Additional problems for practice

These problems are taken from textbooks and assignments and exams of some previous years. These won't be worked out in the classes as other problems with similar concepts have been worked out already. However, these form a part of the exam syllabus. Students are encouraged to form study groups and help each other to work out these as a part of the learning process.

1. The magnetic Induction $\vec{B}$ is defined by the Lorentz force equation $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$ where a charge q moves with a velocity $\vec{v}$. It is found, from two separate experiments, that $\vec{F}/q = 2\hat{e}_z - 4\hat{e}_y$ if $\vec{v} = \hat{e}_x$ and $\vec{F}/q = 4\hat{e}_x - \hat{e}_z$ if $\vec{v} = \hat{e}_y$ Then, find $\vec{B}$ as well as the value of $\vec{F}$ for the case $\vec{v} = \hat{e}_z$

2. A particle moves in a plane along the curve given by $\rho = \exp(a\varphi)$ where ρ and φ are the instantaneous coordinates and a is a constant of appropriate dimensions. Then

   a) sketch its trajectory.
   b) if the acceleration is always directed to the origin, show that its magnitude is proportional to ρ^{-3}.

3. Plane polar coordinates of a particle at time t are given as $\rho = \rho_0 \exp(kt)$ and $\varphi = \omega t$. Determine the angle between the velocity and the acceleration vectors if k and ω are positive constants of appropriate dimensions.

4. A particle moves in a circular path under the action of the force $\vec{F}(\rho,\varphi) = -\frac{a}{\rho^2}\hat{e}_\rho + b\hat{e}_\varphi$ where a and b are positive constants of appropriate dimensions. Then

a) calculate the work done when the particle completes one circle.
b) is it possible of this force to be conservative for some specific values of the constants a and b ?

5. It is found that the height of a particular hill can be expressed as $h(x,y)=10\{2xy-3x^2-4y^2-18x+28y+12\}$ where y is the distance North of and x is the distance East of a certain given landmark. Then,

   a) Where exactly is the top of the hill located?
   b) What is the maximum height of this hill?
   c) How steep is the hill at a point 1m North and 1 m East of a certain landmark?

6. Values of the temperature at different points on a plane surface is given by $T(x,y)=100\exp-\left\{\frac{x^2}{25}+\frac{y^2}{9}\right\}$ Then

   a) Sketch the isotherms.
   b) In which direction should an insect situated at the point (5,3) on this plane move so as to get cooler at the earliest?

7. Consider the potential $U(x,y)=x^2+4y^2$. What is the equation of the equipotentials? Find the magnitude and direction of the gradient at the point (1,1,1) for the surface given by U=5. (All the constants have appropriate dimensions).

8. Evaluate the work done in taking a particle through the closed loop consisting the perimeter of a semicircle and its diameter (=2a) in a plane where a force field $\vec{F}(x,y,z)=x\hat{e}_y$ is experienced. Now repeat this calculation if the force were different, given by $\vec{F}(x,y,z)=k\hat{e}_y$ where k is a constant. Comment on your results.

9. A force field given by $\vec{F}(x,y,z) = xy\hat{e}_x + y^2\hat{e}_y$ exists in a region. A unit mass is taken from the point (0,0) to (1,0), then to (1,1) an finally to (0,1). Then

a) calculate the work done for each segment and the total work done.
b) find the work done if the particle were moved directly from (0,0) to (0,1).
c) Can you make any conclusion about the nature of this force?

10. A potential in a region is given by $\Phi(x,y) = x^2 - y^2$. Then

a) sketch the equipotential lines for this potential.
b) obtain an expression for the corresponding force field.
c) sketch the field lines in the same figure.

11. The force on a particle is given by $\vec{F}(\rho,\varphi) = z\hat{e}_\rho + \rho\hat{e}_z$. Find the corresponding potential $\Phi(\rho,\varphi,z)$. Further, starting from this potential, take its negative gradient and show that we get back the force.