Initial conditions for inflation

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COSMOLOGY

POP *goes the universe*

THE LATEST ASTROPHYSICAL MEASUREMENTS, COMBINED WITH THEORETICAL PROBLEMS, CAST DOUBT ON THE LONG-CHERISHED INFLATIONARY THEORY OF THE EARLY COSMOS AND SUGGEST WE NEED NEW IDEAS

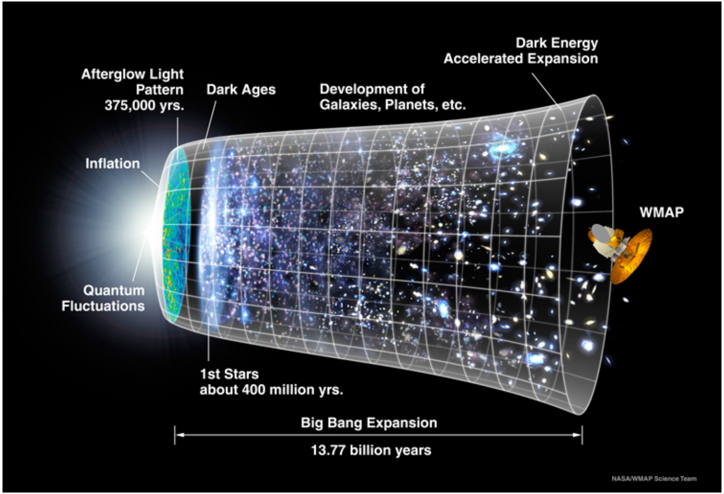
By Anna Ijjas, Paul J. Steinhardt and Abraham Loeb

Photographs by The Voorhes

Outline of the talk

1. Criticisms of inflation
2. Initial conditions for inflationary models
3. Beyond isotropy
4. Beyond homogeneity
5. Summary

Timeline of the universe



Inflation refers to a period of accelerated expansion of the universe[1].

[1] Image from https://map.gsfc.nasa.gov/media/060915/060915_CMB_Timeline75.jpg.

Inflation in trouble?

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Inflationary paradigm in trouble after Planck2013

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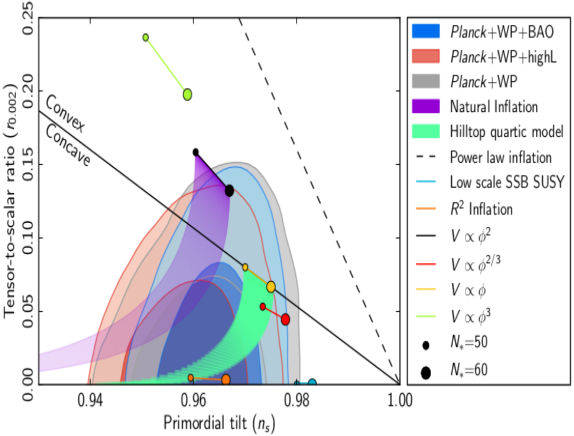
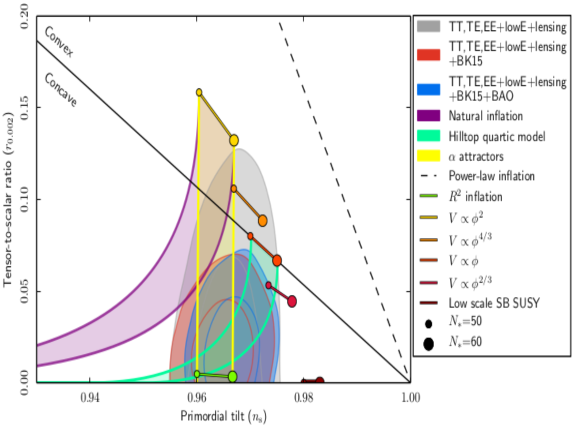
ABSTRACT

Recent results from the *Planck* satellite combined with earlier observations from WMAP, ACT, SPT and other experiments eliminate a wide spectrum of more complex inflationary models and favor models with a single scalar field, as reported by the *Planck* Collaboration. More important, though, is that all the simplest inflaton models are disfavored statistically relative to those with plateau-like potentials. We discuss how a restriction to plateau-like models has three independent serious drawbacks: it exacerbates both the initial conditions problem and the multiverse-unpredictability problem and it creates a new difficulty that we call the inflationary "unlikeliness problem." Finally, we comment on problems reconciling inflation with a standard model Higgs, as suggested by recent LHC results. In sum, we find that recent experimental data disfavors all the best-motivated inflationary scenarios and introduces new, serious difficulties that cut to the core of the inflationary paradigm. Forthcoming searches for B-modes, non-Gaussianity and new particles should be decisive.

