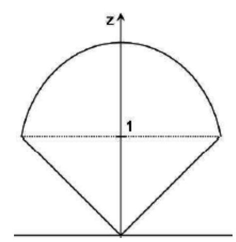
Additional problems for practice : set 02

These problems are taken from textbooks and assignments and exams of some previous years. These won't be worked out in the classes as other problems with similar concepts have been worked out already. However, these form a part of the exam syllabus. Students are encouraged to form study groups and help each other to work out these as a part of the learning process.

1. The force field in a region of space is given by $\vec{F}(r,\theta,\varphi) = -(\hat{e}_r \sin\theta + \hat{e}_\theta \cos\theta)$ Sketch the shape of the corresponding equipotentials.

2. Find the volume of (*a*) a cylinder of radius R and height h and (*b*) a cone of base diameter *R* and semi vertical angle θ using any coordinate system of your choice.

3. An ice cream is designed in the shape shown in the figure and described by a combination of the cone described by $z = \sqrt{(x^2 + y^2)}$ and the hemisphere described by $z = 1 + \sqrt{1 - x^2 - y^2}$. Find its volume using any coordinate system of your choice.



4. Spend some time with this interesting U-tube : A 'U' shaped glass tube has a uniform cross section area of *A* and contains a volume *V* of mercury to some level. The mercury is displaced slightly and released. Disregarding friction, find the period of oscillation in terms of *V*, *A* and *g*.

5. For a lightly damped harmonic oscillator, $\gamma \ll \omega_0,$, show that when the driving frequency for which the steady-state amplitude is one-half, the steady-state amplitude at the resonant frequency is given by $\omega \approx \omega_0 \pm \gamma\sqrt{3}$

6. Show that the quality factor *Q* of a lightly damped harmonic oscillator is equal to the ratio of the steady-state amplitude at the resonant frequency to that at zero driving frequency to that at zero driving frequency for a given value of the forcing amplitude F_0.

7. A particle of mass *m* is in one-dimensional motion under the influence of the potential $V(x)=\frac{k}{2}x^2+\frac{k^2}{x}$ where k is a positive constant of appropriate dimensions. Find the points of equilibrium and comment on stability.

8. The central potential governing the dynamics of a particle is given by and its angular momentum has a magnitude L. Then find the minimum and maximum values possible for r. Also find the radius and the period of the possible circular orbit.

9. A particle of mass *m* moves under an attractive central force $\vec{F}=-k^3\hat{e}_r$ where k is a positive constant of appropriate dimensions. The angular momentum of the system is L.

   a) Write the expression for the effective potential function
   b) If the mass were to move in a circular orbit, find the radius of this orbit and the energy of the particle.

10. The orbit of a particle under a central potential is found to be given by the equation $(x-1)^2+y^2=1$ and the angular momentum of the system is L. Find the magnitude of the force.

11. Show that the orbital period for an earth satellite in a circular orbit, just above the earth's surface is the same as the period of oscillation of the particle dropped into a hole drilled through the earth.

12. A firefly sits at a point close to its rim on one of the tyres (of radius *a*) of a car with the wheels rotating at a constant angular speed ω. Then (i) find an expression for the speed of the firefly with respect to you, a static observer and (ii) sketch the trail of the light emitted by the firefly, as seen by you. Name the geometrical shape.

13. Why does the Halley's comet visit us again and again? Do all comets do so? I have seen it in the year 1986 and you are likely to show it to

your grandchildren in the year 2061. (Nothing will happen to the comet by then; hope the same with humanity!) From this data, find its semi major axis. (It is easier to do this in AU: Astronomical Units; one AU being equal to 1.5×10^{11} m.). Given that its perihelion and aphelion are 0.59 and 35.2 AU respectively, find the eccentricity of the orbit. You will be surprised to see how simple it looks if you express the third law with length measured in AU and time in years.

14. Chandrayaan was India's pride. Several young engineers and scientists (including many who were sitting in the same benches you sit now) were behind the success of this great venture. Just imagine the complications involved in the whole process, the accuracy required (Excuses like 'Sorry, I just missed a negative sign" would have sent the rocket spinning down to hit our heads! Visit the website *https://shanepedia.wordpress.com/out-of-the-world/chandrayaan-i-indias-first-mission-moon/* and sketch a detailed diagram of the orbit modifications done at various specific stages. Check which orbits were changed into which other, at different stages, with quantitative information on the speeds, positions, orbit parameters etc.
