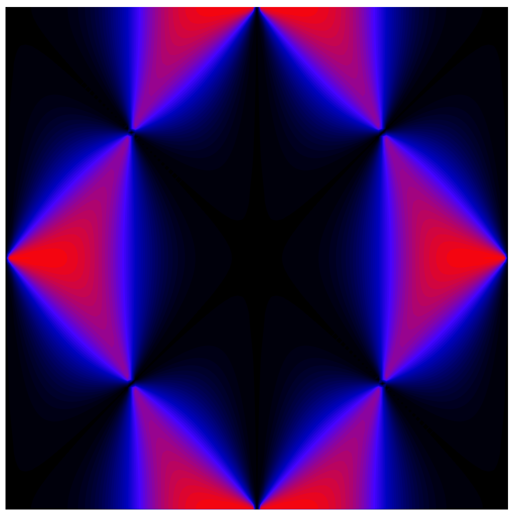
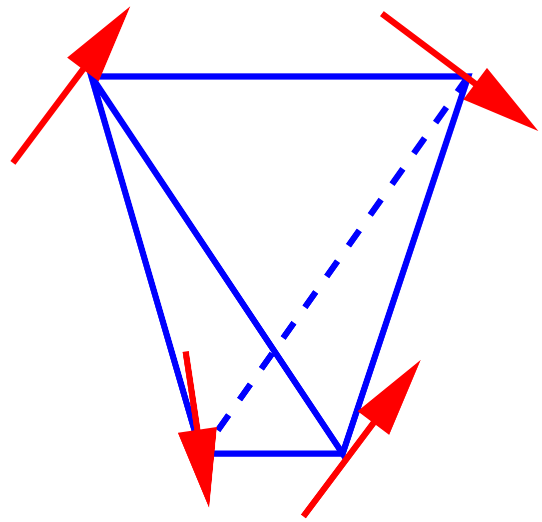
FRUSTRATED MAGNETS & SPIN LIQUIDS





John Chalker

Physics Department, Oxford University

Phases of Matter

In everyday life

Gas **Liquid** **Solid**

Phases of Matter

In everyday life

Gas Liquid Solid

Further possibilities?

plasma, liquid crystal . . .

Phases of Matter

In everyday life

Gas **Liquid** **Solid**

Further possibilities?

Distinguish by

- **Macroscopic response**
- **Microscopic degrees of freedom**

Phases of Matter

In everyday life

Gas **Liquid** **Solid**

Further possibilities?