Constraints on inflation from Planck



Planck 2013 results[1]



Planck 2018 results[2]

[1] Image from P. A. R. Ade *et al*, Astron. Astrophys. **571**, A22 (2014).
[2] Image from Y. Akrami *et al*, arXiv:1807.06211 [astro-ph.CO].

What are the issues with inflation?

The main criticisms raised against inflation can be explained as follows[2]:

- Planck favors models with a single scalar field.
- All the *simplest* models are disfavored relative to those with plateau-like potentials, such as the $f(R) = R + \xi\, R^2$ model.
- The favored inflationary potentials are exponentially unlikely.
- Plateau inflation would only occur if $f(R)$ is precisely cut off at the quadratic order.
- The presence of any initial gradients or inhomogeneities would prevent onset of inflation.

Primary question $\longrightarrow$ Are plateau-like models expected, according to the inflationary paradigm?

[2] A. Ijjas, P. J. Steinhardt, and A. Loeb, Phys. Lett. B **723**, 261 (2013).

Examining the phase space of initial conditions I

We explore single field dynamics on flat FLRW spacetime, for an isotropic and homogeneous situation, in order to investigate the conditions favorable for the onset of inflation.

We study the following system of equations:

$$H^2 = \frac{1}{6\,M_{_{\mathrm{Pl}}}^2}\,\left[\Pi^2 + 2\,V(\phi)\right]\,,$$

$$H^2 + \dot{H} = -\frac{1}{6\,M_{_{\mathrm{Pl}}}^2}\,\left[2\,\Pi^2 - 2\,V(\phi)\right]\,,$$

$$\dot{\Pi} + 3\,H\,\Pi + \frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} = 0\,,$$

where $\Pi = \dot{\phi}$.

This system of equations can be decoupled by rewriting them in terms of the e-folds (N).

Examining the phase space of initial conditions II

The decoupled system of equations is as follows:

$$H^2 = \frac{2}{M_{\rm Pl}^2}\,\frac{V(\phi)}{6-\Gamma^2}\,,$$

$$\epsilon_1 = \frac{\Gamma^2}{2}\,,$$

$$\frac{2}{6-\Gamma^2}\,\frac{{\rm d}\Gamma}{{\rm d}N} + \Gamma = -\frac{{\rm d}{\rm ln}V}{{\rm d}\Phi}\,,$$

where $\Gamma = \dot{\phi}/(M_{\rm Pl}\, H)$, $\Phi = \phi/M_{\rm Pl}$, and $\epsilon_1 = -\dot{H}/H^2$.

The field velocity is bounded by:

$$-\sqrt{6} \lesssim \Gamma \lesssim \sqrt{6}.$$

Inflation requires the condition: $\Gamma^2 < 2$.

Examining the phase space of initial conditions III

The field dynamics is described by the following equation:

$$\frac{1}{\sqrt{6}}\frac{\mathrm{d}}{\mathrm{d}N}\left[\ln\left(\frac{\sqrt{6}+\Gamma}{\sqrt{6}-\Gamma}\right)\right]+\Gamma=-\frac{\mathrm{d}\ln V}{\mathrm{d}\Phi}\equiv\Gamma_{\mathrm{sr}}(\Phi).$$

We divide the evolution of the inflaton into the following three domains:

Kination regime: When a field starts with large initial velocity, *i.e.* $\Gamma\to\sqrt{6}$,

$$\gamma(\Phi)=\left(\sqrt{6}-\Gamma_{\mathrm{ini}}\right)\frac{\mathrm{e}^{\sqrt{6}\Phi}\,V(\Phi)}{\mathrm{e}^{\sqrt{6}\Phi_{\mathrm{ini}}}\,V(\Phi_{\mathrm{ini}})},$$

where $\gamma=\sqrt{6}-\Gamma$. Kination is generically a repeller and cannot be sustained.

Transition regime: When the field leaves kination and enters into inflation, we can assume that $|\Gamma|\gg|\Gamma_{\mathrm{sr}}|$ to obtain

$$\Delta N=\frac{1}{6}\ln\left[\frac{1-\frac{\Gamma_{\mathrm{ini}}^2}{6}\left(\frac{1-\frac{\Gamma_\times}{\Gamma_{\mathrm{ini}}}}{1-\frac{\Gamma_\times\Gamma_{\mathrm{ini}}}{6}}\right)^2}{\left(1-\frac{1-\frac{\Gamma_\times}{\Gamma_{\mathrm{ini}}}}{1-\frac{\Gamma_\times\Gamma_{\mathrm{ini}}}{6}}\right)^2}\right].$$

The number of e-folds spent in this regime depends logarithmically on Γ_{ini}.

