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Explorations in Many-body Physics: Nuclei, Stars and Cold Atoms

I. INTRODUCTION

Many-body problems represent a diverse and complex set of *interacting* physical systems. Examples are numerous: electrons, nucleons, atoms etc. While a study of a particular type of system represents a sub-field of physics, such as atomic, condensed matter, nuclear physics etc, the techniques developed in the course of study are applicable across fields. Dealing with interacting systems can be very challenging as even the simple problem of obtaining the energy spectrum, given a many-body Hamiltonian involves solving a many-body Schödinger equation, which is typically non-trivial [1].

One of the challenging many-body systems are the once encountered in conventional nuclear physics, *the nucleus*. Nuclei are quantum many-body systems governed by the residual strong force and display a remarkably wide range of phenomena. Nuclear interactions are described in terms of pions and nucleons, where the pions are identified as the pseudo-goldstone bosons of the spontaneously broken approximate chiral symmetry of QCD. This description in terms of pions and nucleons is an effective field theory of the underlying strong interactions, QCD, and as is characteristic of all EFTs, this breaks down at a scale where the sub-structure, i.e. quarks and gluons are resolved. Nuclei exist over a wide range of densities, starting with the loosely bound deuteron, which is the only two-body state, to really dense objects like the neutron stars. Fig. 1 shows the nuclear landscape as a function of the number of protons Z and number of neutrons N. The region represented by black squares, referred to as the "valley of stability", contains stable nuclei and moving away from this region, one obtains the unstable isotopes, shaded pink in the figure. The neutron and the proton drip lines represent the limits of nuclear binding. The proton drip line lies closer to the valley of stability due to Coulomb repulsion and is accessible experimentally, while the neutron drip line tends to lie far from the region that is currently accessible through experiments. Hence its exact position still remains an open question. The region in white represents nuclei that are not yet synthesized in the lab. Many of the processes that are of astrophysical interest start with the lighter isotopes and eventually progress close to the drip lines. Current understanding of nuclei far from the valley of stability are based on what is known of the stable isotopes, such as the shell description, magic numbers corresponding to shell closure etc., hence extrapolations in this region tend to be unreliable and opens a wide range of questions and phenomena that remain to be addressed. Experimental facilities that have recently been established will probe nuclei in the unexplored regions of the chart and should provide constraints for the theory [2–4]. Conventional ab initio calculations of nuclear properties usually use the two-nucleon interactions that reproduce the data in the two-body sector accurately. This potential is not unique and experiments contrain only the low-energy part of the interaction. Hence model dependence enters any finite nuclei or nuclear matter calculation. The light nuclei sector already establishes the need for including three-body forces. Hence, one of the current long-term goal in theoretical nuclear physics is to develop a microscopic theory that is capable of describing nuclei across the nuclear chart. Deeper understanding of the physics of nuclei and nuclear matter is important to gain insights into astrophysical processes that go on in the interior of stars.

Another many-body system that has recently gained a lot of interest is the system of ultra-cold atoms and the diverse physics that can be explored as the interactions between the atoms can be easily controlled [5–8]. Advances in laser cooling and trapping have opened up possibilities to study complex many-body systems in a lab through cold atoms experiments. The ability to tune the interaction of the trapped atoms through Feshbach resonances make important connections with the systems encountered in nuclear physics. When the scattering length of the interacting atoms diverge, the notion of a length scale is lost and the physics becomes universal [9–19]. Recent application of effective field theory to bosonic and fermionic atoms in traps and optical lattices have revealed the kind of phenomena that one can expect close to the drip lines, where nuclei with very weak binding exist [20–22]. Similarly the application of Renormalization group based techniques such as the Density Matrix Renormalization Group to cold

