GENERALISED GARFINKLE-VACHASPATI TRANSFORMATION WITH DILATON

SUBHRONEEL CHAKRABARTI (IMSC)

24TH JANUARY 2020

CHENNAI SYMPOSIUM ON GRAVITATION AND COSMOLOGY 2020

IIT MADRAS

- The talk is based on Class. Quantum Grav. **36** (2019) 125008 done in collaboration with Deepali Mishra, Yogesh K. Srivastava and Amitabh Virmani.

- Follow up of an earlier work Gen. Relativ. Gravit. 50 155 by Deepali Mishra, Yogesh K. Srivastava and Amitabh Virmani (Amitabh's talk on CSM-2018).

GARFINKLE-VACHASPATI (GV) TRANSFORMATION

- The GV transformation is a solution generating technique in Einstein gravity.
- Given a spacetime with a metric which admits the following vector k_μ , which is null, Killing and Hypersurface orthogonal w.r.t a scalar S, i.e.

$$k^\mu k_\mu = 0, \qquad \nabla_{(\mu} k_{\nu)} = 0, \qquad \nabla_{[\mu} k_{\nu]} = k_{[\mu} \nabla_{\nu]} S,$$

- One can construct a new solution (matter fields do not transform as long as some conditions are satisfied)

$$g'_{\mu\nu} = g_{\mu\nu} + \mathrm{e}^{-S} \chi k_\mu k_\nu$$

- Where χ is a massless scalar w.r.t the background metric

$$\Box \chi = 0, \qquad k^\mu \partial_\mu \chi = 0.$$

APPLICATIONS AND LIMITATIONS OF GV

- GV allows one to add travelling waves to a given background metric.
- GV has proved to be useful in many contexts, such as adding hair mode to certain class of black holes.

  (Myers et. al.; Dabholkar et. al.; Horowitz, Marolf; Banados et. al.; Hubeny, Rangamani; Banerjee, Mandal, Sen; Srivastava, Jatkar, Sen;…)
- Also proved pivotal in constructing microstates in fuzzball paradigm. (Mathur, Turton ; Bena, Warner; Skenderis, Taylor;…)
- In string theory D1-D5 geometries play a major role in describing black holes.
- However, D1-D5 geometries do not admit a vector that is hypersurface orthogonal.
- Can one "generalise" GV so as to obtain a transformation which will allow us to add hair modes in D1-D5 systems?
- Answered affirmatively in Gen. Relativ. Gravit. 50 155 (Mishra, Srivastava & Virmani).
- The answer gave a solution generating technique in *minimal* 6D SUGRA embedded in type IIB SUGRA on T^4

CASE FOR GGV WITH DILATON

- Type IIB SUGRA is known to exhibit a very powerful symmetry known as S-duality.
- Under this duality, the strong coupling regime gets mapped to the weak coupling regime and vice versa.
- The string coupling is determined by the vev of the dilaton.
- Therefore, to combine GGV with S-duality, one will require to extend the GGV from minimal to non-minimal SUGRA.
- This is also essential since most interesting examples of D1-D5 geometries involve dilaton.
- A priori not at all obvious such a transformation exists.

GGV WITH DILATON (RR SECTOR)

- Consider type IIB SUGRA with only RR 2-form turned on

$$S_{\rm RR} = \frac{1}{16\pi G_{10}} \int \mathrm{d}^{10}x\sqrt{-g}\left[\mathrm{e}^{-2\Phi}[R + 4(\mathrm{d}\Phi)^2] - \frac{1}{12}F_{\mu\nu\rho}F^{\mu\nu\rho}\right]$$

- We embed the 6D SUGRA in 10D as $ds^2_{(s)} = ds^2_6 + e^{\phi} ds^2_4$, where ds^2_4 is the flat 4-torus metric.
- The 6D and 10D dilaton are taken to be the same , i.e $\Phi = \phi$.
- The 6D 2-form field is also taken to be the same as 10D 2-form RR field.
- This leads to the 6D non-minimal SUGRA embedded in type IIB

$$S_6 = \frac{1}{16\pi G_6} \int \mathrm{d}^6x\sqrt{-g}\left[R - (\mathrm{d}\phi)^2 - \frac{1}{12}\mathrm{e}^{2\phi}F_{\mu\nu\rho}F^{\mu\nu\rho}\right]$$

GGV WITH DILATON (RR SECTOR)

- Let $l_{(i)}$ be the 4 torus directions which are spacelike Killing vectors, normalized as $l^\mu l_\mu = e^\phi$.
- Further, let k^μ be a null Killing vector of the 6D metric which satisfies $k^\mu \partial_\mu \phi = 0$.
- Then following transformations generate a new solution in the 6D non-minimal SUGRA under consideration

$$g_{\mu\nu} \to g_{\mu\nu} + \Psi \mathrm{e}^{-\phi}(k_\mu l_\nu + k_\nu l_\mu)$$

$$C \to C - \Psi \mathrm{e}^{-2\phi}(k_\mu l_\nu - k_\nu l_\mu).$$

- This is GGV transformation with dilaton and it is valid under certain conditions satisfied by the scalar Ψ and the background .

