Multi-parameter tests of general relativity using multi-band gravitational wave observations

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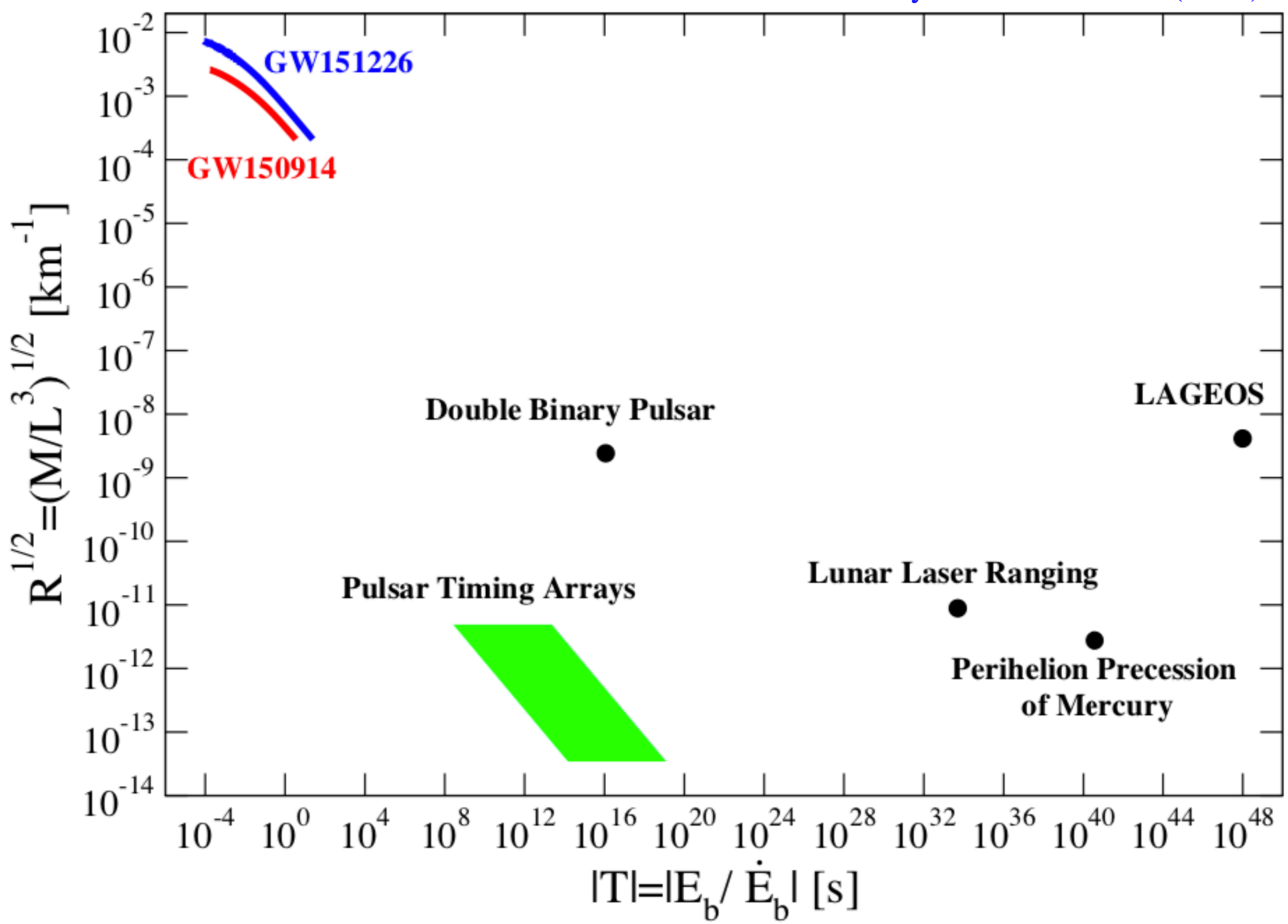
Gravitational Waves as a probe for strong gravitational field regime

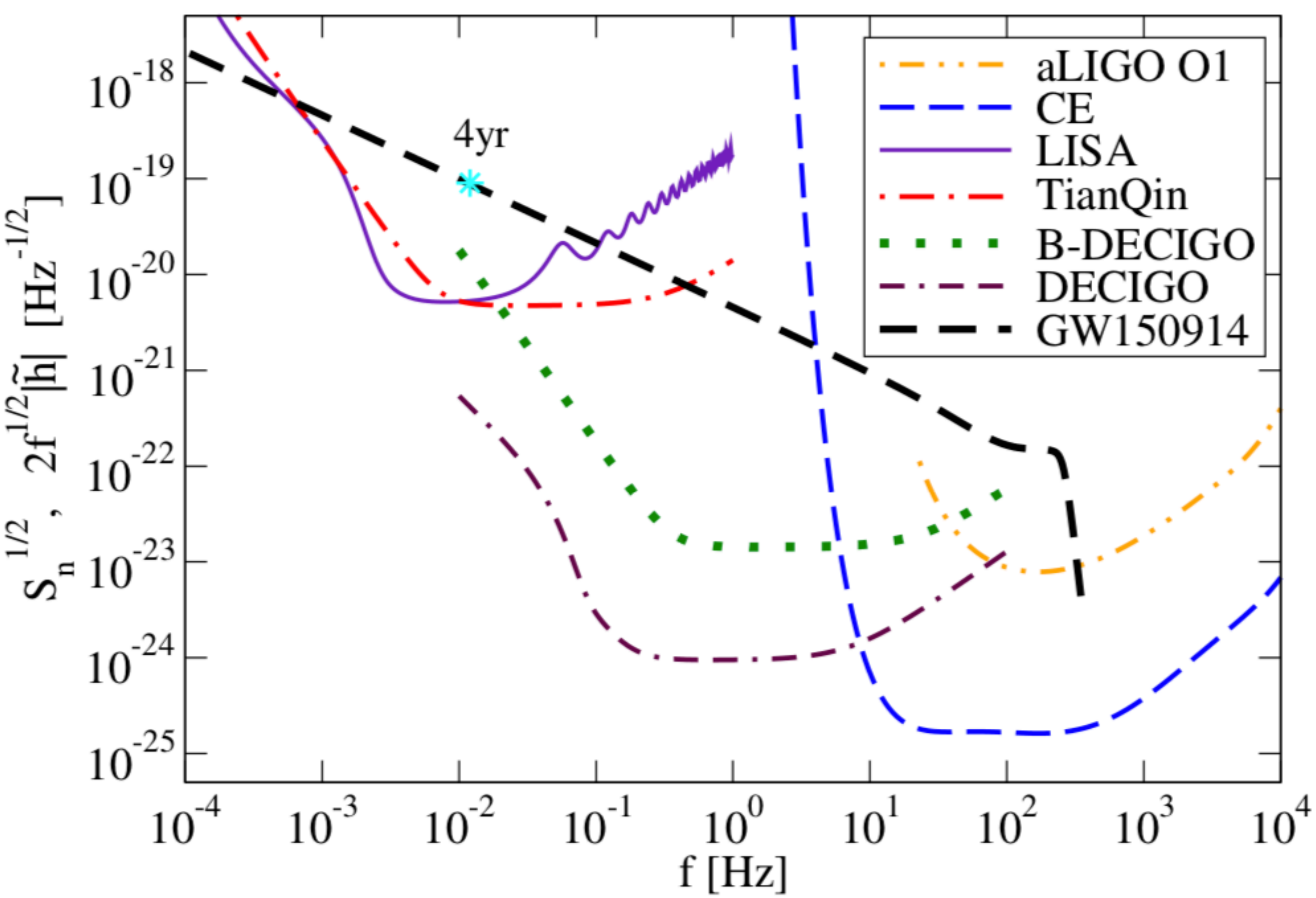
Yunes *et al* PhysRevD.94.084002(2016)



Characteristic curvature vs radiation-reaction time scale for different experiments.