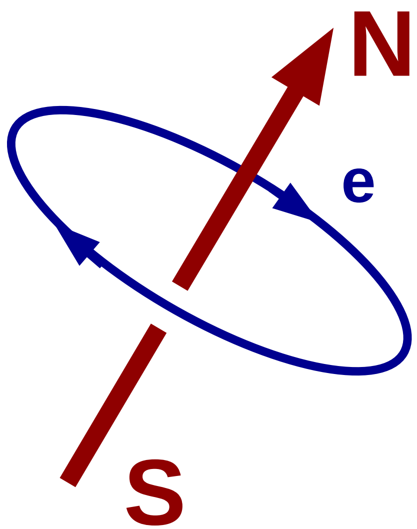
Distinguish by

- **Macroscopic response**
- **Microscopic degrees of freedom**

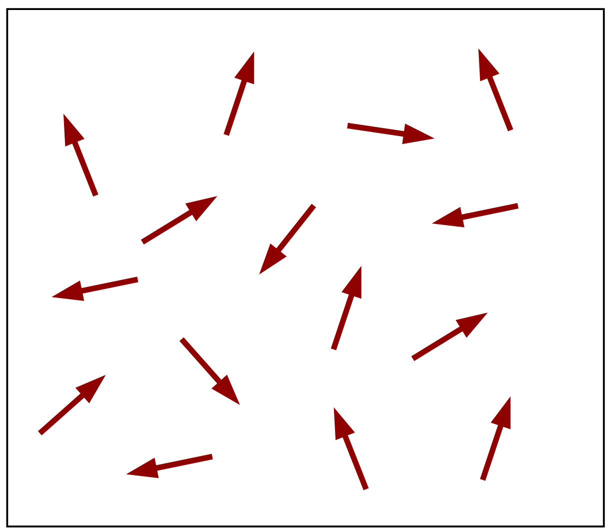
E.g. metals vs electrical insulators

Building blocks for magnetic phases of matter

Atomic magnetic moments



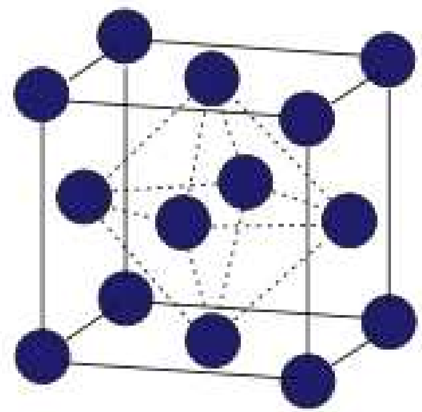
. . . interacting in a solid

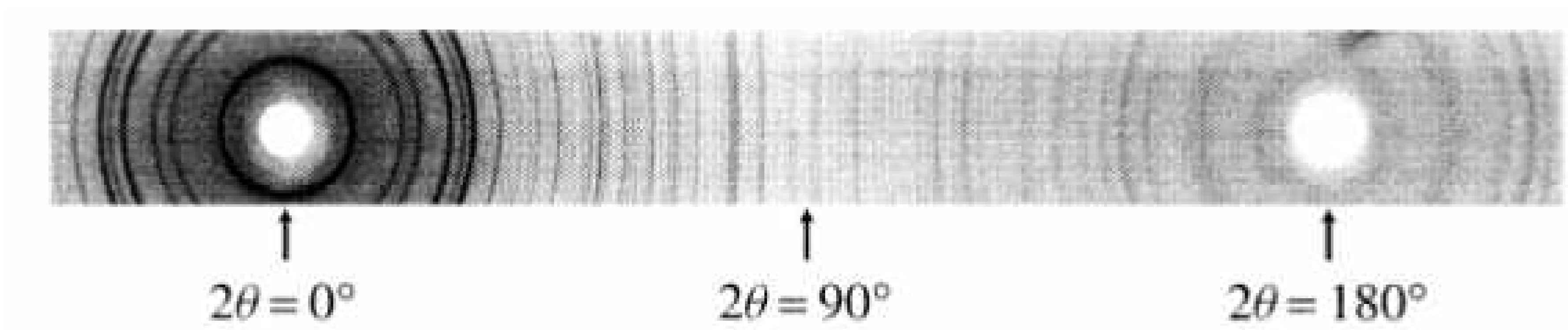


Condensed matter at low temperature

Symmetry breaking as the norm

Crystalline solids

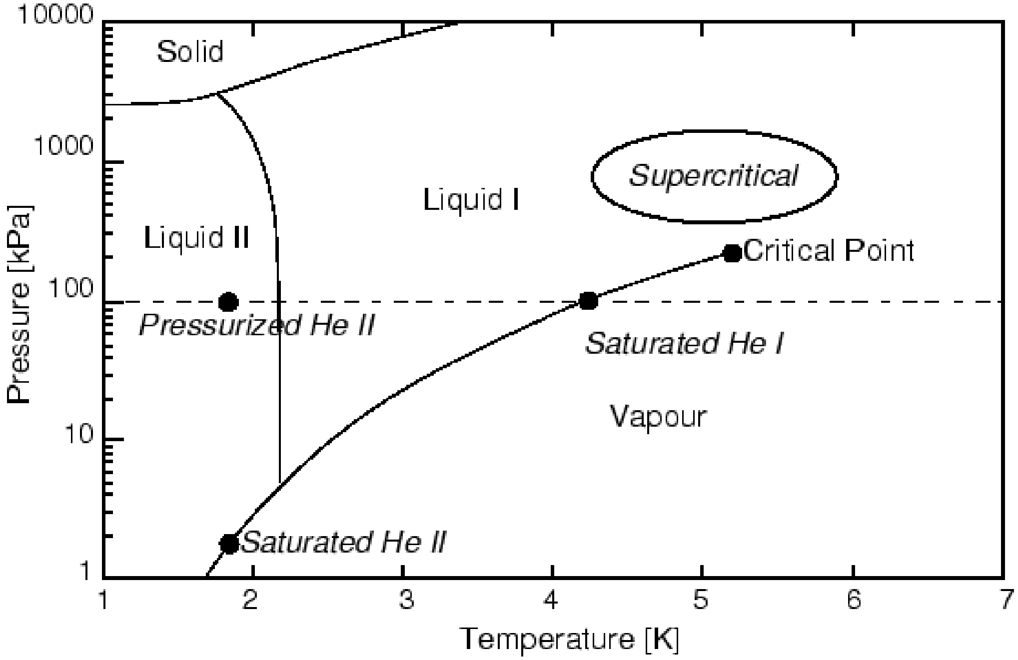


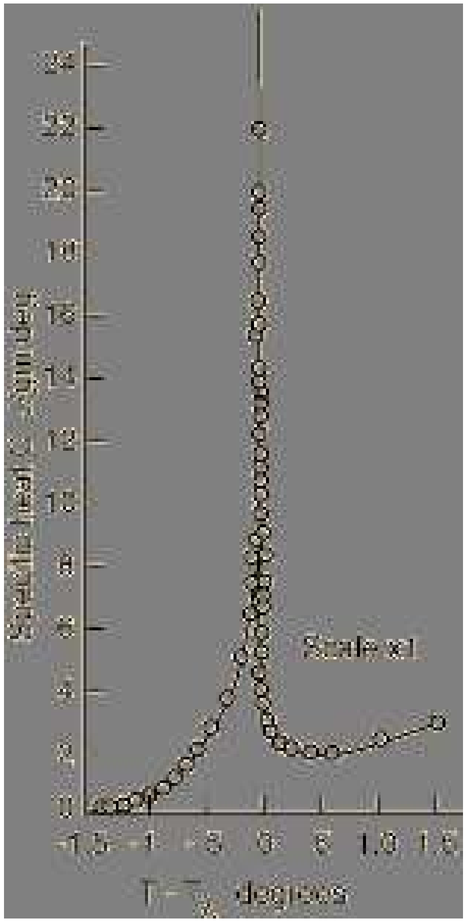


Broken symmetry in Bose liquids

Quantum fluctuations suppress crystalisation

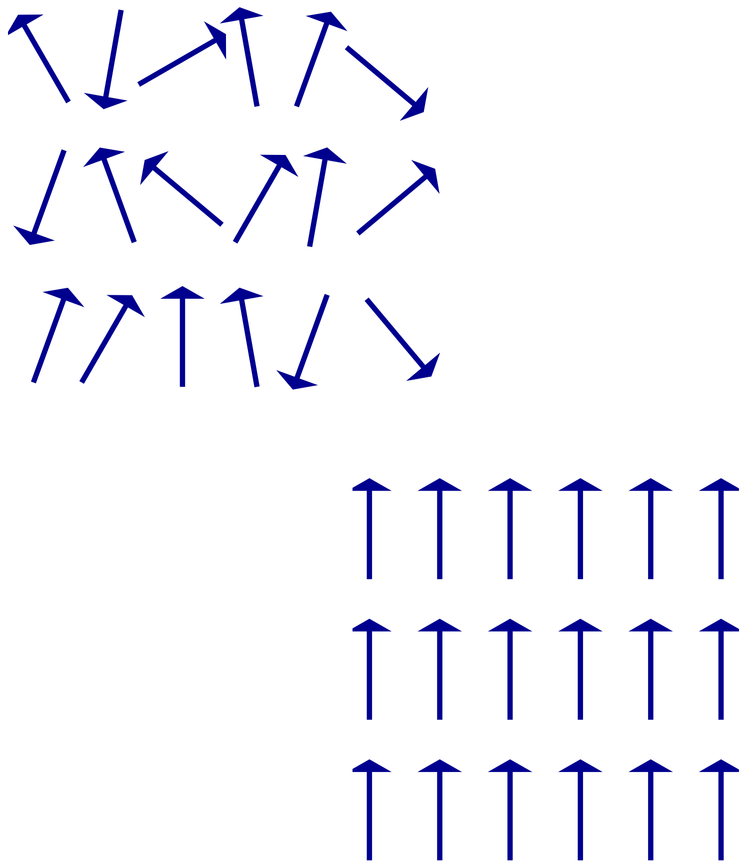
^{4}He phase diagram





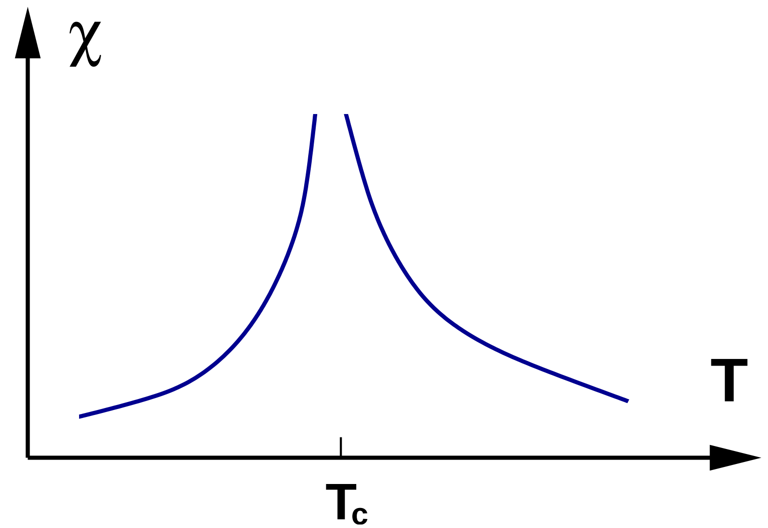
Ordering in ferromagnets

High temperature



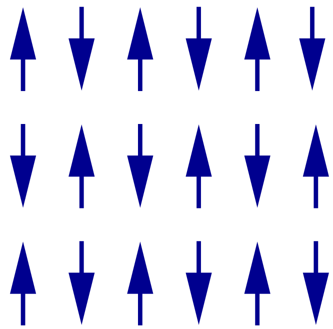
Ground state

Susceptibility

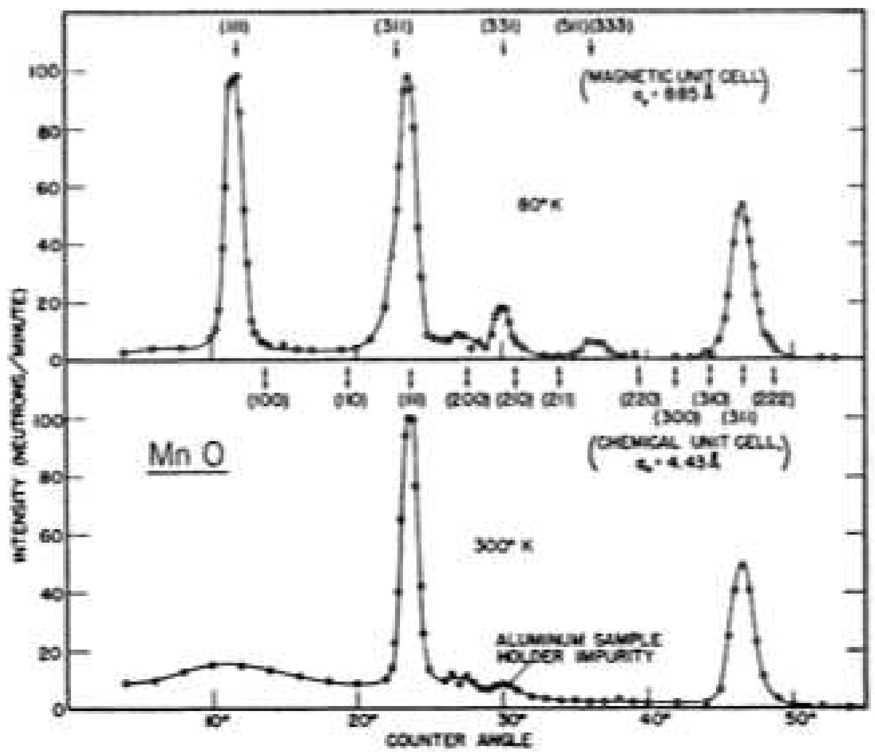


Antiferromagnetic order

Néel order

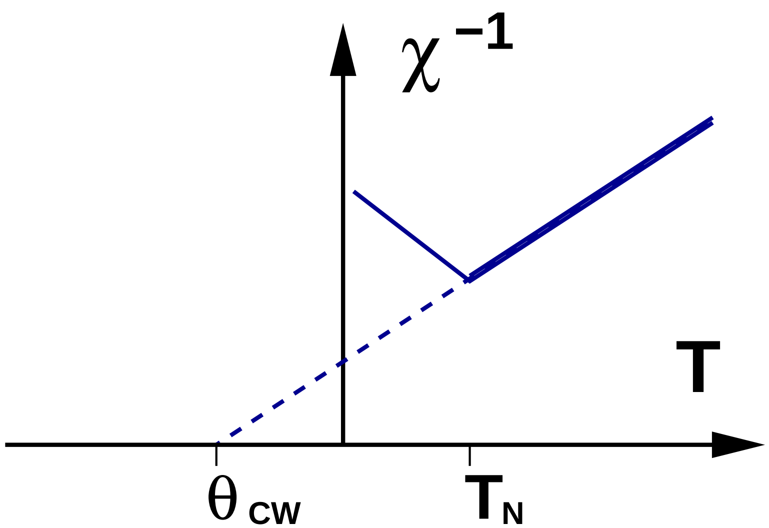


Neutron diffraction



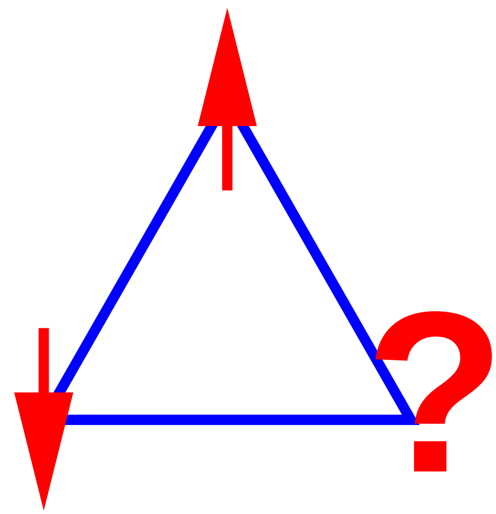
Shull and Smart (1949)

Inverse susceptibility



Can magnetic order be avoided?

