Indian Institute of Technology Madras

Quantum Mechanics for Engineers. PH305. Quiz 1. Sept. 12 2003.

NAME:

ROLL NO.:

Instructions

- Attempt only **any 2 of the "5 Mark"** questions and **any 4 of the "10 Mark"** ones. Clearly mark **here**( ) which **six** questions you should be graded on.
- Please **do not** use pencils or red pens for answering.

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1. Write down the most general possible 2×2 Hermitian matrix. Find its eigenvalues and show that these are real. **(5 Marks)**
2. Consider the three-dimensional Hilbert space spanned by the eigenkets, say, $\{|0\rangle, |1\rangle, |2\rangle\}$, of an observable A. Let $|\phi_1\rangle = |0\rangle + (1+i)|1\rangle + 2i|2\rangle$ be the state of a system described by such a Hilbert space. Normalize it. If measurements of A^2 are made what are the possible final states and with what probabilities are these obtained? **(10 Marks)**.
3. Is it true that $[A, A^\dagger] = 0$ for a general operator A? Is it true for a Hermitian operator? Is it true for a unitary operator? **(5 Marks)**.
4. Is it true that if $[A, B] = 0$, and $[B, C] = 0$, then $[A, C] = 0$? Justify any position you may take. **(5 Marks)**.

5. Explain with the help of S_x and S_z measurements why the state

$$\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

cannot be thought of as having 50% $|+\rangle$ and 50% $|-\rangle$ states? **(10 Marks)**.

6. Consider the state $|+\rangle$, eigenstate of S_z with spin up. Find the uncertainties associated with measurements of S_x and S_y and verify if the uncertainly relation is satisfied. **(10 Marks)**.

7. Show that $[x, p^2] = 2i\hbar p$. What is $[x, p^3]$? **(10 Marks)**.

8. Consider the subspace of periodic functions in $L^2[0, 1]$, that is those functions that satisfy $\phi(0) = \phi(1)$. Find the eigenfunctions and eigenvalues of the momentum operator $-i\hbar d/dx$ on this Hilbert space. **(10 Marks)**.