Examining the phase space of initial conditions IV

Inflationary regime: We can now consider $\Gamma^2 \ll 6$ to obtain the following equation:

$$\frac{\mathrm{d}\Gamma^2}{\mathrm{d}N} + 6\,\Gamma^2 \simeq -6\,\frac{\mathrm{d}\ln V}{\mathrm{d}N}.$$

This can be solved to obtain:

$$\Gamma^2(N) = \left(\Gamma_\times^2 + \left.\frac{\mathrm{d}\ln V}{\mathrm{d}N}\right|_{N_\times}\right)\mathrm{e}^{-6\Delta\bar{N}} - \frac{\mathrm{d}\ln V}{\mathrm{d}N} + \mathcal{C}(N),$$

where $\Delta\bar{N} = N - N_\times$.

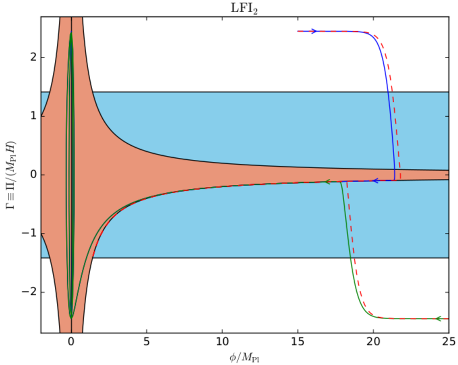
In order to verify our approximate analytical solutions, we study the effect of these initial conditions numerically.

We start with a grid of initial conditions in phase space $(\Phi_{\rm ini}, \Gamma_{\rm ini})$, and numerically integrate the above system of equations in terms of the number of e-folds for each trajectory in phase space.

Along each numerically integrated trajectory, the number of e-folds spent in an inflationary regime is estimated.

Large Field Inflation I

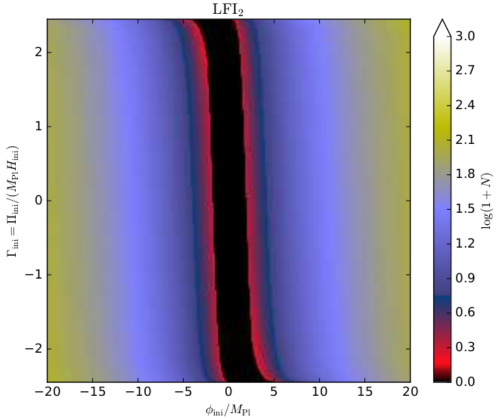
We consider the following potential: $V(\phi) = M^4 \left(\phi/M_{\mathrm{Pl}}\right)^p$.



Two numerically integrated phase space trajectories (solid blue and solid green curves) starting deep inside the kination regime, *i.e.* when $\Gamma^2 \simeq 6$, have been plotted for the case $p = 2$[3]. The approximate analytical solutions (red dashed curves) have also been shown. The horizontal blue region corresponds to inflation, while the orange region is the domain for which $|\Gamma| \leq |\Gamma_{\mathrm{sr}}|$, where $\Gamma_{\mathrm{sr}} = -\mathrm{d}\ln V/\mathrm{d}\Phi$. The slow roll attractor is the boundary $\Gamma = \Gamma_{\mathrm{sr}}$.

[3] D. Chowdhury, J. Martin, C. Ringeval, and V. Vennin, Phys. Rev. D **100**, 083537 (2019).