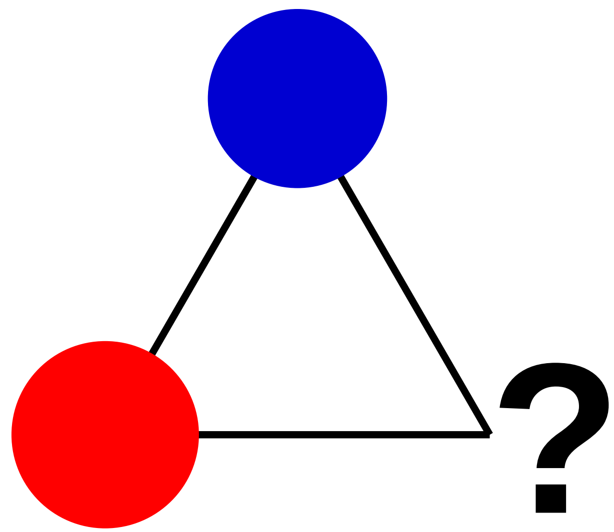
Frustration and degeneracy



Anderson 1956, Villain 1979

Can magnetic order be avoided?

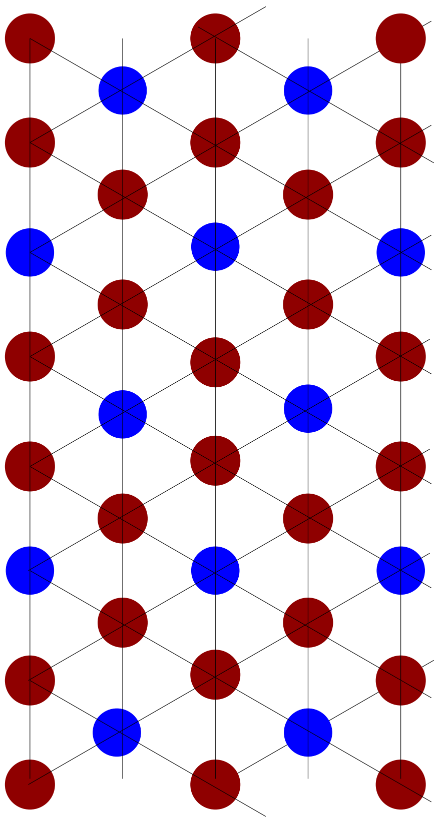
Frustration and degeneracy



Anderson 1956, Villain 1979

Triangular lattice ground states

Six ordered states



Triangular lattice ground states

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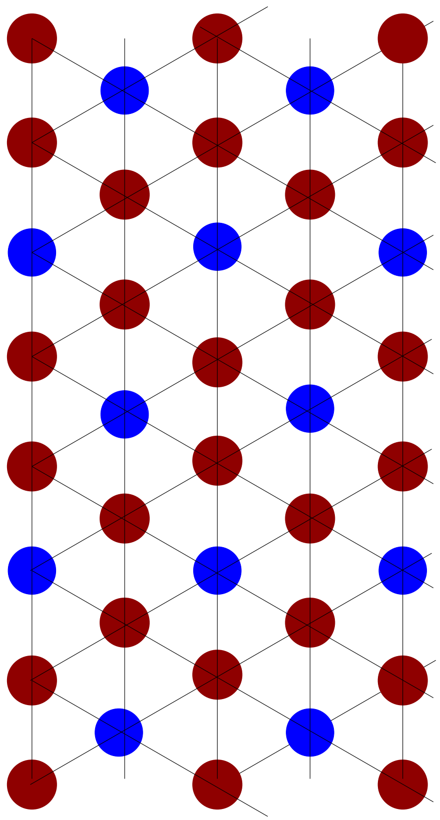
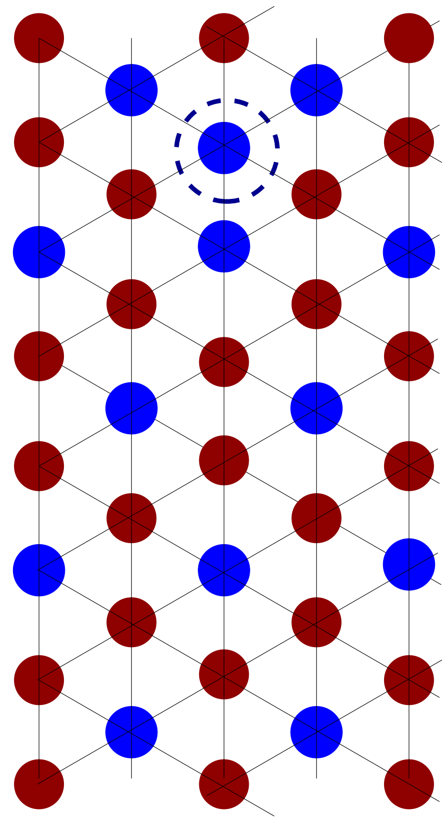


with defects at no energy cost



Triangular lattice ground states

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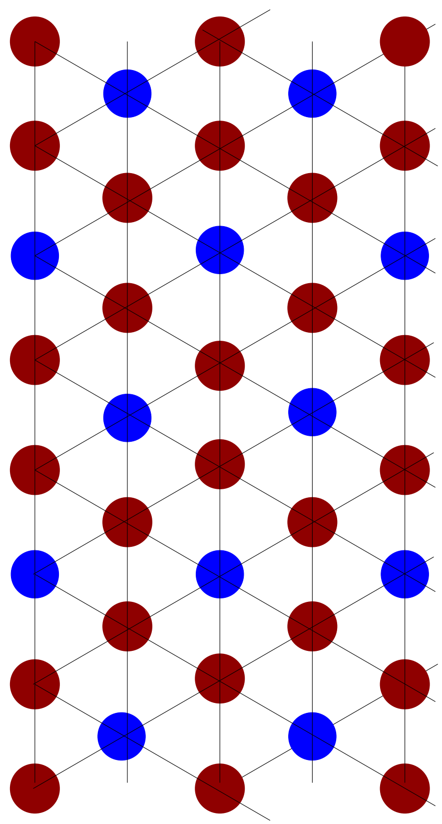
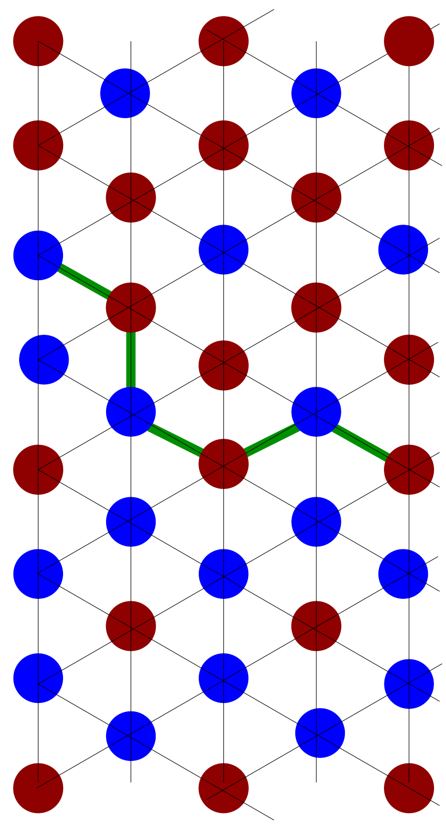


and domain walls at no energy cost



Triangular lattice ground states

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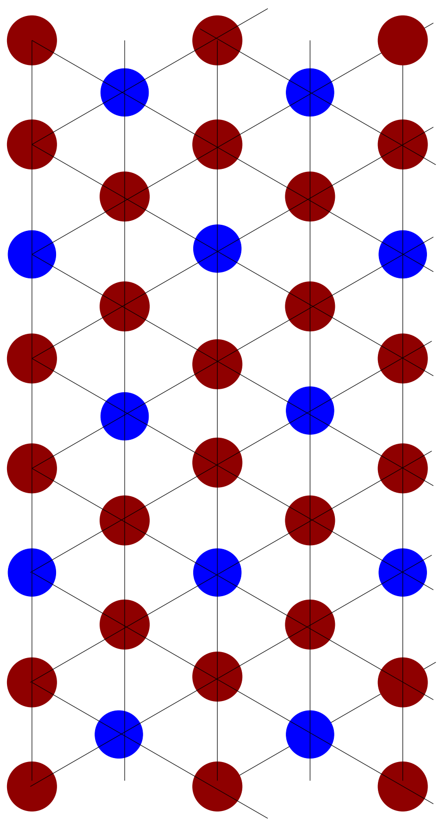

