Classroom

In this section of *Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

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The Real Effects of Pseudo Forces

A clear comprehension of the Newtonian notion of force remains elusive to many students due to lack of experience with observations in a non-inertial frame of reference. In this article is described a software that can be easily run on a desktop computer and which acquaints the student-user with the Newtonian notion of force. Fundamental principles of causality and determinism in Newtonian mechanics are elucidated.

1. Introduction

The widespread improvement in computational facilities in various educational institutions has made posssible the development of various computer based physics educational software which aid in demonstrating some of the more subtle and important physical phenomena which are otherwise imperceptible in our daily lives. A prime example of one such concept is that of motion in non-inertial frames. In fact, it was pointed out only a decade ago in a prominent paper[1] that ".... not only students, but also professional physicists to quite a large extent do not have a full understanding of the concept of force

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.....". This difficulty arises primarily due to the fact that for a vast majority of phenomena in everyday life, qualitative and quantitative consequences of the non-inertial nature of the frame of reference fixed to the earth is not recognised by most students of Newtonian mechanics.

If the observer's frame of reference is non-inertial, the acceleration (effect) seen by him cannot be explained by him only in terms of the 'physical' causes.

The Galileo–Newton law of inertia selects out a class of inertial frames of reference with respect to which uniform motion (including one at zero momentum) is self-sustaining. It emphasizes the fact that in this frame, no force (cause) is required to 'explain' an object's mechanical state of rest (or of uniform motion) which is completely determined by the initial conditions of the position and momentum of the mechanical system.

The Newton's second law searches for a 'cause' to explain any departure from equilibrium, and quantitatively establishes a linear proportionality between acceleration and the physical interaction which causes this departure. The equation of motion $\vec{F} = m\vec{a}$ thus expresss the linear proportionality between the cause $\vec{F}$ and its effect $\vec{a}$ and furthermore it identifies the constant of proportionality as the object's mass. That this proportionality holds good in the inertial frame is an important aspect, and this property is contained in the definition of an inertial frame as one in which Newton's laws hold.

2. Computer Simulation

In order to focus on the relationship of the 'principle of causality and determinism' with the 'inertial' nature of the frame of reference, we discuss below the computer simulation of a situation in which the effects of making observations in a non-inertial frame of reference are observable and dramatic. If the observer's frame of reference is non-inertial, the acceleration (effect) seen by him cannot be explained by him only in terms of the 'physical' causes. The important thing is that if one does not take into account the fact that an observer on earth is in fact in a non-inertial frame of reference, then

It is well known that laboratory experiments aimed at demonstrating such situations are not quite easy to set up [1], since observable effects of the non-inertial nature of an earth-fixed frame of reference are rather weak.

for such an observer either

- Newton's laws do not hold good or
- the observer's perception of forces(causes) that change an equilibrium state as seen by her/him would be different from that of an observer in an inertial frame of reference.

This would directly affect his interpretation of fundamental interactions (such as gravitational and/or electromagnetic) in nature. It is well known that laboratory experiments aimed at demonstrating such situations are not quite easy to set up[1], since observable effects of the non-inertial nature of an earth-fixed frame of reference are rather weak.

We consider two astronauts, A and B, conducting an experiment inside a space vehicle in a region where all gravitational effects may be neglected, since we can consider the space vehicle to be in a state of free-fall. The space vehicle is however considered to be in a state of rotation (provided to give directional stability) at a constant angular speed ω about an axis through it. We shall refer to this axis as the Z-axis. The astronauts A and B are diametricallly opposite to each other inside the vehicle and we examine the motion of a tiny tool thrown by A inside the space ship. After the tool is thrown by A, there is no physical force acting on it. In the absence of any such real force, the astronauts (having studied Newton's first law!) expect the tool to traverse along a straight line at a constant momentum it acquired when it was thrown. They however find the tool to traverse along unimaginable curves and must conclude that in their frame of reference, Newton's laws fail!. The causality relation (acceleration is proportional to the 'physical' force acting on the object) does not hold in the accelerated frame of reference.

An equation of motion $\vec{F'} = m\vec{a'}$ can however be used in the astronaut's frame of reference if the left hand side of this equation contains all the mathematical constructs of the accelerated (in this case rotating) frame of reference. There is no simple proportionality between the real, physical force and the acceleration seen by the astronauts. In fact, the astronauts see real effects of forces that are a combination of real physical interactions and also pseudo-forces. In other words, they see effects that are real, of causes that include pseudo-causes.