Multiband visibility of Binary Black Holes(BBH)

Carson *et al* arXiv:1905.13155 (2019)



- LISA(Space based detector) is sensitive in the milli-hertz regime.
- Cosmic Explorer(CE) (3G- ground based detector) is sensitive in the kilo-hertz regime.
- Stellar mass BBH systems like GW150914 would be detected by LISA with SNR $\rho > 3$ and in CE band with $\rho \sim 10^3$.

Sesana PhysRevLett.116.231102 (2016)

Multi-band visibility of Binary Black Holes(BBH)

- Such high SNR detections in the CE band can put stringent bounds masses and spins of the BBH system.

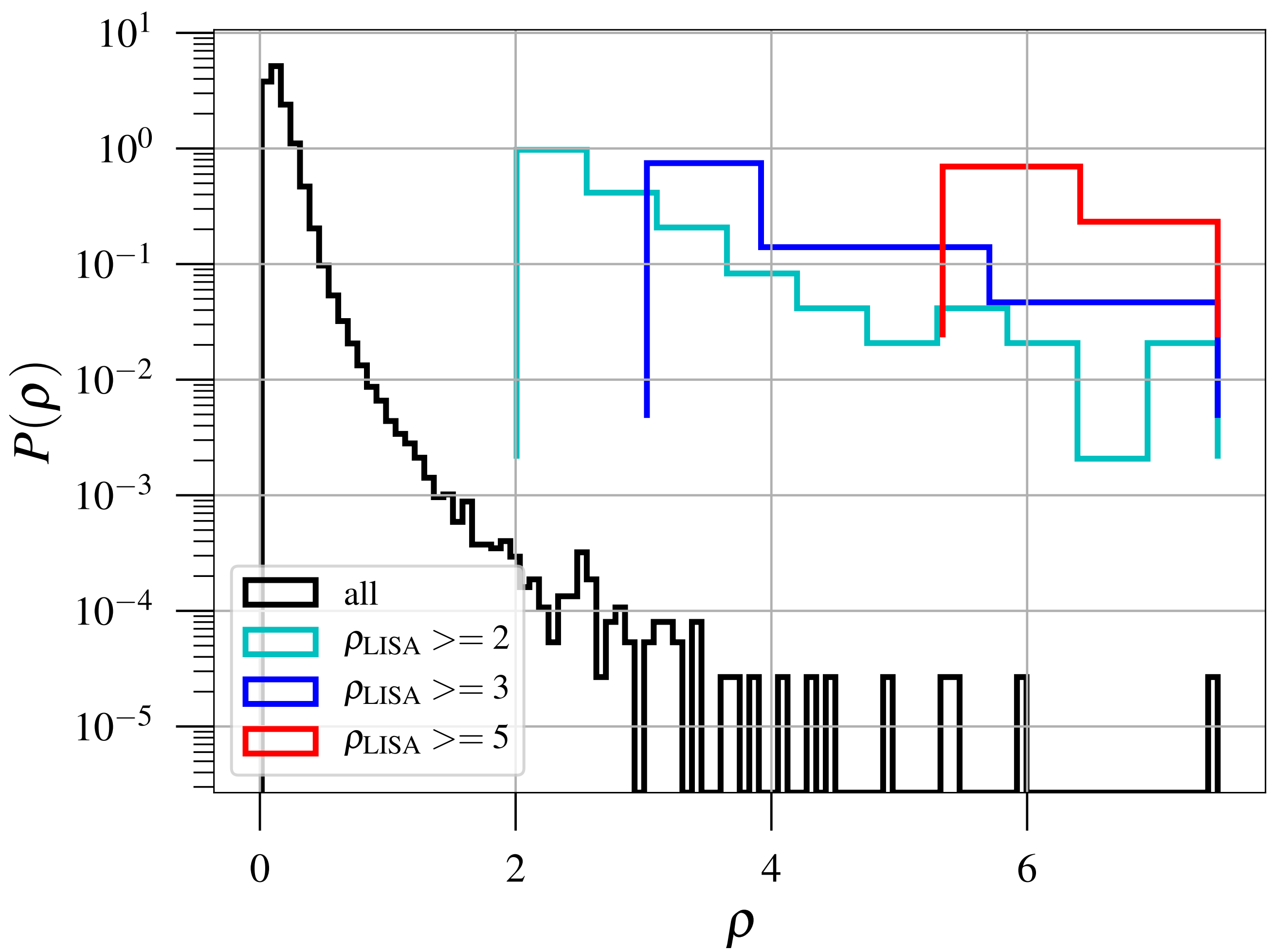
- This allows us to go back and search for those signals from stellar mass BBHs in LISA data that would have been flagged with sub-threshold SNR.

- Lowering of SNR threshold in LISA band also increases the number of events detected jointly by LISA and CE.

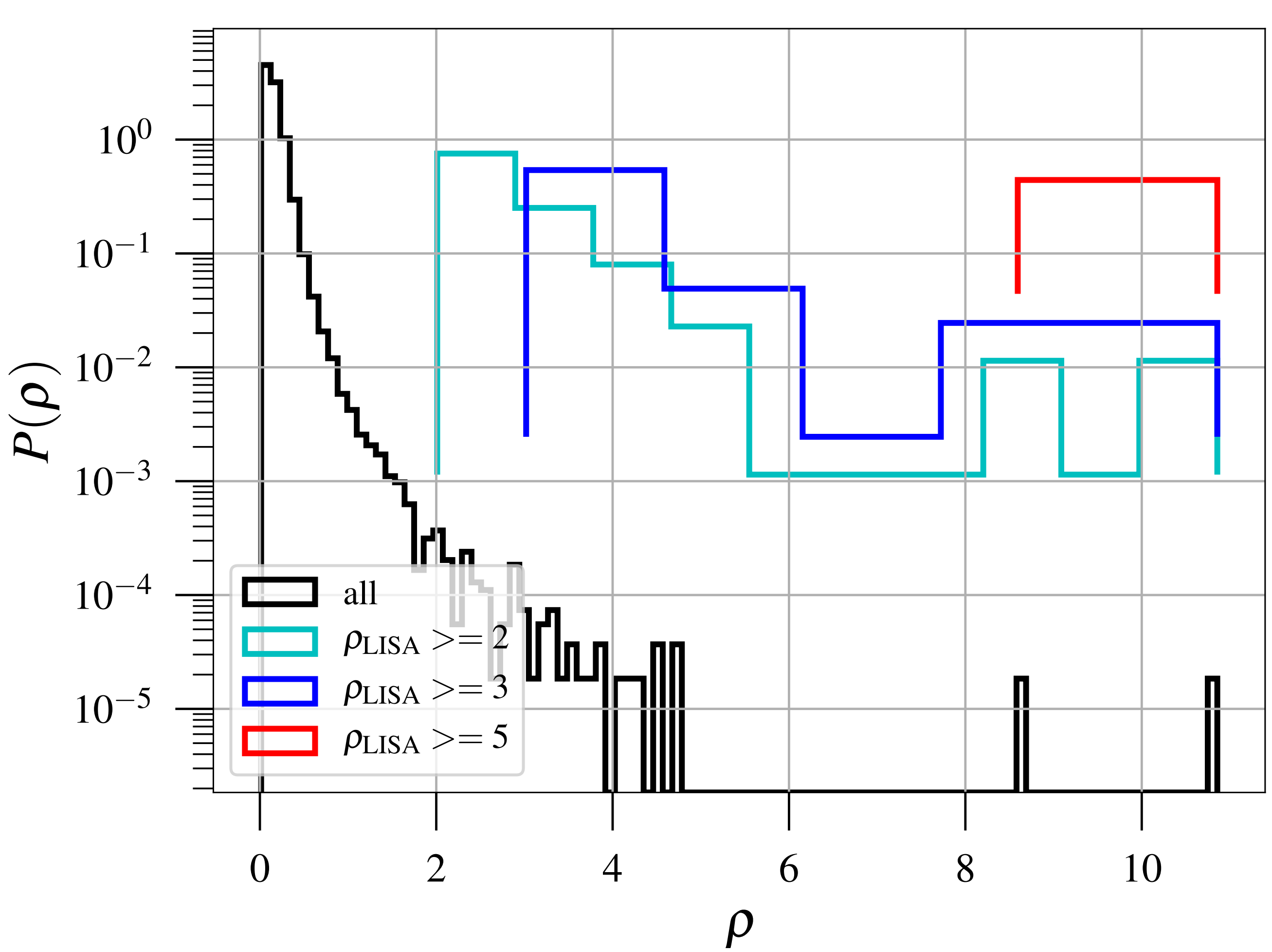
- We demonstrate the power of such multi-band observation technique of GWs on parametrised tests of GR.

