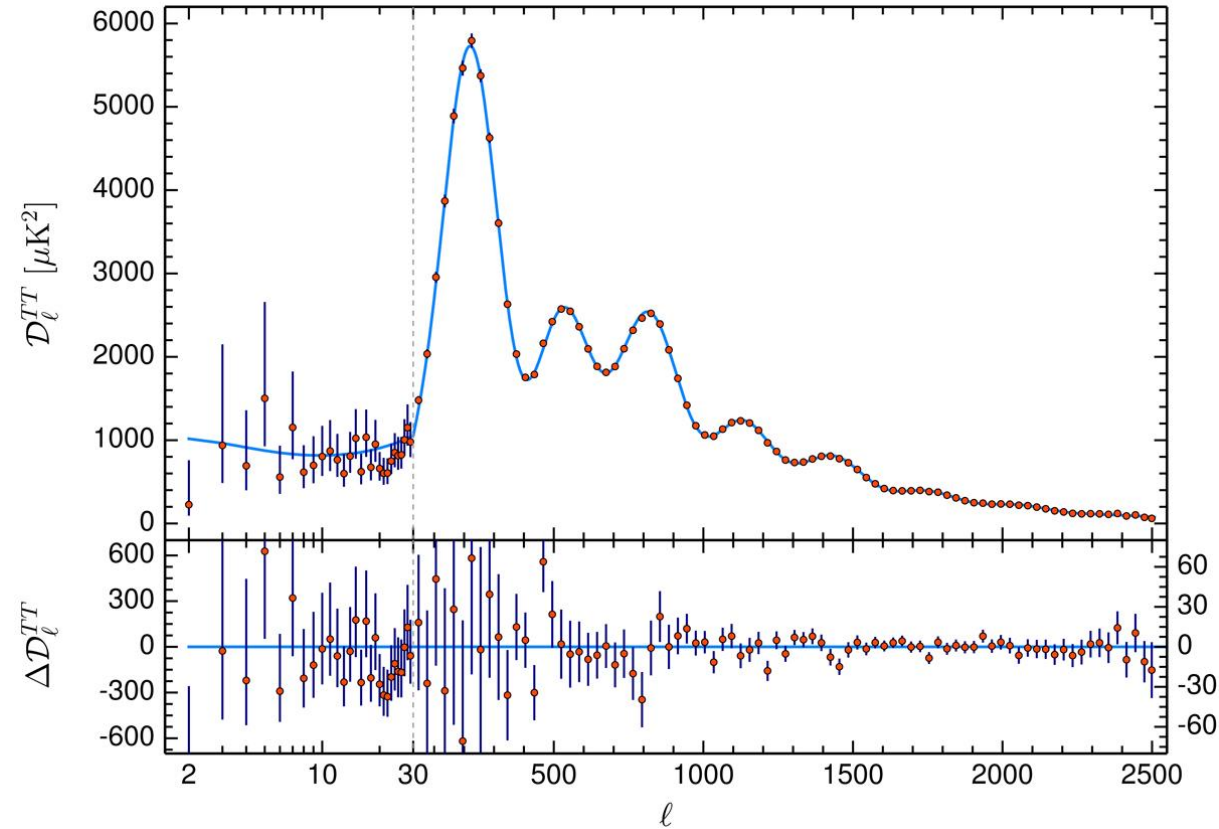
Measurement of H_0 from early Universe



six independent parameters of **ΛCDM** model.

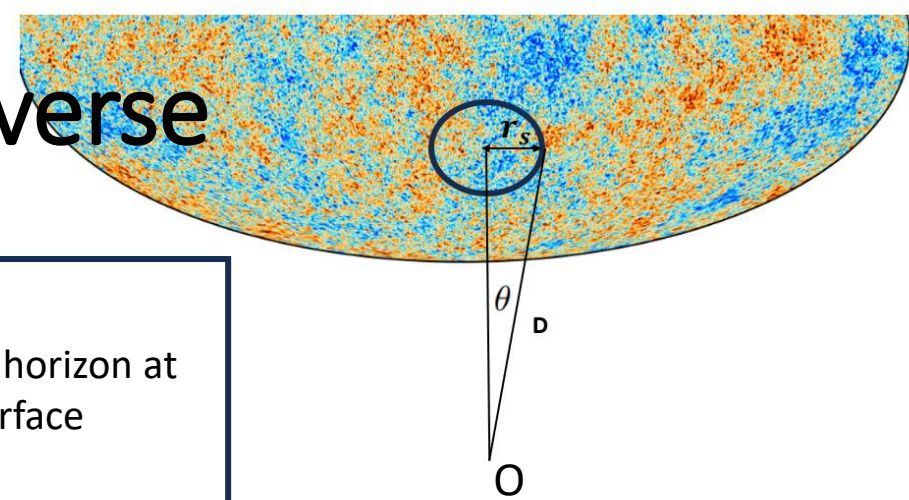
Parameter	Combined
$\Omega_b h^2$	0.02233 ± 0.00015
$\Omega_c h^2$	0.1198 ± 0.0012
$100\theta_{MC}$	1.04089 ± 0.00031
τ	0.0540 ± 0.0074
$\ln(10^{10} A_s)$	3.043 ± 0.014
n_s	0.9652 ± 0.0042
$\Omega_m h^2$	0.1428 ± 0.0011
H_0 [km s ⁻¹ Mpc ⁻¹] ...	67.37 ± 0.54
Ω_m	0.3147 ± 0.0074
Age [Gyr]	13.801 ± 0.024
σ_8	0.8101 ± 0.0061
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$..	0.830 ± 0.013
z_{re}	7.64 ± 0.74
$100\theta_*$	1.04108 ± 0.00031
r_{drag} [Mpc]	147.18 ± 0.29

Derived parameters



Planck 2018 measurements assuming ΛCDM model give, $H_0 = 67.37 \pm 0.54$ km/sec/Mpc

Measurement of H_0 from early Universe



Angular size of sound horizon at last scattering surface: **precisely determined from the peaks in CMB** (0.03 % Precision)

$$\theta = \frac{r_s}{D}$$

comoving sound horizon at last scattering surface

comoving distance to the last scattering surface.

$$r_s = \int_{z_L}^{\infty} \frac{c_s(z')}{H(z')} dz'$$

Based on pre-recombination physics or **early universe physics**.

$$c_s = \frac{1}{\sqrt{3(1 + 3\Omega_b/\Omega_r)}}$$

$$D(z_L) = \int_0^{z_L} \frac{dz}{H(z)}$$

This involves **late universe physics**, depending on dark energy model i.e. $H(z)$ or $E(z)$.

$$H(z) = H_0 \times E(z)$$

H_0 can be extrapolated for a given model at $H(z=0)$.

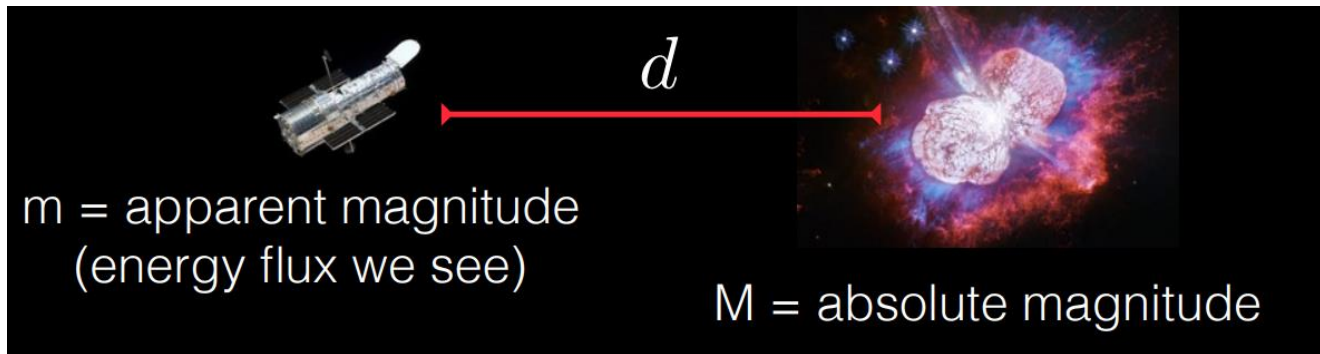
$$z_L = 1100$$

Redshift of recombination or last scattering surface

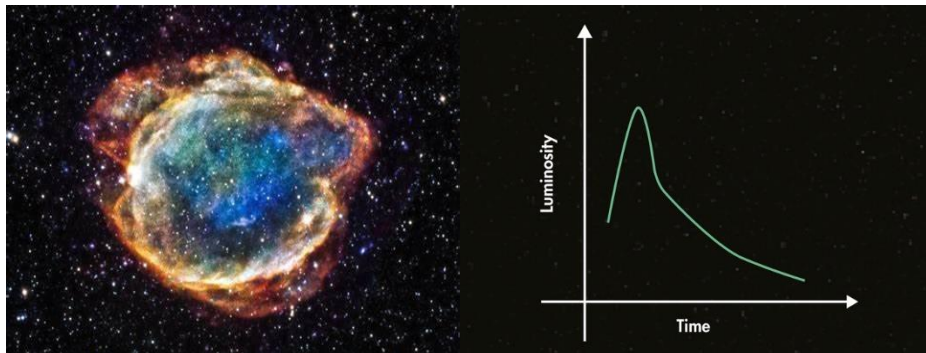
$$E(z) = \sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}$$

For LCDM model

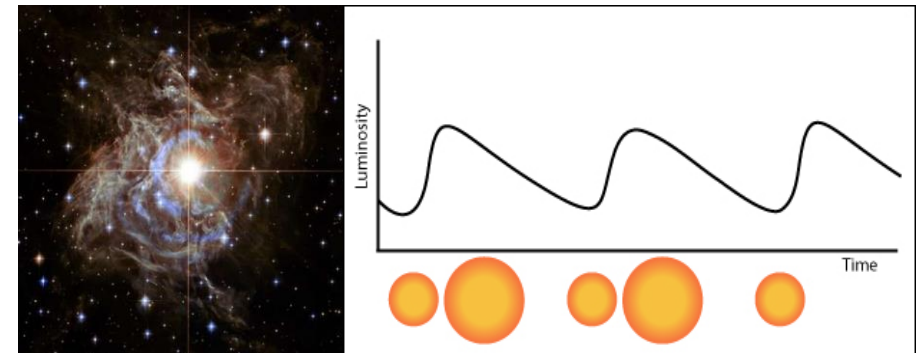
Measurement of H_0 from Late Universe



Observing standard candles to calibrate distances to galaxies and using Hubble's law to calculate H_0 .



Type Ia Supernovae: Thermonuclear explosion of white dwarf stars reaching Chandrasekhar mass Limit.



Cepheid variables: Pulsating stars with a definite period luminosity relation

Measurement of H_0 from Late Universe

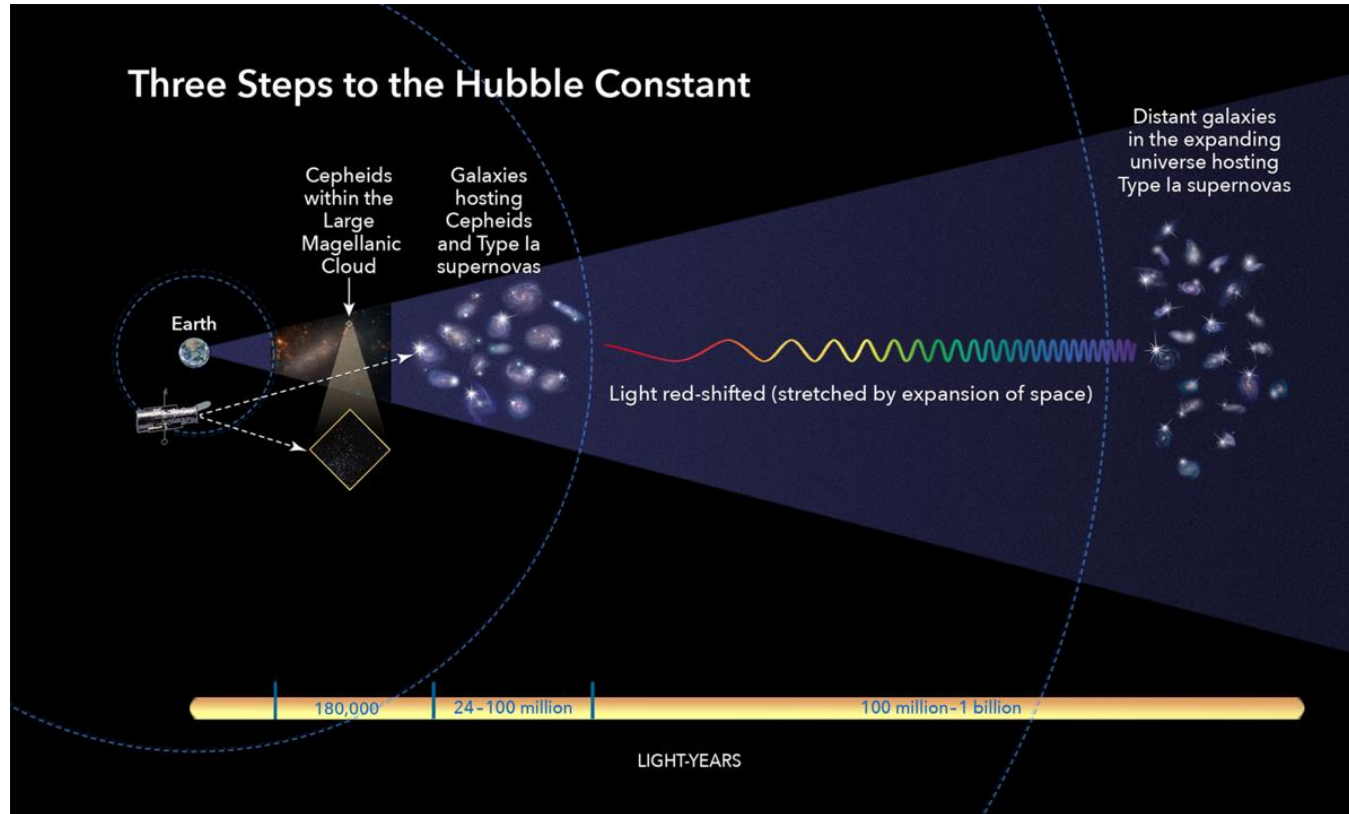


Image Credit: NASA

Cosmic Distance Ladder : calibrating distances to galaxies farther away upto redshift ~ 0.1

- The **SHOES** Program (Supernovae and H_0 for the Equation of State of dark energy) measured $H_0 = 73.3 \pm 1.04$ km/sec/Mpc (*Riess et al 2022*).
- This drives the H_0 tension $\sim 5\sigma$
- In fact various other local measurements, apart from SHOES also give $H_0 > 70$ km/sec/Mpc, indicating tension with the Planck (LCDM) value (*Freedman 2021, Anand et al 2021, Shajib et al 2023, Pesce et al 2020 ...*).