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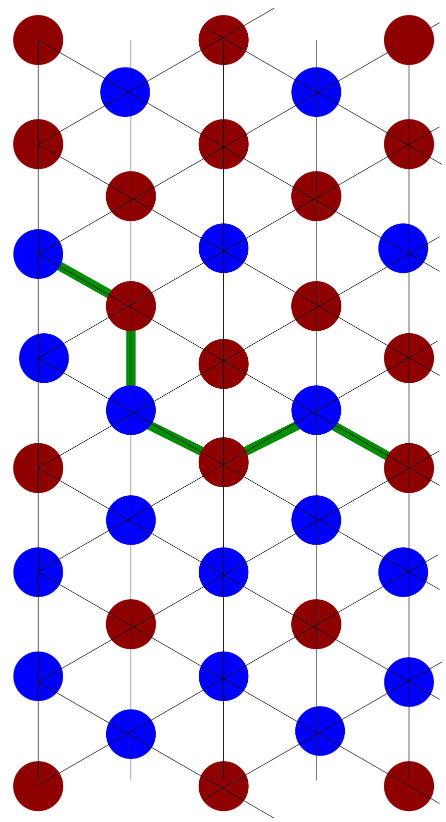
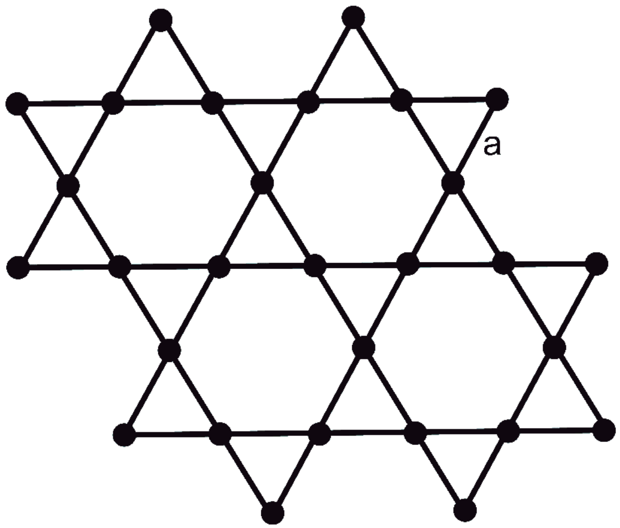


. . . so fluctuations prevent order

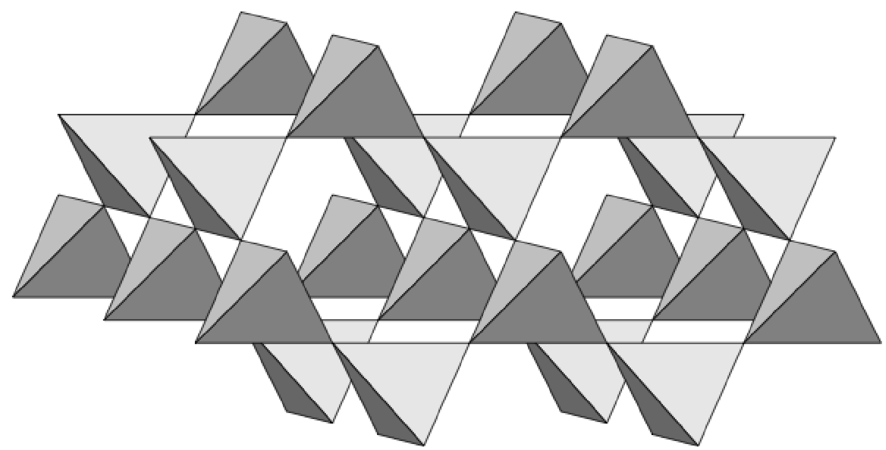
Examples of frustrated lattices

Building block: corner-sharing frustrated units

2D: kagome lattice

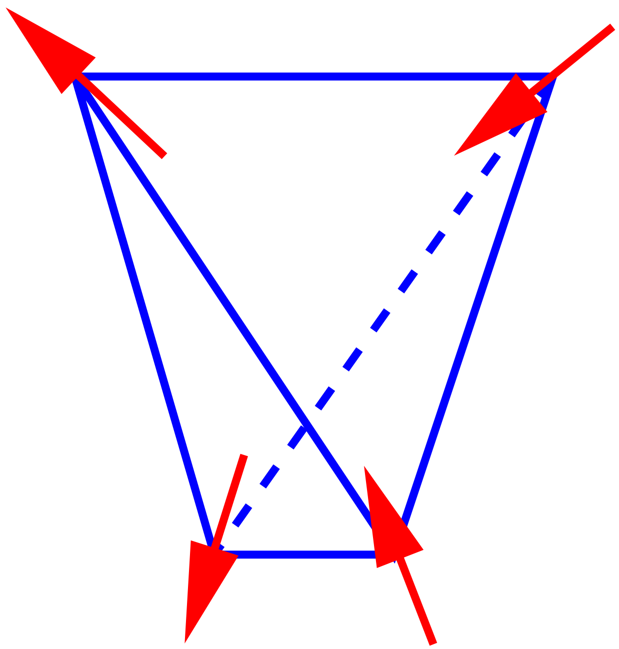


3D: pyrochlore lattice



Frustration and residual entropy

Spin ice



Anisotropy +

ferromagnetic exchange

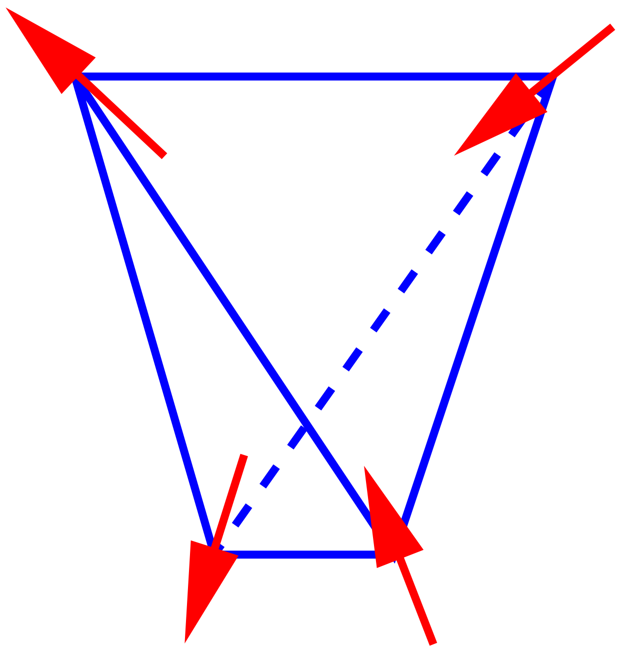
$Dy_2Ti_2O_7$

& $Ho_2Ti_2O_7$

Ground states: ‘two-in, two-out’

Frustration and residual entropy

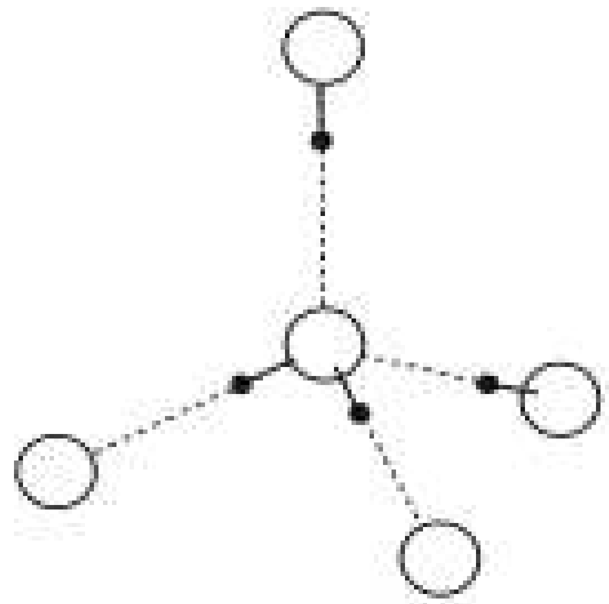
Spin ice



Anisotropy +

ferromagnetic exchange

Water ice



Pauling 1935

Ground states: 'two-in, two-out'