The causality relation (acceleration is proportional to the 'physical' force acting on the object) does not hold in the accelerated frame of reference.

In the rotating frame of reference the force $\vec{F'}$ is not merely the 'physical' force $\vec{F}$, but must be replaced by [2,3]

$$\vec{F'} = \vec{F} - m\vec{\omega} \times (\vec{\omega} \times \vec{r_r}) - 2m\cancel{\vec{\omega} \times} (\vec{\omega} \times \frac{\delta \vec{r_r}}{\delta t}). \quad (1)$$

The vector $\vec{r_r}$ with the subscript 'r' denotes the position vector of the object in motion with respect to the rotating frame of reference. It is pertinent to add here that an inertial frame of reference develops a different notion of force compared to a non-inertial one. The operational definition of a real/physical force is intimately tied up to the cause-effect relationship in an inertial frame of reference. In (1) $\vec{r_r}$ is the position vector of the object thrown relative to rotating frame of reference, S_r. The time derivative operator $(\frac{\delta}{\delta t})$ pertains to the rotating frame S_r and is related to the corresponding operator $(\frac{\mathrm{d}}{\mathrm{d}t})$ of the inertial frame S according to the following operator equivalence:

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\delta}{\delta t} + \vec{\omega}\times \quad (2)$$

The second term in (1) is the centrifugal force and the third term is the Coriolis force. Understanding (2) is of central importance to set up the equation of motion. Equations (1) and (2) are simple consequences of a non-

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relativistic transformation from an inertial to a rotating frame of reference.

Parenthetically, one may also note that under the Lorentz transformations, the concept of time itself is to be modified and space and time variables get scrambled. In contrast to this, in the present non-relativistic transformations the notion of time in the inertial and the rotating frames remains essentially the same, but the derivative of a vector with respect to the time parameter in the two frames is related according to (2). It is easy to see that (1) follows immediately if, as a first step, the operator of (2) is applied to $\vec{r_r}$ and then as a second step, the same operator is applied once again to the result. In order to obtain a solution for the trajectory of the tool thrown by A in the rotating coordinate system S_r, we consider the Z-axis of S_r to be through the centre of the plane of the space vehicle and along $\vec{\omega}$, and we consider the astronauts A and B to be located at Cartesian coordinates (0,−R,0) and (0,+R,0) respectively. The tool is considered to be thrown by astronaut A at an initial velocity in the XY-plane and (1) must now be integrated to get the trajectory of the tool as would be seen by the astronauts in the rotating frame.

If $\vec{r_r}(t)$ denotes the instantaneous position of the tool in the rotating (spaceship's) frame of reference then,

$$\vec{r_r}(t) = x_r(t)\hat{e}_{x,r} + y_r(t)\hat{e}_{y,r} \qquad (3)$$

$\hat{e}_{x,r}$,$\hat{e}_{y,r}$ being the Cartesian base vectors in the rotating frame of reference. In the frame S_r these base vectors are *not* functions of time. The acceleration $\vec{a_r}(t)$ in the rotating frame is given by

$$\vec{a_r}(t) = \frac{\mathrm{d}^2\vec{r_r}}{\mathrm{d}t^2} = (\vec{\omega}.\vec{\omega})\vec{r_r} + 2(\frac{\mathrm{d}\vec{r_r}}{\mathrm{d}t}) \times \vec{\omega}, \qquad (4)$$

which is the second order differential equation we must now solve in order to determine the trajectory of the tool thrown by A. By projecting the left hand side of

(4) on $\hat{e}_{x,r}$ and $\hat{e}_{y,r}$ separately and equating the two respectively with the corresponding projections of the right hand side of (4) we get the following two coupled differential equations:

$$\frac{\mathrm{d}^2 x_r}{\mathrm{d}t^2} = 2\frac{\mathrm{d}y_r}{\mathrm{d}t}\omega + \omega^2 x_r, \quad (5)$$

and

$$\frac{\mathrm{d}^2 y_r}{\mathrm{d}t^2} = -2\frac{\mathrm{d}x_r}{\mathrm{d}t}\omega + \omega^2 y_r. \quad (6)$$

These equations are differentiated once again to get two more equations now for the third order deivatives.