S_8 Tension

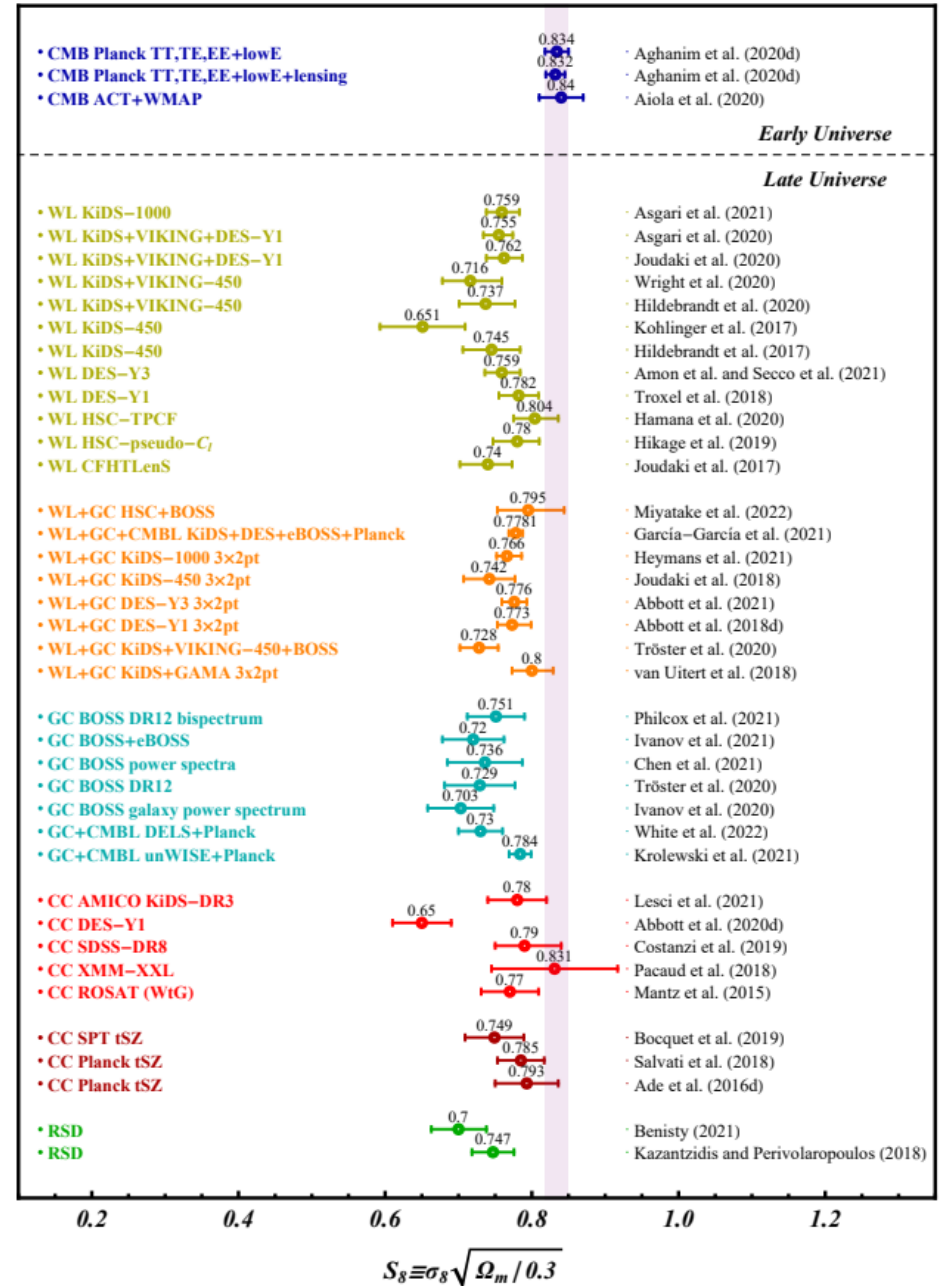
$$S_8 = \sigma_8 \left(\frac{\Omega_M}{0.3} \right)^{0.5}$$



A measure of amplitude of matter clustering in late universe

σ_8 is the variance of density field smoothed over $8h^{-1}$ Mpc

2-3 σ tension at present



(Abdalla et al 2022)

How to address Hubble Tension?

Review of solutions

arXiv > astro-ph > arXiv:2103.01183

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[Submitted on 1 Mar 2021 (v1), last revised 5 Jun 2021 (this version, v3)]

In the Realm of the Hubble tension — a Review of Solutions

[Eleonora Di Valentino](#), [Olga Mena](#), [Supriya Pan](#), [Luca Visinelli](#), [Weiqiang Yang](#), [Alessandro Melchiorri](#), [David F. Mota](#), [Adam G. Riess](#), [Joseph Silk](#)

arXiv > astro-ph > arXiv:2107.10291

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[Submitted on 21 Jul 2021 (v1), last revised 15 Oct 2021 (this version, v2)]

The H_0 Olympics: A fair ranking of proposed models

[Nils Schöneberg](#), [Guillermo Franco Abellán](#), [Andrea Pérez Sánchez](#), [Samuel J. Witte](#), [Vivian Poulin](#), [Julien Lesgourgues](#)

arXiv > astro-ph > arXiv:2203.06142

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Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 11 Mar 2022 (v1), last revised 24 Apr 2022 (this version, v3)]

Cosmology Intertwined: A Review of the Particle Physics, Astrophysics, and Cosmology Associated with the Cosmological Tensions and Anomalies

[Elcio Abdalla](#), [Guillermo Franco Abellán](#), [Amin Aboubrahim](#), [Adriano Agnello](#), [Ozgur Akarsu](#), [Yashar Akrami](#), [George Alestas](#), [Daniel Aloni](#), [Luca Amendola](#), [Luis A. Anchordoqui](#), [Richard I. Anderson](#), [Nikki Arendse](#), [Marika Asgari](#), [Mario Ballardini](#), [Vernon Barger](#), [Spyros Basilakos](#), [Ronaldo C. Batista](#), [Elia S. Battistelli](#), [Richard Battye](#), [Micol Benetti](#), [David Benisty](#), [Asher Berlin](#), [Paolo de Bernardis](#), [Emanuele Berti](#), [Bohdan Bidenko](#), [Simon Birrer](#), [John P. Blakeslee](#), [Kimberly K. Boddy](#), [Clecio R. Bom](#),

Possible Resolutions to Hubble Tension

Aim: Modifying the LCDM picture without disturbing the well constrained peaks of CMB.

Fixed by CMB ← $\theta = \frac{r_s}{\int_0^{z_L} \frac{dz'}{H(z')}} = \frac{H_0 r_s}{\int_0^{z_L} \frac{dz'}{E(z')}}$

Modifying early universe

- Decreasing sound horizon r_s (changing pre-recombination physics).
- H_0 is increased in order to fix θ , without changing late universe physics.

Modifying late universe

- Modifying $H(z)$ or $E(z)$ (with non-trivial dark energy models), keeping comoving distance to last scattering surface unchanged.
- Pre-recombination physics is not disturbed.

Early universe solutions

- Reducing the comoving sound horizon

$$r_s = \int_{z_L}^{\infty} \frac{c_s(z)}{H(z)} dz$$

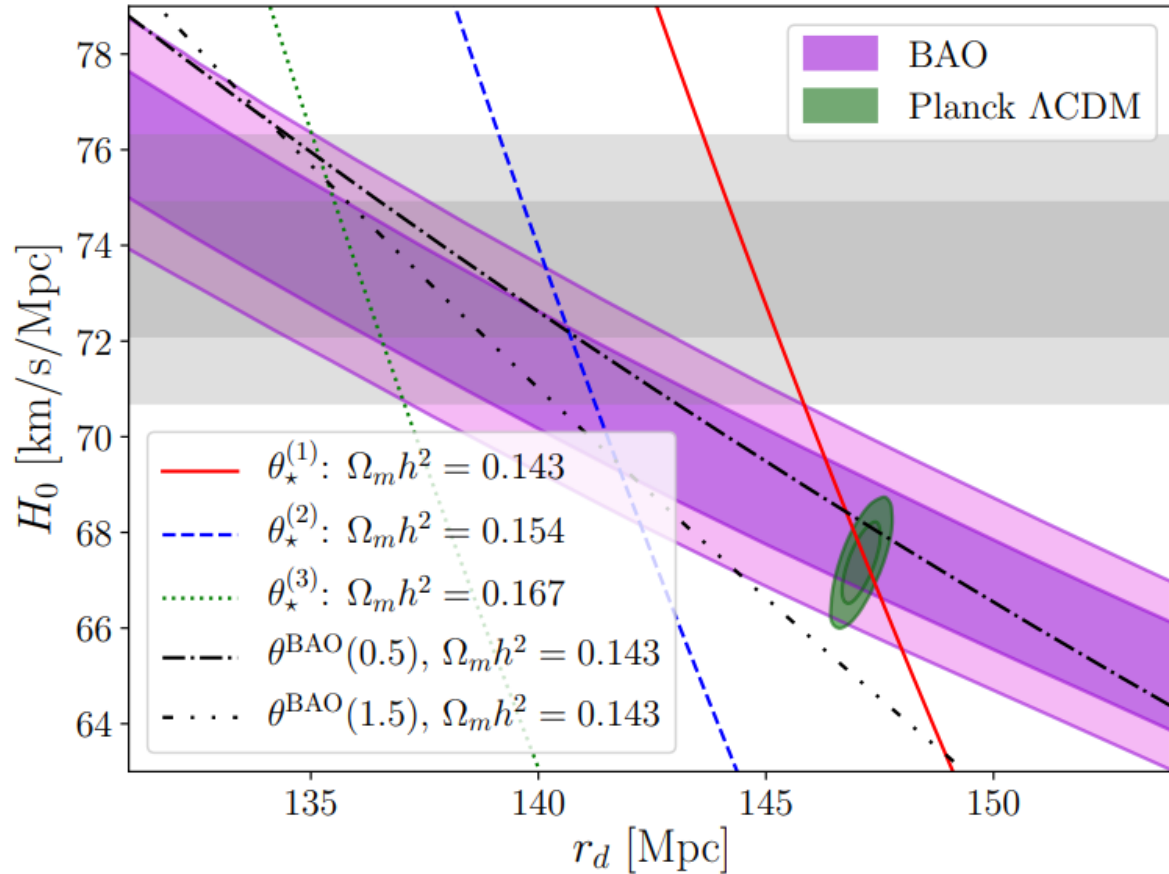
Altering recombination history

- Primordial magnetic fields
(Jedamzik et al 2020)
- Non-standard recombination
(Chiang et al 2018)
- Varying fundamental constant
(Sekiguchi et al 2020, Hart et al 2020)

$\uparrow H(z) \implies \uparrow \rho(z)$ since $H(z) \propto \sqrt{\rho(z)}$

- Extra radiation (N_{eff})
(Kreisch et al 2019, Sakstein et al 2019, Archidiacono et al 2020, Anchordoqui et al 2019, Gonzalez et al 2020....)
- Energy injection around matter radiation equality:
Early Dark Energy (EDE), Early Modified Gravity (EMG)
(Karwal et al 2016, Poulin et al 2018, Braglia et al 2021)

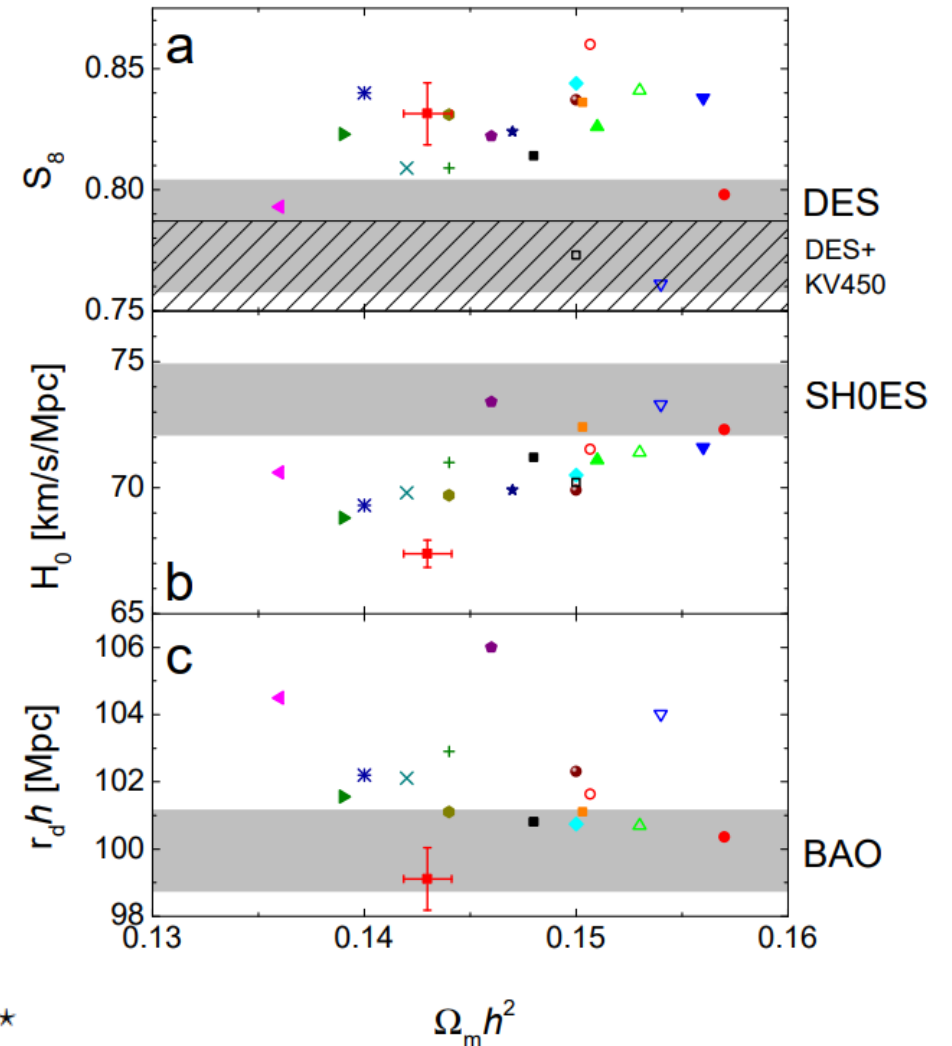
But there exist some issues *(Based on Jedamzik et al 2020)*



$$\theta_{\star} \equiv \frac{r_{\star}}{D(z_{\star})}$$

$$\theta_{\perp}^{\text{BAO}}(z_{\text{obs}}) \equiv \frac{r_d}{D(z_{\text{obs}})}$$

$$r_d = 1.0184 r_{\star}$$



So early universe solutions **alone** cannot resolve the H_0 tension!