Multi-band visibility of Binary Black Holes(BBH)

SNR distribution of stellar mass binary black holes in the LISA and CE bands using GWTC-1 mass and redshift distribution.



Gaussian spin distribution



Uniform spin distribution

Parametrised tests of General Relativity(GR)

In this framework, deviation from GR is parametrised in terms on the Post Newtonian(PN) deformation parameters.

$$h(f) = A f^{\frac{-7}{6}} e^{i \Psi(f)}$$

→ Frequency domain gravitational waveform for inspiraling compact binaries.

$$\Psi(f) = 2\pi f t_c + \phi_c + \frac{\pi}{4} + \sum_k [\phi_k v^{\frac{(k-5)}{3}}]$$

Where, $(k = 0, 2, 3, 4, 5, 5l, 6, 6l, 7)$

→ ϕ_k's are characterised by total mass M and spins χ_1, χ_2 of the source.

$$\hat{\phi}_k \rightarrow \hat{\phi}_k(1 + d\hat{\chi}_k)$$

→ deformation parameters, $d\hat{\chi}_k$ which are zero in GR, are introduced at each Post-Newtonian(PN) order.

Arun *et al*, Class.Quant.Grav.23:L37-L43,2006
Arun *et al*, Phys.Rev.D74:024006,2006
Yunes *et al*, Phys.Rev.D80:122003,2009
Li *et al*, Phys. Rev. D 85, 082003 (2012)

Parametrised tests of General Relativity(GR)

- Most general test of GR : All the 8 deformation coefficients $\hat{d\chi}_k$ up to 3.5PN are simultaneously measured.

- Due to high degeneracy of $\hat{d\chi}_k$'s with the source parameters M and $\chi_{1,2}$, above test does not yield meaningful constraints.

- Instead, one deformation parameter is estimated at a time together with other source parameters, assuming all others are set to GR value i.e zero, leading to 8 separate one-parameter-tests. LVC PhysRevD.100.104036 (2019)

- The extremely good bounds from one-parameter-tests are artificially stringent and it will be incorrect to rule theories out based on these bounds.

- We demonstrate that this hindrance can be overcome by looking into the GW signals from an astrophysical population of stellar mass BBHs, at multiple frequency bandwidths and coherently combine the information.

Methodology: Fisher Analysis

$\{ D_L, t_c, \phi_c, M_c, \eta, \chi_1, \chi_2, \{d\chi_k\}\}$ → Parameter space, 15D: Γ

GR: 7 D

Non-GR:8 D

$P(\Delta\theta_i) \propto e^{-\Gamma_{ab}\Delta\theta^a\Delta\theta^b}$ → Likelihood under the assumption of Gaussian noise and high Signal to Noise Ratio.

Fisher matrix from a single detector

$$\Gamma_{ab} = \langle \frac{\partial h}{\partial \theta_a} | \frac{\partial h}{\partial \theta_b} \rangle$$

→ Noise weighted inner product of derivatives of the waveform.

$$C_{ab} = (\Gamma^{-1})_{ab}$$

→ Covariance matrix corresponding to the 15D parameter space.

$$\sigma_a = \sqrt{C_{aa}}$$

→ $1-\sigma$ bounds on the parameters θ_a that is marginalised over rest of the parameters.

Methodology: Fisher Analysis

Fisher matrix from multiple detectors

$$\tilde{\Gamma}_{ab} = \Gamma_{ab}^{CE} + \Gamma_{ab}^{LISA}$$ → In order to obtain the multi-band Fisher matrix we simply add both the Fisher matrices from the individual detectors.

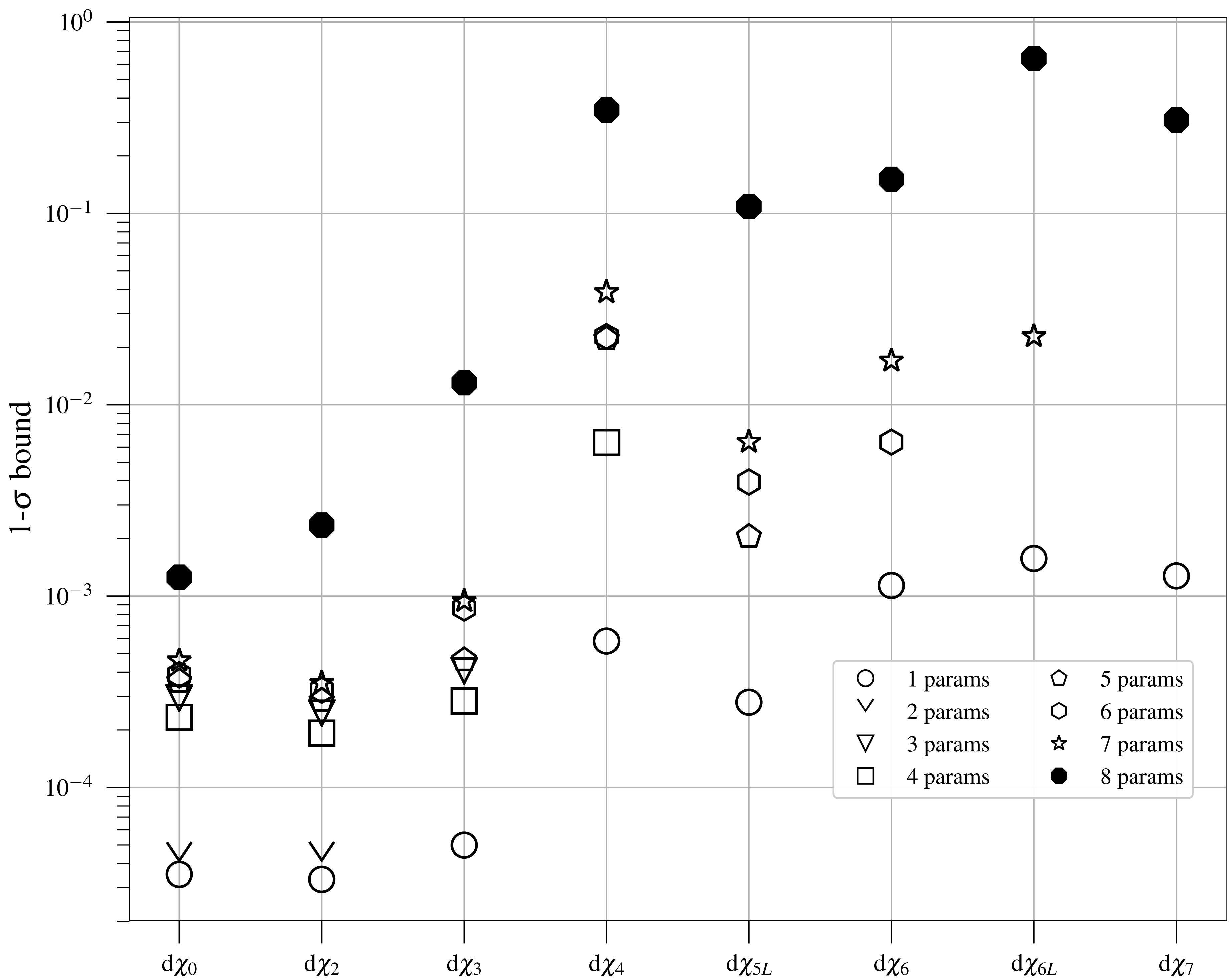
$$(\sigma_a)_n = \sqrt{(\Gamma^{-1}_{aa})_n}$$ → $1 - \sigma$ bounds on the parameters for the nth event in the population.