CONDITIONS ON Ψ AND THE BACKGROUND

- The transversality condition of the background: $k^{\mu} F_{\mu\nu\rho} = -d(e^{-\phi} k)_{\nu\rho}$.
- Wave equation for Ψ w.r.t background metric: $\nabla_{\mu}(e^{-2\phi} g^{\mu\nu} \nabla_{\nu} \Psi) = 0$.
- Compatibility of Ψ with Killing symmetries: $k^{\mu}\partial_{\mu}\Psi = 0$, $k^{\mu} l_{\mu} \Psi = 0$.

- With these conditions and the definition of the GGV transformation one can explicitly check that the equations of motions are satisfied by the deformed solution as well.
- The presence of the dilaton now allows us to use S-duality and write down GGV transformations in the NS-NS sector as well.

NS-NS SECTOR

- In Einstein frame, S-duality acts as: $g^E_{\mu\nu} \to g^E_{\mu\nu}$, $\Phi \to -\Phi$, $C_{\mu\nu} \to B_{\mu\nu}$, (Recall $ds^2_E = e^{-\frac{\Phi}{2}} ds^2_S$) .
- Now for the type IIB action with only Kalb-Rammond field turned on

$$S_{\rm NS} = \frac{1}{16\pi G_{10}} \int \sqrt{-g} e^{-2\Phi} \left[R + 4(\mathrm{d}\Phi)^2 - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right]$$

- With 6D embedding $ds^2_{(s)} = e^{-\phi} ds^2_6 + ds^2_4$ and $\Phi = -\phi$, one can obtain a new solution as

$$g_{\mu\nu} \to g_{\mu\nu} + \Psi(k_\mu l_\nu + k_\nu l_\mu),$$

$$B_{\mu\nu} \to B_{\mu\nu} - \Psi(k_\mu l_\nu - k_\nu l_\mu).$$

- Where the transversality condition now is: $k^\mu H_{\mu\nu\rho} = -(dk)_{\nu\rho}$ and the rest of the conditions look same.

APPLICATIONS TO A CLASS OF D1-D5 SYSTEM

- Consider type IIB string theory compactified on $S^1 \times T^4$, and let $y \in S^1$, $z_i \in T^4$.
- We consider travelling wave deformations along the torus directions for a class of background given by

(Giusto, Mathur, Saxena)

$$
\begin{aligned}
ds^2 = & -\frac{1}{h}(dt^2 - dy^2) + \frac{Q_p}{hf}(dt - dy)^2 + hf\left(\frac{dr^2}{r^2 + (\gamma_1+\gamma_2)^2\eta} + d\theta^2\right) \\
& + h\left(r^2 + \gamma_1(\gamma_1+\gamma_2)\eta - \frac{Q_1 Q_5 (\gamma_1^2 - \gamma_2^2)\eta \cos^2\theta}{h^2 f^2}\right)\cos^2\theta\, d\psi^2 \\
& + h\left(r^2 + \gamma_2(\gamma_1+\gamma_2)\eta + \frac{Q_1 Q_5 (\gamma_1^2 - \gamma_2^2)\eta \sin^2\theta}{h^2 f^2}\right)\sin^2\theta\, d\phi^2 \\
& + \frac{Q_p(\gamma_1+\gamma_2)^2\eta^2}{hf}(\cos^2\theta\, d\psi + \sin^2\theta\, d\phi)^2 \\
& - \frac{2\sqrt{Q_1 Q_5}}{hf}(\gamma_1 \cos^2\theta\, d\psi + \gamma_2 \sin^2\theta\, d\phi)(dt - dy) \\
& - \frac{2\sqrt{Q_1 Q_5}(\gamma_1+\gamma_2)\eta}{hf}(\cos^2\theta\, d\psi + \sin^2\theta\, d\phi)\, dy + \sqrt{\frac{H_1}{H_5}}(dz^i dz^i),
\end{aligned}
$$