Opinion

Seven Hints That Early-Time New Physics Alone Is Not Sufficient to Solve the Hubble Tension

Sunny Vagnozzi

Special Issue

Modified Gravity Approaches to the Tensions of Λ CDM

Edited by

Dr. Eleonora Di Valentino, Prof. Dr. Leandros Perivolaropoulos and Dr. Jackson Levi Said

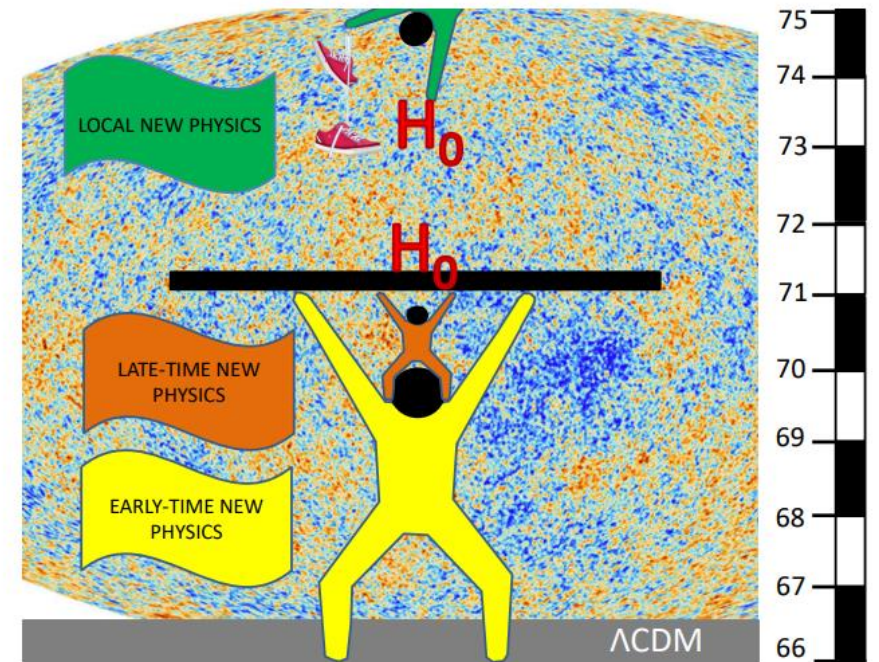


Image Credit: Cristina Ghirardini

For more details
read the following

Late universe solutions

Modifying late time expansion history without disturbing comoving distance to LSS.

$$D(z_L) = \int_0^{z_L} \frac{dz}{H(z)}$$



Expansion rate in late universe: crucial dependence on dark energy model

Simplest solution: an extension of LCDM model with a dark energy component (equation of state w)

$$H(z) = H_0 \sqrt{\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + (1 - \Omega_{r0} - \Omega_{m0})(1+z)^{3(1+w)}}$$

To resolve Hubble tension, $\uparrow H_0 \implies (1+w) < 0$

Phantom
dark energy

But simplest phantom models are now ruled out by observations as they worsen σ_8/S_8 tension.

Need for a non-trivial dynamical dark energy?

A dark energy field whose equation of state evolves with time $w(z)$: But what else?

arXiv > astro-ph > arXiv:2201.11623

Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 27 Jan 2022]

Simultaneously solving the H_0 and σ_8 tensions with late dark energy

Lavinia Heisenberg, Hector Villarrubia-Rojo, Jann Zosso

According to *Heisenberg et al 2022*, late dark energy models must exhibit **PHANTOM DIVIDE** behavior to simultaneously alleviate H_0 and S_8 tension.

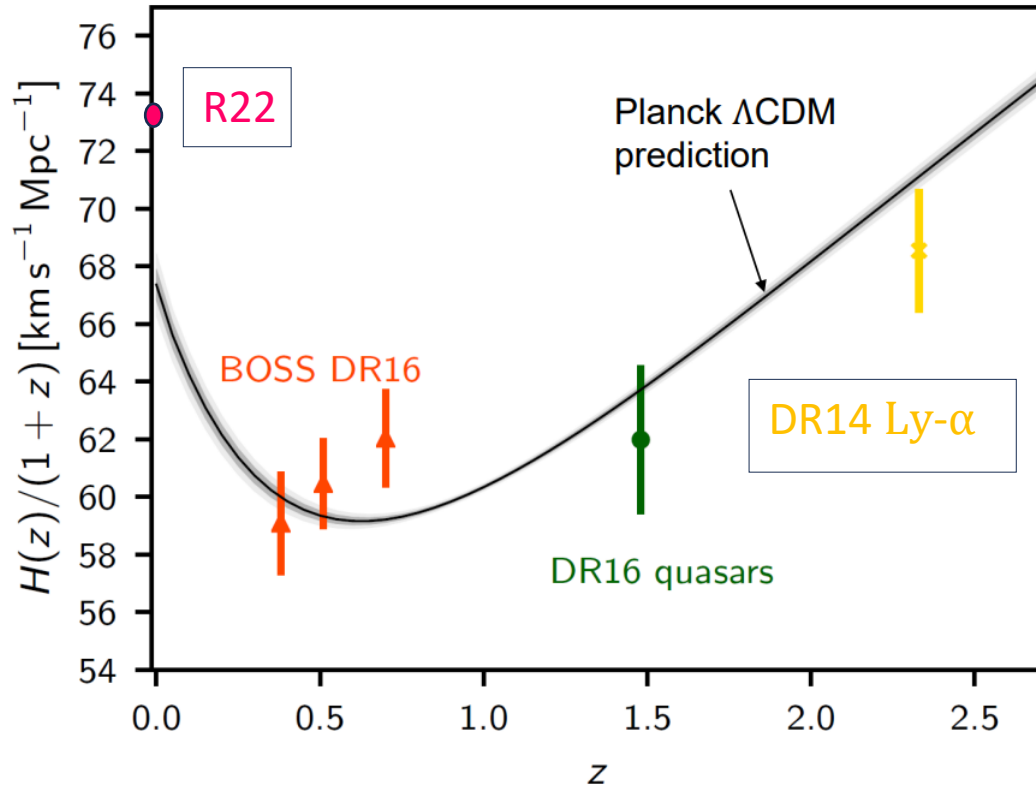
A dynamics which can give rise to a phantom divide behavior?



One way to achieve this is to have a negative dark energy density during some epoch at high redshifts

Are there any signatures of negative dark energy density in observational data?

Hints for negative Dark energy? BAO Ly- α Anomaly



A&A 629, A85 (2019)

Baryon acoustic oscillations at $z = 2.34$ from the correlations of Ly α absorption in eBOSS DR14

Victoria de Sainte Agathe¹, Christophe Balland¹, Héliion du Mas des Bourboux², Nicolás G. Busca¹, Michael

A&A 629, A86 (2019)

Baryon acoustic oscillations from the cross-correlation of Ly α absorption and quasars in eBOSS DR14

Michael Blomqvist¹, Héliion du Mas des Bourboux², Nicolás G. Busca³, Victoria de Sainte Agathe³, James

$\sim 2\sigma$ tension in the measurement of $H(z)$ at $z \sim 2.3$ from prediction of ΛCDM .

Hints for negative Dark energy? BAO Ly- α Anomaly

LETTERS

MODEL-INDEPENDENT EVIDENCE FOR DARK ENERGY EVOLUTION FROM BARYON ACOUSTIC OSCILLATIONS

V. Sahni¹ , A. Shafieloo^{2,3} , and A. A. Starobinsky^{4,5}

Published 2014 September 19 • © 2014. The American Astronomical Society. All rights reserved.

[The Astrophysical Journal Letters, Volume 793, Number 2](#)

Citation V. Sahni *et al* 2014 *ApJL* 793 L40

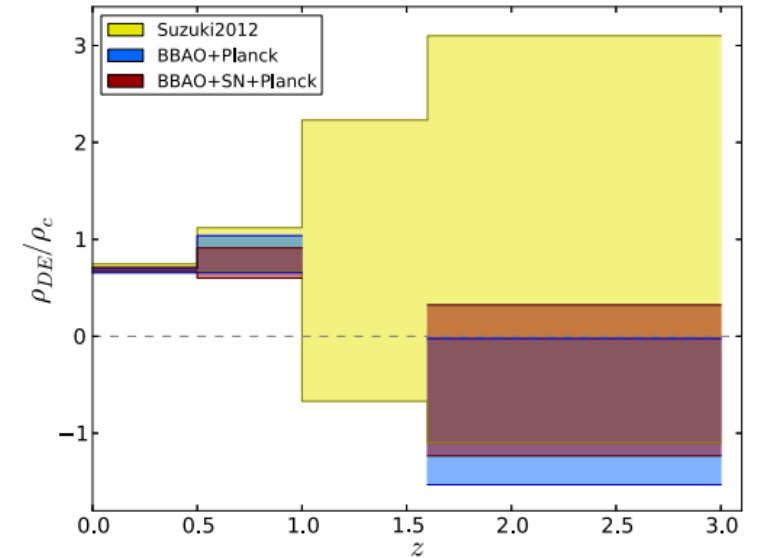
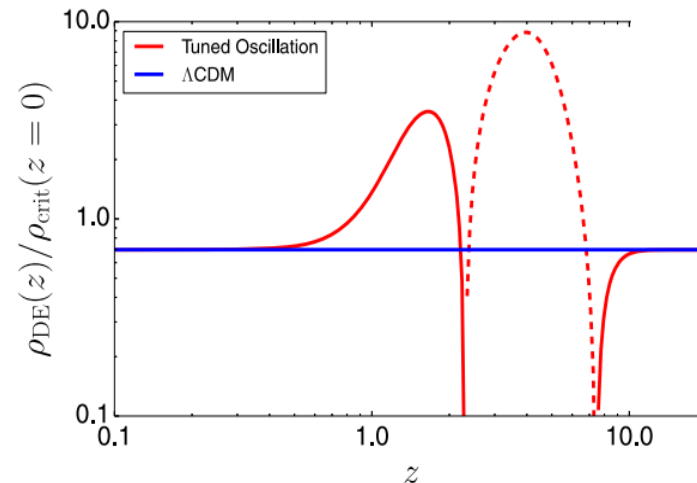
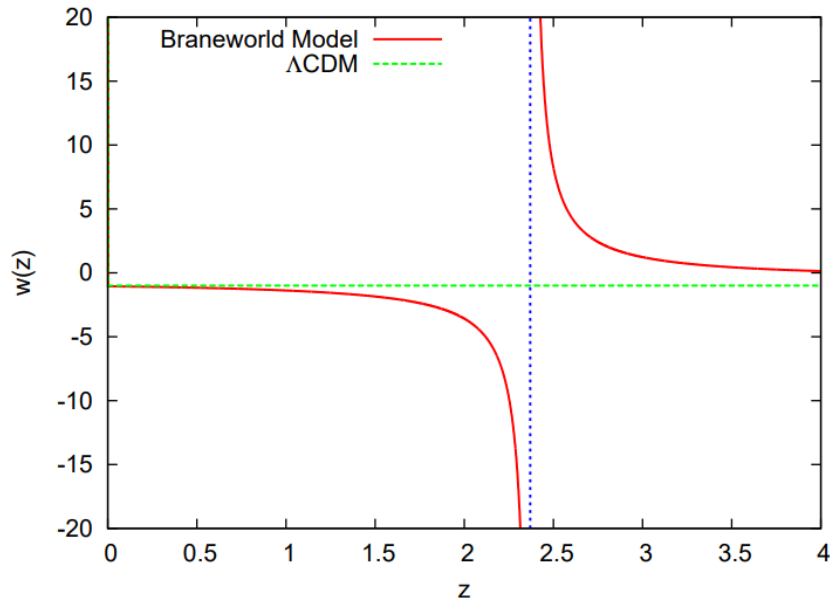
DOI 10.1088/2041-8205/793/2/L40

Cosmological implications of baryon acoustic oscillation measurements

Éric Aubourg *et al.* (BOSS Collaboration)

Phys. Rev. D **92**, 123516 – Published 14 December 2015

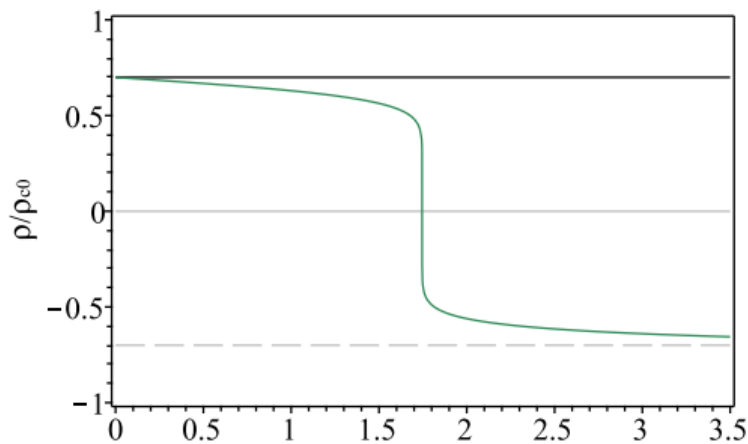
Negative energy density at high redshifts ($z > 2$) can explain the feature.