$$\frac{1}{\sigma^2} = \frac{1}{{\sigma_1}^2} + \frac{1}{{\sigma_2}^2} + \ldots\ldots + \frac{1}{{\sigma_N}^2}$$ → The resultant 1 – σ bounds on the parameters after 'stacking'.

N is the number of sources in the joint population detected by LISA and CE.

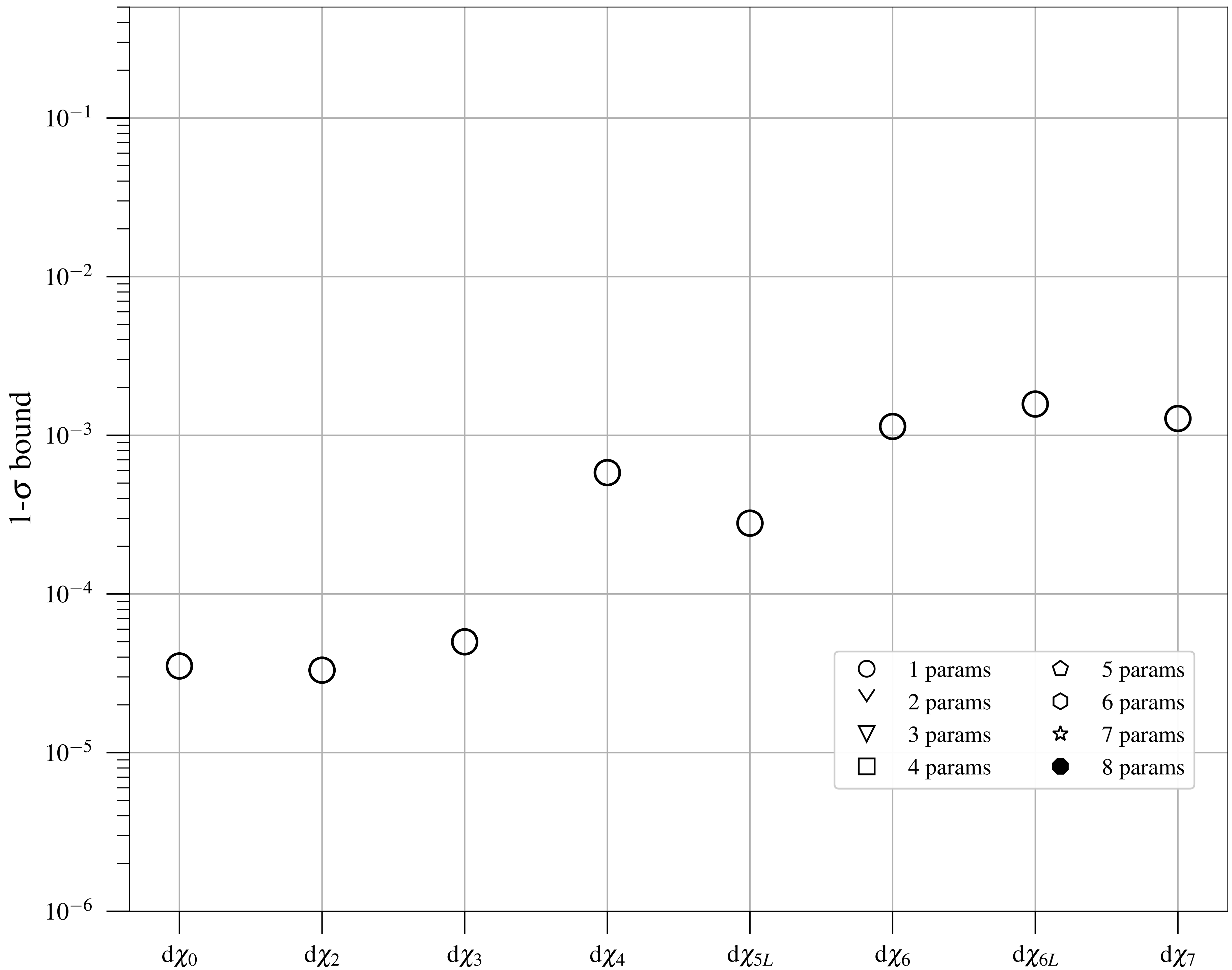
Results