$$
\begin{aligned}
C^{(2)} = & -\frac{\sqrt{Q_1 Q_5}\cos^2\theta}{H_1 f}(\gamma_2\, dt + \gamma_1\, dy)\wedge d\psi - \frac{\sqrt{Q_1 Q_5}\sin^2\theta}{H_1 f}(\gamma_1\, dt + \gamma_2\, dy)\wedge d\phi \\
& + \frac{(\gamma_1+\gamma_2)\eta Q_p}{\sqrt{Q_1 Q_5} H_1 f}(Q_1\, dt + Q_5\, dy)\wedge(\cos^2\theta\, d\psi + \sin^2\theta\, d\phi) \\
& - \frac{Q_1}{H_1 f} dt \wedge dy - \frac{Q_5 \cos^2\theta}{H_1 f}(r^2 + \gamma_2(\gamma_1+\gamma_2)\eta + Q_1)\, d\psi \wedge d\phi,
\end{aligned}
$$

$$
e^{2\Phi} = \frac{H_1}{H_5},
$$

$$
Q_1 = \frac{g\,\alpha'^3}{V} n_1, \qquad Q_5 = g\,\alpha'\, n_5, \qquad Q_p = \frac{g^2 \alpha'^4}{V R_y^2} n_p
$$

$$
\gamma_1 = -am, \qquad \gamma_2 = a\left(m + \frac{1}{k}\right)
$$

$$
a = \frac{\sqrt{Q_1 Q_5}}{R_y}, \qquad Q_p = -\gamma_1 \gamma_2 \qquad \eta = \frac{Q_1 Q_5}{Q_1 Q_5 + Q_1 Q_p + Q_5 Q_p}
$$

$$
f = r^2 + a^2(\gamma_1+\gamma_2)\eta(\gamma_1 \sin^2\theta + \gamma_2 \cos^2\theta),
$$

$$
H_1 = 1 + \frac{Q_1}{f}, \quad H_5 = 1 + \frac{Q_5}{f}, \quad h = \sqrt{H_1 H_5}.
$$

DEFORMED SOLUTION

- With a co-ordinate change $u = t + y$, it turns out $k = \frac{\partial}{\partial u}$ is an appropriate null Killing vector.
- The torus directions provide us with the spacelike Killing vectors: $l^{(i)} = \frac{\partial}{\partial z^i}$.
- The background satisfies the transversality condition as well.
- The Killing compatible scalar function can be obtained by solving the wave function

$$\Psi_i(r,v) = \sum_{n=-\infty}^{\infty} c_n^i \left(\frac{r^2}{r^2\left(1 + a^2 \frac{(Q_1+Q_5)}{Q_1 Q_5} m\left(m + \frac{1}{k}\right)\right) + \frac{a^2}{k^2}} \right)^{\frac{|n|k}{2}} \mathrm{e}^{-\mathrm{i}n\frac{v}{R_y}}$$

- The GGV can now be applied, it turns out, it just corresponds to making the following shift in metric and form fields

$$du \to du + \Psi_i dz^i$$

- When $Q_1 = Q_5$, the answer matches with the GGV without dilaton result obtained earlier.

DEFORMED SOLUTION

- The deformed solution has flat asymptotics, which can be made manifest by suitable change of co-ordinates.
- One can extract the ADM mass and the angular momenta

$$M = \frac{\pi}{8G}(Q_1 + Q_5)(2\pi R) + P_{y'} \qquad J_\phi = \frac{n_1 n_5}{2}(m + \frac{1}{k}) \qquad J_\psi = -\frac{n_1 n_5}{2} m$$

$$P_{y'} = \frac{n_1 n_5}{R}\left[m(m + \frac{1}{k}) + \frac{Q_1}{4a^2}\frac{1}{2\pi R}\int_0^{2\pi R} dy' f_i f_i\right]$$

- One can check the determinant of the deformed and undeformed metric remains the same.
- The scalar function $\Psi(r, v)$ also is finite everywhere.
- Therefore potential singularity of deformed solution can exist only where the background becomes singular.
- The background is known to be smooth everywhere, which shows the deformed solution also remains smooth.