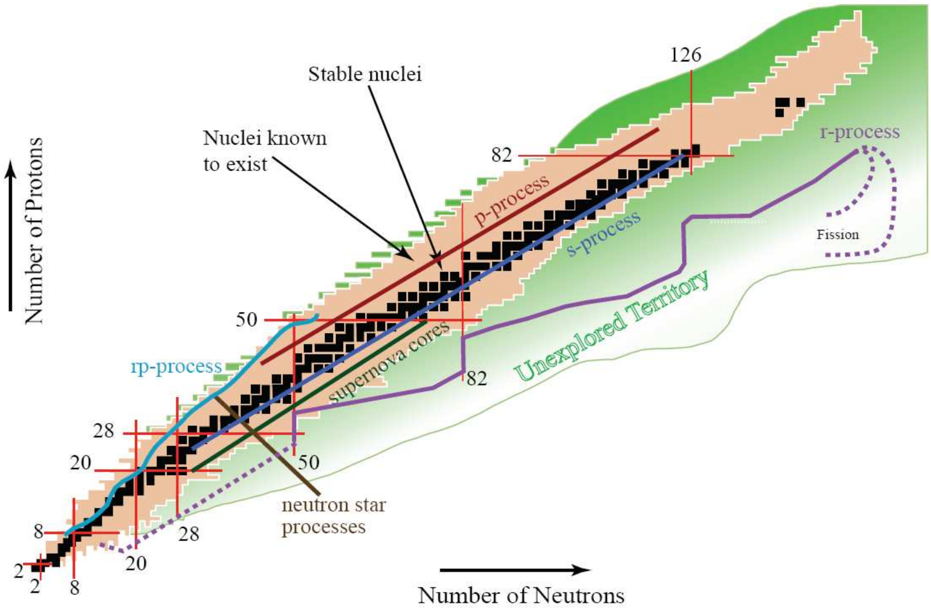

FIG. 1: Table of Nuclides

atoms systems specially in low dimensions have unparalleled accuracy in predicting ground state phases [5, 70–72]. This field, although very active, is relatively new and lots of new phenomena specially in the strong interaction regime are waiting to be explored.

Therefore, developing theoretical techniques that can be used to render many-body problems tractable form an important line of research that brings together different sub-fields of physics, combined with the current advances in computational and experimental physics. The following summarizes the recent progress in the field of theoretical nuclear structure [43, 51–53] and cold atoms [5, 70–72] using techniques based on the Renormalization Group principle and proposes few of the many open questions that could be addressed in the near future.

II. NUCLEAR STRUCTURE - RECENT DEVELOPMENTS

Nuclei have been objects of intense study for more than seventy years, yet the fundamental question of describing finite nuclei starting from microscopic forces has remained an open challenge. While QCD is the correct theory of strong interactions, it is non-perturbative at low energies and hence a description of nuclei using quark and gluon degrees of freedom is not feasible. The conventional approach to nuclear structure involves the modeling of the two-nucleon interaction using meson exchanges that are then used as inputs for many-body calculations. There are several high precision potentials available for modeling the interaction between two nucleons, such as Nijmegen, Bonn, AV$-$18 etc., which reproduce the data in the two-body sector, i.e. the deuteron binding energy and the scattering phase shifts up to $E_{lab} \approx 350\,\text{MeV}$, very accurately. All these potentials use the one-pion exchange interaction for the long-range part, but the details of the intermediate range attraction and the short-distance repulsion are different in different potential models. When such conventional two-body interactions are used as input in few- and many-body sectors, the results become model dependent. A related issue is that the proper description of few- and many-body systems require many-body forces. This can already be seen at the level of the simplest few-body system, the triton, where the three-body binding energy cannot be reproduced with a two-body force alone. Further, the presence of strong short-range correlations from conventional interactions means that solving the many-body Schrodinger equation is feasible only for light nuclei as the computational costs scale rapidly with the number of nucleons A. Therefore, we seek a *model independent, systematic* and *tractable* calculation of nuclear properties.

The requirement of a model-independent and systematic description is fulfilled by the effective field theory (EFT) approach to nuclear interactions [23–30] that are based on very general physical principles, i.e, low-energy processes do not depend on *the details* of the short-distance physics due to insufficient resolution (this scale being set by the wavelength of the probe or the energy scales of the interaction). An EFT program can be established in the following steps:

1. Define the most general Lagrangian with *the relevant degrees of freedom* (d.o.f.) i.e., d.o.f. relevant at the given momentum scale Λ such that the Lagrangian is consistent with the global and local symmetries of the underlying theory.