Using IMRPhenomD waveform



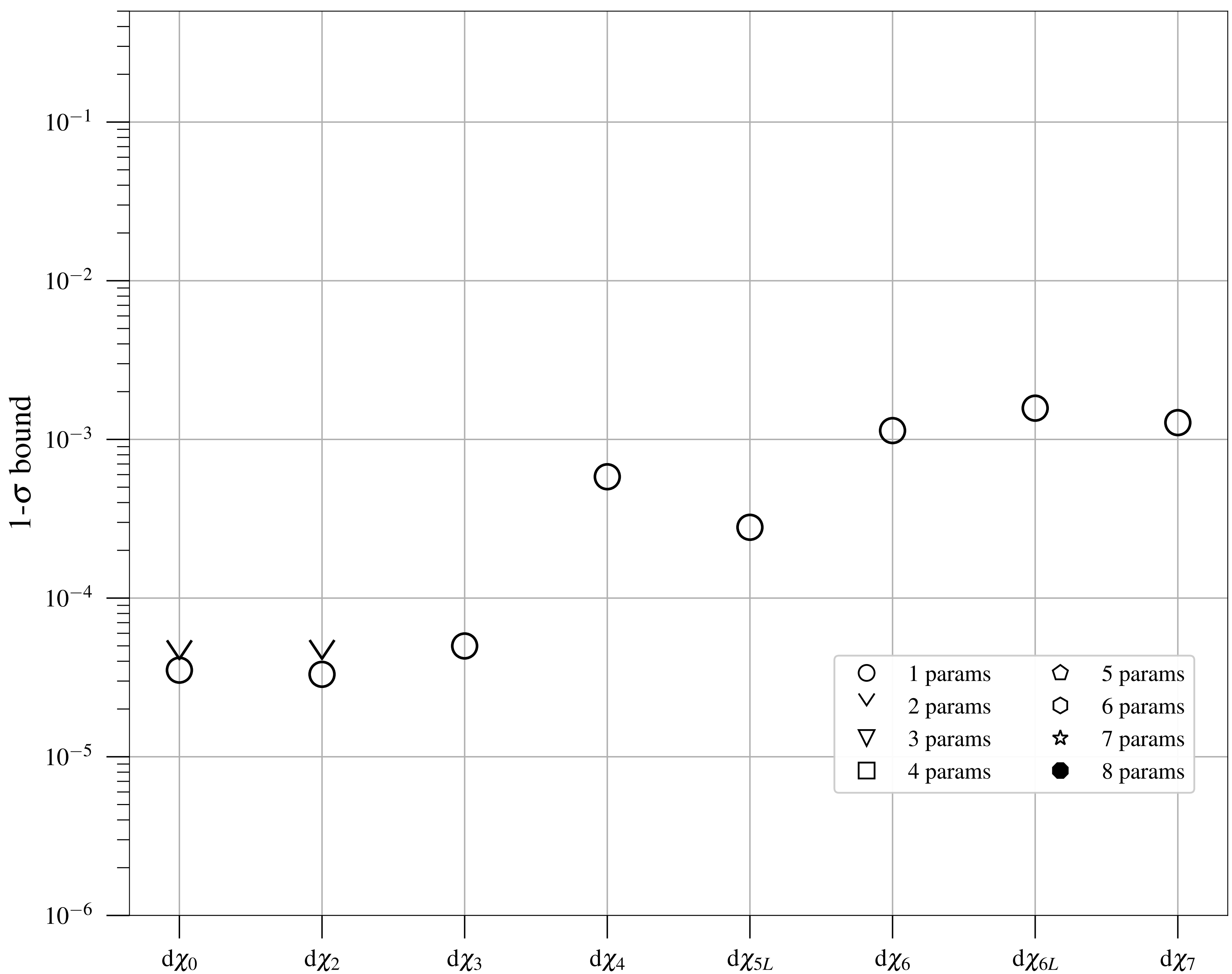
$1-\sigma$ bounds for different multi-parameter tests with Gaussian spin distribution on the detected population

Results



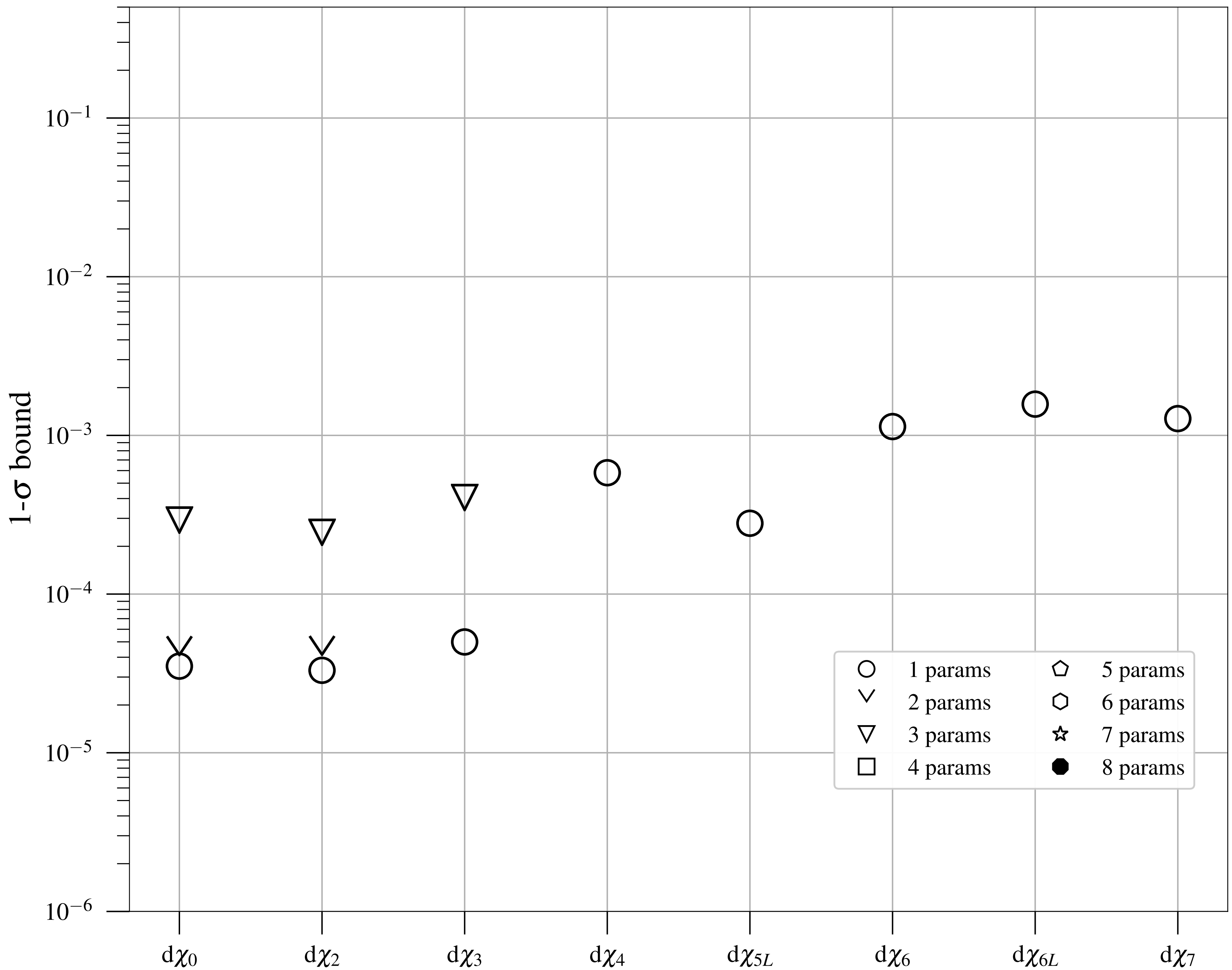
$1-\sigma$ bounds for one-parameter-at-a-time test with Gaussian spin distribution.