$$\frac{\mathrm{d}^3 x_r}{\mathrm{d}t^3} = 2\frac{\mathrm{d}^2 y_r}{\mathrm{d}t^2}\omega + \omega^2\frac{\mathrm{d}x_r}{\mathrm{d}t}, \quad \frac{\mathrm{d}^3 y_r}{\mathrm{d}t^3} = -2\frac{\mathrm{d}^2 x_r}{\mathrm{d}t^2}\omega + \omega^2\frac{\mathrm{d}y_r}{\mathrm{d}t}. \quad (7)$$

The above equations can be decoupled by going over to the fourth order differential equations:

$$\frac{\mathrm{d}^4 x_r}{\mathrm{d}t^4} + 2\omega^2\frac{\mathrm{d}^2 x_r}{\mathrm{d}t^2} + \omega^4 x_r = 0, \frac{\mathrm{d}^4 y_r}{\mathrm{d}t^4} + 2\omega^2\frac{\mathrm{d}^2 y_r}{\mathrm{d}t^2} + \omega^4 y_r = 0. \quad (8)$$

The solutions to (7)are:

$$x_r(t) = (A_x + B_x t)\cos(\omega t) + (C_x + D_x t)\sin(\omega t), \quad (9)$$

and

$$y_r(t) = (A_y + B_y t)\cos(\omega t) + (C_y + D_y t)\sin(\omega t). \quad (10)$$

Here, the A's, B's, C's and D's are various contants of integration determined by the initial conditions. To solve each fourth order differential equation we need the values at $t = 0$ of x_r and y_r, and their derivatives with respect to time upto order three. This gives us the eight initial conditions we need to determine the eight unknowns $A_x, B_x, C_x, D_x, A_y, B_y, C_y$ and D_y. For example, if we consider the equation in x_r , $x_r(0)$ and $\frac{\mathrm{d}x_r}{\mathrm{d}t}|_{t=0}$

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are given by the two known initial conditions which are the x-coordinate and the speed along x direction. We can however determine $\frac{d^2x_r}{dt^2}|_{t=0}$ and $\frac{d^3x_r}{dt^3}|_{t=0}$ by using the original coupled equations. It is easily seen that $\frac{d^2x_r}{dt^2}|_{t=0}$ is given by (5) knowing $\frac{dy_r}{dt}|_{t=0}$ and $x_r(0)$. Differentiating equations (5) and (6) once more we get the two third order differential equations(7)which will give us $\frac{d^3x_r}{dt^3}|_{t=0}$ and similarly $\frac{d^3y_r}{dt^3}|_{t=0}$ in terms of the now known lower order derivatives. Thus all the eight arbitrary constants can be completely determined.

If the initial velocity is expressed as

$$\vec{v_r}(t=0) = v_{0,x}\hat{e}_{x,r} + v_{0,y}\hat{e}_{y,r} \tag{11}$$

then, the path of the tool is given by:

$$x_r(t) = (v_{0,x} + R\omega)t\,\cos(\omega t) + (v_{0,y}t - R)\sin(\omega t), \tag{12}$$

$$y_r(t) = (v_{0,y}t - R)cos(\omega t) - (v_{0,x} + R\omega)t\; sin(\omega t), \tag{13}$$

The solutions for the equation of motion of the tool in an inertial frame of reference are

$$x_i(t) = (v_{0,x} + R\omega)t, \tag{14}$$

$$y_i(t) = -R + v_{0,y}t. \tag{15}$$

In several texts on this subject, the centrifugal term is often discussed in some detail. However, except in specialised texts, the Coriolis term is not given equal importance. In order to highlight the consequence of this term, it is instructive to focus attention on it. In our software therefore, we have introduced the option to completely ignore the centrifugal term and study the consequence of the Coriolis term alone. Of course, in a rotating frame of reference, both the terms would always be present – the centrifugal term always, and the Coriolis term would be zero only when the object under observation is at rest in that frame.

3. Trajectory in the Rotating and the Inertial Frame

We illustrate the trajectory of a tool thrown by the astronaut A in the rotating spaceship by a few examples below.