2. Declare a regularization and renormalization scheme.

3. Establish a well defined power-counting.

An EFT Lagrangian for nuclear physics includes the relevant low-energy physics explicitly in terms of the low-energy degrees of freedom, usually nucleons, pions and possibly delta's, while the high-energy part is coarse-grained and replaced by contact interactions (delta functions in position space, which are renormalized). This leads to low-energy couplings that capture the short-distance physics that is needed for the low-energy description. The expansion parameter for the nuclear EFT is the ratio of the momentum and the pion mass to the chiral symmetry scale (roughly rho mass). For practical purposes, this series has to be truncated at some finite order and the truncation error can be systematically traced in calculated observables. Effective field theories therefore establish a link between conventional nuclear physics and the theory of strong interactions, QCD, and serve as a systematic, model-independent approach to nuclear interactions and when combined with the recent Renormalization Group (RG) approach to nuclear structure, make the nuclear many-body problem *tractable* [43, 47–49, 51–53].

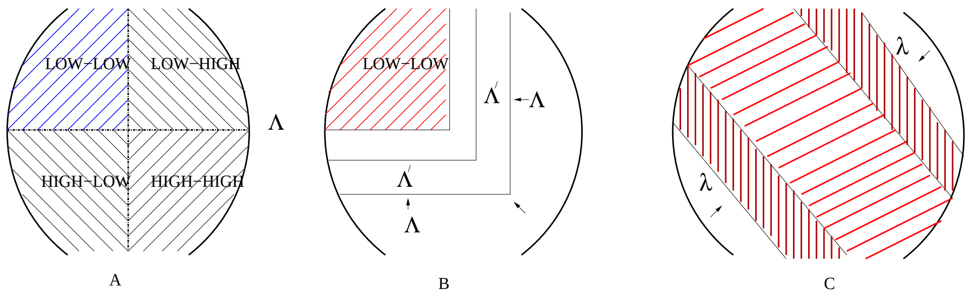

FIG. 2: Schematic of the Renormalization group evolution. (A) Un-evolved Hamiltonian (B) $V_{\mathrm{low}\,k}$ evolution (C) V_{SRG} evolution

Major simplifications to the conventional many-body calculations arise due to the de-coupling of the low- and high-momentum states as a result of RG running of the resolution scale. Much progress has already been made in developing EFT inter-nucleon forces [31–40]. The EFT based interactions are softer than the conventional ones as the details of the intermediate and the high-energy states are replaced by renormalized contact interactions and their derivatives. However, we have shown that even the most sophisticated chiral EFT potentials [41, 42], which have a natural low cut-off of typically $\approx 3 - 3.5\,\mathrm{fm}^{-1}$ and reproduce the two-nucleon data to the same accuracy as the conventional ones, are not soft enough and can be further softened, i.e., the low- and the high-momentum states can be further decoupled using the Renormalization Group approach to the NN interaction [51]. This decoupling using RG can be achieved in several ways leading to the following two sets of low-momentum interactions, referred to as $V_{\mathrm{low}\,k}$ and V_{SRG} [43–45] as illustrated in Fig. 2. In addition there are several other ways of obtaining an effective interaction at low-momenta notably, the V_{UCOM} interaction developed by Roth et al [58]. In the last couple of years a lot of work has been done to improve the properties of the low-momentum interaction. We have investigated a smooth cut-off RG set up that has better convergence properties in few and many-body calculations, including no-core shell model calculations [48–50, 52]. Recently, the similarity renormalization group (SRG) approach has received a lot of attention [59–65]. The SRG approach achieves the decoupling of low- and high-momentum modes using unitary transformations of the Hamiltonian. This has the unique feature of driving the high-momentum states towards the diagonal, which makes it different from the RG methods so far seen, but the results, e.g convergence properties in few-body systems are similar to those of $V_{\mathrm{low}\,k}$. Unitary transformations of the Hamiltonian have technical advantages for the consistent evolution of many-body operators. When these low-momentum interactions are used as input in finite nuclei and infinite nuclear matter calculations, physical observables should be independent of the cut-off. Any residual dependence sets the scale for missing many-body forces in a given approximation.

