On the challenges in the choice of the non-conformal coupling function in inflationary magnetogenesis

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Talk based on *arXiv:2111.01478 [astro-ph.CO]*

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Outline

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  - Generation of primordial magnetic field
  - Electro-magnetic (EM) power spectra
- Choice of coupling function for slow-roll inflationary models
- Construction of coupling function for models generating features in scalar power spectrum
  - Models with feature over large scale
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- Conclusion

Observational evidence

- In galaxies, magnetic field of strength $\sim 10^{-6}$ G on scales of $1 - 10$ Kpc [1]
- In clusters of galaxies, the strength is $\sim 10^{-7} - 10^{-6}$ G on scales of 10Kpc $-$ 1 Mpc[2]
- In intergalactic medium(IGM) voids $\geq 10^{-16}$ G on scales above 1 Mpc [3]

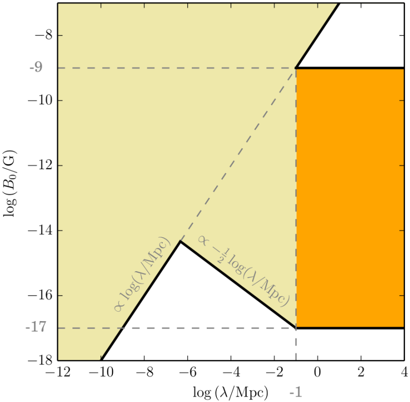
The origin of the seed magnetic field could be astrophysical or cosmological!

[1] Beck R 2001 Space Sci. Rev. 99 243–60,Beck R and Wielebinski R 2013 Planets, Stars and Stellar Systems vol 5, ed T D Oswalt and G Gilmore (Dordrecht: Springer) p 641

[2] Clarke T E, Kronberg P P and Böhringer H 2001 Astrophys. J. 547 L111–4,Govoni F and Feretti L 2004 Int. J. Mod. Phys. D 13 1549–94

[3] Neronov A and Vovk I 2010 Science 328 73

Constraints on inter galactic magnetic field (IGMF)



Constraints on B_0 , the magnetic field strength today, as a function of the comoving scale λ.[4]

[4] Tommi Markkanen, Sami Nurmi, Syksy Räsänen and Vincent Vennin, JCAP06(2017)035

Inflation

- Inflation is a period of accelerated expansion of the early universe, introduced to solve horizon and flatness problem[5].

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- The equation of motion for inflaton field is

$$\ddot{\phi} + 3H\dot{\phi} + V_{\phi} = 0,$$

where $H = \dot{a}/a$ is Hubble parameter and a is the scale factor.

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  where $H = \dot{a}/a$ is Hubble parameter and a is the scale factor.
- The slow roll parameters are defined as

$$\epsilon_1 = -\frac{H_N}{H}$$

  and

$$\epsilon_{i+1} = \frac{\mathrm{dln}\epsilon_i}{\mathrm{d}N}$$

Here, e-fold $N = \int \mathrm{d}t\, H = \ln\left(\frac{a(t)}{a_i}\right)$

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Generation of primordial magnetic field (PMF)

$$S[A^{\mu}] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

- Standard electromagnetic action is invariant under conformal transformations.
- FLRW metric is always conformally flat: $g_{\mu\nu}^{\mathrm{FLRW}} = \Omega^2\eta_{\mu\nu}$
- Hence $B \propto 1/a(t)^2$ and spectrum is scale dependednt. [6]

[6] Kandaswamy Subramanian,Rep. Prog. Phys. 79 (2016) 076901 (47pp)

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$$S[A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\sqrt{-g}J(\phi)^2F_{\mu\nu}F^{\mu\nu}$$

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In Coulomb gauge ($A_\eta = 0$ and $\partial_i A^i = 0$)

$$\bar{A}_k'' + 2\,\frac{J'}{J}\,\bar{A}_k' + k^2\,\bar{A}_k = 0.$$

If we write $\bar{A}_k = \mathcal{A}_k/J$, then this equation reduces to[7]

$$\mathcal{A}_k'' + \left(k^2 - \frac{J''}{J}\right)\mathcal{A}_k = 0.$$

[7] Jerome Martin, Jun'ichi Yokoyama, JCAP 01 (2008) 025

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The power spectra associated with the magnetic and electric fields are defined to be

$$\begin{aligned}
\mathcal{P}_{\rm B}(k) &= \frac{k^5}{2\,\pi^2}\,\frac{J^2}{a^4}\,|\bar{A}_k|^2 = \frac{k^5}{2\,\pi^2\,a^4}\,|\mathcal{A}_k|^2,\\
\mathcal{P}_{\rm E}(k) &= \frac{k^3}{2\,\pi^2}\,\frac{J^2}{a^4}\,|\bar{A}_k'|^2 = \frac{k^3}{2\,\pi^2\,a^4}\,\left|\mathcal{A}_k' - \frac{J'}{J}\,\mathcal{A}_k\right|^2.
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EM power spectra