Pauling's entropy estimate

One tetrahedron

Total number of states: 16

Fraction that are ground states: $\frac{6}{16}$

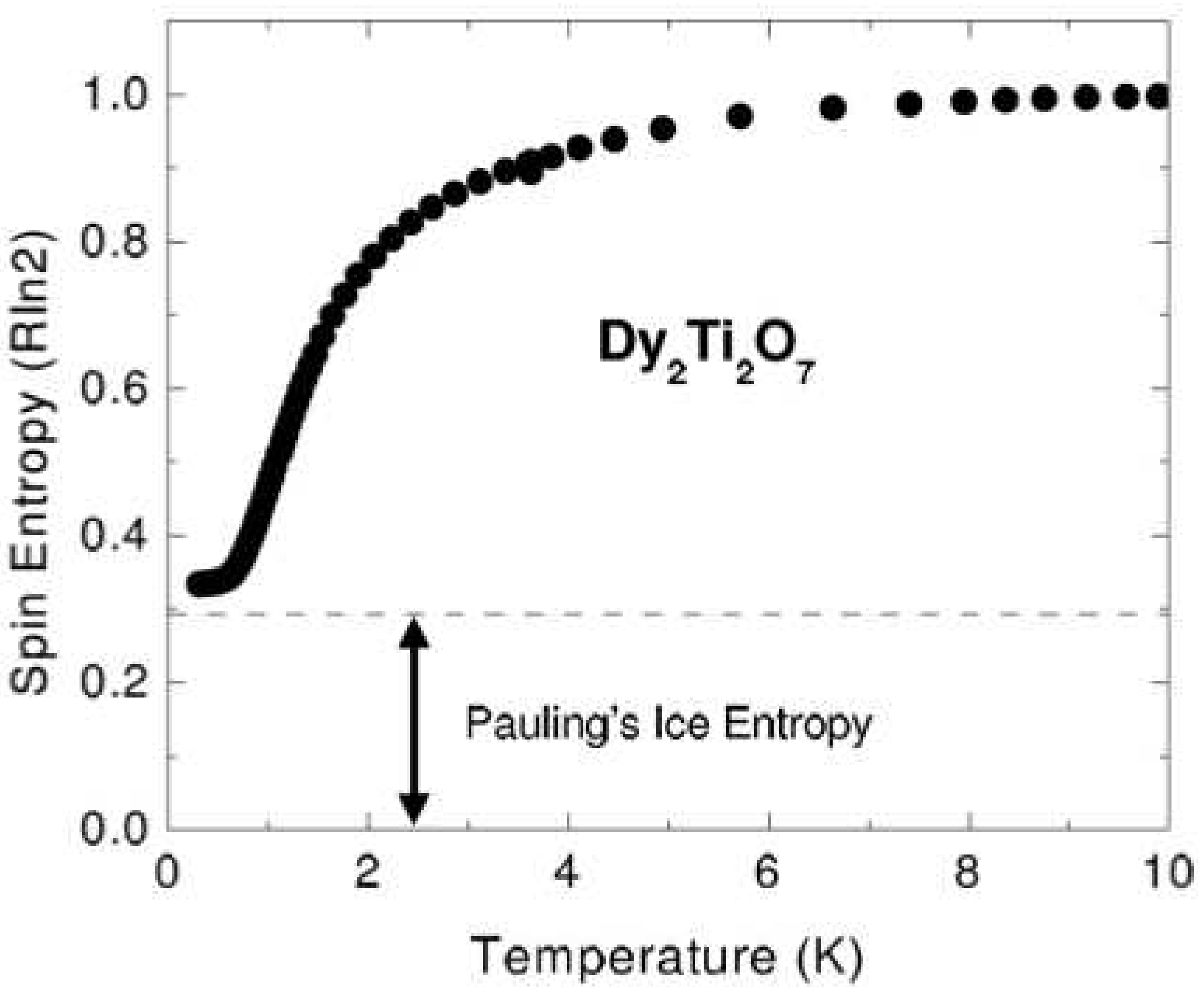
Pyrochlore lattice

Estimate for number of ground states:

$$(total \# states) \times \left(\frac{6}{16}\right)^{(\#\text{ tetrahedra})} = 2^{(\#\text{ spins})} \times \left(\frac{6}{16}\right)^{(\#\text{ spins}/2)}$$

$$= \left(\frac{3}{2}\right)^{(\#\text{ spins}/2)}$$

Pauling entropy in experiment



$Dy_2Ti_2O_7$, **Ramirez *et al*, Nature 399, 333 (1999).**

Spin liquids

States of matter

Gas

uncorrelated atoms

Liquid

short-range correlations

Solid

crystalline order

Spin liquids

States of matter

Gas

uncorrelated atoms

Liquid

short-range correlations

Solid

crystalline order

States of spin systems

Paramagnet

uncorrelated spins

Spin liquid

Néel state

periodic spin order

Spin liquids

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States of spin systems

Paramagnet

uncorrelated spins

Spin liquid

topological order

& fractionalised excitations

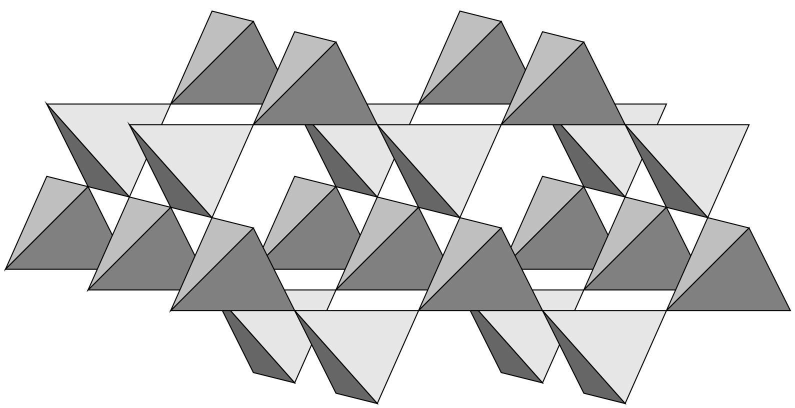
Néel state

periodic spin order

Correlations induced by ground state constraints

Local constraints

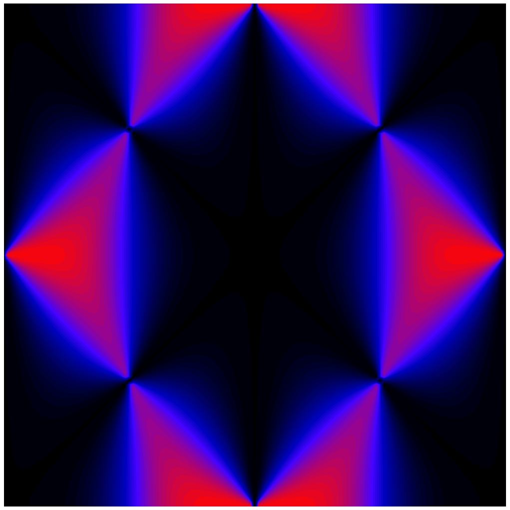
$$\sum_{tet} \mathbf{S}_i = 0$$



Long range correlations

Sharp structure in

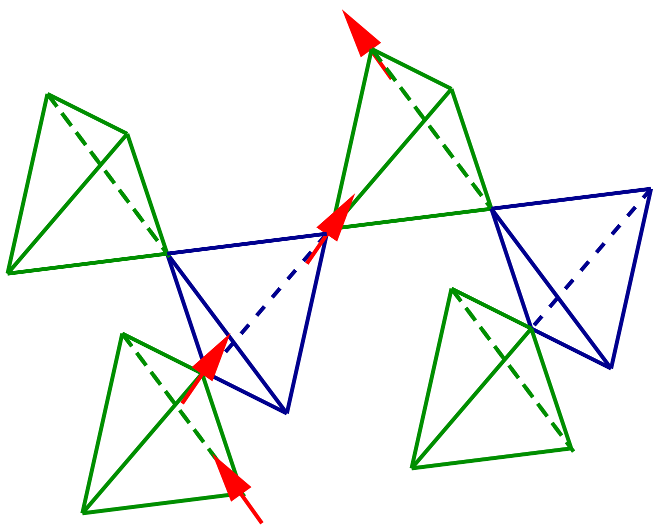
$$\langle \mathbf{S}_{-\mathbf{q}} \cdot \mathbf{S}_{\mathbf{q}} \rangle$$

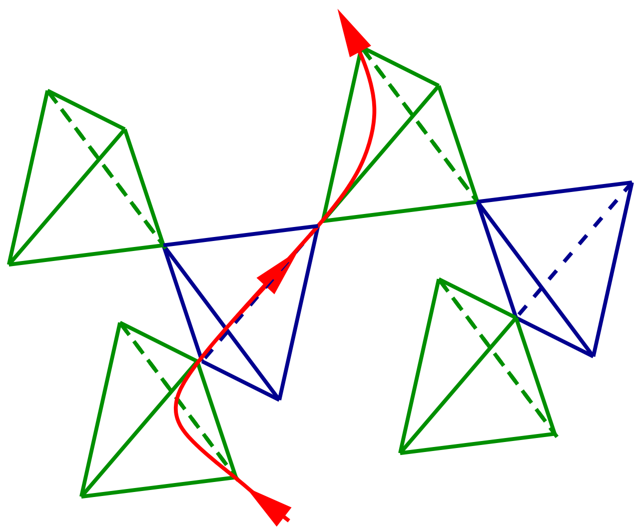


Gauge theory of ground state correlations

Youngblood *et al* (1980), Huse *et al* (2003), Henley (2004)

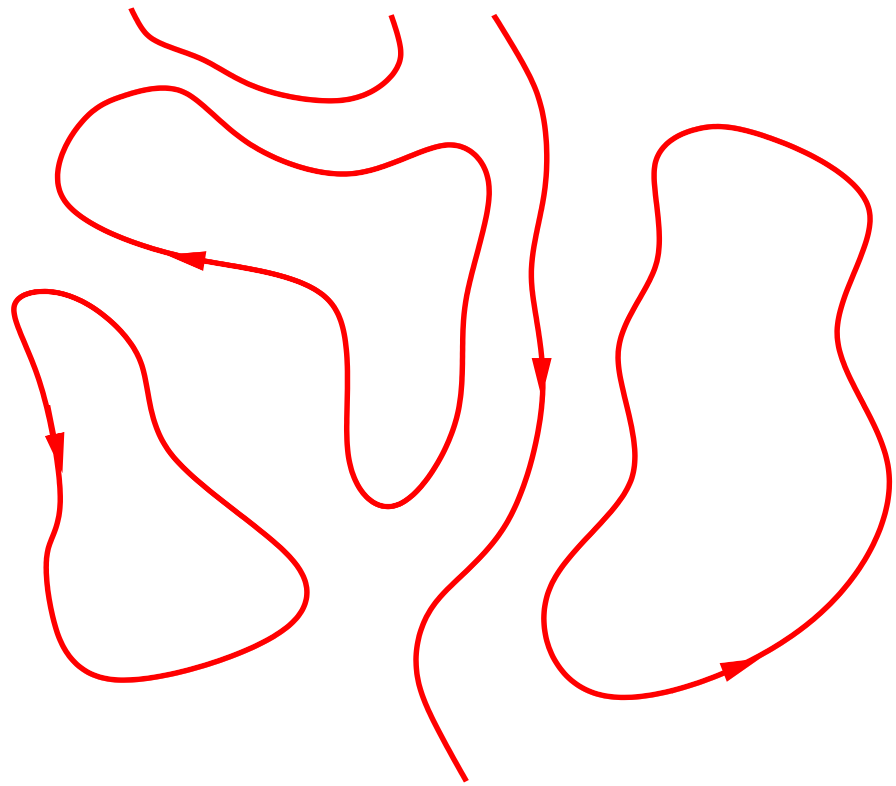
Map spin configurations . . . **. . . to vector fields** $\mathbf{B}(\mathbf{r})$