When low-momentum interactions are used as inputs for a many-body calculation important simplifications result, as the conventional sources of non-perturbative physics, i.e., the short-range repulsion and the tensor forces that are singular at short-distance and hence require iteration, have been smeared out. Shallow-bound states, which remain in free-space, get Pauli-blocked at finite densities. As a result, in nuclear matter, for instance, using the RG based two-body interactions together with the leading chiral EFT three-body force as input, renders the calculation of bulk properties perturbative, at least in the particle-particle channel [47]. The role of three-body forces has been increasingly emphasized in the light nuclei sector. In nuclear matter, the three-body force is essential for saturation, contrary to conventional wisdom and it turns out that it is perturbative for low values of the cut-off [46], this scale being set by the Fermi momentum k_{f}. Therefore, a Born series expansion in medium makes sense and expensive G-matrix re-summations can be avoided.

We have able to quantify the notion of "perturbativeness" of the particle-particle channel in symmetric nuclear matter by studying the magnitudes of the eigenvalues of the operator $G_0 V$, referred to as the "Weinberg Eigenvalues", where G_0 is the two-body Green's function as the cut-off is lowered [51]. This results in major reductions in the magnitudes of the Weinberg eigenvalues and hence improved convergence properties [51]. We have also been able to demonstrate that in addition to capturing the scale dependent evolution of physics, the eigenvalues preserve the non-perturbative physics in medium, such as pairing close to the Fermi surface in the particle-particle channel [53]. As a result, the eigenvalues can be used as a tool to analyze the evolution of scale dependent physics as well as identify the non-perturbative pieces.

While RG approaches are insensitive to the starting interaction, using EFT based interactions allow for systematic improvement of the input. Therefore, the candidate approach to nuclear structure uses low-momentum potentials starting from chiral EFT interactions. Such an input would allow computations that combine the systematics of an effective field theory and the good convergence properties of low-momentum interactions.

A. Ongoing and Future Possibilities in Nuclear Structure

In infinite nuclear matter a power counting for low-momentum interactions that allow for controlled approximations still needs to be established. Such a power counting will contribute to the ongoing efforts by several groups to set up a universal nuclear energy density functional. One of the important steps is a better understanding of the physics of the particle-hole channel. We are currently trying to get an understanding of the notion of "perturbativeness" for the particle-hole channel. This is accomplished by first investigating the convergence of a perturbative RPA correlation energy calculation to its exact value using the two-body low-momentum particle-hole interaction as input, similar to the finite nuclei calculations by Roth et al. where the perturbativeness of the particle-hole channel for bulk properties has been established [54–57] using V_{UCOM} interactions. Analogous to the particle-particle channel in infinite nuclear matter, one can trace the behavior of the particle-hole Weinberg eigenvalues as a function of the cut-off. Such an analysis coupled with the cut-off dependence of the RPA results will set the scale for the missing many-body forces as well as lead to a better understanding of the role played by the many-body forces in the particle-hole channel.

A related problem is the Pion condensation in infinite nuclear matter that is controlled by the particle-hole physics and is of importance in neutron star cooling rates. Hence investigating the critical density for the onset of the condensation using RG based interactions should yield systematic, model independent results. It would be interesting to analyze the role of many-body forces in Pion condensation by studying the cut-off dependence of the calculated critical densities.

A systematic microscopic description of the crust of neutron stars has remained an open challenge. The outer crust, which is at relatively low density, consists of stable nuclei, which become progressively neutron rich radially inwards as the density of matter increases, until a critical density is reached where the neutrons begin to "drip". Near the neutron drip lines, the binding is weak and the neutron star consists of quasi-nuclei, i.e. clusters of protons and neutrons embedded in a sea of neutrons, where the protons form a lattice to minimize Coulomb repulsion, until they finally dissolve at about nuclear matter density to form uniform matter of neutrons, protons and leptons in beta equilibrium in the inner layers of the crust. Extensive studies have tried to capture the transition from finite nuclei to uniform matter in neutron star crusts [66–68]. This passage is through a phase of inhomogeneous structures such as rods, plates etc., giving the name *Pasta Phase* for this density regime. Although, attempts have been made to perform microscopic studies of these inhomogeneous structures based on realistic NN interactions [69, 73], the results are model dependent. This motivates a study of the neutron star crust using RG based low-momentum interactions as input. Such a study would contribute to a better understanding of nuclei close to the drip lines where nuclear binding is weak.