For the choice of coupling function $J(\eta) \propto a(\eta)^n$(in de-Sitter $a = -1/H_I\eta$)

The magnetic and electric power spectra are[8]

$$\mathcal{P}_{\rm B}(k) = \frac{H_{\rm I}^4}{8\,\pi}\,\mathcal{F}(m)\,(-k\,\eta_{\rm e})^{2\,m+6},$$

where,

$$\mathcal{F}(m) = \frac{1}{2^{2\,m+1}\cos^2(m\,\pi)\,\Gamma^2(m+3/2)},$$

and

$$m = \begin{cases} n, & \text{for } n < -\frac{1}{2}, \\ -n-1, & \text{for } n > -\frac{1}{2}. \end{cases}$$

$$\mathcal{P}_{\rm E}(k) = \frac{H_{\rm I}^4}{8\,\pi}\,\mathcal{G}(m)\,(-k\,\eta_{\rm e})^{2\,m+4},$$

where,

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and

$$m = \begin{cases} n, & \text{for } n < \frac{1}{2}, \\ 1-n, & \text{for } n > \frac{1}{2}. \end{cases}$$

---

[8] Debika Chowdhury, L. Sriramkumar, Rajeev Kumar Jain, Phys.Rev.D 94 (2016) 8, 083512

$$n_{\rm B} = \begin{cases} 2\,n + 6, & \text{for } n < -\frac{1}{2}, \\ 4 - 2\,n, & \text{for } n > -\frac{1}{2}, \end{cases} \qquad n_{\rm E} = \begin{cases} 2\,n + 4, & \text{for } n < \frac{1}{2}, \\ 6 - 2\,n, & \text{for } n > \frac{1}{2}. \end{cases}$$

The specturm of magnetic field evidently is scale invariant when, $n \simeq -3$ or when $n \simeq 2$.

[9] $\rho_{\rm I}/M_{\rm Pl}^4 \sim H_{\rm I}^2/M_{\rm Pl}^2$

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For $n = -3$, $n_{\rm E} = -2$ and

$$\mathcal{P}_{\rm E}(k) \propto (-k\eta_{\rm e})^{-2}$$

This leads to backreaction $(\mathcal{P}_{\rm B}(k) + \mathcal{P}_{\rm E}(k) \gg \rho_{\rm I})$[9].

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$$\mathcal{P}_{\rm B}(k) = \frac{9\,H_{\rm I}^4}{4\,\pi^2}, \quad \mathcal{P}_{\rm E}(k) = \frac{H_{\rm I}^4}{4\,\pi^2}\,(-k\,\eta_{\rm e})^2.$$

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Helical magnetic field

$$S[A^\mu] \;\; = \;\; -\frac{1}{16\,\pi}\int \mathrm{d}^4x\,\sqrt{-g}\left[J^2(\phi)\,F_{\mu\nu}\,F^{\mu\nu} - \frac{\gamma}{2}\,I^2(\phi)\,F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right],$$

where $\widetilde{F}^{\mu\nu} = (\epsilon^{\mu\nu\alpha\beta}/\sqrt{-g})\,F_{\alpha\beta}$

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where $\widetilde{F}^{\mu\nu} = (\epsilon^{\mu\nu\alpha\beta}/\sqrt{-g})\,F_{\alpha\beta}$
The equation of motion

$$\mathcal{A}_k^{\sigma\,\prime\prime} + \left(k^2 + \frac{2\,\sigma\,\gamma\,k\,J'}{J} - \frac{J''}{J}\right)\mathcal{A}_k^{\sigma} = 0.$$

where $\sigma = \pm 1$ represents positive and negative helicity.

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The power spectra of the magnetic and electric fields

$$\mathcal{P}_{\rm B}(k) \quad = \quad \frac{k^5}{4\,\pi^2}\,\frac{J^2}{a^4}\,\left[\left|\bar{A}_k^+\right|^2 + \left|\bar{A}_k^-\right|^2\right] = \frac{k^5}{4\,\pi^2\,a^4}\,\left[\left|\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^-\right|^2\right],$$

$$\mathcal{P}_{\rm E}(k) \quad = \quad \frac{k^3}{4\,\pi^2}\,\frac{J^2}{a^4}\,\left[\left|\bar{A}_k^{+\prime}\right|^2 + \left|\bar{A}_k^{-\prime}\right|^2\right]$$

$$\quad = \quad \frac{k^3}{4\,\pi^2\,a^4}\,\left[\left|\mathcal{A}_k^{+\prime} - \frac{J'}{J}\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^{-\prime} - \frac{J'}{J}\mathcal{A}_k^-\right|^2\right].$$