It is clear that the orbiting spaceship being in a state of 'free fall' is in a zero-g (zero-gravity) field and once the tool is given the initial velocity when the astronaut A throws it, the tool cannot experience any of the known physical forces: gravity, electromagnetic, nuclear strong/weak. As mentioned in Section 2, one would expect therefore that the tool would traverse uniformly in accordance with Newton's I law of motion along a straight line. It is important to note, however, that this would hold essentially in an inertial frame of reference, and in none other. The trajectories are illustrated in the following five cases:

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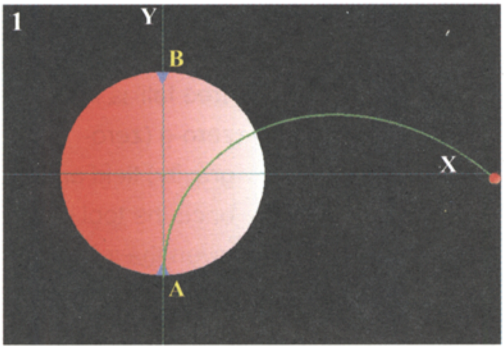
Case(1): *Figure* 1 shows the trajectory of the tool thrown by the astronaut A toward B in the rotating frame.

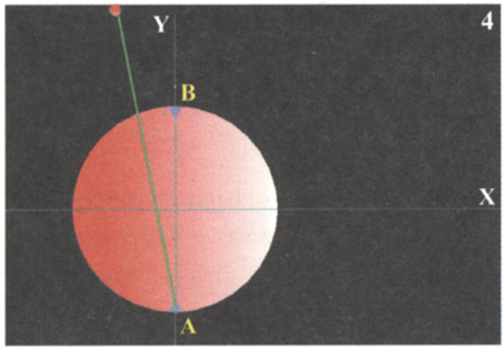
The initial conditions for this case are: Radius R = 10 m, angular speed $\omega = 0.15$ rad/sec and the X- and Y-components of the throw velocity are 0 m/s and 5 m/s respectively.

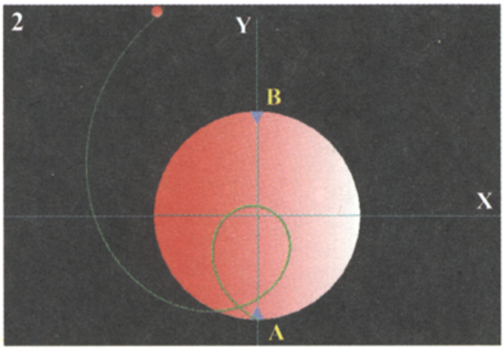
The tool is thrown by astronaut A directly toward B, but as seen by the observers in the rotating frame it goes nowhere near B and instead curves away to the right. Thus contrary to what one would expect in an inertial frame, a direct throw is not sucessful.

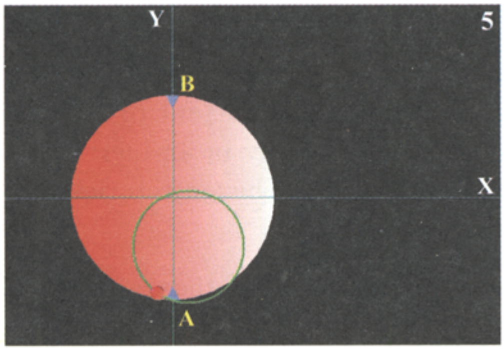
Case(2): *Figure* 2 shows the trajectory of the tool thrown by the astronaut A toward B in the rotating frame for a second set of initial conditions.

The initial conditions for this case are: Radius R = 10 m, angular speed $\omega = 0.95$ rad/sec and the X- and Y-components of the throw velocity are -10 m/s and 5 m/s respectively.









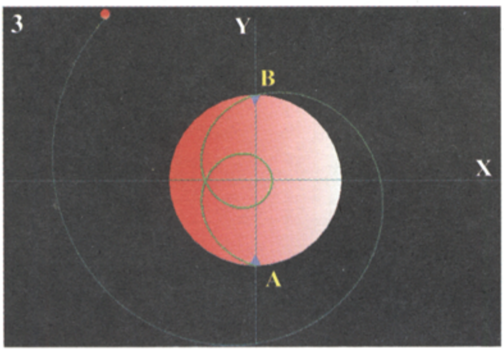


Figure 1 Trajectory seen in the rotating frame of the object thrown by A toward B (Case 1).

Figure 2. Trajectory seen in the rotating frame of the object thrown by A toward B (Case 2).

Figure 3. Trajectory seen in the rotating frame of the object thrown by A toward B (Case 3).

Figure 4. Trajectory of the object thrown by A toward B as viewed from an inertial frame of reference (Case 4).

Figure 5. Trajectory of the object thrown by A toward B seen in the rotating frame of reference, but determined with the neglect of the centrifugal force (Case 5).