Large Field Inflation II

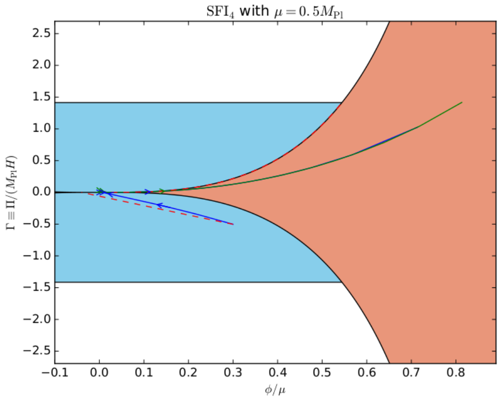


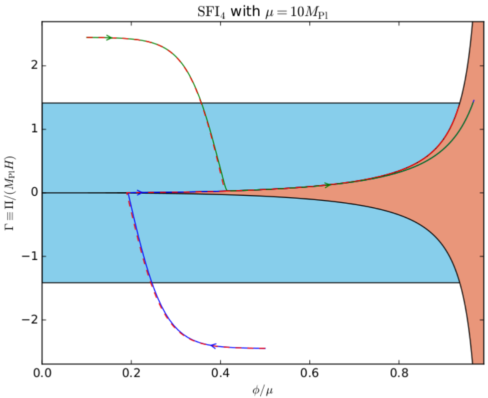
The number of e-folds of inflation achieved along phase space trajectories starting with different initial conditions for the case $p = 2$ has been plotted[4]. Evidently, almost all the initial conditions lead to inflation.

[4] D. Chowdhury, J. Martin, C. Ringeval, and V. Vennin, Phys. Rev. D **100**, 083537 (2019).

Small Field Inflation I

We consider the following potential: $V(\phi) = M^4 \left[1 - (\phi/\mu)^p\right].$

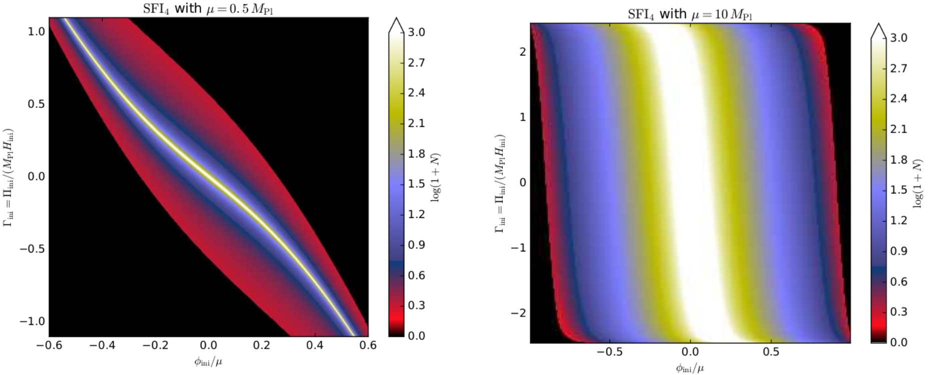




Phase space trajectories for the case $p = 4$ have been plotted for $\mu > M_{\rm Pl}$ (on the left) and $\mu < M_{\rm Pl}$ (on the right)[5]. For the case $\mu > M_{\rm Pl}$ (on the left), almost all the initial conditions relax towards slow roll without any fine tuning.

[5] D. Chowdhury, J. Martin, C. Ringeval, and V. Vennin, Phys. Rev. D **100**, 083537 (2019).

Small Field Inflation II



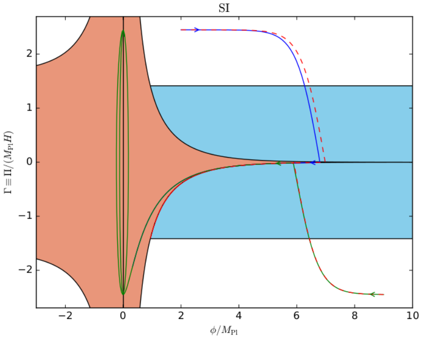
The number of e-folds of inflation for $p = 4$ achieved along phase space trajectories starting with different initial conditions have been plotted for the cases $\mu > M_{\mathrm{Pl}}$ (on the left) and $\mu < M_{\mathrm{Pl}}$ (on the right)[6]. Almost all the initial conditions lead to inflation for $\mu > M_{\mathrm{Pl}}$ while some fine tuning is required for $\mu < M_{\mathrm{Pl}}$.

On the basis of Bayesian evidence, Planck data disfavors the scenarios having $\mu < M_{\mathrm{Pl}}$ compared to the ones with $\mu > M_{\mathrm{Pl}}$.

[6] D. Chowdhury, J. Martin, C. Ringeval, and V. Vennin, Phys. Rev. D **100**, 083537 (2019).