DECOUPLING LIMIT

- The undeformed geometry is known to develop a large AdS region when $\frac{\sqrt{Q_1 Q_5}}{R_y^2} \ll 1$.
- To obtain the decoupled metric we scale the co-ordinates as

$$\bar{u} = \frac{u}{R_y} \ , \ \bar{v} = \frac{v}{R_y} \ , \ \bar{r} = \frac{r}{a}$$

- Then take $R_y \to \infty$ with Q_1 , Q_5 fixed. The result is a metric describing the inner region of $AdS_3 \times S^3 \times T^4$
- One can play a similar game with the deformed metric to obtain a metric which can be cast as a standard form of asymptotic $AdS_3 \times S^3 \times T^4$ (one also needs to scale $\bar{\Psi}_i = \frac{R_y}{\sqrt{Q_5}} \Psi_i$)

$$P_{y'} = \frac{n_1 n_5}{R}\left[m(m + \frac{1}{k}) + \frac{1}{8\pi}\int_0^{2\pi} d\bar{y} \bar{f}_i \bar{f}_i\right] \qquad J_\phi = \frac{n_1 n_5}{2}(m + \frac{1}{k}) \qquad J_\psi = -\frac{n_1 n_5}{2} m$$

DEFORMED STATES IN THE D1-D5 CFT

- Let the undeformed gravity configuration with ADM momentum $\frac{n_1 n_5}{R}\left[m(m+\frac{1}{k})\right]$ be described by $|\psi\rangle$ in D1-D5 CFT.
- Then the normalized deformed state is $$|\Psi\rangle = \exp\left[-\frac{n_1 n_5}{4}\sum_{n>0} n(\mu_n^i)^* \mu_n^i\right] \exp\left[\sum_{n>0} \mu_n^i J_{-n}^i\right] |\psi\rangle$$
- Here J_{-n}^i are the modes of four $U(1)$ currents of the D1-D5 CFT.
- The matching between gravity and CFT fixes

$$\mu_n^i = \frac{1}{n}\frac{\sqrt{Q_1}}{a} c_n^i$$

F1-P SYSTEM

- This is extensively studied, so no really new physics is expected.
- Nonetheless serves as a testing ground for GGV in NS sector.
- Chiral Null model for NS sector of type II SUGRA is described by

$$\mathrm{d}s^2 = H^{3/4}(-\mathrm{d}u\mathrm{d}v + K\mathrm{d}v^2 + 2A_i\mathrm{d}x^i\mathrm{d}v) + H^{-1/4}(\mathrm{d}x^i\mathrm{d}x^i + \mathrm{d}z^j\mathrm{d}z^j)$$

$$B_{uv} = \frac{H}{2}, \qquad B_{vi} = -HA_i, \qquad e^{2\phi} = H,$$

$$x^i \in \mathbb{R}^4$$

- Level-matched F1-P configurations (with smearing over 4-torus) require

$$H^{-1} = 1 + \frac{Q}{L_y}\int_0^{L_y} \frac{\mathrm{d}v}{|x - F(v)|^2}, \qquad A_i = -\frac{Q}{L_y}\int_0^{L_y} \frac{\mathrm{d}v\dot{F}_i(v)}{|x - F(v)|^2}, \qquad K = \frac{Q}{L_y}\int_0^{L_y} \frac{\mathrm{d}v\dot{F}_i(v)\dot{F}_i(v)}{|x - F(v)|^2}$$

F1-P SYSTEM

- With ∂_u as spacelike Killing and torus directions as our null Killing, we can once again apply GGV.
- The deformed solution is another chiral null model with the only modification being

$$ds^2 = H^{3/4}(-dudv + Kdv^2 + 2A_i dx^i dv - \Psi_{(j)} dz^j dv) + H^{-1/4}(dx^i dx^i + dz^j dz^j) \qquad B_{vz_j} = H\Psi_{(j)} \qquad \Box\Psi_{(j)} = 0$$

- One can think of the deformed solution as getting additional components for gauge fields A_i (along the torus directions).
- This however does not satisfy the level-matching condition. But an ordinary GV can be applied to add appropriate momentum to ensure level-matching.
- Alternatively one can do a co-ordinate change and a gauge transformation of the B-field to rewrite the deformed solution as a pp-wave added to F1-P solution, which is also obtained as an ordinary GV with the scalar function

$$\chi = H^{-1}$$

FUTURE DIRECTIONS

- SUSY properties of deformed solutions?
- GGV like transforms for other SUGRA, D=11 SUGRA has a proposal now. (Mishra, Srivastava, Virmani)
- A deeper understanding of GGV transforms and their classification?
- GV was very helpful in constructing hair modes for 4d-5d black holes. Hair removal was crucial to restore equality of entropy from geometry and counting. (Banerjee, Mandal, Sen; Srivastava, Jatkar, Sen)
- Similar question can be asked in context of type IIB on 4-torus, where GGV will play a role in constructing hair modes for the torus directions – Work in progress (S.C ,Suresh Govindarajan, Shanmugapriya Prakasham, Yogesh K. Srivastava, Amitabh Virmani) .

Thank you!