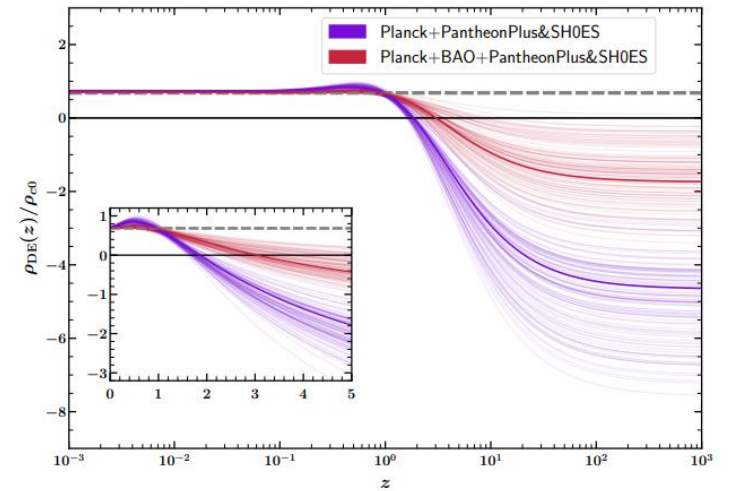
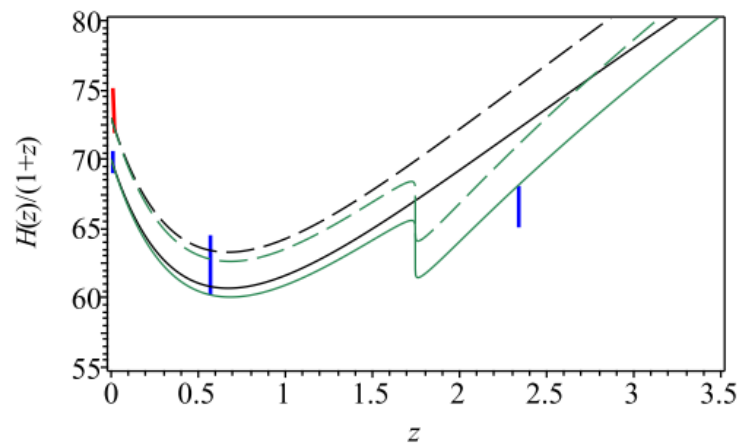
Hints for negative Dark energy?

Dark energy models with a negative energy density feature at high redshifts, give a good fit to observation data (BAO, SN, H0, Planck)

- Graduated dark energy (*Akarsu et al 2020*)
- Negative cosmological constant (plus extra component) (*Calderon et al 2021, Sen et al 2021*)
- Sign switching cosmological constant (*Akarsu et al 2021, Akarsu et al 2023,*)
- Omnipotent dark energy (*Adil et al 2023*)



(*arXiv:1912.08751*)



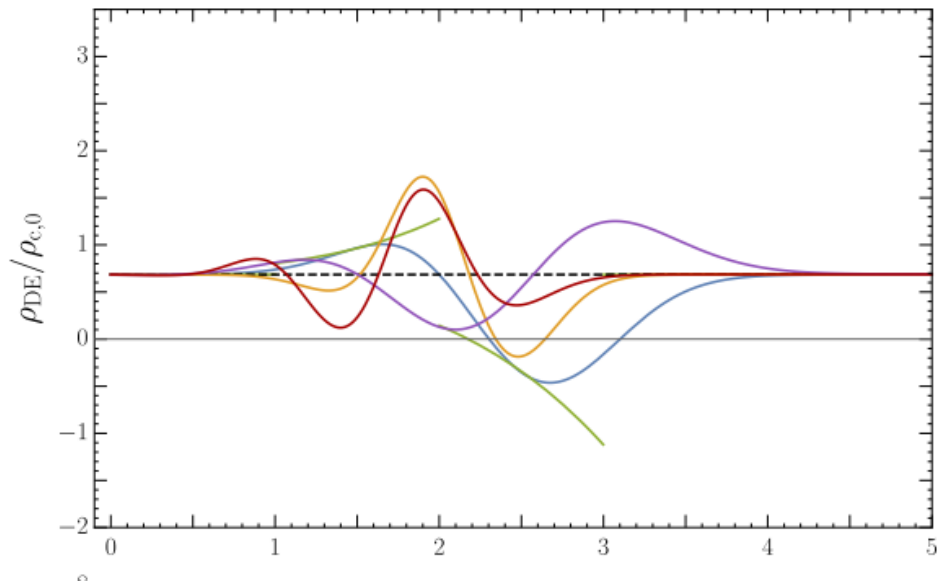
(*arXiv:2306.08046*)

Hints for negative Dark energy?

Observational Reconstructions hint towards negative energy density at high redshifts

Inevitable manifestation of wiggles in the expansion of the late Universe

Özgür Akarsu, Eoin Ó Colgáin, Emre Özülker, Somyadip Thakur, and Lu Yin
Phys. Rev. D **107**, 123526 – Published 20 June 2023



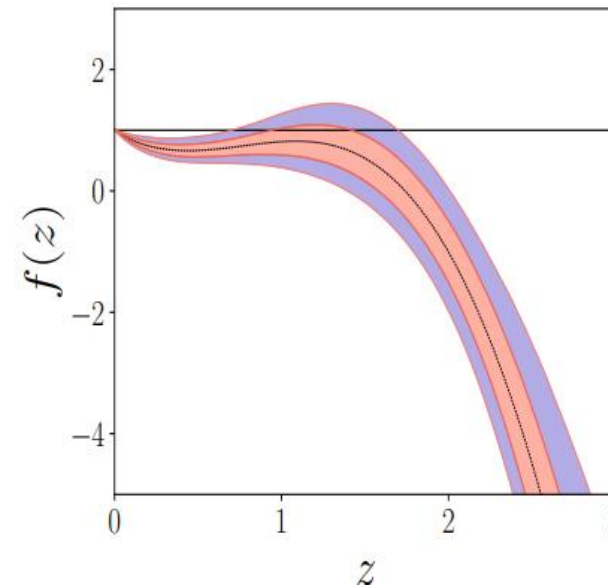
Beyond Λ CDM with low and high redshift data: implications for dark energy

Koushik Dutta (Saha Inst.), Ruchika (Jamia Millia Islamia), Anirban Roy (SISSA, Trieste), Anjan A. Sen (Jamia Millia Islamia), M.M. Sheikh-Jabbari (IPM, Tehran)

Aug 20, 2018

10 pages

Published in: *Gen.Rel.Grav.* 52 (2020) 2, 15



$$3H^2(z) = \rho_m + \rho_{\text{DE}} = \rho_{m0}(1+z)^3 + \rho_{\text{DE}0}f(z)$$

Our Approach

Towards a possible solution to the Hubble tension with Horndeski gravity

Yashi Tiwari, Basundhara Ghosh, Rajeev Kumar Jain

arXiv: 2301.09382

Motivation

- A late universe solution to address Hubble tension
- Dynamical dark energy which can exhibit interesting features like negative dark energy, phantom crossing.
- To motivate the model from a **Lagrangian perspective** in the framework of generalized scalar-tensor theories.
- But lets first talk a bit about **Horndeski theory**.

Horndeski theory

- The Lagrangian constructed out of metric tensor and scalar field, such that **equations of motion are second order**.

(Kobayashi et al 2011, Kobayashi 2019)

$$\mathcal{L} = \sum_{i=2}^5 \mathcal{L}_i,$$

$$\mathcal{L}_2 = G_2(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X)\square\phi,$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X}(\phi, X)\left[(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,X}(\phi, X)\left[(\square\phi)^3 - 3\square\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right],$$

$$G_{i,Y} = \partial G_i / \partial Y$$

$$X = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi$$

$$G_4 = \frac{M_{Pl}^2}{2} \longrightarrow \text{Einstein-Hilbert action}$$

$$G_2 = X - V(\phi), G_4 = \frac{M_{Pl}^2}{2} \longrightarrow \text{Single-field inflation or Quintessence dark energy}$$

Other subclasses: non-minimal coupling, Galileons, derivative couplings ...

Applications: Primordial black-hole formation, Black-hole physics, CMB anomalies, non-trivial dark energy models...

Horndeski theory (Background equations)

(Matsumoto et al 2017)

$$\left. \begin{aligned} 3H^2 &= \kappa^2(\rho_\phi + \rho_M) \\ -3H^2 - 2\dot{H} &= \kappa^2(p_\phi + p_M) \end{aligned} \right\} \text{Friedmann Equations}$$

$$\frac{1}{a^3} \frac{d}{dt}(a^3 \mathcal{J}) = \mathcal{P}_\phi \longrightarrow \text{Evolution of scalar field}$$

$$\begin{aligned} \rho_\phi &= 2XG_{2,X} - G_2 + 6X\dot{\phi}HG_{3,X} - 2XG_{3,\phi} - 6H^2G_4 + 24H^2X(G_{4,X} + XG_{4,XX} - 12HX\dot{\phi}G_{4,\phi X} \\ &\quad - 6H\dot{\phi}G_{4,\phi} + 2H^3X\dot{\phi}(5G_{5,X} + 2XG_{5,XX}) - 6H^2X(3G_{5,\phi} + 2XG_{5,\phi X}) + \frac{3H^2}{\kappa^2} \end{aligned}$$

$$\begin{aligned} \mathcal{J} &= \dot{\phi}G_{2,X} + 6HXG_{3,X} - 2\dot{\phi}G_{3,\phi} + 6H^2\dot{\phi}(G_{4,X} + 2XG_{4,XX}) - 12HXG_{4,\phi X} \\ &\quad + 2H^3X(3G_{5,X} + 2XG_{5,XX}) - 6H^2\dot{\phi}(G_{5,\phi} + XG_{5,\phi X}), \end{aligned}$$

$$\begin{aligned} \mathcal{P}_\phi &= G_{2,\phi} - 2X(G_{3,\phi\phi} + \ddot{\phi}G_{3,\phi X}) + 6(2H^2 + \dot{H})G_{4,\phi} + 6H(\dot{X} + 2HX)G_{4,\phi X} \\ &\quad - 6H^2XG_{5,\phi\phi} + 2H^3X\dot{\phi}G_{5,\phi X}. \end{aligned}$$

$$\kappa^2 = 1/M_{\text{Pl}}^2$$

Horndeski theory (Perturbations)

$$\mathcal{S}_2 = \int dt d^3x a^3 \left[Q_S \left(\overbrace{\dot{\mathcal{R}}^2 - \frac{c_S^2}{a^2} (\partial_i \mathcal{R})^2}^{\text{scalar}} \right) + Q_T \left(\overbrace{\dot{h}_{ij}^2 - \frac{c_T^2}{a^2} (\partial_k h_{ij})^2}^{\text{tensor}} \right) \right],$$

(Felice et al 2011, Bellini et al 2014)

Stability Conditions (for a consistent theory)

$$c_S^2 > 0$$

To avoid gradient
instability

$$c_T^2 > 0$$

$$Q_S > 0$$

To avoid ghost
instability

$$Q_T > 0$$

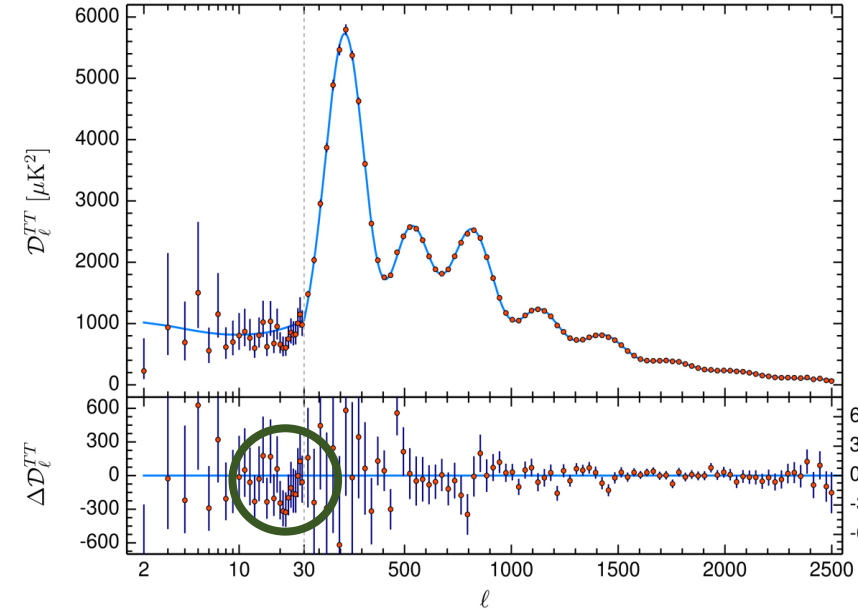
where $c_S, Q_S, c_T, Q_T = \mathcal{F}(G_i, G_{i,Y}, \phi, \dot{\phi})$

An example !!