'two-in two out' groundstates . . . **. . . map to divergenceless** $\mathbf{B}(\mathbf{r})$

Ground states as flux loops

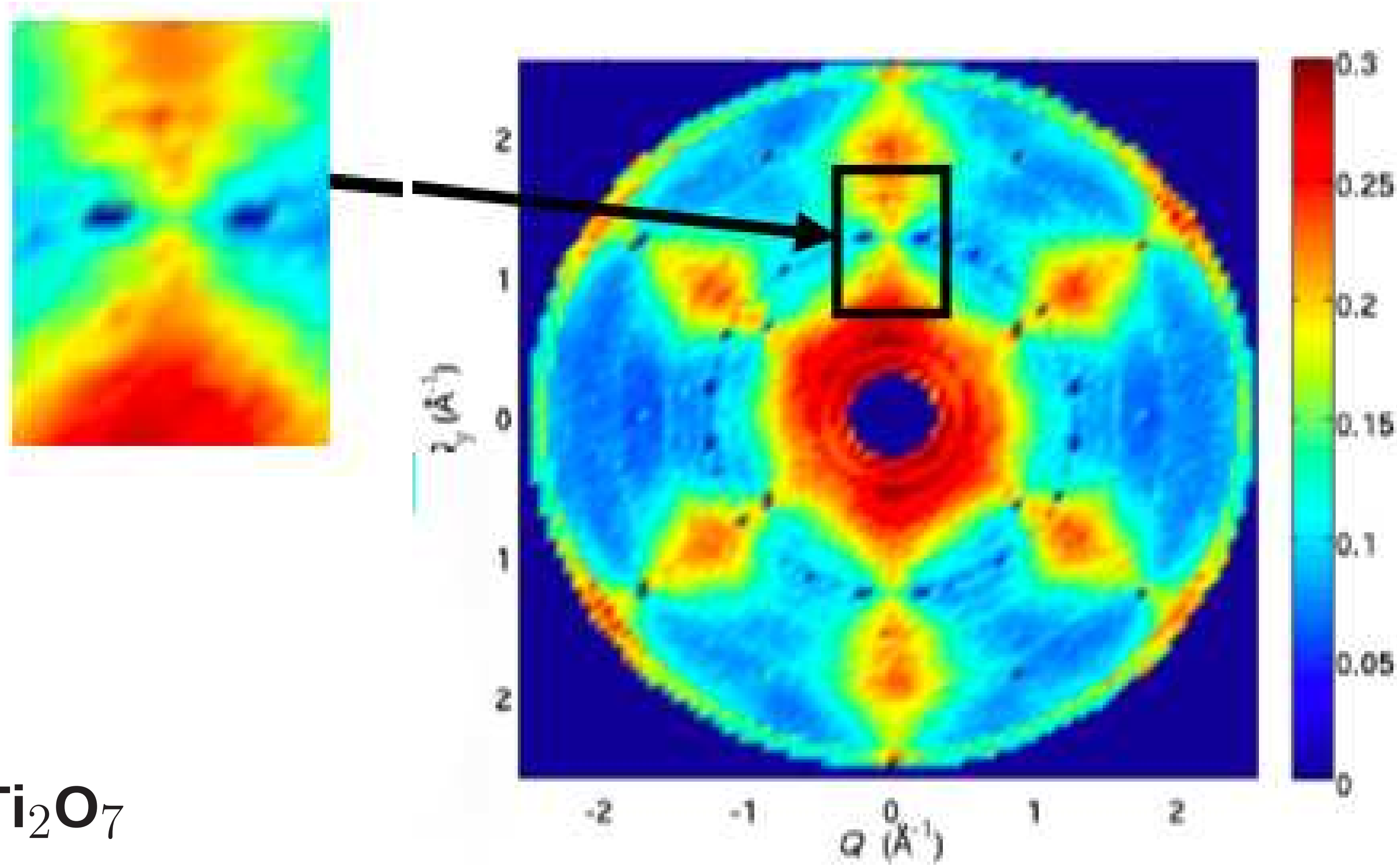


Entropic distribution: $P[\mathbf{B}(\mathbf{r})] \propto \exp(-\kappa \int \mathbf{B}^2(\mathbf{r}) \mathrm{d}^3\mathbf{r})$

Power-law correlations: $\langle B_i(\mathbf{r}) B_j(\mathbf{0}) \rangle \propto r^{-3}$

Low T correlations from neutron diffraction

Fennell, Bramwell and collaborators (2009)

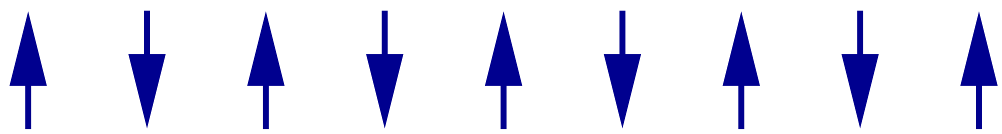


$Ho_2Ti_2O_7$

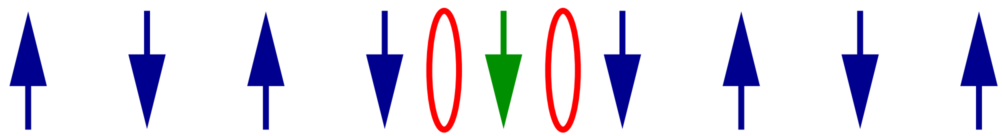
Classical fractionalised excitations

Fractionalisation in one dimension

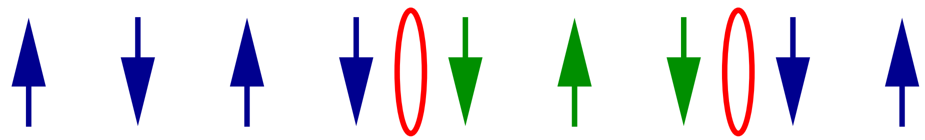
Ground state



An excited state



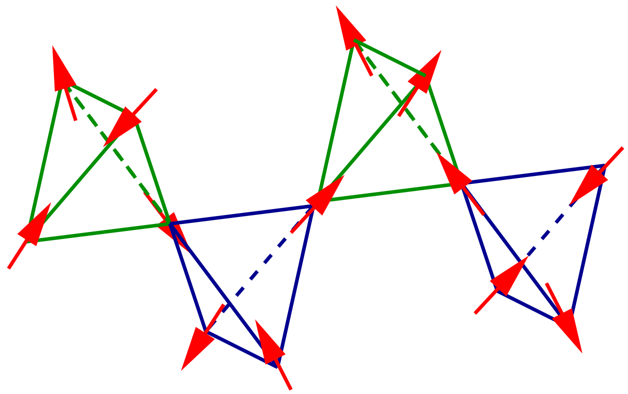
. . . two separated excitations



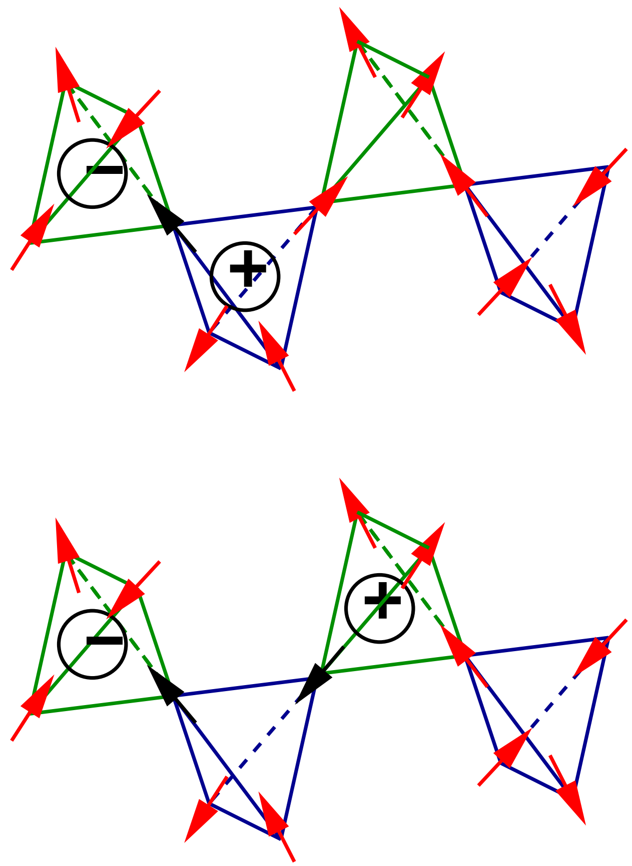
Fractionalisation in spin ice

Monopole excitations

Ground state



Excited states

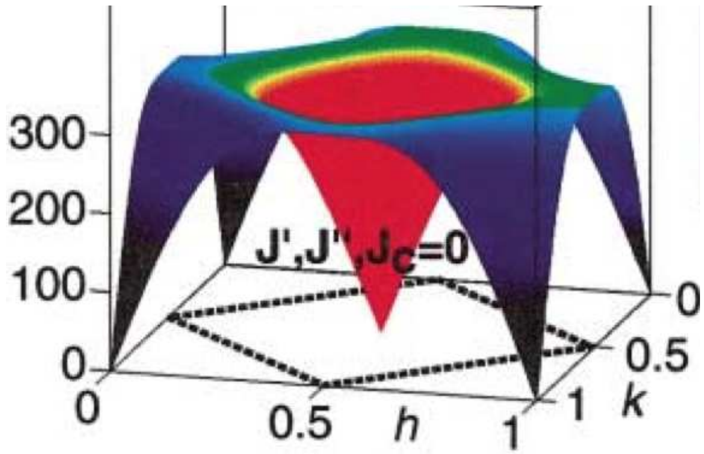


Castelnovo, Moessner and Sondhi (2008)