For $n = 2$,

$$\begin{aligned}
\mathcal{P}_{\rm B}(k) &= \frac{9\,H_{\rm I}^4}{4\,\pi^2}\,f(\gamma),\\
\mathcal{P}_{\rm E}(k) &= \frac{9\,H_{\rm I}^4}{4\,\pi^2}\,f(\gamma)\,\biggl[\gamma^2 - \frac{\sinh^2(2\,\pi\,\gamma)}{3\,\pi\,\left(1+\gamma^2\right)\,f(\gamma)}\,(-k\,\eta_{\rm e})\\
&\quad + \frac{1}{9}\,\left(1 + 23\,\gamma^2 + 40\,\gamma^4\right)\,\left(-k\,\eta_{\rm e}\right)^2\biggr],
\end{aligned}$$

where the function $f(\gamma)$ is given by

$$f(\gamma) = \frac{\sinh\left(4\,\pi\,\gamma\right)}{4\,\pi\,\gamma\,\left(1 + 5\,\gamma^2 + 4\,\gamma^4\right)}.$$

For $\gamma = 1, \quad f(\gamma) \simeq 10^3$

Power spectrum in slow roll inflationary models

- In terms of e-folds, the coupling function is given by

$$J(N) = \left(\frac{a}{a_e}\right)^n = \exp\left[n\left(N - N_{\rm e}\right)\right]$$

Recall,

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0,$$

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Recall,

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- The quadratic potential

$$V(\phi) = \frac{m^2}{2}\,\phi^2.$$

we can arrive at the form of $J(N)$ that we desire if we choose $J(\phi)$ to be

$$J(\phi) = \exp\left[-\frac{n}{4\,M_{_{\rm Pl}}^2}\left(\phi^2 - \phi_{\rm e}^2\right)\right].$$

- The small field model described by the potential

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^q\right]$$

and we shall focus on the case wherein $q = 2$.
We choose the coupling function $J(\phi)$ to be

$$J(\phi) \simeq \left(\frac{\phi}{\phi_{\mathrm{e}}}\right)^{n\,\mu^2/2M_{_{\mathrm{Pl}}}^2} \exp\left[-\frac{n}{4\,M_{_{\mathrm{Pl}}}^2}\,(\phi^2 - \phi_{\mathrm{e}}^2)\right].$$

- The first Starobinsky model described by the potential

$$V(\phi) = V_0 \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{_{\rm Pl}}}\right)\right]^2.$$

We can choose $J(\phi)$ in the model to be

$$J(\phi) = \exp\left\{-\frac{3\,n}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{_{\rm Pl}}}\right) - \exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi_{\rm e}}{M_{_{\rm Pl}}}\right) - \sqrt{\frac{2}{3}}\left(\frac{\phi}{M_{_{\rm Pl}}} - \frac{\phi_{\rm e}}{M_{_{\rm Pl}}}\right)\right]\right\}.$$

Recall,

$$\mathcal{P}_{\rm B}(k) \simeq \frac{9\,H_{\rm I}^4}{4\,\pi^2} \simeq \frac{9\,\pi^2}{16}\,(r\,A_{\rm s})^2\,,$$
$$\mathcal{P}_{\rm E}(k) \simeq \frac{\mathcal{P}_{\rm B}(k)}{9\,M_{\rm Pl}^4}\left(\frac{k_*}{k_e}\right)^2$$

where r = Tensor to scalar ratio and $A_{\rm s}$ = Amplitude of scalar power spectrum

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SR Model	r	$\mathcal{P}_{_{\rm B}}(k)/M_{\rm Pl}^4$
QP	1.6×10^{-1}	6.27×10^{-19}
SFM	5.79×10^{-2}	8.21×10^{-20}
FSM	4.8×10^{-3}	5.64×10^{-22}

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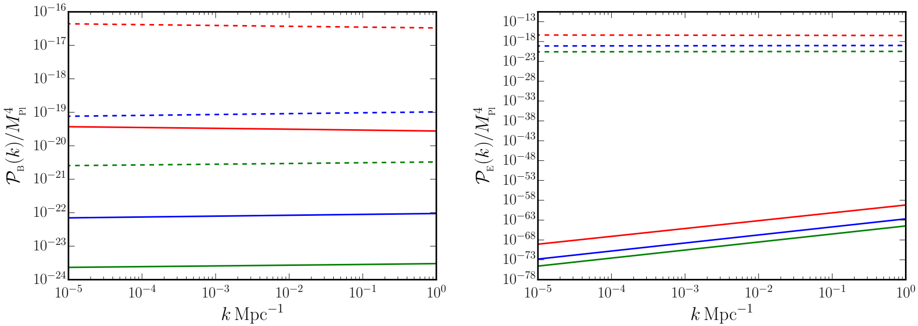
In the helical case, when $n = 2$,

$$\frac{\mathcal{P}_{_{\rm B}}(k)}{M_{_{\rm Pl}}^4} \simeq \frac{9\,\pi^2}{16}\,(r\,A_{\rm s})^2\,f(\gamma), \quad \frac{\mathcal{P}_{_{\rm E}}(k)}{M_{_{\rm Pl}}^4} \simeq \frac{\mathcal{P}_{_{\rm B}}(k)}{M_{_{\rm Pl}}^4}\,\gamma^2.$$

To avoid backreaction, $f(\gamma)\,(1+\gamma^2) \lesssim 10^{10} \implies \gamma \lesssim 2.5$