Access by Indian Institute of Science, B

Understanding large scale CMB anomalies with the generalized nonminimal derivative coupling during inflation

Yashi Tiwari, Nilanjandev Bhauimik, and Rajeev Kumar Jain
Phys. Rev. D **107**, 103513 – Published 12 May 2023



GNMDC

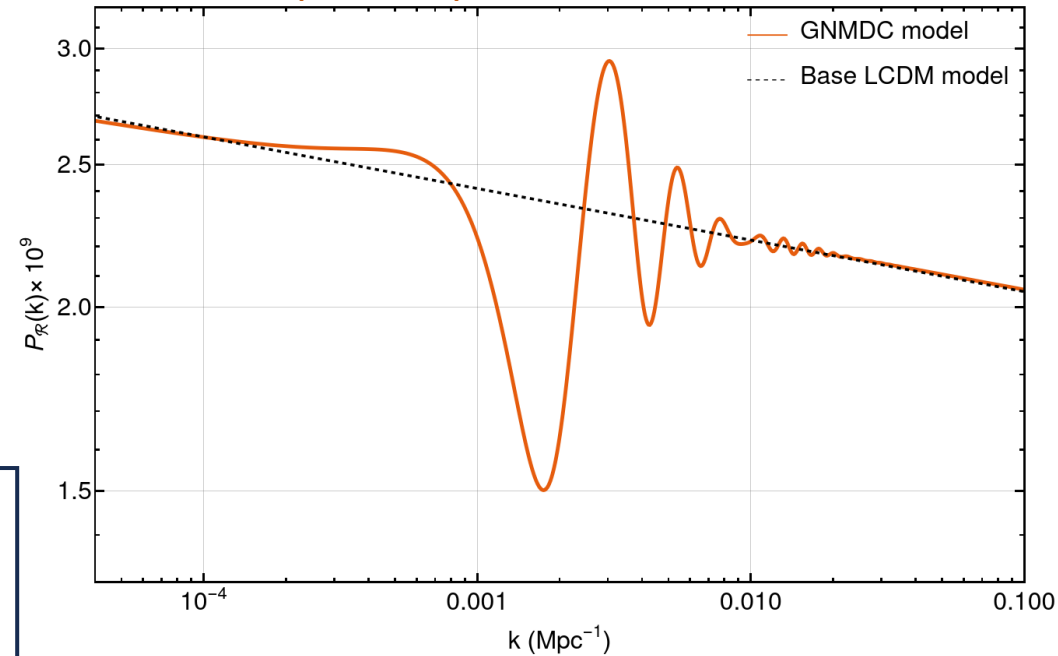
$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{2} \left(g^{\mu\nu} - \theta(\phi) G^{\mu\nu} \right) \partial_\mu \phi \partial_\nu \phi - V(\phi) \right],$$

This can be obtained from
Horndeski Lagrangian using

$$G_2 = X - V(\phi), G_4 = \frac{M_{Pl}^2}{2}, G_5 = G_5(\phi)$$

$$\theta(\phi) = -2G_{5,\phi}$$

Primordial power spectrum



(Similar features as studied by Jain et al. 2008, Hazra et al. 2010)

Building a dark energy model in the
framework of Horndeski gravity

Model Specifications

- A dynamical scalar field as dark energy (No cosmological constant).

$$\mathcal{L}_\phi = G_4(\phi)\mathcal{R} - G_3(\phi, X)\square\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

↑ **Non-minimal coupling (NMC)** **kinetic term (K)** ↑

↓ **Self-interaction (SI)
(Galileon)** **Scalar field potential** ↓

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\mathcal{L}_\phi + \mathcal{L}_M \right) \longrightarrow \text{Total action}$$

Background equations are solved giving initial conditions on $\phi, \dot{\phi}, H$ at high redshift.
 The case with $c_1 = c_2 = c_3 = 0$ corresponds to Quintessence .

We choose

$$ds^2 = -dt^2 + a(t)^2 d\bar{x}^2,$$

$$G_3(\phi, X) = c_1\phi + c_2X$$

$$G_4(\phi) = \frac{1}{2} + c_3\phi$$

$$X = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi$$

$$V(\phi) = V_0\phi$$

$$\kappa^2 = 1 \quad \longleftarrow \text{Kept fixed}$$

$$G_5 = 0$$

c_1, c_2 and c_3 are the free parameters, controlling strengths of coupling terms.

Model Specifications

Effective energy density of dark energy field

$$\rho_\phi = \underbrace{\frac{1}{2}\dot{\phi}^2 + V(\phi)}_{\text{Canonical kinetic + potential}} - \underbrace{6c_3\phi H^2 - 6c_3H\dot{\phi}}_{\text{Due to NMC, leads to negative energy density in past : phantom crossing (for } c_3 > 0)} - \underbrace{c_1\dot{\phi}^2 + 3c_2H\dot{\phi}^3}_{\text{Due to Self interactions}}$$

pressure

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) + 6c_3\phi H^2 + 4c_3\phi\dot{H} + 2c_3\ddot{\phi} + 4c_3H\dot{\phi} - c_1\dot{\phi}^2 - c_2\ddot{\phi}\dot{\phi}^2$$

It will be shown later that **both** nonminimal coupling and self interactions are needed to appropriately address the H_0 tension.

$$G_3(\phi, X) = c_1\phi + c_2X$$

$$G_4(\phi) = \frac{1}{2} + c_3\phi$$

$$X = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi$$

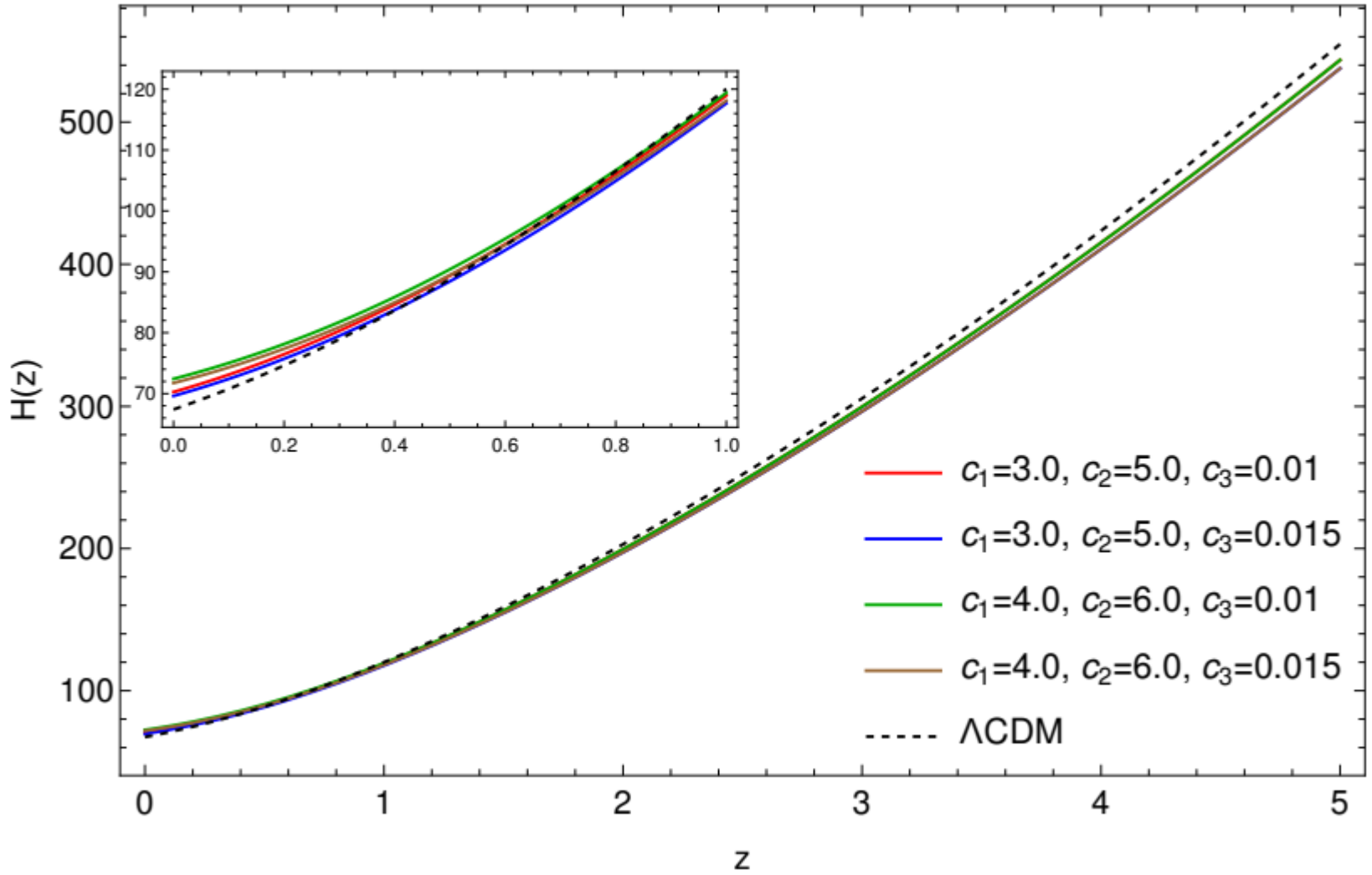
$$V(\phi) = V_0\phi$$

$$w_\phi = \frac{p_\phi}{\rho_\phi}$$

↓
Equation of state

$$\frac{d\rho_\phi}{dz} = \frac{3(1 + w_\phi)\rho_\phi}{1 + z}$$

Distinct Features



- At high redshifts
 $H(z) < H_{\Lambda\text{CDM}}(z)$
- At low redshifts,
 $H(z) > H_{\Lambda\text{CDM}}(z)$



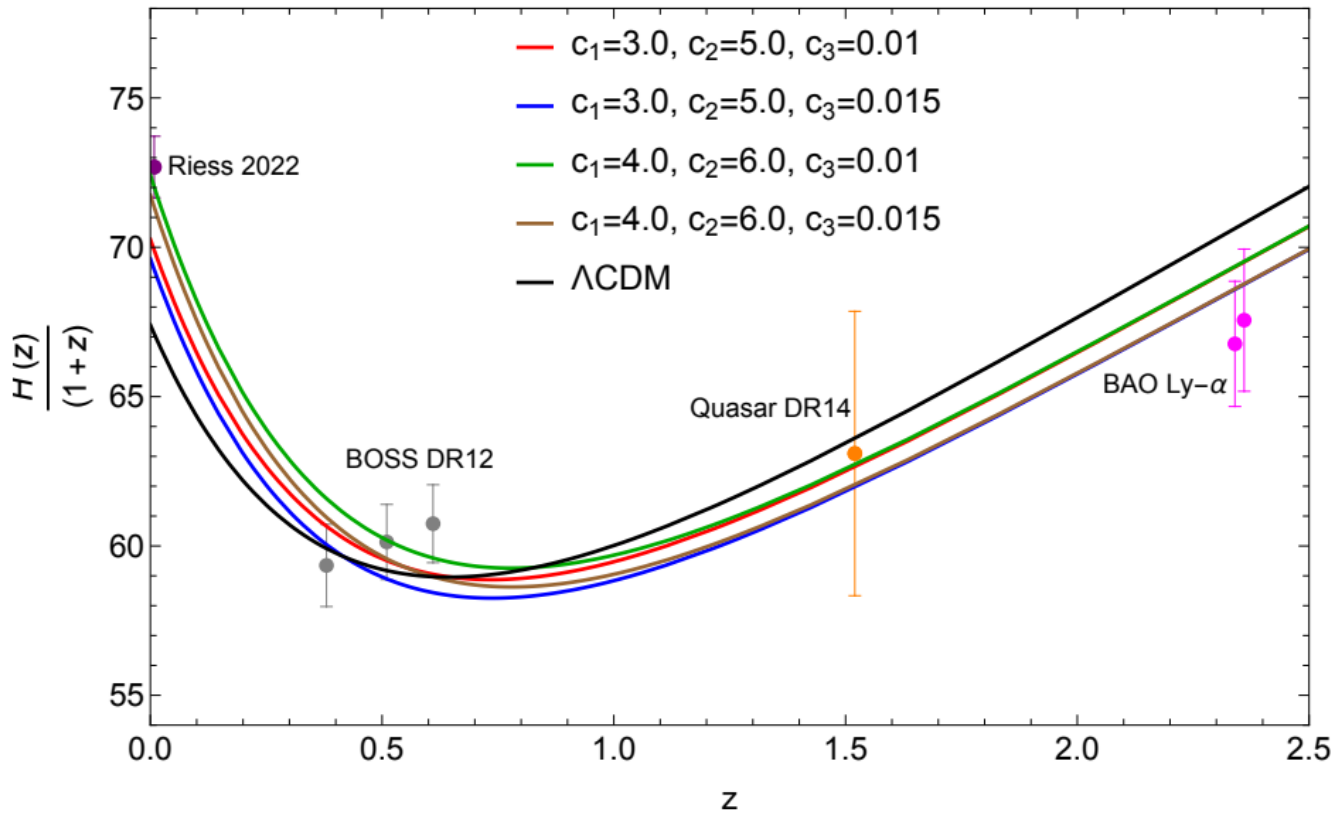
Necessary for D to remain undisturbed

$$D(z_L) = \int_0^{z_L} \frac{dz}{H(z)}$$

i.e. to ensure CMB peaks remain undisturbed

(Refer: Adil et al 2023)