Dynamics: ordered vs. fractionalised magnets

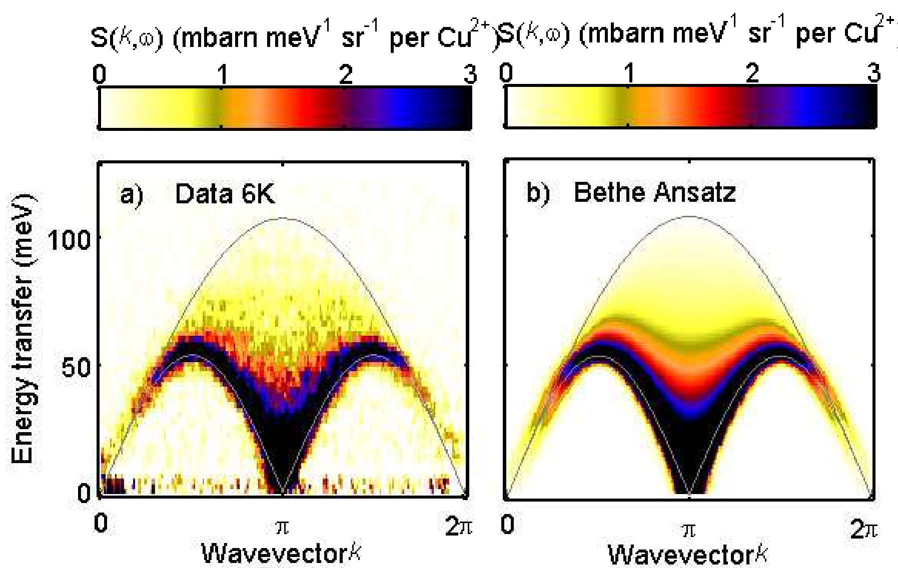
Compare dynamic structure factors

Magnon dispersion in Néel state



undoped cuprate, Coldea *et al.* (2001)

Spinon continuum in Heisenberg chain

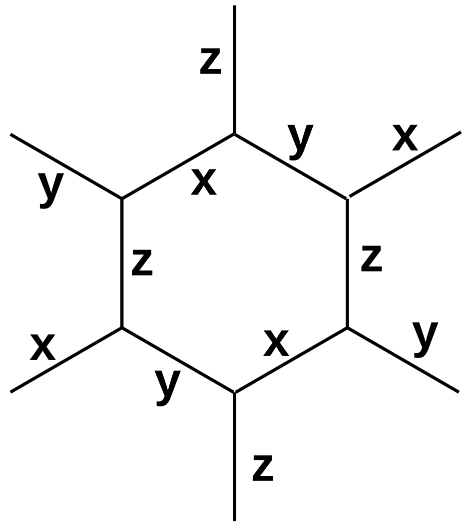


$KCuF_3$, Caux *et al.* & Lake *et al.* (2013)

Kitaev's honeycomb model

Spin $S = 1/2$ quantum magnet

with strong 'spin-orbit' anisotropy



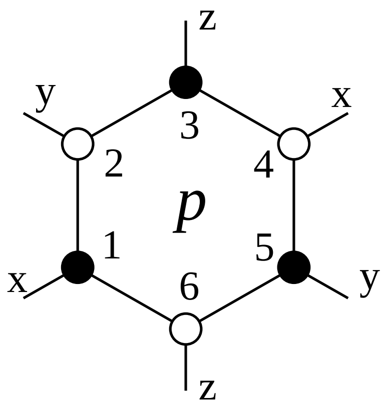
$$\mathcal{H} = -J \left\{ \sum_{\mathrm{x-bonds}} \sigma_i^x \sigma_j^x + \sum_{\mathrm{y-bonds}} \sigma_i^y \sigma_j^y + \sum_{\mathrm{z-bonds}} \sigma_i^z \sigma_j^z \right\}$$

A. Kitaev, Ann. Phys. 321, 2 (2006)

Suggested realisation: G. Jackeli and G. Khaliullin, PRL 102, 017205 (2009)

Emergent degrees of freedom

Static fluxes



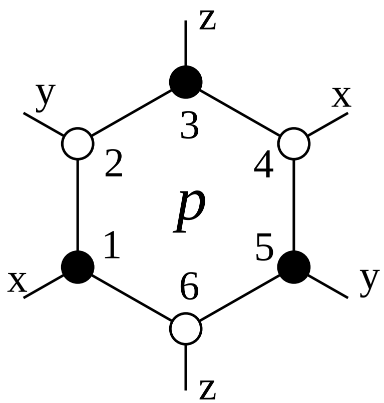
$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

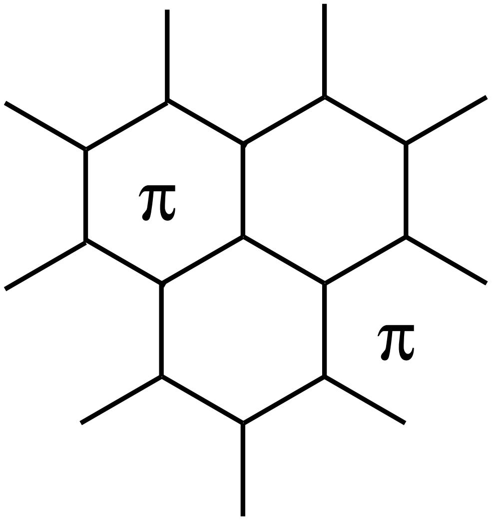
$$[W_p, H] = 0$$

$$[W_p, W_q] = 0$$

Emergent degrees of freedom

Static fluxes . . . and . . . free fermions





$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

$$[W_p, H] = 0$$

$$[W_p, W_q] = 0$$

Tight binding model

hopping magnitudes J

signs set by Z_2 **fluxes**

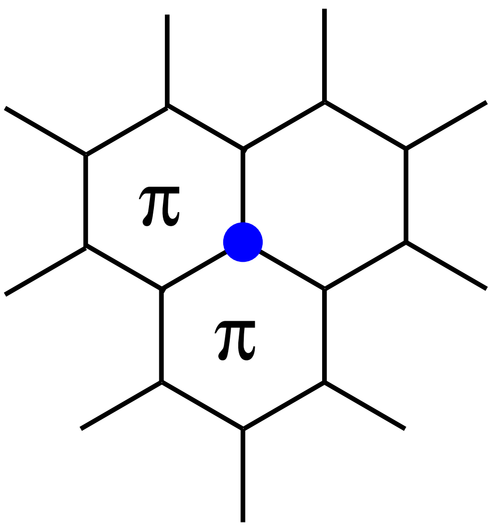
Spin correlations ultra-short-range: $\langle \sigma_j^\alpha \sigma_k^\alpha \rangle = 0$ **for** $|\mathbf{r}_j - \mathbf{r}_k| > 1$

Baskaran *et al.* **(2007)**

Computing the dynamic response

$$S^{\alpha\beta}(\mathbf{r},t) = \langle 0|e^{i\mathcal{H}t}\sigma^{\alpha}_{\mathbf{r}}e^{-i\mathcal{H}t}\sigma^{\beta}_{0}|0\rangle$$

