Quiz III
PH 5080 - Statistical Physics
Course Instructor: Ashwin Joy

Date: March 31, 2017
Time duration: 8:00 am - 8:50 am (50 minutes)
Total Marks: 25
Electronic devices such as smart-phones, calculators are not allowed.

Part I (16 Marks)
Pick the correct answer/s from the choices given next to each question. Choosing an incorrect option will lead to a zero score.

1. The energy fluctuations $\langle E^2 \rangle_c$ of a system in contact with a heat reservoir scales as
   (a) $N^{1/2}$.
   (b) $N^{-1/2}$.
   (c) $N$.
   (d) $N^0$.

2. The heat capacity of a gas at constant pressure $C_P = \partial (E + PV) / \partial T$ is
   (a) $(1/2)Nk_B$.
   (b) $Nk_B$.
   (c) $(3/2)Nk_B$.
   (d) $(5/2)Nk_B$.
   **Hint**: You may use the Gibbs partition function $Z = (\beta P)^{-N-1}\lambda^{-3N}$ where $\lambda = h/\sqrt{2\pi mk_BT}$.

3. For a system at constant temperature $T$ and pressure $P$, the average volume
   (a) $\langle V \rangle = -\beta(\partial \ln Z / \partial P)$.
   (b) $\langle V \rangle = -\beta \ln Z / \partial P$.
   (c) $\langle V \rangle = -\beta^{-1}(\partial Z / \partial P)$.
   (d) $\langle V \rangle = -\beta^{-1}(\partial \ln Z / \partial P)$.
   Here $Z$ is the Gibbs partition function and $\beta^{-1} = k_BT$.

4. A system at constant temperature and chemical potential will have
   (a) fluctuations in energy only.
   (b) fluctuations in both energy and volume but not in number of particles.
   (c) fluctuations in energy, volume and number of particles.
   (d) fluctuations in energy and number of particles only.

5. Consider the Ising model Hamiltonian, $-\mathcal{H} = H \sum_i s_i + J \sum_{i=1}^{N-1} s_is_j$. The exchange interaction $J > 0$ favors low temperature
   (a) para-magnetic state with average magnetization $M = 0$.
   (b) anti-ferromagnetic ordering with average magnetization $M = 0$.
   (c) ferromagnetic ordering with average magnetization $M = \pm 1$ as $H \to 0^\pm$.
   (d) disorder.
6. Ignoring boundary effects and taking $N$ to be large, the ground state energies of a 2-d Ising model for a square lattice and a triangular lattice are
(a) $-2NJ$ and $-3NJ$ respectively.
(b) $-4NJ$ and $-3NJ$ respectively.
(c) $-2NJ$ and $-6NJ$ respectively.
(d) $-4NJ$ and $-6NJ$ respectively.
Note that here $J > 0$ is the exchange interaction.

7. For an Ising model with nearest neighbors interactions and free energy density $f_b = F/N$, we have
(a) $f_b > 0$.
(b) $f_b < 0$.
(c) $\partial^2 f_b / \partial T^2 |_H \geq 0$.
(d) $\partial^2 f_b / \partial T^2 |_H \leq 0$.

8. $N$ spins in external magnetic field $\vec{H}$ provide a Gibbs ensemble with the partition function $Z(N, H, T) = \sum_{\{\sigma_i\}} \exp(\beta \vec{H} \cdot \vec{M})$, where $M = \mu_0 \sum_{i=1}^N \sigma_i$ and $\{\sigma_i = \pm 1\}$. The susceptibility $\chi(T) = \partial M / \partial H |_{H=0}$ is given as
(a) $\mu_0$.
(b) $\mu_0^2$.
(c) $\beta \mu_0^2$.
(d) $N \beta \mu_0^2$.

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**Part II (9 Marks)**

1. Molecular oxygen has a net magnetic spin $\vec{S}$ of unity, that is, $S^z$ is quantized to -1, 0, or +1. The Hamiltonian for an ideal gas of $N$ such molecules in a magnetic field $\vec{H} \parallel \hat{z}$ is

$$\mathcal{H} = \sum_{i=1}^N \left[ \frac{\vec{p}_i^2}{2m} - \mu H S_i^z \right]$$

where $\{\vec{p}_i^*\}$ are the center of mass momenta of the molecules. The corresponding coordinates $\{\vec{q}_i^*\}$ are confined to a volume $V$. (Ignore all other degrees of freedom.)

(a) Treating $\{\vec{p}_i^*, \vec{q}_i^*\}$ classically, but the spin degrees of freedom as quantized, calculate the partition function, $Z(N, V, T, H)$.

(b) Find the average magnetic dipole moment, $\langle M \rangle / V$, where $M = \mu \sum_{i=1}^N S_i^z$.

(c) What are the probabilities for $S_i^z$ of a specific molecule to take on values of $-1, 0, +1$ at a temperature $T$?