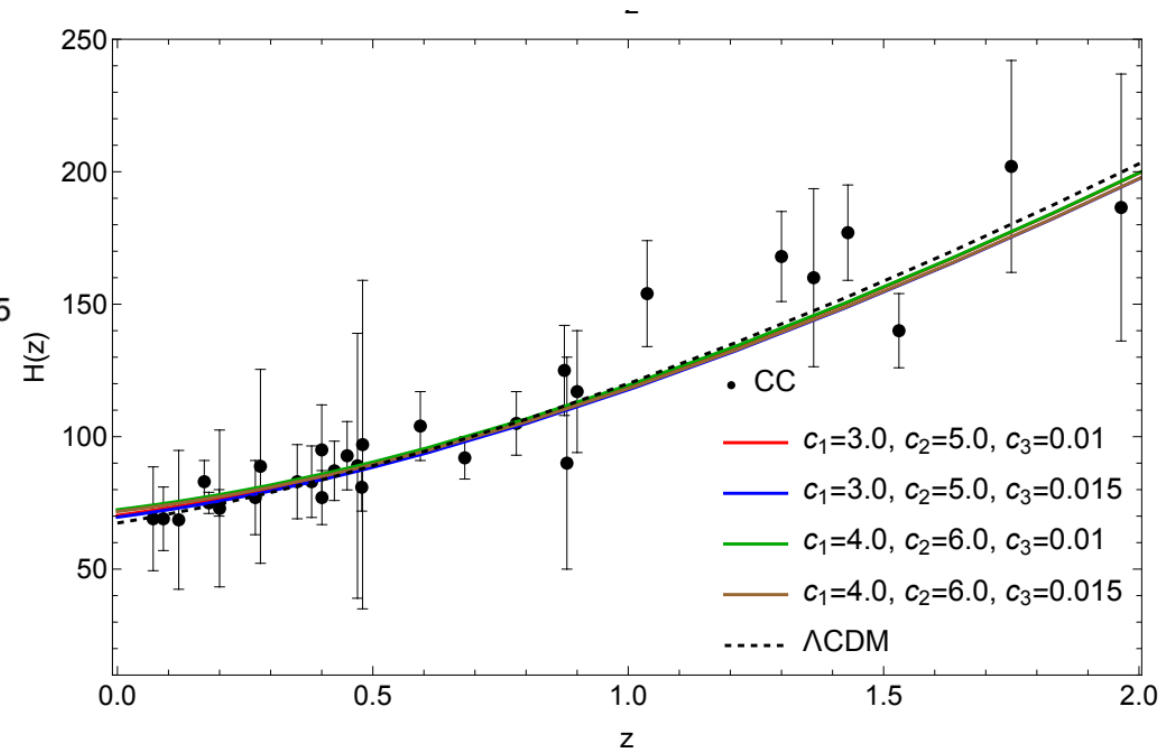
Distinct Features



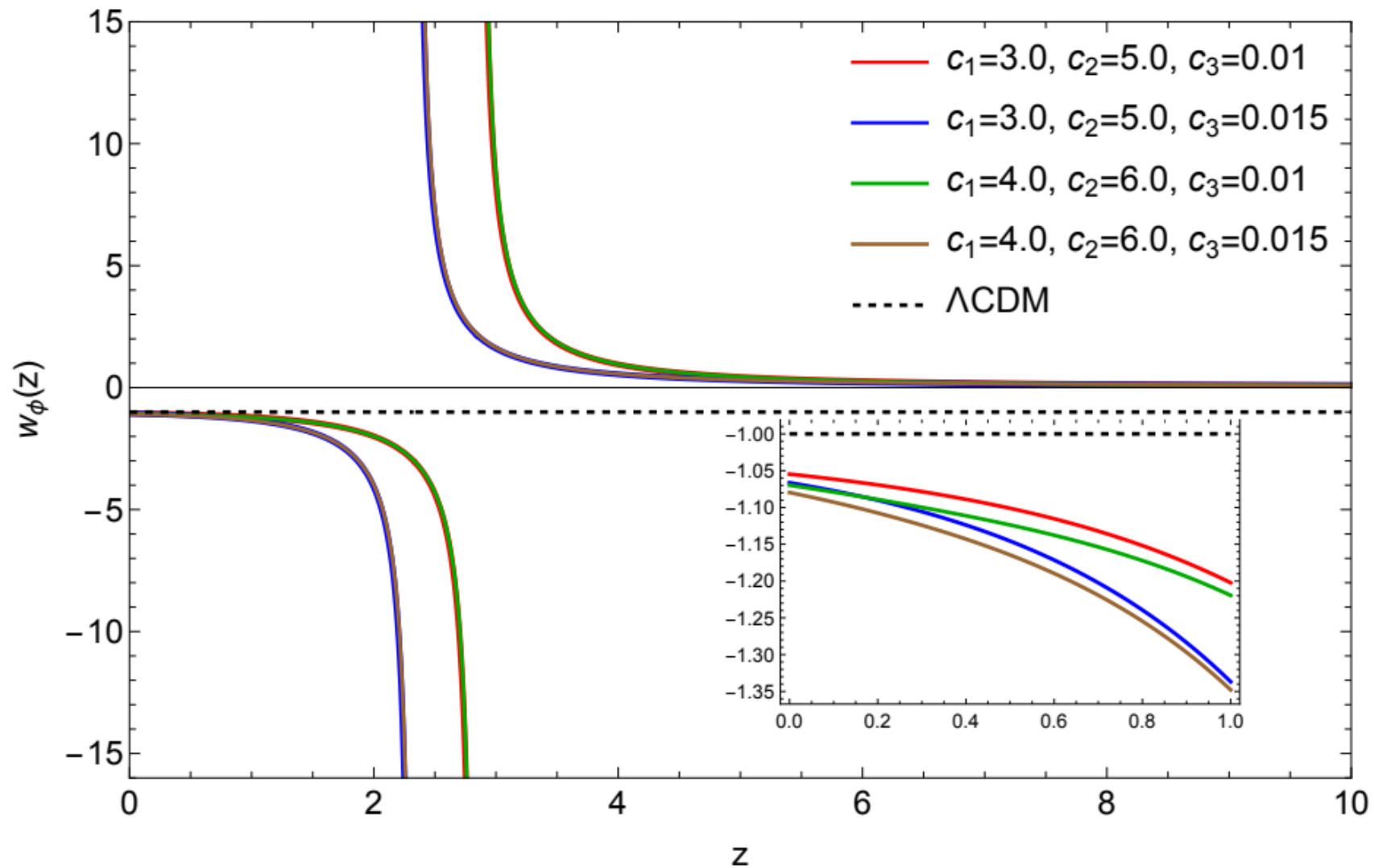
Evolution of $H(z)$ with 33 Cosmic chronometer (CC) data

Negative energy density ($c_3 > 0$) can explain the BAO Lyman-alpha anomalous $H(z)$ measurement at $z \sim 2.3$.

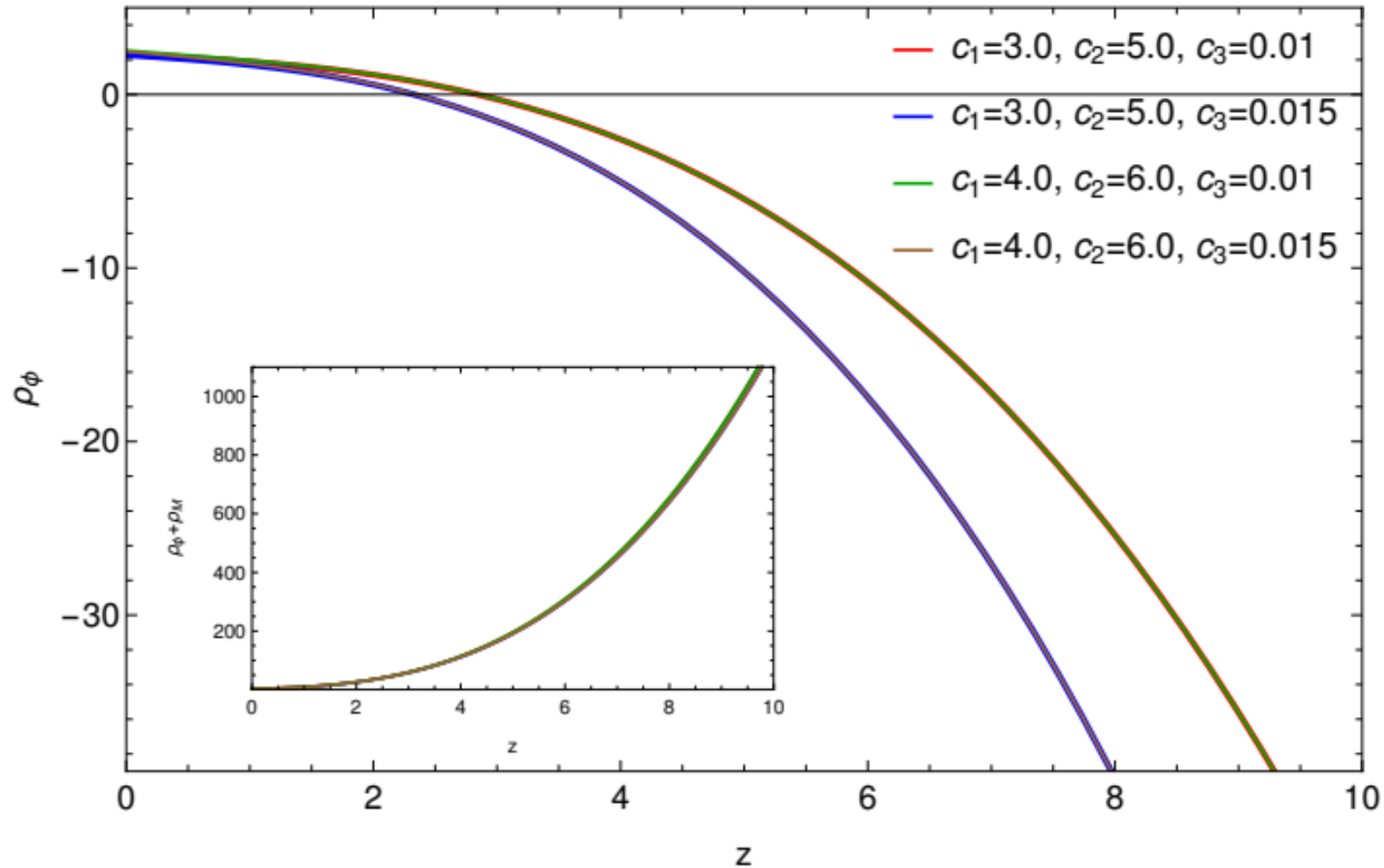
(Aubourg et al 2014, Sahni et al 2014, Poulin et al 2018, Adil et al 2023)



Distinct Features : Equation of state



Distinct Features : Energy density of scalar field



- Energy density of the scalar field is negative at high redshifts.
- The **total** energy density of universe is always **positive**.

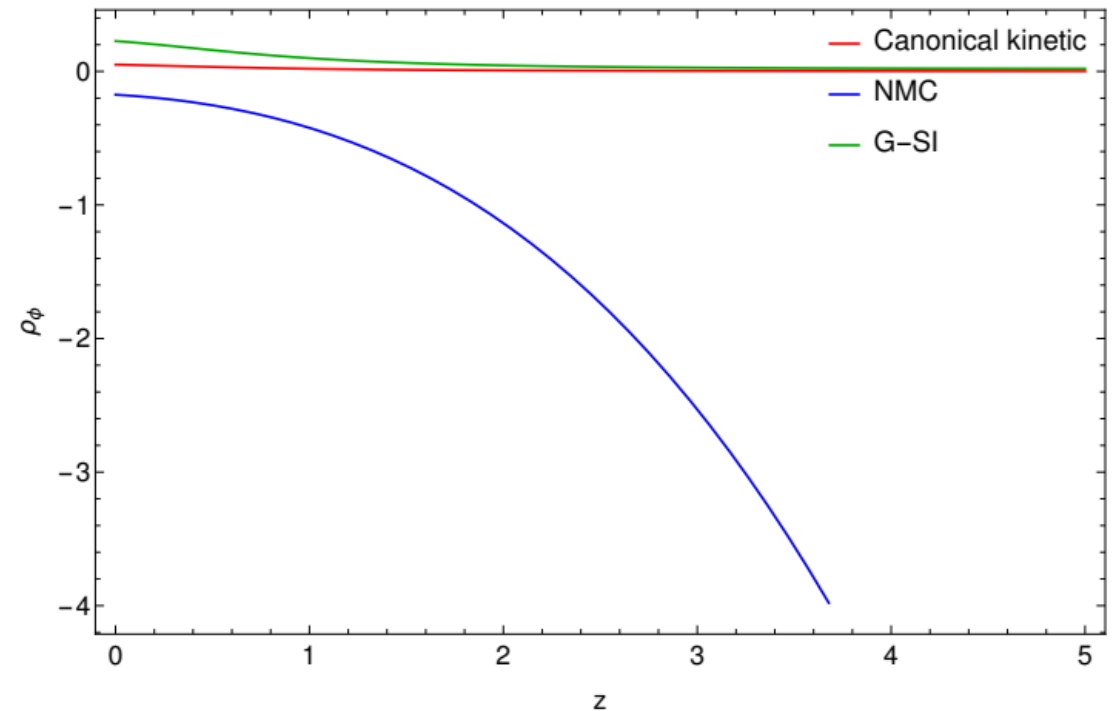
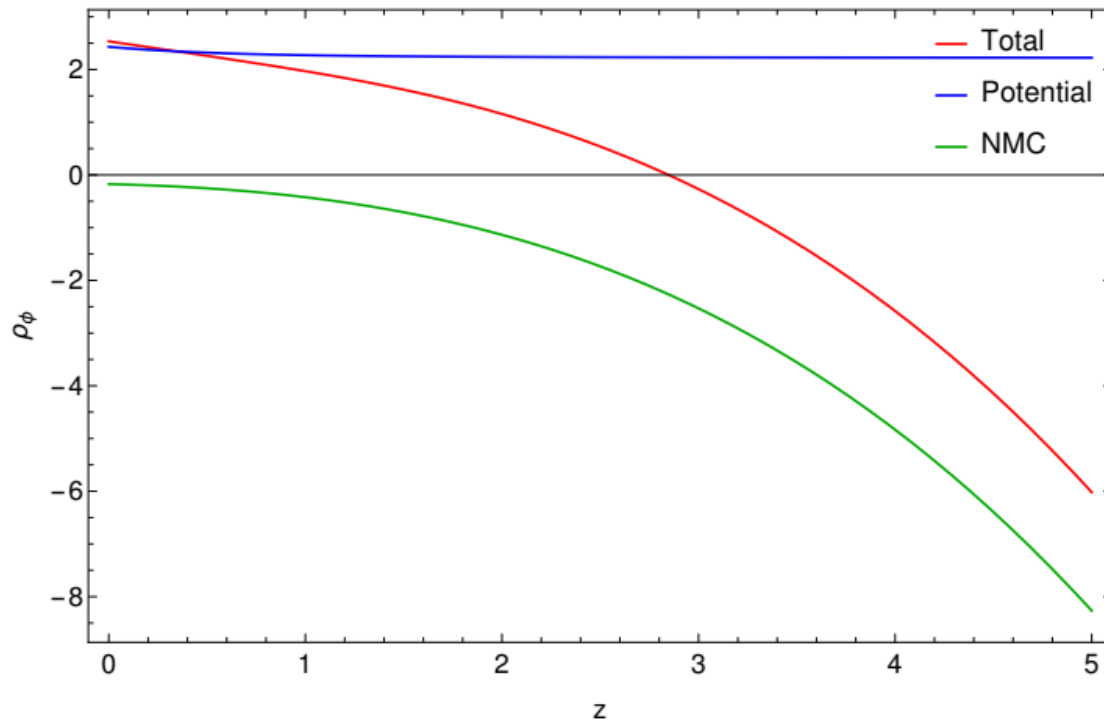
Distinct Features : Energy density of scalar field