σ^{β}_{0} **adds two fluxes and odd number of fermions to** $|0\rangle$



'just' free fermion time evolution

in presence of added fluxes

Baskaran ***et al.*** **(2007)**

Gross features of $S(Q,\omega)$

Gapless phase: $J_x = J_y = J_z$

- Fractionalisation

  $\Rightarrow$ broad response

  — correlations short range

- Energy cost for flux addition

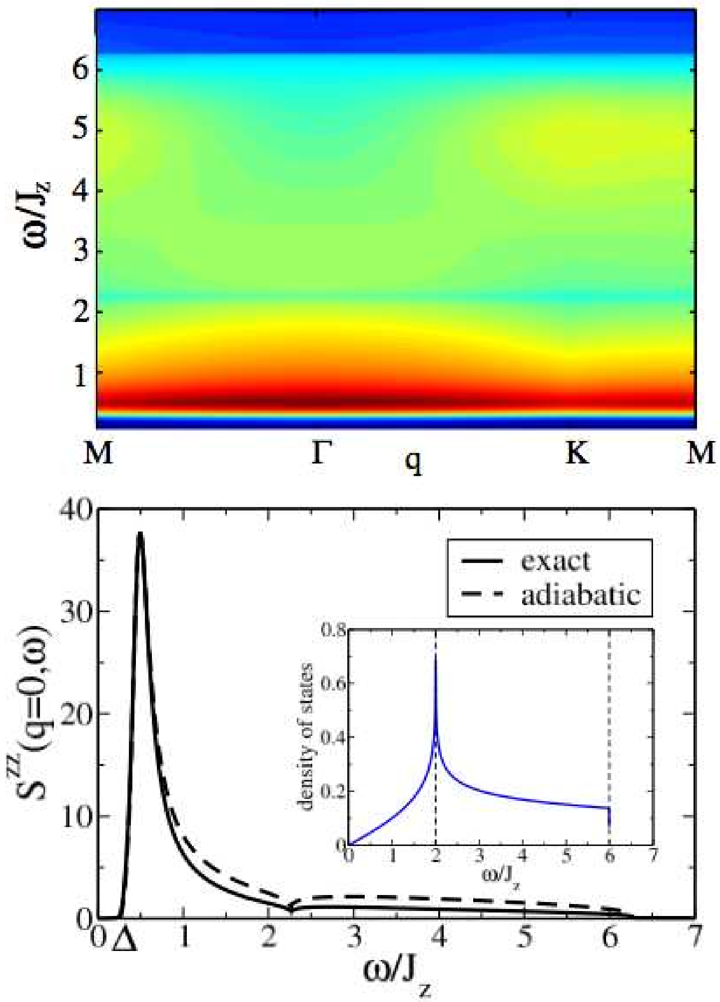
  $\Rightarrow$ gapped response

- $S(Q,\omega)$ is imperfect image

  of fermion density of states

  — influence of fluxes on dynamics

  but $\sim$ 98% of wt single pcle

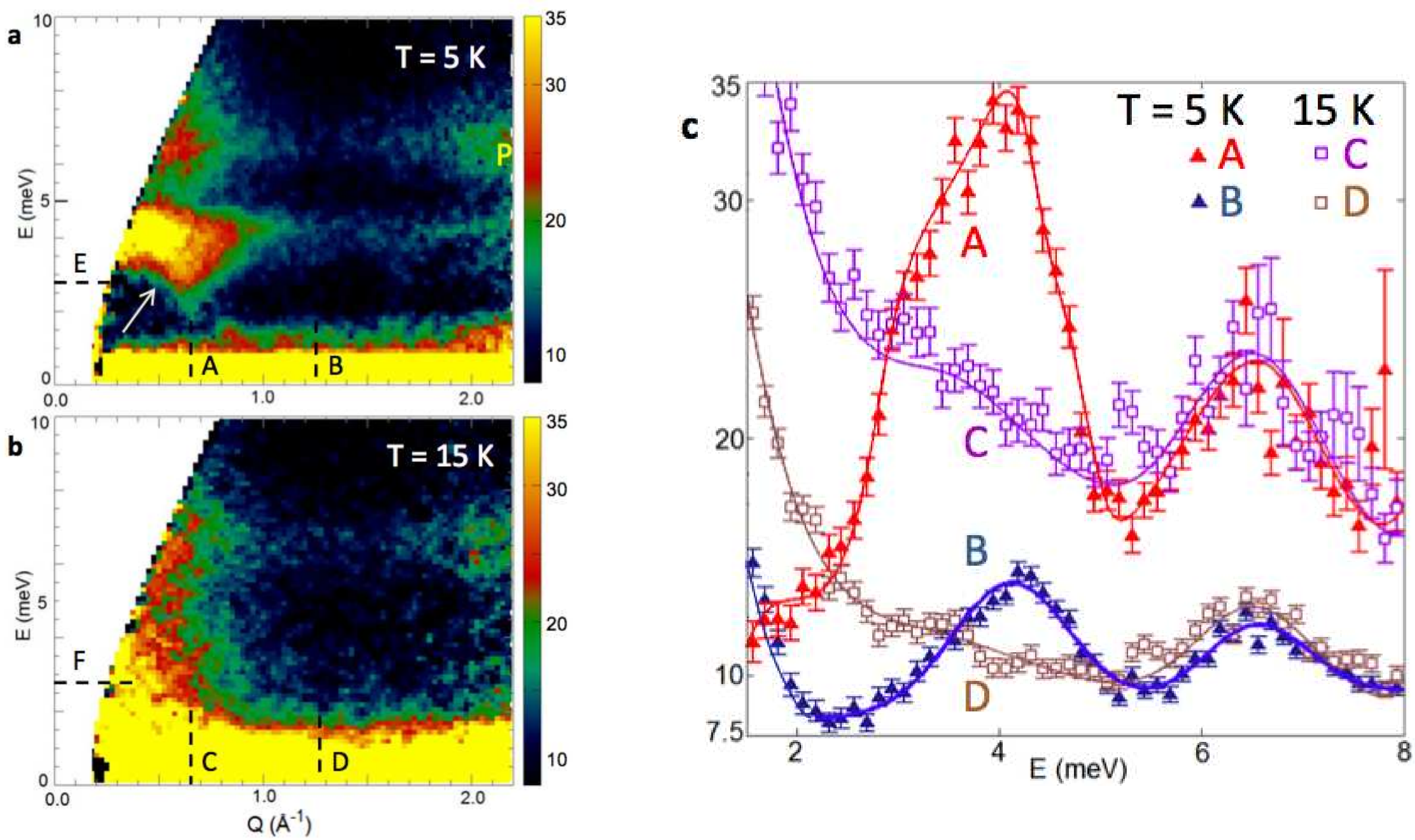


Knolle, Kovrizhin, JTC & Moessner (2014)

Data on α-RuCl$_3$

Nagler group, Nature Materials (2016)

Inelastic neutron scattering from powder sample $T_N \approx 14K$



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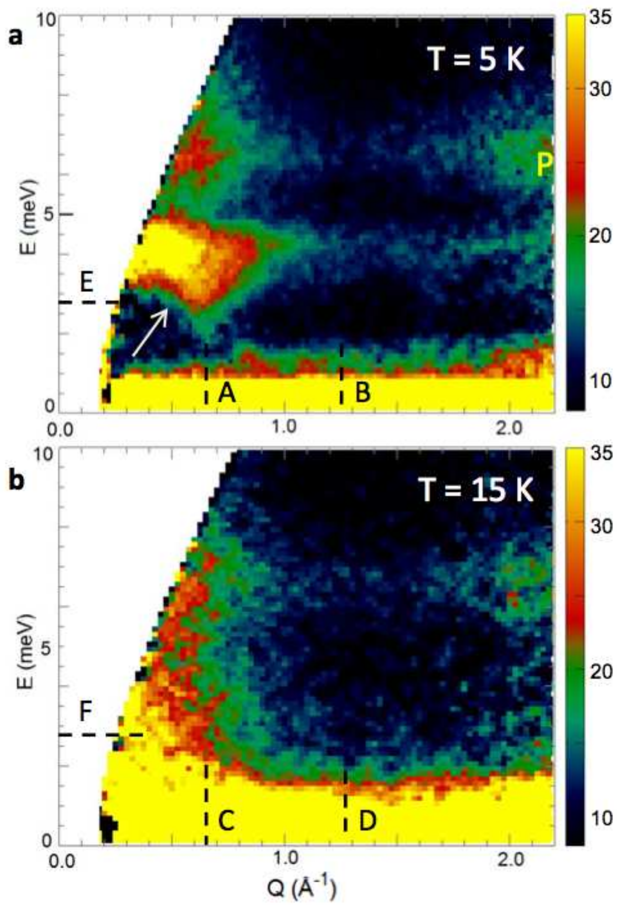

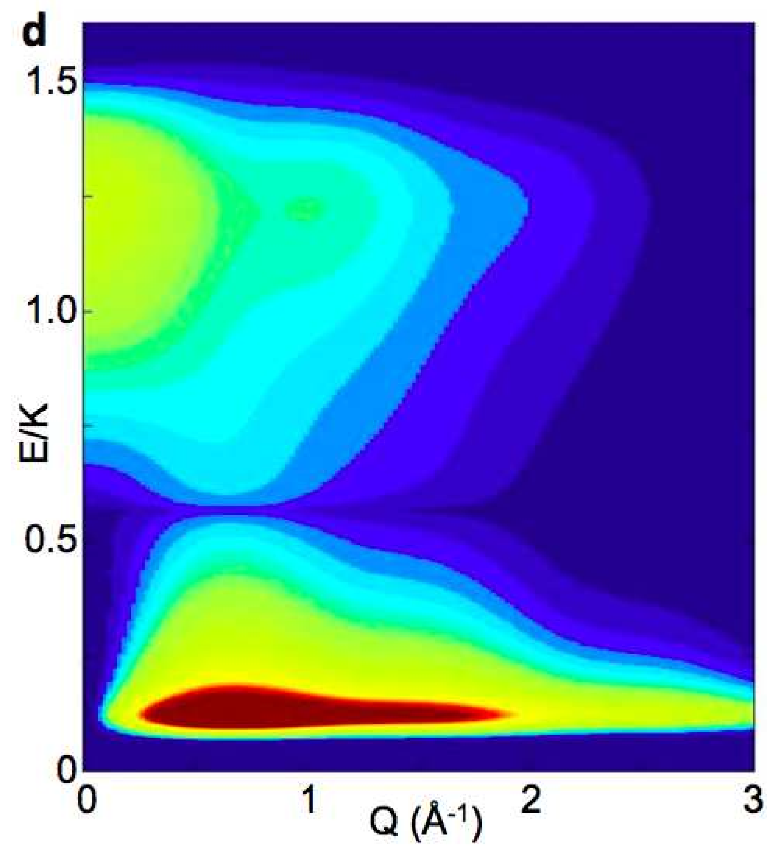

Theory: pure Kitaev interactions

Summary

Geometric frustration

promotes classical degeneracy

destabilises conventional ordered states

Spin liquids

strongly fluctuating and highly correlated

emergent degrees of freedom

exotic excitations