Additional problems for practice: set 03

These problems are taken from textbooks and assignments and exams of some previous years. These won't be worked out in the classes as other problems with similar concepts have been worked out already. However, these form a part of the exam syllabus. Students are encouraged to form study groups and help each other to work out these as a part of the learning process.

1. A vector field is given by $\vec{A}(x,y,z) = x^2\hat{e}_x + y^2\hat{e}_y + z^2\hat{e}_z$. Show that the total flux of this over a closed surface of a cylinder of cross section given by $x^2 + y^2 = 16$ and bound by planes z=0 and z =3. Hint: Use of cylindrical polar system would make direct integration easier. However, do you actually have to carry out the direct integration to get the flux? Think.

2. Find the divergence of the vector field given by $\vec{V}(r,\theta,\varphi) = U\cos\theta\left(\frac{a}{r^3} - 1\right)\hat{e}_r + U\sin\theta\left(\frac{a}{2r^3} + 1\right)\hat{e}_\theta$

3. Compute the divergence of the function

$$\vec{v}(r,\theta,\varphi) = (r\cos\theta)\hat{e}_r + (r\sin\theta)\hat{e}_\theta + (r\sin\theta\cos\varphi)\hat{e}_\varphi$$

   Check Gauss's theorem for this function using the inverted hemisphere (*i. e.* with its flat surface on top, on the xy plane with its center at the origin) of radius R.

4. Consider the electrostatic field $\vec{E}(r,\theta,\varphi) = \frac{\hat{e}_r}{r^2}$.

   a) Find the flux through a sphere of radius R and volume V, centered at the origin.
   b) Find the divergence of this vector $\vec{\nabla}\cdot\vec{A}$ and the total flux $\iiint_V \left(\vec{\nabla}\cdot\vec{A}\right)dV$.
   c) Apply Gauss's divergence theorem. Do you find anything strange here?

5. The electrostatic potential in the region $0 < r < \infty$ is given by $\Phi(r,\theta,\varphi) = \left(\frac{k}{r^2}\cos\theta\right)$ where k is a positive constant of appropriate dimensions.

   a) Find the corresponding electrostatic field in the region.

   b) Evaluate the volume charge density in the region.

6. The electrostatic field in a region is found to be $\vec{E}(r,\theta,\varphi) = kr^3\hat{e}_r$ (with $k > 0$) Show that the charge density in the region is $5k\varepsilon_0 r^2$. Also find the total charge contained in a sphere of radius R kept at the region with its center at the origin. Do this in two ways: (a) by direct integration of the charge density and (b) using Gauss's theorem.

7. Consider the vector field $\vec{A}(r,\theta,\varphi) = kr\hat{e}_r$ (with $k > 0$) in a region of space.

   a) Find the net flux of this field through the shell region enclosed by two concentric spherical surfaces with radii a and b, (with $b > a$), centered at the origin. Do this smartly, without actually evaluating the flux integral.

   b) Evaluate the charge density in the region if this vector represents an electrostatic field.

8. Use Gauss' law in electrostatics to find the electrostatic field at any point due to the following objects and sketch plots as a function of the distance, in each case:

   (a) A sphere of finite radius and a uniform volume charge density.

   (b) An infinite line of charge with a uniform linear charge density.

   (c) An infinite sheet of charge with a uniform surface charge density.

Note: *'Introduction to Electrodynamics' by Griffiths is an excellent reference for divergence as well as electrostatics. Several copies are available in the central library.*