Results



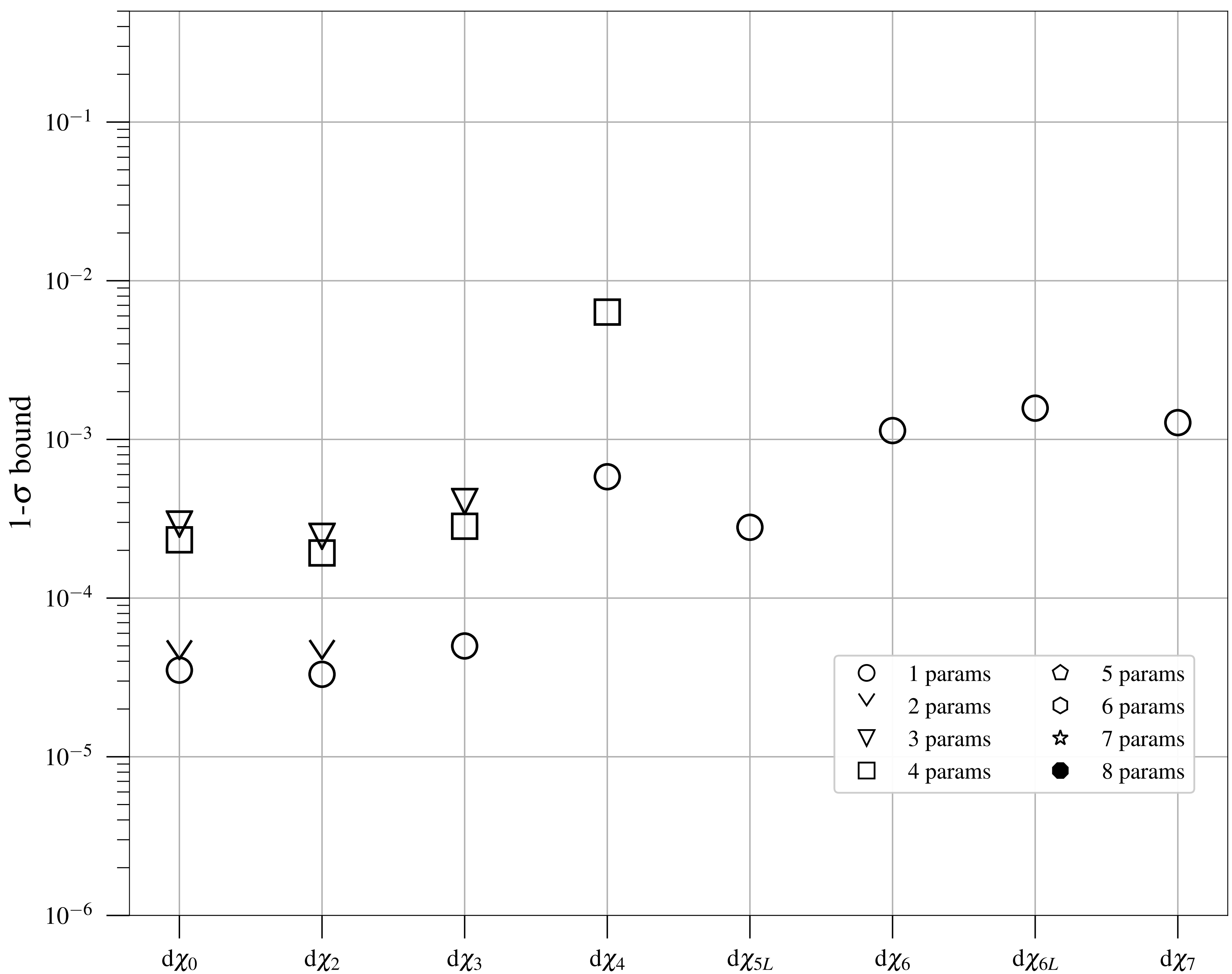
$1-\sigma$ bounds for two-parameter test with Gaussian spin distribution.

Results



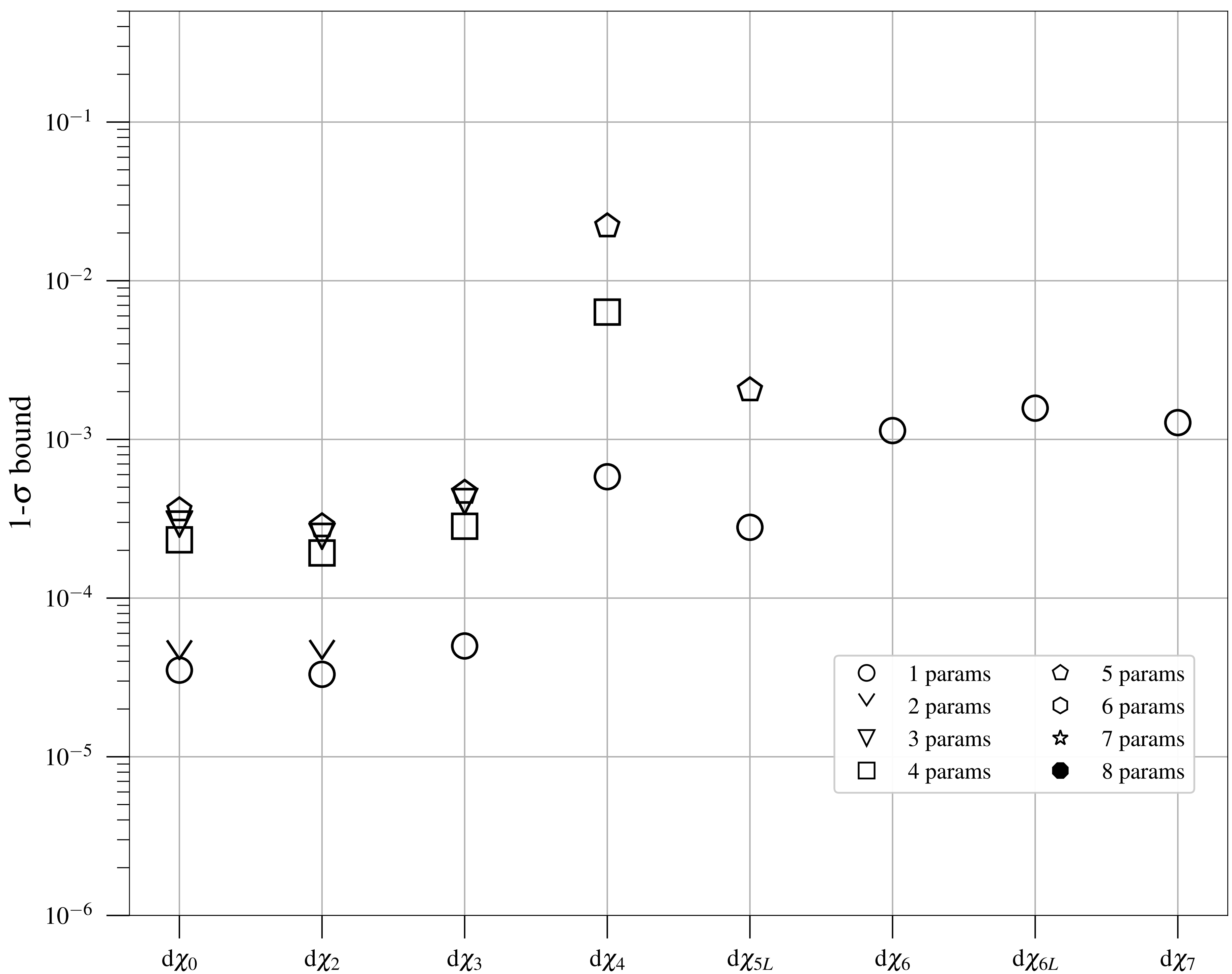
$1-\sigma$ bounds for three-parameter test with Gaussian spin distribution.