As seen by the observers in the rotating frame, the object takes a weird curved path. Astronaut A must therefore adjust his initial throw velocity suitably if the tool has to reach B.

As seen by the observers in the rotating frame, the object takes a weird curved path. Astronaut A must therefore adjust his initial throw velocity suitably if the tool has to reach B.

Case(3): *Figure* 3 shows the trajectory of the tool thrown by the astronaut A toward B in the rotating frame for a third set of initial conditions.

The initial conditions for this case are: Radius $R = 10$ m, angular speed $\omega = 0.95$ rad/sec and the X- and Y-components of the throw velocity are -10.0325 m/s and 2.755 m/s respectively.

In this case, the tool is seen to reach B after making a loop inside the circular spaceship. Thus A has found one suitable throw velocity. Other suitable throw velocities are of course possible.

Case(4): *Figure* 4 shows the trajectory of the tool thrown by astronaut A as viewed from an inertial frame of reference. The initial conditions for the case shown are the same as those used for the case in *Figure* 3. As expected, the trajectory seen by observers in this frame will be a straight line but the astronauts and the spaceship are seen to rotate in this case.

Case(5): *Figure* 5 shows the trajectory of the tool thrown by astronaut A as seen in the rotating frame of reference if the centrifugal force were to be neglected in the analysis. This trajectory is obviously not possible physically and is shown only to highlight the otherwise weak effect of the Coriolis force. The initial conditions for the case shown are the same as those used for the case in *Figure* 3. It is seen that the Coriolis force acting alone would cause the tool to take a circular path as the Coriolis force is at any instant perpendicular to the direction of the intantaneous velocity. *Figure* 5 shows such a trajectory for a particular throw velocity.

In case (3), the tool is seen to reach B after making a loop inside the circular spaceship.

Most students enjoy experimenting with the initial

While the students experiment with different parameters for the throw velocity, they also discover the sensitivity of the solutions to the initial conditions, and thus develop a feel for the same.

conditions at their disposal in this menu-driven program to figure out how the trajectory of the object thrown can be made to intersect the point B diametrically opposite to A so that the other astronaut will be able to catch the tool. While the students experiment with different parameters for the throw velocity, they also discover the sensitivity of the solutions to the initial conditions, and thus develop a feel for the same.

The Coriolis force in particular, is difficult to comprehend since its effect is relatively weak in our daily life, though several macroscopic effects such as direction of the cyclonic winds [2], the rotation of plane of oscillation of the Foucault pendulum [4], certain oceanic currents [5], etc. are due to the Coriolis effect.These effects become still weaker closer to the equator, making them less dramatic in places like India compared to countries in Europe.

4. Concluding Remarks

An essential point illustrated by the current software is that an object in motion observed in a rotating frame of reference is seen to depart from uniform motion along a straight line even in the absence of any real physical force. Thus to the observer in the rotating frame, the observation of departure from uniform rectilinear motion is real. The cause for this departure however does not involve any physical force but is rather a consequence of the fact that (s)he is not in an inertial frame. These are the *real* effects of *pseudo* forces seen by an observer in an accelarated frame of reference.

5. System Requirements

Since the software developed was coded in Java, the programs are expected to be platform independent. The users however need Java2 or higher to be installed on their machines. Java2 is available for free from Sun Microsystems. Since the graphics can be demanding on system memory, we recommend that the programs be run on a system that has atleast 128MB of RAM and a standard desktop configuration for everything else.

We gratefully acknowledge the facilities generated using a grant from the Department of Science and Technology, Government of India, New Delhi, which were used for the present work. Academic institutions desirous of using the software described in this work may contact P C Deshmukh.

Suggested Reading

[1] H A Daw, *American Journal of Physics*, Vol. 55, pp.1010-1014, 1987.

[2] H Goldstein, *Classical Mechanics*, Pearson education , India, 2002.

[3] A D Davis, *Classical Mechanics*, Academic Press College Division, Orlando, 1986.

[4] V Barger and M Olsson, *Classical Mechanics: A Modern Perspective*, McGraw Hill, New York, 1973.

[5] G Sposito, *Introduction to Classical Dynamics*, John Wiley, New York, 1976.

I maintain there is much more wonder in science than in pseudoscience. And in addition, to whatever measure this term has any meaning, science has the additional virtue, and it is not an inconsiderable one, of being true.

Carl Sagan
(1934-1996)