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) - \underbrace{6c_3\phi H^2 - 6c_3H\dot{\phi}}_{\text{(NMC)}} - \underbrace{c_1\dot{\phi}^2 + 3c_2H\dot{\phi}^3}_{\text{(G-SI)}}$$

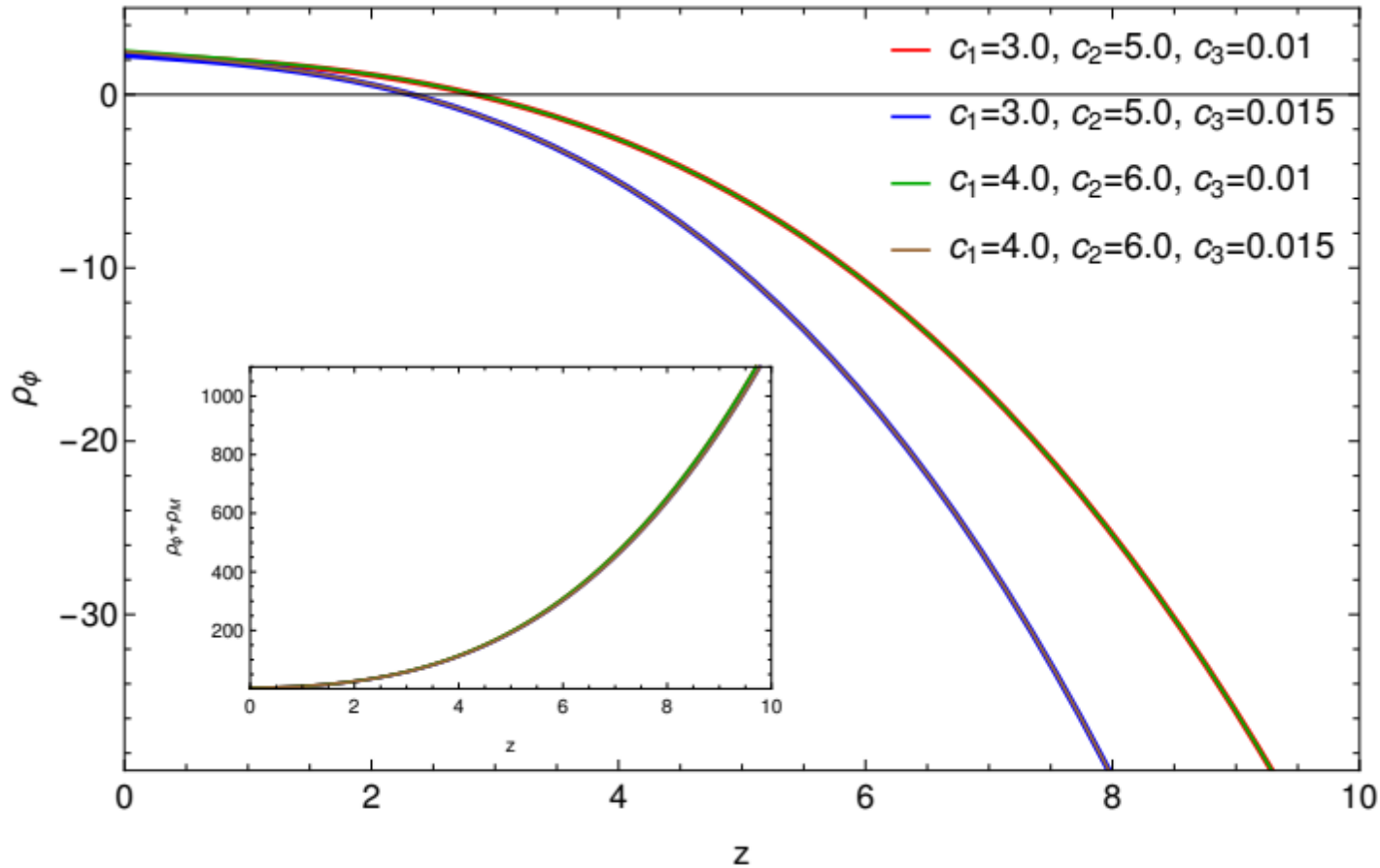
Dominated at high z as H ↑ as z ↑.
Dominated at low z as $\dot{\phi}$ increases.

$$G_3(\phi, X) = c_1\phi + c_2X$$

$$G_4(\phi) = \frac{1}{2} + c_3\phi$$



Distinct Features : Phantom Crossing

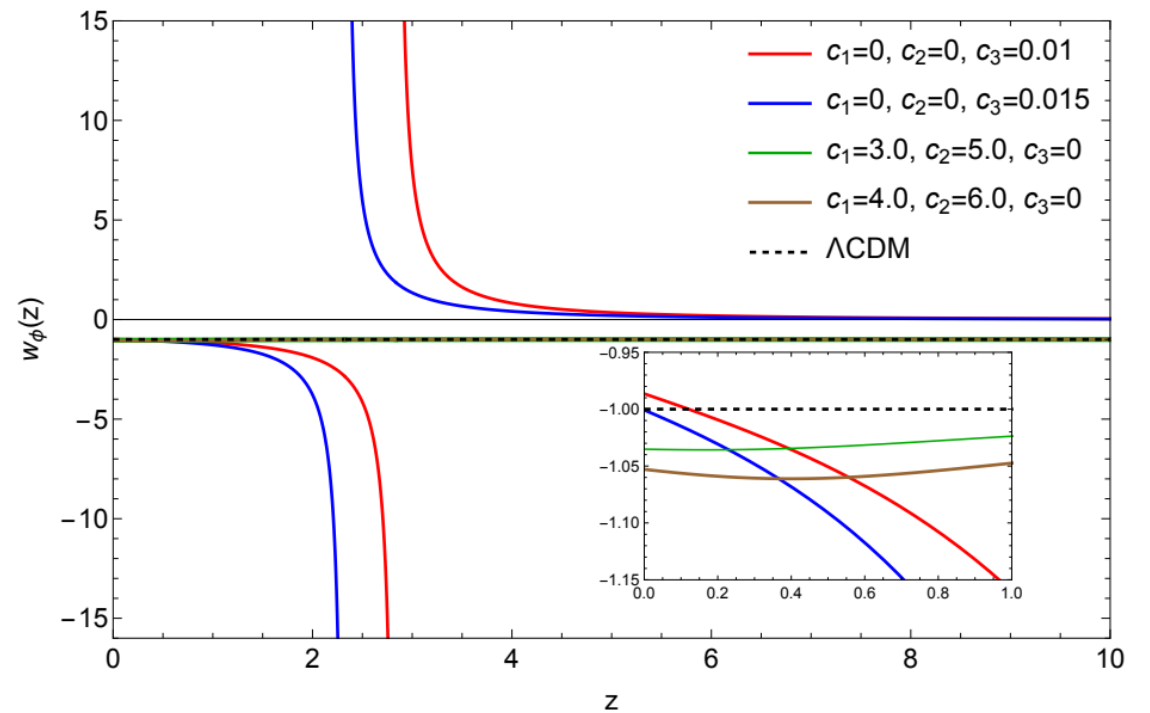
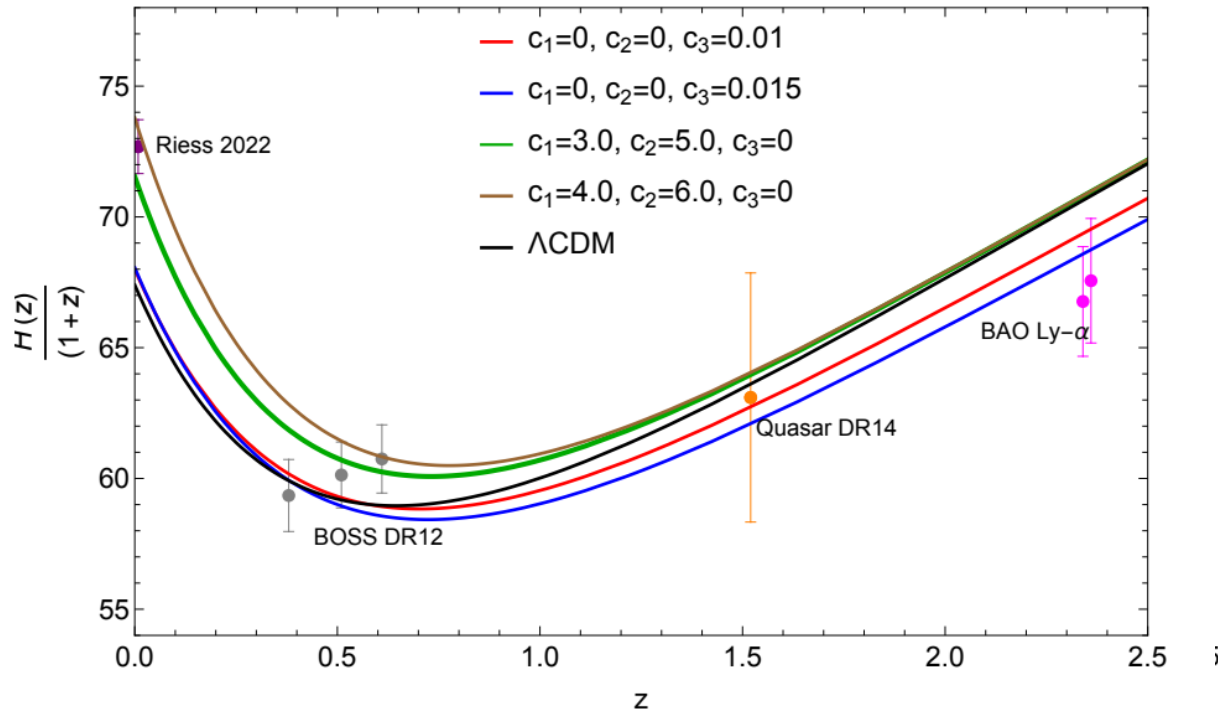


The evolution of energy density of scalar field from negative to positive values leads to a **phantom crossing**. This can be understood as follows,

$$\frac{d\rho_\phi}{dz} = \frac{3(1+w_\phi)\rho_\phi}{1+z}$$

$$\text{As } \frac{d\rho_\phi}{dz} < 0, \quad \begin{aligned} \rho_\phi < 0 &\Rightarrow w_\phi > -1 \\ \rho_\phi > 0 &\Rightarrow w_\phi < -1 \end{aligned}$$

Effect of interactions on dynamics (Separately)



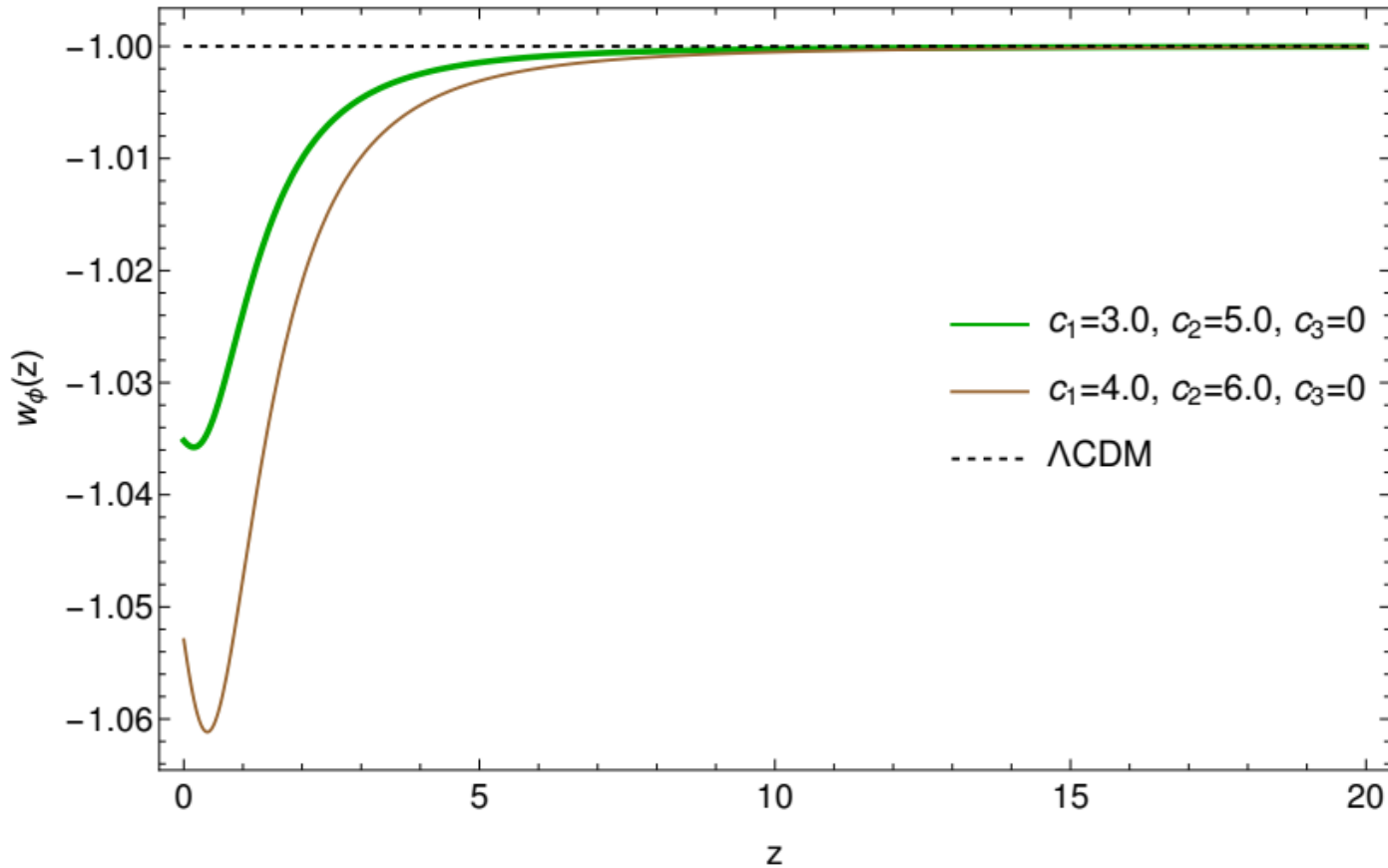
Case : $G_3(\phi, X) = 0$; $G_4 = \frac{1}{2} + c_3\phi$

$G_3(\phi, X) = c_1\phi + c_2X$

- Only non-minimal coupling
- Negative energy density at high redshifts.
- Phantom crossing (a necessary condition!)
- May not achieve very large H_0 .