Starobinsky Inflation I

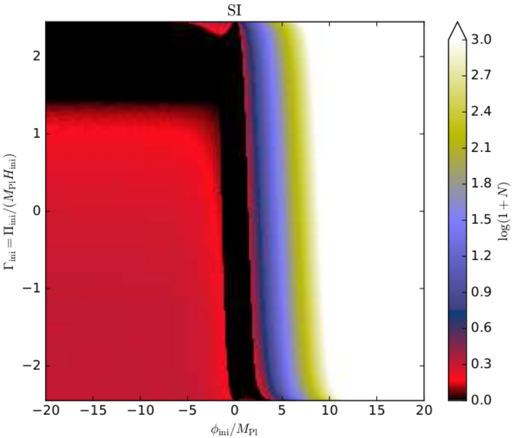
In terms of Bayesian evidence, the most favored models by Planck are the so-called plateau models. A typical example of plateau inflation is the Starobinsky model, which has the following form for its potential: $V(\phi) = M^4 \left[1 - \exp\left(-\sqrt{2/3}\,\phi/M_{\rm Pl}\right)\right]^2$.



Two numerically integrated phase space trajectories (solid blue and solid green curves) starting deep inside the kination regime, *i.e.* when $\Gamma^2 \simeq 6$, have been plotted for the Starobinsky model[7].

[7] D. Chowdhury, J. Martin, C. Ringeval, and V. Vennin, Phys. Rev. D **100**, 083537 (2019).

Starobinsky Inflation II



The number of e-folds of inflation achieved along phase space trajectories starting with different initial conditions has been plotted[8]. Evidently, nearly all the positive field values lead to inflation.

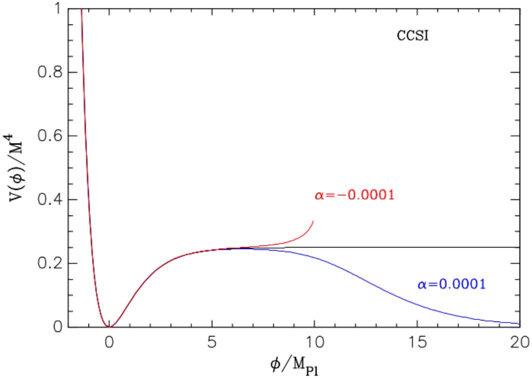
[8] D. Chowdhury, J. Martin, C. Ringeval, and V. Vennin, Phys. Rev. D **100**, 083537 (2019).

Starobinsky inflation with cubic corrections I

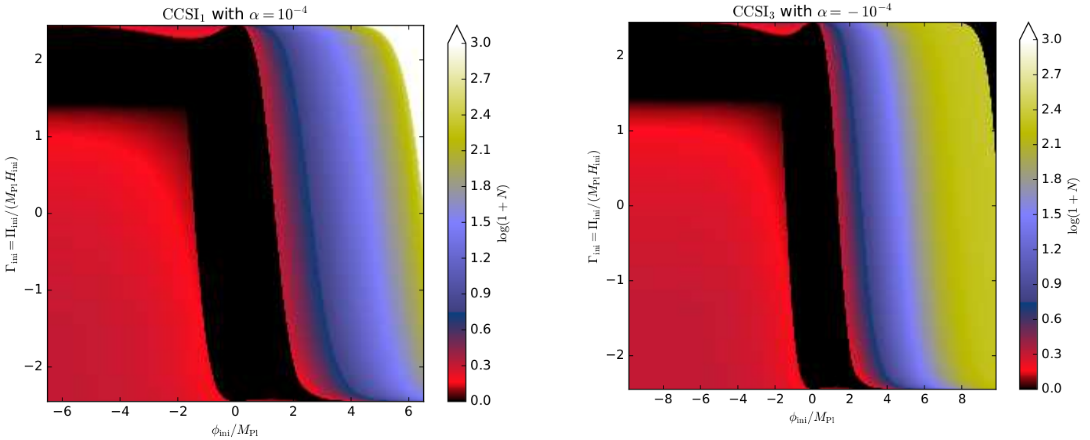
This model is given by

$$V(\phi) = M^4\left(1 - \mathrm{e}^{-y}\right)^2 \frac{1 + \sqrt{1 + 3\alpha\left(\mathrm{e}^y - 1\right)} + 2\alpha\left(\mathrm{e}^y - 1\right)}{\left[1 + \sqrt{1 + 3\alpha\left(\mathrm{e}^y - 1\right)}\right]^3},$$

where $y = \sqrt{2/3}\,\phi/M_{\mathrm{Pl}}$.



Starobinsky inflation with cubic corrections II



The number of e-folds of inflation achieved along phase space trajectories starting with different initial conditions has been plotted[9].

[9] D. Chowdhury, J. Martin, C. Ringeval, and V. Vennin, Phys. Rev. D **100**, 083537 (2019).

