Additional problems for practice: set 04

These problems are taken from textbooks and assignments and exams of some previous years. These won't be worked out in the classes as other problems with similar concepts have been worked out already. However, these form a part of the exam syllabus. Students are encouraged to form study groups and help each other to work out these as a part of the learning process.

1. The force field in a region of space is given by $\vec{F}(x,y,z) = \{yz\hat{e}_x + zx\hat{e}_y + xy\hat{e}_z\}$
   a) Find the corresponding potential.
   b) Find the curl of this vector explicitly.
   c) Use divergence theorem to to find the flux through any closed surface within the region.

2. The force field in a region of space is given by $\vec{F}(x,y,z) = F_0\{yz\hat{e}_x + zx\hat{e}_y + xy\hat{e}_z\}$
   a) Find the corresponding potential.
   b) Using the divergence theorem, find the flux through any closed surface in the region.

3. A vector field is given by $\vec{A}(x,y,z) = \{x^2\hat{e}_x + y^2\hat{e}_y + z^2\hat{e}_z\}$. Evaluate the surface integral $\int_S \vec{A}\cdot d\vec{S}$ over the surface of a cylinder with base $x^2 + y^2 = 16$, with top and bottom surfaces given by $z = 0$ and $z = 3$.

4. Examine whether the vectors given below are conservative, by taking the curl.
   a) $\vec{A}(x,y,z) = (\sin y + z)\hat{e}_x + (x\cos y - z)\hat{e}_y + (x\cos y - z)\hat{e}_y$
   b) $\vec{4} = x^2\ \hat{e}_x + y^2\ \hat{e}_y + z^2\ \hat{e}_z$
   c) $\vec{A}(\rho,\varphi,z) = (\rho^2 + z^2)\{\rho\hat{e}_\rho + z\hat{e}_z\}$
   d) $\vec{A}(r,\theta,\varphi) = f(r)\sin\varphi\hat{e}_r$

5. A physical quantity is represented by the vector $\vec{A}(x,y) = -k\left\{y\hat{e}_x - x\hat{e}_y\right\}$ where k is a constant of appropriate dimensions. Find the circulation $\oint_C \vec{A}(x,y)\cdot dl$ around the curve C, a circle of radius a in the xy-plane with origin as center.

6. The velocity field for a steady flow of an incompressible inviscid fluid is given by $v_x = \frac{-c^2 y}{\rho^2};\ v_y = \frac{c^2 x}{\rho^2}$ and $v_y = 0$ where c is a constant of appropriate dimensions and $\rho = \sqrt{(x^2 + y^2)}$. Then

   a) show that this flow is irrotational.
   b) find the 'velocity potential'
   c) Sketch the streamlines of the flow.

7. The electrostatic potential in a region of space 0 < π < ∞ is given by $\Phi(r,\theta,\varphi) = \left(\frac{k}{r^2}\right)\cos\theta$ where k is a positive constant of appropriate diemnsions. Find the corresponding electrostatic field as well charge density in the region.

8. The velocity of an incompressible inviscid fluid in a region free from sources and sinks is given by $\vec{v}(\rho,\varphi,z) = v_\rho \hat{e}_\rho - 2kz\hat{e}_z$ where *k* is a positive constant of proper dimensions. Determine the functional form of v_ρ given that $v_\rho = 0$ at the origin of the coordinates. Sketch the radial component as a function of the distance from the origin.

9. The velocity field of an irrotational flow of an incompressible nonviscous liquid is given as $\vec{v}(x,y,z) = \left\{-ay\hat{e}_x + ax\hat{e}_y + b\hat{e}_z\right\}$, Find the pressure at any point (*x*,*y*,*z*) in the fluid.