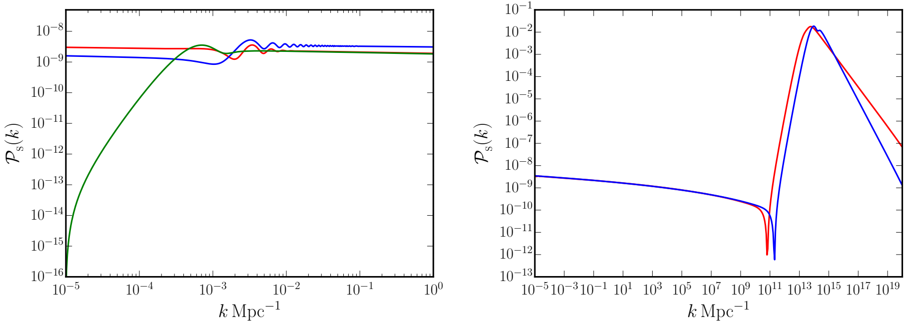


The spectra of the magnetic (on the left) and electric (on the right) for the quadratic potential (in red), the small field model (in blue) and the first Starobinsky model (in green) in both the non-helical (as solid lines) and helical (as dashed lines) cases.[10]

[10] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

Models generating Features in scalar power spectrum

- Introducing a step to the well know slow-roll models
- Model with sudden change in slope of potential between two slow roll regions
- Models with an inflection point in the potential



The scalar spectra with features over the CMB scales (on the left) for the QP with a step (in red), the second Starobinsky model (in blue), the first punctuated inflation model (in green) and the spectra with a peak at small scales (on the right) in the ultra slow roll (in red) and the second punctuated inflation(in blue) models.

Coupling function for models generating features in scalar power spectrum

Features over large scales

- **Introducing a step by hand in the potential** which has the form

$$V_{\rm step}(\phi) = V(\phi)\left[1 + \alpha \tanh\left(\frac{\phi - \phi_0}{\Delta\phi}\right)\right]$$

where, evidently, ϕ_0, α and $\Delta\phi$ denote the location, the height and the width of the step.

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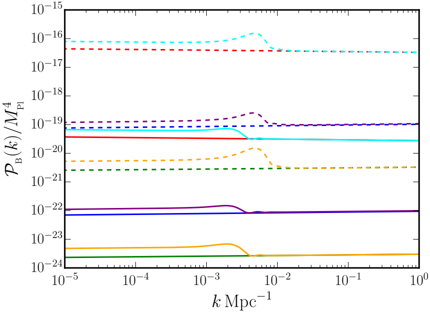
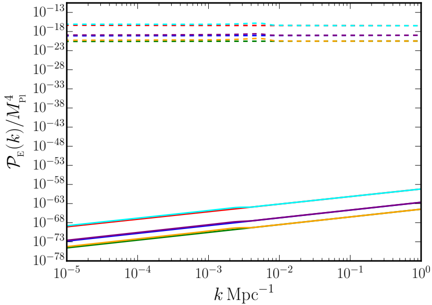


The spectra of the magnetic field for potential with step for (in cyan), the small field model (in purple) and the first Starobinsky model (in orange) in both the non-helical (as solid lines) and helical (as dashed lines) cases.[11]



The spectra of the Electric field for potential with step for (in cyan), the small field model (in purple) and the first Starobinsky model (in orange) in both the non-helical (as solid lines) and helical (as dashed lines) cases.[11]

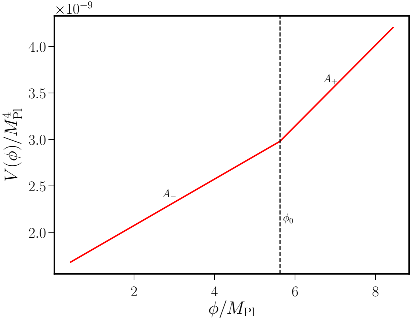
[11] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

Model with change in slope in potential

Second Starobinsky model[12],

$$V(\phi) = \begin{cases} V_0 + A_+\,(\phi - \phi_0), & \text{for } \phi > \phi_0, \\ V_0 + A_-\,(\phi - \phi_0), & \text{for } \phi < \phi_0. \end{cases}$$

.