$G_4(\phi) = \frac{1}{2} + c_3\phi$

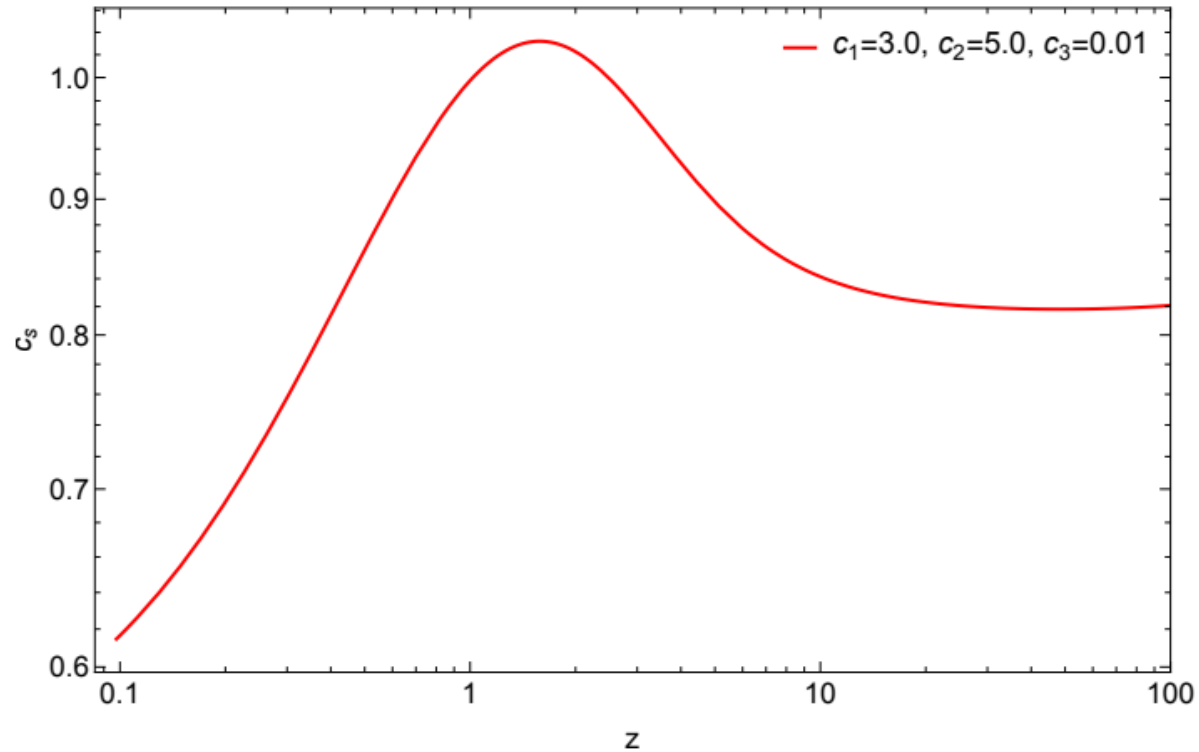
Effect of interactions on dynamics (Separately)



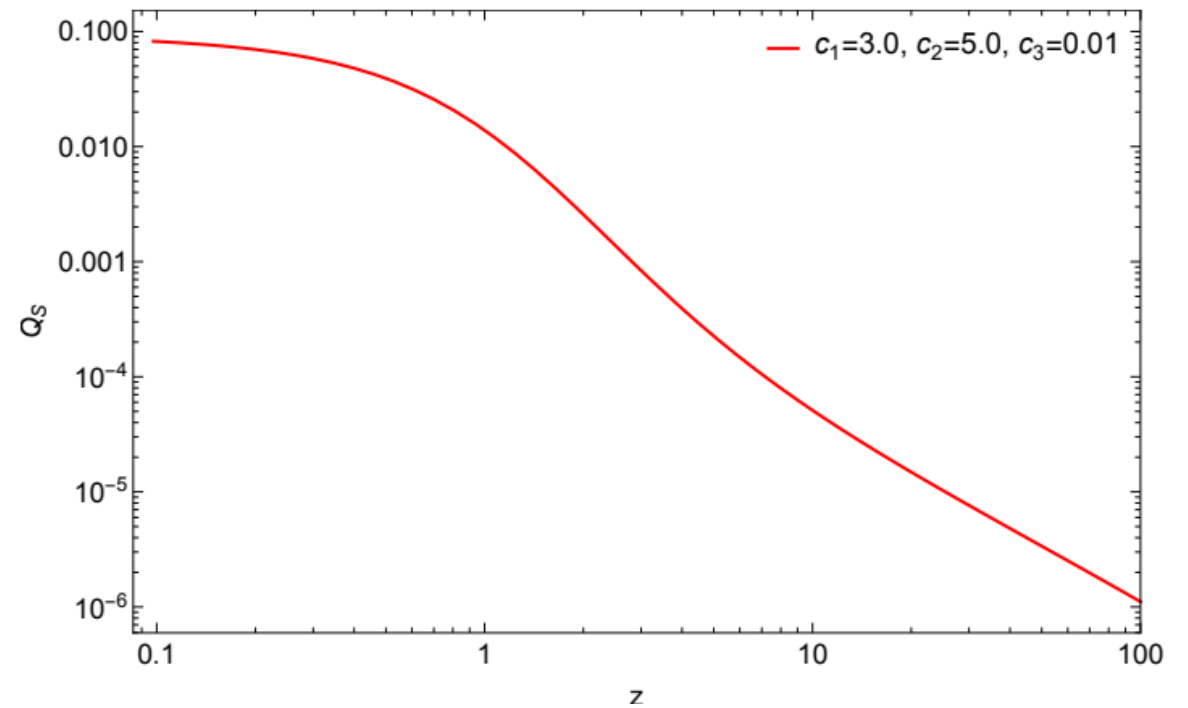
Case : $G_3(\phi, X) = c_1\phi + c_2X$; $G_4 = \frac{1}{2}$; $c_3 = 0$

- Only self interactions.
- Phantom switch at low redshifts.
- Energy density is always positive.
- Large values of H_0 can be attained by tuning c_1, c_2 .
- No Phantom crossing
- Behaves like hockey-stick model (Not preferred by observations)

Stability Conditions: Towards consistent model building



No gradient instability



Ghost free theory

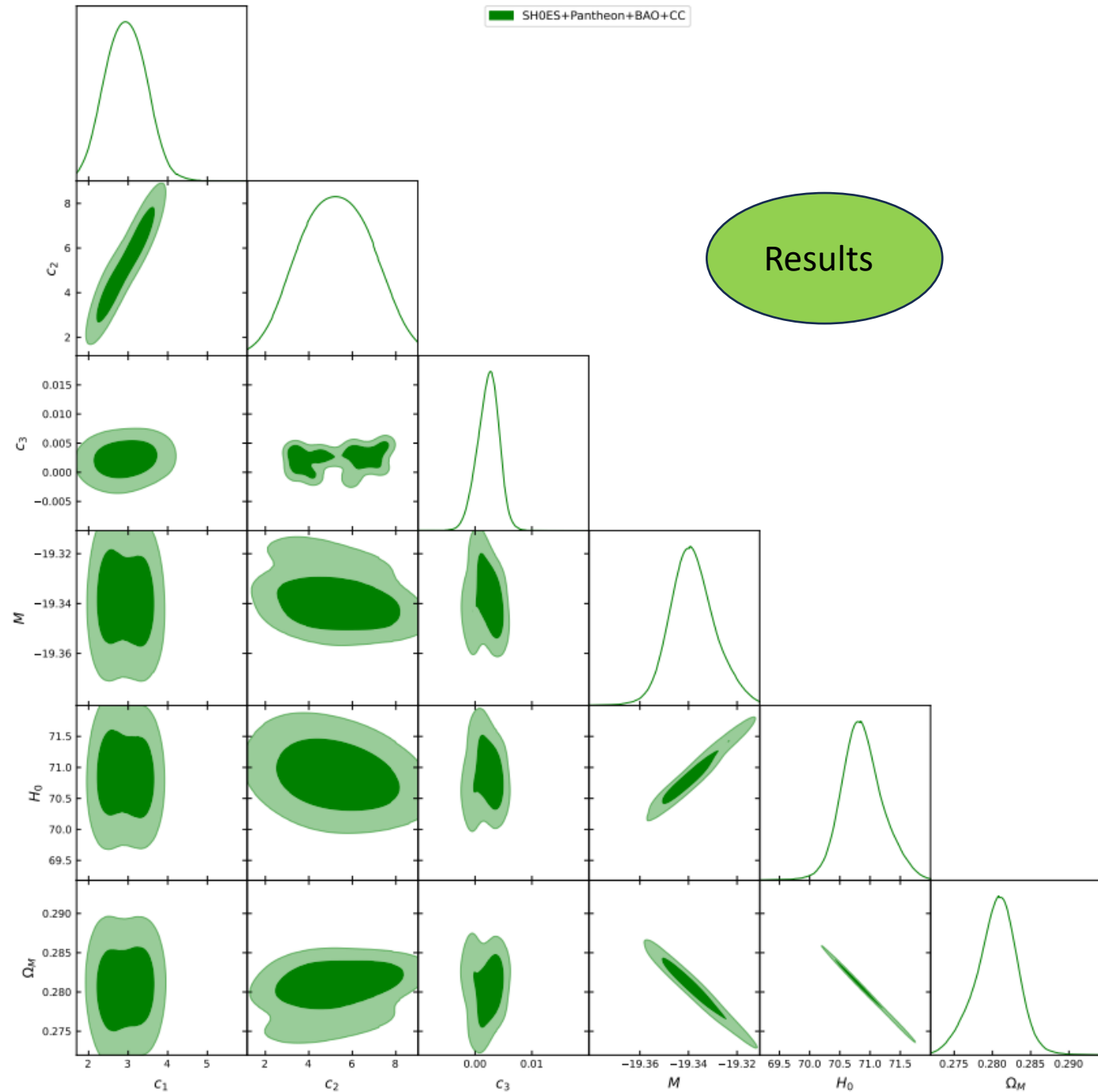
Constraints on Parameter space from Observations

- We employ Markov Chain Monte Carlo technique to obtain constraints on the parameter space.
- Uniform priors are provided on three model parameters and on absolute magnitude of Supernovae (M).
- We use the following observation data for our analysis:

Parameters	Priors
c_1	[2.0,8.0]
c_2	[2.0,10.0]
c_3	[-0.02,0.02]
M	[-19.5,-19.0]

1. SHOES: Modelled with a Gaussian likelihood on $M = -19.2435 \pm 0.0373$ (*Riess et al 2021, Camarena et al 2021*)
2. 1048 SNIa Pantheon Sample in redshift range $0.01 < z < 2.3$. (*Scolnic et al 2017*)
3. Six $H(z)$ measurement from BAO. (*Alam et al 2020, also compiled in Table I of Tiwari et al 2023*)
4. 33 Cosmic chronometer (CC) measurement of $H(z)$. (*as compiled in Table III of Gomez-Valent et al 2023, using covariance matrix method as discussed in Moresco et al 2020*)

For more details visit: [arXiv 2301.09382](https://arxiv.org/abs/2301.09382)



Parameter	68% limits
c_1	2.93 ± 0.45
c_2	5.2 ± 1.5
c_3	$0.0023^{+0.0021}_{-0.0017}$
M	$-19.3383^{+0.0081}_{-0.010}$
H_0	$70.87^{+0.29}_{-0.37}$
Ω_M	$0.2805^{+0.0029}_{-0.0024}$

- Data prefers a positive c_3 (1σ) which indicates a preference of negative dark energy at high redshifts.
- Inclusion of CMB data will tighten the constraints on the parameter space.
- As of now the present model reduces the tension with SHOES measurement to $\sim 2\sigma$.

Conclusion and Future Prospects

- We exploit the phenomenology of Horndeski theory to build dark energy model in order to address Hubble Tension.
- Interesting features like negative energy density at high redshifts, phantom crossing, etc can be obtained in such a setup.
- Constrains are obtained on parameter space by Supernovae, BAO and CC data.
- Next step is to study the perturbation theory for such Horndeski models: obtaining power spectra and confronting with CMB data. *(in progress)*
- Studying the implications of such models towards resolution of other cosmological tensions like **growth tension**. *(in progress)*

Thank you
