Indian Institute of Technology Madras
Quantum Mechanics for Engineers. PH305.
Quiz 2. Oct. 29 2003.

Instructions

- Answer **all** four questions. They are of equal weightage.
- Please **do not** use pencils or red pens for answering.
- Look at the board for useful information.

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1. A particle is trapped in an infinite one-dimensional square well potential of width a.

   (a) If the initial state is:

   $$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|\phi_0\rangle + \frac{1}{\sqrt{2}}|\phi_1\rangle,$$

   where $|\phi_0\rangle$ and $|\phi_1\rangle$ are the (normalized) ground and first excited states of the well. Find the auto-correlation function defined as:

   $$f(t) = |\langle\psi(t)|\psi(0)\rangle|^2,$$

   where $|\psi(t)\rangle$ is the state after a time t. What time will this reach a maximum, and what is this maximum?

   (b) Prove that for *any* initial state there exists a time, that is independent of this state, at which the initial state exactly reappears. Prove this by finding such a time. How does this compare with part (a)?

2. A stream of particles of energy $E > 0$ impinge from $-\infty$ on a delta function *barrier* $V(x) = v_0\delta(x)$, where v_0 is a positive constant. Find the transmission and reflection coefficients.

3. Consider the Hamiltonian $\hat{H} = g\,\hat{a}^{\dagger 2}\hat{a}^2/2$, where g is a constant with dimensions of energy. Find the eigenvalues and eigenfunctions of $\hat{H}$. Write down the Heisenberg equations of motion for the raising and lowering operators. (This is a popular and useful Hamiltonian in quantum optics.)

4. An infinite square well of width a is suddenly halved in width. What is the probablity that a particle trapped in the original ground state transits to the new ground state? Is energy spent or derived in this process?