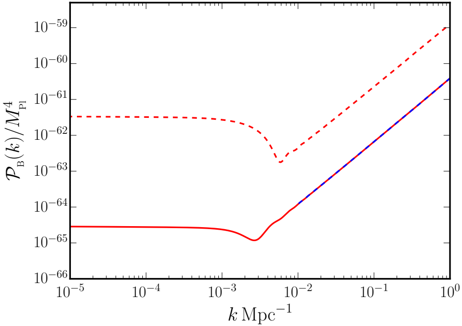
For numerical analysis

$$\begin{aligned} V(\phi) &= V_0 + \frac{1}{2}\,(A_+ + A_-)\,(\phi - \phi_0) \\ &\quad + \frac{1}{2}\,(A_+ + A_-)\,(\phi - \phi_0)\tanh\left(\frac{\phi - \phi_0}{\Delta\phi}\right), \end{aligned}$$

[12] A. A. Starobinsky, JETP Lett. 55, 489 (1992)

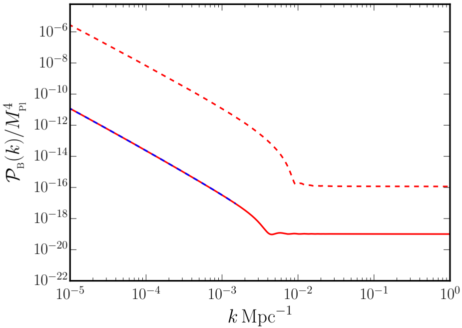
$$J_+(\phi) = J_{0+} \exp\left\{-\frac{n}{2\,M_{_{\rm Pl}}^2}\left[\left(\phi_+ - \phi_0 + \frac{V_0}{A_+}\right)^2 - \left(\phi_{\rm i} - \phi_0 + \frac{V_0}{A_+}\right)^2\right]\right\}$$

.



A linear fit (indicated in dashed blue) to the non-helical power spectra over the small scales lead to the spectral indices $n_{_{\rm B}} = 1.75$. For the values of the parameters we have worked with, the analytical estimates $(n_{_{\rm B}} = -4\,(A_- - A_+)/A_+)$ for these indices prove to be $n_{_{\rm B}} = 1.71$.

$$J_-(\phi) \quad = \quad J_{0-}\, \exp\left\{-\frac{n}{2\, M_{_{\rm Pl}}^2} \left[\left(\phi_- - \phi_0 + \frac{V_0}{A_-}\right)^2 - \left(\frac{V_0}{A_-}\right)^2 - 2\, N_0\, M_{_{\rm Pl}}^2\right]\right\},$$

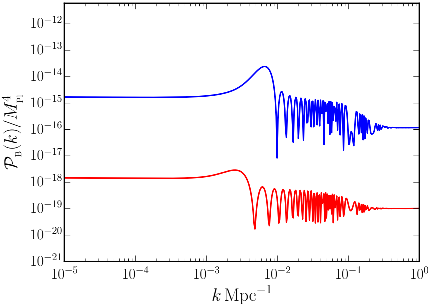


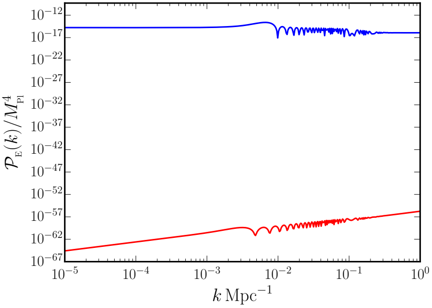
A linear fit (indicated in dashed blue) to the non-helical power spectra over the large scales lead to the spectral indices $n_{_{\rm B}} = -2.78$[13]. For the values of the parameters we have worked with, the analytical estimates $(n_{_{\rm B}} = 4\,(A_- - A_+)/A_-)$ for these indices prove to be $n_{_{\rm B}} = -2.92$.

[13] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

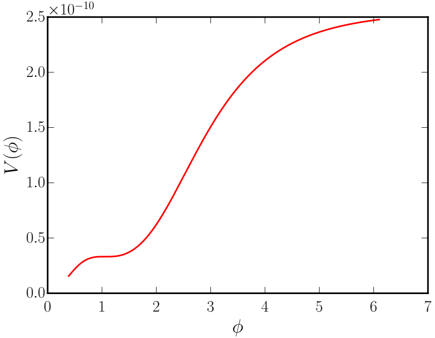
Ironing out the feature

$$J(\phi) = \frac{J_1}{2\,J_{0+}}\left[1+\tanh\left(\frac{\phi-\phi_0}{\Delta\phi_1}\right)\right] J_+(\phi) + \frac{J_1}{2\,J_{0-}}\left[1-\tanh\left(\frac{\phi-\phi_0}{\Delta\phi_1}\right)\right] J_-(\phi),$$





Features due to models with inflection point in the potential



Features over large scales

- First punctuation inflation model

$$V(\phi) = \frac{m^2}{2}\,\phi^2 - \frac{2\,m^2}{3\,\phi_0}\,\phi^3 + \frac{m^2}{4\,\phi_0^2}\,\phi^4.$$

Features over small scales

- Ultra slow roll model

$$V(\phi) = V_0 \left\{\tanh\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right) + A\,\sin\left[\frac{1}{f_\phi}\tanh\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right)\right]\right\}^2.$$

- Second punctuated inflation model

$$\begin{aligned} V(\phi) &= V_0\left[c_0 + c_1\,\tanh\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right)\right. \\ &\quad \left. + c_2\,\tanh^2\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right) + c_3\,\tanh^3\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right)\right]^2. \end{aligned}$$

