Indian Institute of Technology Madras
Quantum Mechanics for Engineers. PH350.
Final 28 Nov. 2003.

Instructions

- Answer all questions. Part A has Eight questions while Part B has Six.
- The marks are indicated in bold at the end of the questions. Maximum marks=**60**. No choices.
- Do **NOT** mix up answers to parts A and B. Clearly indicate in your answer sheets, the parts.
- Please **do not** use pencils or red pens for answering.

Part A

1. Consider state of a two-level system $|\psi\rangle = a|0\rangle + b|1\rangle$ (not necessarily normalized), where $|0\rangle$ and $|1\rangle$ is an orthonormal basis.

   (a) What is $\langle\psi|$? **(1)**

   (b) What is $\langle\psi|\psi\rangle$? **(1)**

   (c) Consider an opertor A in the space, whose representation in the above basis is
   $$A = \begin{pmatrix} 3 & 1+i \\ 1-i & 2 \end{pmatrix}.$$
   Can A qualify for a physical observable? Why OR Why not? **(1)**

   (d) If A is measured in the given state, what is its expectation value? **(2)**

   (e) What are the possible values of A on measurement? **(2)**

   (f) What are the final states after the measurement of A, and what are the probabilities for reaching these? **(3)**

2. A spin-half particle is known to be a pure state with equal probability of being found in $|S_z+\rangle$ and $|S_z-\rangle$. How many such possible states are there? State any one. **(3)**

3. One hundred spin-half particles are passed through three Stern Gerlach apparati, first oriented along the z, the second along x and the third along the z directions. After each measurement the spin $-\hbar/2$ component is blocked. Enumerate the outcomes of the three measure ments. Is you answer determinsitic, or statistical? **(3)**

4. If A and B are Hermitian operators then

   (a) $A + B$ is Hermitian. True or False.

   (b) AB is Hermitian. True or False. **(2)**

5. If A and B are Unitary operators then

   (a) $A + B$ is Unitary. True or False.

   (b) AB is Unitary. True or False. **(2)**

6. Encircle one of the possibilities:

   (a) The Hamiltonian has to be an Unitary / Hermitian operator.

   (b) The time-evolution operator is an Unitary / Hermitian operator.

   (c) The finite displacement operator is an Unitary / Hermitian operator. **(3)**

7. A particular Hamiltonian is found to be an unitary operator. What are the possible energies that the system may have (in arb. units)? **(3)**

8. J_x, J_y, J_z are angular momentum operators, while x and p are position and momentum operators. Find the following commutators:

   (a) $[J_x, [J_y, J_z]]$

   (b) $[x^2, p^2]$

   (c) $[x, J_x]$

   (d) $[\sin(x), p]$ **(4)**

Part B

1. A particle of mass M rotates freely on a circle of radius R.

   (a) Write the Lagrangian of the system.

   (b) Write down the Hamiltonian of the system, identify the conjugate variables.

   (c) Using the fact that angular momentum generates rotations, find the energy eigenvalues and corresponding eigenfunctions for this particle. **(5)**

2. Consider the one-half harmonic oscillator potential: $V(x) = m\omega^2 x^2/2$ for $x > 0$ and $V(x) = \infty$ for $x \leq 0$, that is for a potential that is harmonic on the positive half of the real line and for which there is a hard wall at the origin.

   (a) What are the eigenenergies and corresponding eigenfunctions?

   (b) Let an initial state be
   $$|\psi(0)\rangle = |z\rangle - |-z\rangle$$
   where $|z\rangle$ is a coherent state of the corresponding full harmonic oscillator. Argue that this is an allowed initial state for the one-half harmonic oscillator, for $Re(z) \geq 0$.

   (c) This initial state is subjected to the one-half harmonic potential. Find the state at any later time t. **(5)**

3. A particle (mass m) is in the groundstate of an harmonic oscillator potential well (frequency ω), when at time $t = 0$ the potential is suddenly removed. Examine the fate of the particle subsequently. In particular find its position and momentum distributions at later times. Also sketch them. **(5)**

4. Prove by any means the so-called Weyl commutation relations:
   $$\exp(-i\hat{p}a/\hbar)\exp(i\hat{q}b/\hbar) = \exp(-iab/\hbar)\exp(i\hat{q}b/\hbar)\exp(-i\hat{p}a/\hbar)$$
   where a and b are constants with appropriate units. How would you interpret this? **(5)**

5. A one-dimensional simple harmonic oscillator is subjected to a perturbation

$$\lambda H_1 = b\,x,$$

where b is a real constant.

(a) Calculate the energy shift of the ground state to the lowest nonvanishing order.

(b) Solve this problem exactly and compare with your result above.

(5)

6. Solve for the reflection and transmission coefficients of the one-dimensional barrier potential: $V(x) = 0$ for $x > |a|$, and $V(x) = V_0$ for $x \leq |a|$, where V_0 and a are constants with appropriate units. Consider both the cases, when the incoming particles are more and less energetic than V_0. **(5)**