Examining anisotropic initial conditions

We consider the following Bianchi I metric:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\gamma_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j,$$

where

$$\gamma_{ij} = \begin{pmatrix} \mathrm{e}^{2\beta_1(t)} & 0 & 0 \\ 0 & \mathrm{e}^{2\beta_2(t)} & 0 \\ 0 & 0 & \mathrm{e}^{2\beta_3(t)} \end{pmatrix}.$$

For this metric, the energy density of the shear behaves as $\rho_\sigma \propto a^{-6}$.

If the energy density of the inflaton, *i.e.* ρ_ϕ, initially surpasses ρ_σ, the universe undergoes inflation and isotropizes.

If $\rho_\sigma \gg \rho_\phi$ initially, the universe does not undergo accelerated expansion. For a slowly rolling field, ρ_ϕ is approximately constant and starts to dominate ρ_σ after a brief transitory period, leading to inflation. If the kinetic energy of the field initially dominates the potential energy, both ρ_ϕ and ρ_σ decay rapidly as a^{-6}, again leading to the onset of inflation.

Therefore, presence of an initial shear does not stop inflation from occurring[10].

[10] D. Chowdhury, J. Martin, C. Ringeval, and V. Vennin, Phys. Rev. D **100**, 083537 (2019).

Examining inhomogeneous initial conditions I

We consider the effective density approximation wherein we study an inhomogeneous scalar field on an isotropic and homogeneous FLRW background, neglecting the backreaction of the field inhomogeneities on the metric[11]. The effect of the inhomogeneities is only to modify the value of the Hubble parameter.

We can express the scalar field as follows:

$$\phi(t, \boldsymbol{x}) = \phi_{_0}(t) + \Re\left[\delta\phi(t)\, \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}/a(t)}\right].$$

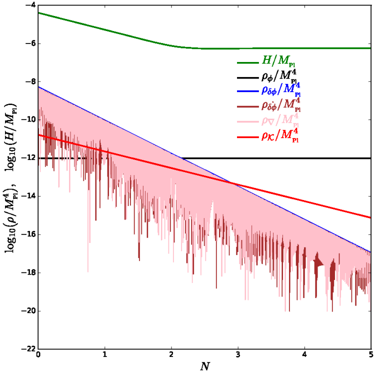
The corresponding Klein-Gordon equation is split into two equations – for the zero mode and the inhomogeneous mode.

We also assume the size of the inhomogeneities to be much smaller than the Hubble radius, *i.e.* $k \gg a\, H$.

The energy density of the inhomogeneities is defined as $\rho_{_{\delta\phi}} = \rho_{_{\dot{\delta\phi}}} + \rho_{_\nabla}$, with $\rho_{_{\dot{\delta\phi}}} = \dot{\delta\phi}^2/2$ and $\rho_{_\nabla} = k^2\delta\phi^2/(2a^2)$. The energy density of the homogeneous mode is, as usual, given by $\rho_\phi = \dot{\phi}_{_0}^2/2 + V(\phi_{_0})$.

[11] D. S. Goldwirth and T. Piran, Phys. Rev. Lett. **64**, 2852 (1990); Phys. Rept. **214**, 223 (1992).

Examining inhomogeneous initial conditions II



Evolution of the Hubble parameter and the energy densities of the various components obtained by numerical integration of the Klein-Gordon and the Friedmann-Lemaître equations have been plotted[12]. We have chosen inflation to be driven by the Starobinsky potential, with $M = 0.001\, M_{\rm Pl}$. The initial conditions have been set such that the inhomogeneities dominate initially. We find that, however, ρ_ϕ takes over rapidly so that the universe becomes homogeneous and inflation starts.

[12] D. Chowdhury, J. Martin, C. Ringeval, and V. Vennin, Phys. Rev. D **100**, 083537 (2019).

Summary

- We find that fine tuning of initial conditions is not necessary for large field inflation.
- For small field models, fine tuning depends strongly on the ratio $\mu/M_{\rm Pl}$.
- In the Starobinsky model, which exemplifies plateau inflation, slow roll inflation occurs without fine tuning.
- The inflationary paradigm is thereby vindicated by the Planck data, since Planck favors plateau potentials which do not have any problem with initial conditions.
- We find that inflation can occur despite presence of an initial anisotropy.
- Under the effective density approximation, we find that initial inhomogeneities do not prevent the onset of inflation. In order to study the most general case of how initial inhomogeneities affect the onset of inflation, one has to resort to complete numerical integration of the full Einstein equations.

Thank you!