- For first punctuated inflation model,

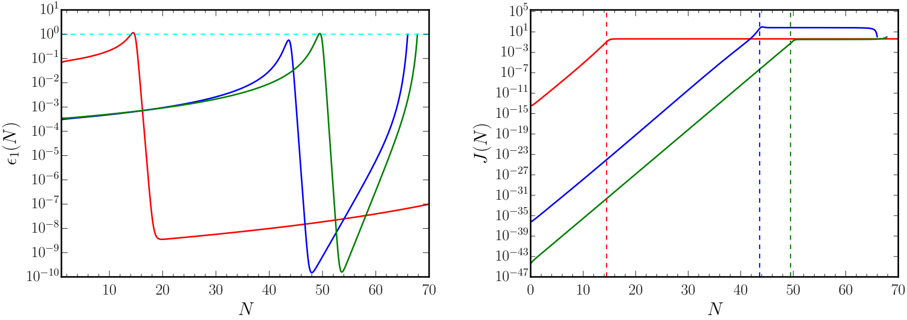
$$J(\phi) = \exp\left\{n\left[a_1\left(\frac{\phi^2-\phi_{\rm e}^2}{M_{_{\rm Pl}}^2}\right) + b_1\left(\frac{\phi-\phi_{\rm e}}{M_{_{\rm Pl}}}\right)\right]\right\}$$

- For ultra slow roll inflation model,

$$J(\phi) = \exp\left\{n\left[a_2\left(\frac{\phi^4-\phi_{\rm e}^4}{M_{_{\rm Pl}}^4}\right) + b_2\left(\frac{\phi^3-\phi_{\rm e}^3}{M_{_{\rm Pl}}^3}\right) + c_2\left(\frac{\phi^2-\phi_{\rm e}^2}{M_{_{\rm Pl}}^2}\right) + d_2\left(\frac{\phi-\phi_{\rm e}}{M_{_{\rm Pl}}}\right)\right]\right\},$$

- For second punctuated inflation inflation model,

$$J(\phi) = \exp\left\{n\left[a_3\left(\frac{\phi^6-\phi_{\rm e}^6}{M_{_{\rm Pl}}^6}\right) + b_3\left(\frac{\phi^5-\phi_{\rm e}^5}{M_{_{\rm Pl}}^5}\right) + c_3\left(\frac{\phi^4-\phi_{\rm e}^4}{M_{_{\rm Pl}}^4}\right) + d_3\left(\frac{\phi^3-\phi_{\rm e}^3}{M_{_{\rm Pl}}^3}\right) + e_3\left(\frac{\phi^2-\phi_{\rm e}^2}{M_{_{\rm Pl}}^2}\right) + f_3\left(\frac{\phi-\phi_{\rm e}}{M_{_{\rm Pl}}}\right)\right]\right\},$$

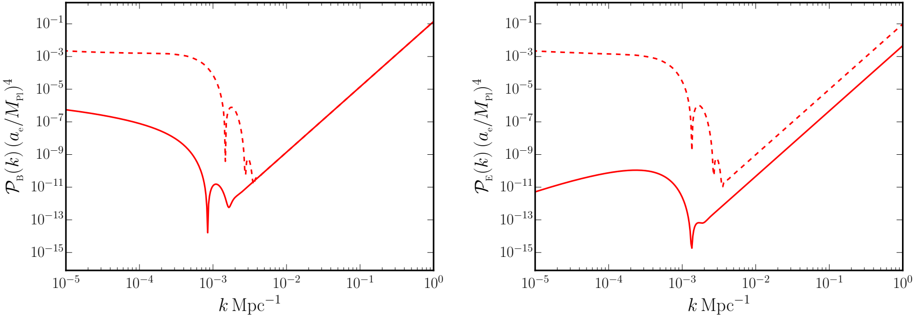


The evolution of ϵ_1 and $J(N)$ for the first and second punctuated inflation model and ultra slow model(in solid red, green and blue, respectively) as a function of the e-fold N. [14]

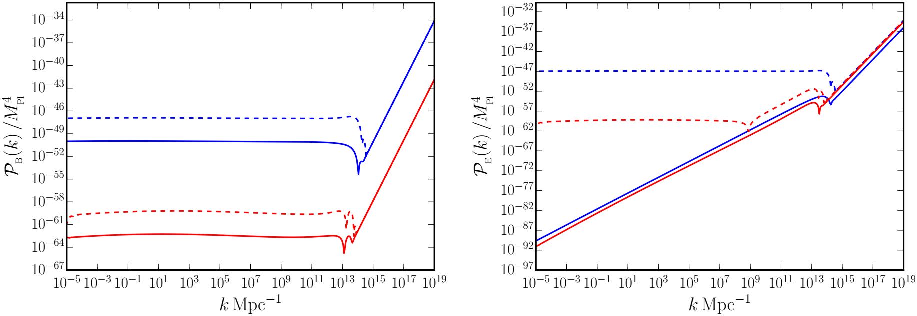
[14] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

EM power spectra

- For first punctuated inflation model,[15]



[15] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]