Results



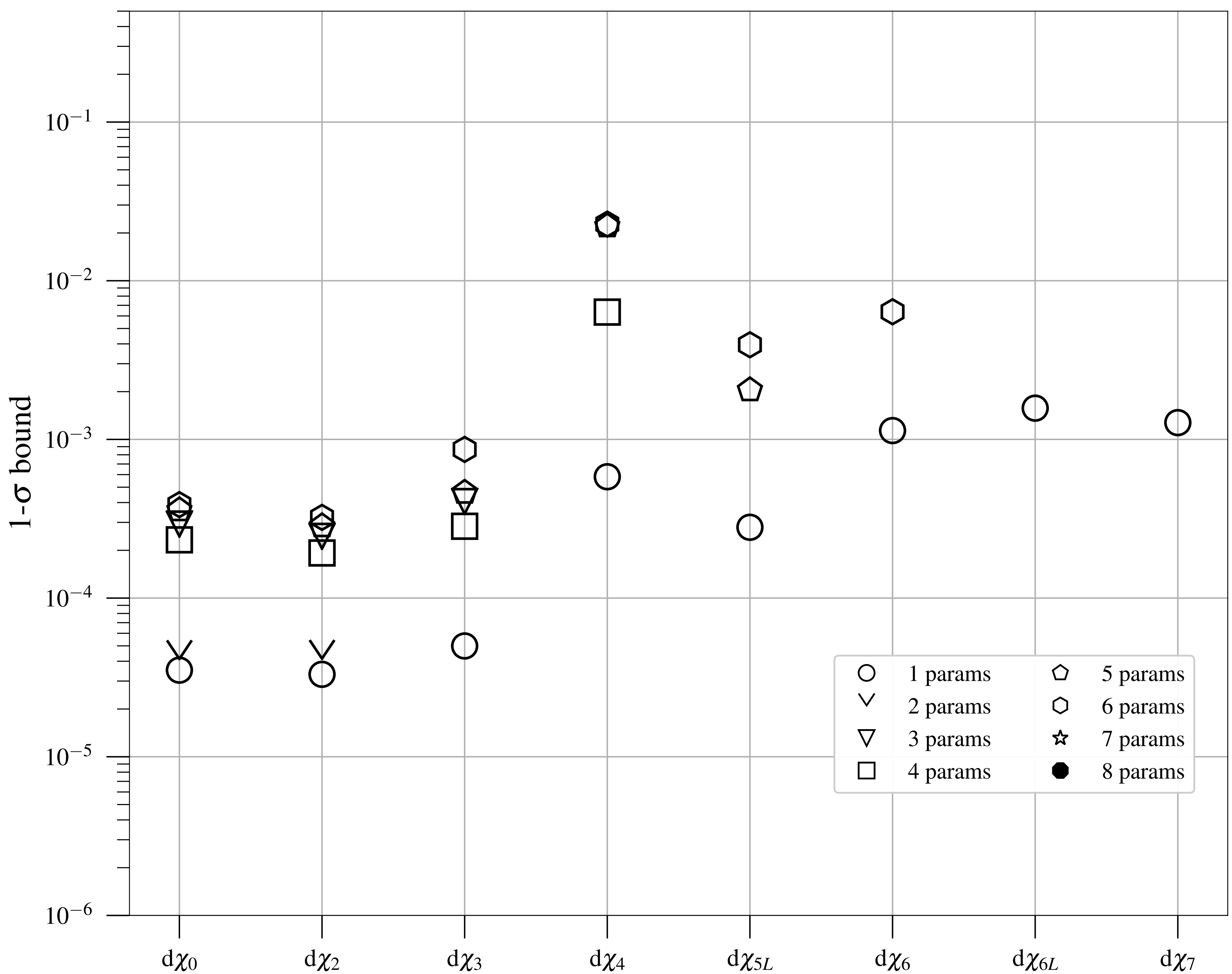
$1-\sigma$ bounds for four-parameter test with Gaussian spin distribution.

Results



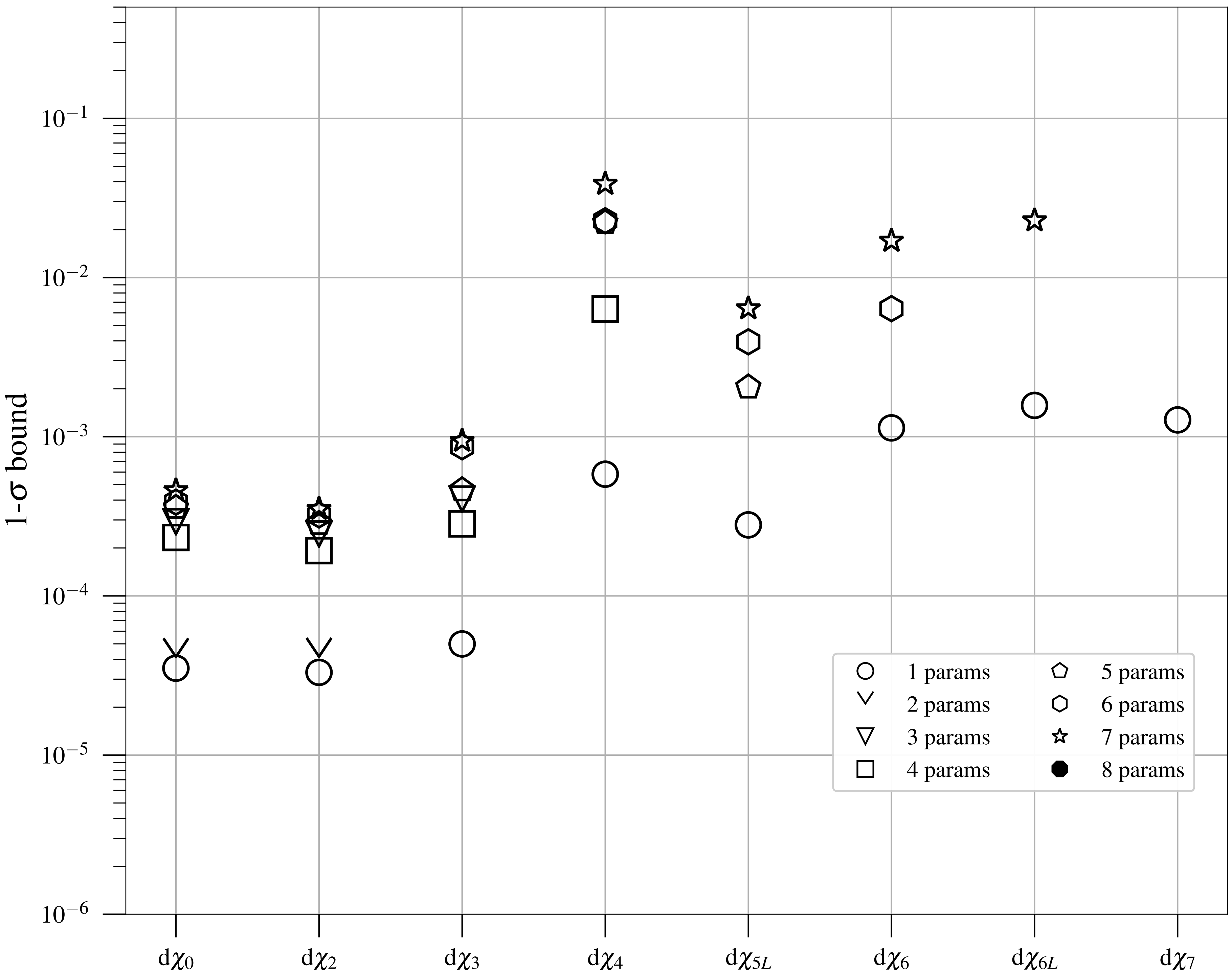
$1-\sigma$ bounds for five-parameter test with Gaussian spin distribution.