In infinite nuclear matter, there is a BEC-BCS crossover as a function of density. At low-density, neutrons and protons form deuterons, which get Pauli blocked close to saturation densities and the bound states exist close to the Fermi surface as cooper pairs. Therefore, it would be interesting to study pairing in infinite matter at finite temperature, in particular neutron matter, where at low-densities, the s-wave scattering length diverges. We are currently investigating infinite neutron matter using low-momentum interactions as input within the Nozi‘ere Schmitt-Rink scheme [74]. Future work would allow for including higher order correlations systematically.

It would also be interesting to apply these RG techniques developed in the nuclear context to other systems where a microscopic interatomic potential is involved, such as the He atoms. Although RG techniques have been used widely in condensed matter systems, the added experience from the nuclear interactions might prove fruitful.

III. COLD ATOM PHYSICS

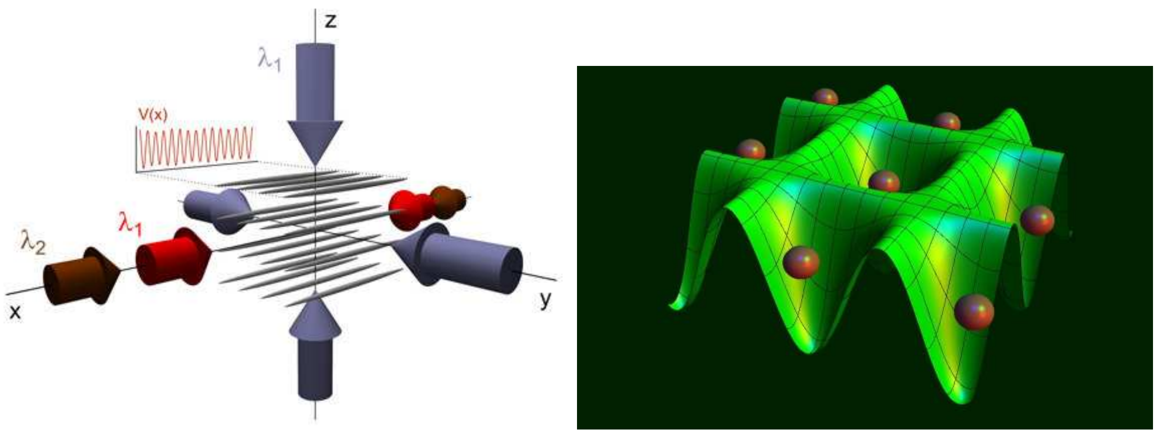

FIG. 3: Optical Lattice: Left panel shows the laser configuration and the right panel is an illustration of a three-dimensional lattice.

The physics of strongly interacting system can be studied in a controlled manner in a lab using systems of ultra-cold atoms. In these experiments, the gases are cooled to low-temperatures using laser cooling techniques and when an ultra-cold atomic gas is loaded on to an optical lattice, the atoms distribute themselves between the different wells, as seen in Fig. 3. In a lattice, the atoms can either tunnel to the neighboring sites or continue to reside on the same site, their dynamics being determined by the depth of the optical lattice, which can be tuned easily in experiments. The lattices formed in experiments involve three counter-propagating laser beams and are three-dimensional. But by increasing the laser intensity along one direction, the tunneling in that direction can be suppressed. As a result one can produce lower dimensional systems very easily. This opens the possibility of realizing diverse condensed matter systems in a lab and also studying the physics of lower dimensional systems in the strong interaction regimes. An added advantage that such simulations using cold atoms have over conventionally realizable condensed matter systems is that they are defect free. Of course it is possible to introduce the defects by hand in order to study disordered systems for instance. Therefore the extreme control and tunability that cold atoms experiments offer, make them a new tool that is indispensable for the study of the many diverse phenomena present in condensed matter.

