PH 350. Quantum Mechanics for Engineers. PS-1.

Aug. Friday the 13th 2004.

Review of Classical Mechanics

1. Consider a one-dimensinal harmonic oscillator (mass m, frequency ω) whose potential energy is $m\omega^2 q^2/2$. Write the (a) Lagrangian (b) Hamiltonian (c) Hamiltonian equations of motion. Solve the equations of motion by any means.

2. For the previous problem, factorize the Hamiltonian as follows: take $a = \sqrt{\frac{m\omega}{2}} \left(q + i \frac{p}{m\omega} \right)$ where ($i = \sqrt{-1}$) such that $H = \omega\, a^* a$. Find the Poisson bracket $\{a, a^*\}$. Find the equation for the time evolution of a, solve it, and hence solve the harmonic oscillator.

3. Find the time period T of the Harmonic oscillator in (1). Let $f(q)$ be the function such that $f(q)\, dq$ is the fraction of the time T spent between q and $q + dq$. Find $f(q)$ for some fixed particle energy E. Sketch $f(q)$ for various energies. Similarly define $g(p)$ such that $g(p)\, dp$ is the fraction of time spent in the momentum region p to $p + dp$. Find $g(p)$.

4. Generalize the previous problem for any bounded one-dimensional potential $V(q)$.

5. Evaluate the Poisson Brackets: $\{q, p^2\}$, $\{q, f(p)\}$, $\{p, g(q)\}$. Here f and g are arbitrary (once-differentiable) functions.

6. A bead is in a circular wire that is initially alinged with the $x - z$ plane, with gravity acting in the $-\hat{z}$ direction, and its axis passing through the z axis. This is now rotated about the z axis at a constant speed ω. Write the Lagrangain, the Hamiltonian and the equations of motion.

7. Consider a free particle in 3 space. Using the cartesian, cylindrical and spherical polar coordinates write the Lagrangian, the Hamiltonian and the generalized momenta.

8. A particle is trapped inside a smooth vertical cone (half angle α) with its apex downward in the direction of the gravitational field. Using cylindrical coordinates find the Lagrangian. Find the conserved quantities, and the equation of motion of the particle in the cone.

9. Show that $Q = \ln\left(\frac{\sin(p)}{q}\right)$, $P = q\cot(p)$ is a canonical transformation. Find the corresponding generating functions $F_1(q, Q)$ and $F_2(q, P)$.

10. Consider the following generating function in two-dimensions: $F_2 = xP_x + yP_y + \theta(xP_y - yP_x)$. Show that the canonical transformation generated can be interpreted as a rotation by angle θ for small θ. (Thus angular momentum is said to generate rotations).