The spectra of the magnetic (on the left) and electric (on the right)fields in the ultra slow roll inflationary model (in red) and the second punctuated inflationary model(in blue) in the non-helical (as solid lines) and helical (as dashed lines)cases.[16]

[16] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

EM power spectra

Analytical estimate

- The choice of coupling function $J(\eta) \propto a^2$
- J is constant for the ultra slow roll region, so $J''/J \sim 0$
  The solution for $\mathcal{A}_k$(non helical)in the slow roll region

$$\mathcal{A}_k^{\mathrm{I}}(\eta) = \frac{1}{\sqrt{2\,k}} \left(1 - \frac{3\,i}{k\,\eta} - \frac{3}{k^2\,\eta^2}\right) \mathrm{e}^{-i\,k\,\eta}.$$

  In the ultra slow roll (usr) region

$$\mathcal{A}_k^{\mathrm{II}}(\eta) = \frac{1}{\sqrt{2\,k}} \left(\alpha_k\,\mathrm{e}^{-i\,k\,\eta} + \beta_k\,\mathrm{e}^{i\,k\,\eta}\right).$$

- Matching the solutions at the onset of USR, $(\eta = \eta_1)$, we get the coefficients α_k and β_k.

The power spectra
when $k\eta_1 \ll 1$

$$\mathcal{P}_{\rm B}(k) \simeq \frac{9\,H_{\rm I}^4}{4\,\pi^2}\left[\frac{a(\eta_1)}{a(\eta_{\rm e})}\right]^4,$$
$$\mathcal{P}_{\rm E}(k) \simeq \frac{H_{\rm I}^4}{4\,\pi^2}\left(\frac{4\,k}{k_1}\right)^2\left[\frac{a(\eta_1)}{a(\eta_{\rm e})}\right]^4.$$

when $k\eta_1 \gg 1$

$$\mathcal{P}_{\rm B}(k) \propto k^4 \,\text{and}\, \mathcal{P}_{\rm E}(k) \propto k^4$$

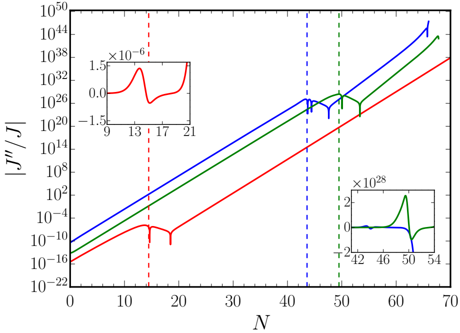
Similarly for the helical case when $k\eta_1 \ll 1$

$$\mathcal{P}_{\rm B}(k) \simeq \frac{9\,H_{\rm I}^4}{4\,\pi^2}\,f(\gamma)\left[\frac{a(\eta_1)}{a(\eta_{\rm e})}\right]^4,$$
$$\mathcal{P}_{\rm E}(k) \simeq \frac{9\,H_{\rm I}^4}{4\,\pi^2}\,f(\gamma)\,\gamma^2\left[\frac{a(\eta_1)}{a(\eta_{\rm e})}\right]^4,$$

and when $k\eta_1 \gg 1$

$$\mathcal{P}_{\rm B}(k) \propto k^4 \,\text{and}\, \mathcal{P}_{\rm E}(k) \propto k^4$$

Analytical VS Numerical



At suitably early times when $k \gg \sqrt{|J''/J|}$,

$$\mathcal{A}_k^{\rm I}(\eta) = \frac{1}{\sqrt{2\,k}}\,{\rm e}^{-i\,k\,\eta}.$$

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[17] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

At late times when $k \ll \sqrt{|J''/J|}$,

$$\mathcal{A}_k^{\rm II}(\eta) = \frac{1}{\sqrt{2\,k}}\,\left[\alpha_k + \beta_k\,\eta\right],$$

where the coefficients α_k and β_k are to be determined by matching the above solutions and their derivatives at the time η_k corresponding to $k = \sqrt{|J''/J|}$. Now,

$$\alpha_k = (1 + i\,k\,\eta_k)\,{\rm e}^{-i\,k\,\eta_k}, \quad \beta_k = -i\,k\,\eta_k\,{\rm e}^{-i\,k\,\eta_k},$$

and hence, at late times, we have

$$\mathcal{A}_k^{\rm II}(\eta) = \frac{1}{\sqrt{2\,k}}\,\left[1 - i\,k\,(\eta - \eta_k)\right]\,{\rm e}^{-i\,k\,\eta_k}.$$

In the limit $(-k\,\eta_{\mathrm{e}}) \ll 1$ to be

$$\begin{aligned}
\mathcal{P}_{\mathrm{B}}(k) &= \frac{H_{\mathrm{I}}^4}{4\,\pi^2}\,(-k\,\eta_{\mathrm{e}})^4\,\left(1 + k^2\,\eta_k^2\right), \\
\mathcal{P}_{\mathrm{E}}(k) &= \frac{H_{\mathrm{I}}^4}{4\,\pi^2}\,(-k\,\eta_{\mathrm{e}})^4.
\end{aligned}$$