In addition to simulating various condensed matter systems, cold atoms in optical lattices allow for the study of few-body physics in a controlled manner [9–19]. In any well of the optical lattice, the interaction between any two atoms is determined by their S-wave scattering length to leading orders at low energies. Using Feshbach resonances, the scattering length can be tuned and in the limit where the scattering length becomes large compared to the range of the interaction, the notion of a length scale is lost and the physics becomes universal. This limit of large scattering lengths is reached through a very specific combination of parameters of the underlying theory and is usually referred to as fine-tuning. While the scattering length can be tuned in cold atoms systems through external magnetic fields, fine-tuning can also be accidental such as the large S-wave scattering lengths for the two-nucleon systems, ^{4}He atoms etc. Therefore an understanding of few-body physics in a controlled manner, improves our understanding of low-energy nuclear physics, in particular of weakly bound finite nuclei that exist close to the drip lines [89, 90].

Cold atoms systems are many-body systems therefore the arsenal of techniques available in other fields can be very well applied here. Recent years have witnessed several successful applications such as the usual many-body perturbation theory, numerical techniques like the Quantum Monte Carlo, the Density Matrix Renormalization Group etc., although most of the techniques are adapted to address the lattice structure and geometry. In lower dimensional systems, for instance in one dimension, the Density Matrix Renormalization Group (DMRG) offers good numerical accuracy while computing ground state quantities and corresponding correlations at low computational costs, although its application to higher dimensions is still unsatisfactory [5]. The DMRG technique was introduced for one dimensional spin chains [8] and its successful application to one dimensional cold atoms systems paved the way for several variants, such as time dependent formulations, extension to two dimensional classical systems etc. An example of its successful application to condensed matter systems is the prediction of the ground state phases for interacting atoms in 1D optical lattices. We have recently studied cold bosons in one dimensional optical lattices and predicted the ground state phases within the Bose Hubbard model and its extensions [70–72]. In order to make connection with the experiments, one needs to isolate signatures of the different ground state phases that can be measured in a lab, preferably in the presence of an external harmonic trap in addition to the optical lattice. This has interesting consequences: the trap destroys translational invariance and results in the phases co-existing [5, 70, 72, 91]. Of special interest is the extended Bose Hubbard model, which has a nearest neighbor repulsion in addition to the on-site term and our study has demonstrated the possibility of observing a super solid phase besides a host of other exotic ground state phases [71]. We are currently trying to establish signatures of the ground state phases, specially the super solid phase in the presence of a trap [72]. Given the vast diversity of systems and geometries that can be studied in cold atoms experiments, lots of new possibilities and exotic phases are waiting to be explored in the lab in the near future. Hence theoretical predictions should be reliable, accurate and systematic.

A. Ongoing and Future possibilities in Cold Atoms Physics

In order to study systems in the strong interaction regime it is important to develop accurate numerical techniques at low computational costs. While DMRG has been very successful in one dimension in obtaining ground state energies and correlations, it would be very useful if its accuracy can be carried over to higher dimensions, specially three. The most successful applications of the DMRG technique to higher dimensions involve a recasting of the higher dimensional problem into approximate lower dimensions, as has been in done in quantum chemistry and recently in nuclear physics [75–85]. A direct generalization of the algorithm to two or three dimensions results in higher computational costs and loss of numerical accuracy. Therefore it would be useful to investigate the possibility of extending the DMRG algorithm to higher dimensions. Such an extension would open up the possibility of studying diverse problems. In particular, studying the ground state phases in the presence of an external harmonic trap in three dimensions and picking out experimental signatures will allow better simulation of experimental conditions.

Investigating the application of coupled cluster method to lattice problems could be another interesting direction for future work. Coupled clusters, currently used in quantum chemistry and recently in nuclear physics, is not restrictive to lower dimensions. It has been successfully applied to atomic and spin systems [86]. Some exploratory applications to cold atoms confined in traps and lattices have been done by Cederbaum et al. [87, 88] and hence a detailed study would be fruitful.

Effective field theory has already been applied to study universal few-body physics of cold atoms in traps and lattices. Extending EFT techniques to pin down the ground state phases of cold atoms in lattices would result in systematic predictions and also give deeper insights into the many-body dynamics that lead to the formation of a particular phase. Such a description will combine the systematics of an EFT with the computational power of the available numerical techniques resulting in a better understanding of the physics.

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