Using the behaviour of J''/J at late times , we estimated $(k^2\,\eta_k^2) \lesssim 10^{-2}$

J(R) coupling

$$S[A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\,\sqrt{-g}\,J^2(R)\,F_{\mu\nu}\,F^{\mu\nu}$$

If we chose a coupling function such that $J(R) \propto \eta^{-n}$, we can expect scale invariant spectra for the magnetic field when $n = -3$ and $n = 2$.

$$R = 6\,\frac{a''}{a^3}$$

J(R) coupling

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$$R = 6\,\frac{a''}{a^3} = 6\,H^2\,(2 - \epsilon_1)$$

$$R \propto \eta^{2\,\epsilon_1}$$

J(R) coupling

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$$R = 6\,\frac{a''}{a^3} = 6\,H^2\,(2 - \epsilon_1)$$

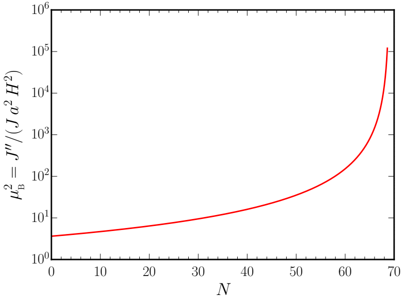
$$R \propto \eta^{2\,\epsilon_1}$$

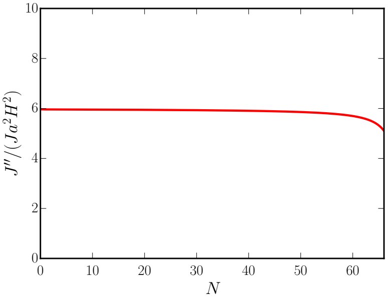
$$J(R) = \left(\frac{R(\eta)}{R(\eta_\mathrm{e})}\right)^\alpha,$$

In terms of the conformal time

$$J(\eta) \simeq \left(\frac{\eta}{\eta_\mathrm{e}}\right)^{2\,\epsilon_1\,\alpha},$$

If we choose $\alpha = -n/(2\,\epsilon_1)$, we can recover $J(R) \propto \eta^{-n}$.

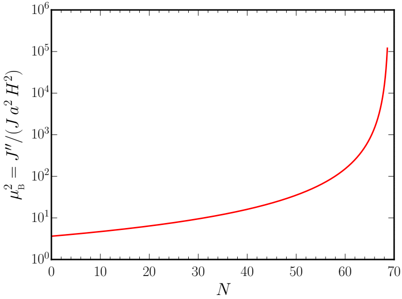


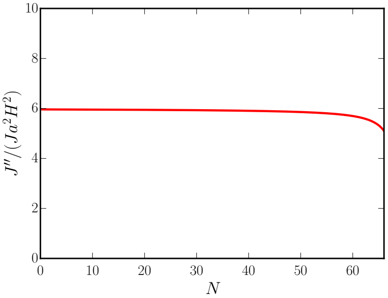


The evolution of the quantity $\mu_{\rm B}^2 = J''/(J\,a^2\,H^2)$, as a function of e-folds N for $J(R)$ coupling (on left) and $J(\phi)$ coupling(on right) for quadratic potential model. We have set $\alpha = -1/\epsilon_{1*} \simeq -10^2$, where ϵ_{1*} is the value of the first slow roll parameter when the pivot scale k_* leaves the Hubble radius. The pivot scale k_* leaves the Hubble radius at the e-fold of $N = 18.63$. We find that $\mu_{\rm B}^2 \simeq 6$ near $N \simeq 18$.[18]

$$J \propto R^{\alpha(N)} \text{ and } \alpha(N) = \frac{2\,(N-N_{\rm e})}{\ln\left[H^2\,(2-\epsilon_1)/H_{\rm e}^2\right]}$$

[18] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]





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$J \propto R^{\alpha(N)}$ and $\alpha(N) = \frac{2\,(N-N_{\rm e})}{\ln\left[H^2\,(2-\epsilon_1)/H_{\rm e}^2\right]}$

Breaks invariance of action under general coordinate transformation.

[18] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

Conclusion

- Although a nearly scale invariant primordial scalar power spectrum generated in slow roll inflationary models, is remarkably consistent with the CMB data, it has been repeatedly noticed that certain features in the scalar power spectrum can improve the fit to the data.
- When strong departures from slow roll arise, these deviations also led to features in the spectra of electromagnetic fields.
- In certain scenarios, it is also possible that the strengths of the magnetic fields are considerably suppressed on large scales.
- While it seems possible to remove the strong features in the spectra of the electromagnetic fields allowing us to arrive at nearly scale invariant spectra of required strengths, it is achieved at the terrible cost of extreme fine-tuning.
- When the electromagnetic fields are coupled to the scalar curvature, we found that it is challenging to obtain nearly scale invariant magnetic fields of the desired shapes and strengths even in slow roll inflation.

Thank You