Results



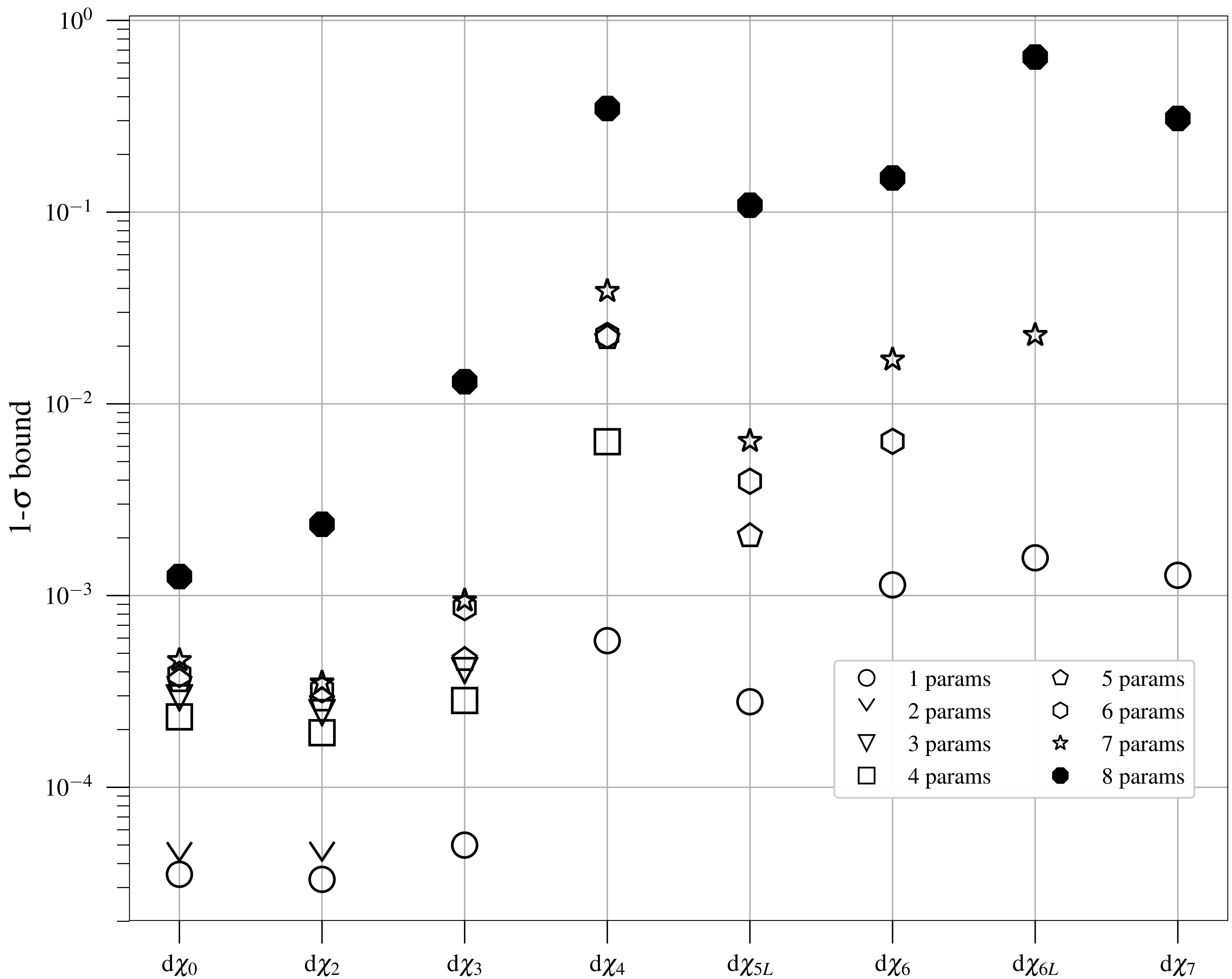
$1-\sigma$ bounds for six-parameter test with Gaussian spin distribution.

Results



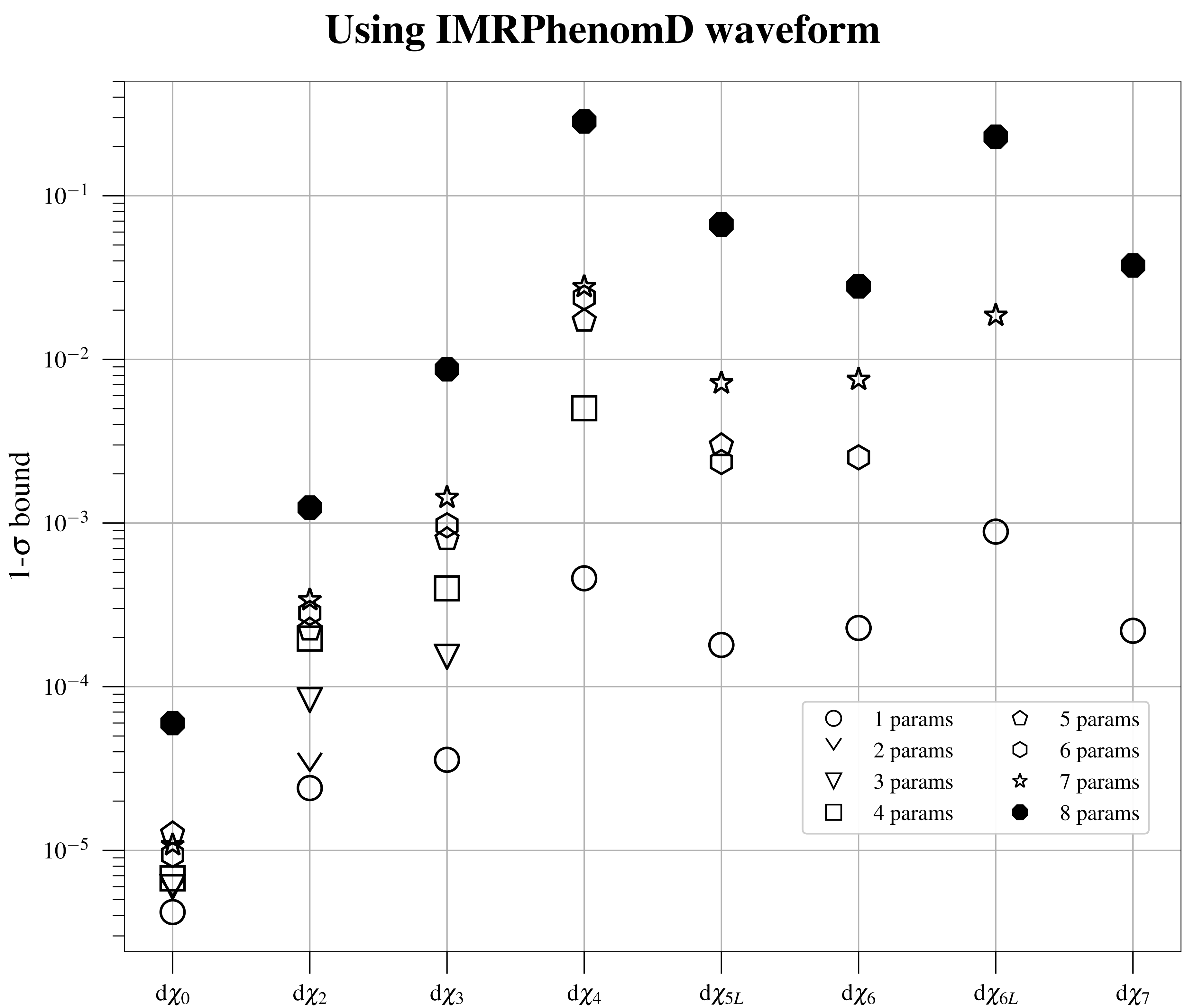
$1-\sigma$ **bounds for seven-parameter test with Gaussian spin distribution.**

Results



$1-\sigma$ bounds for eight-parameter test with Gaussian spin distribution.

Results



$1-\sigma$ bounds for different multi-parameter tests with uniform spin distribution on the detected population

Conclusions

- We obtain a realistic astrophysical population of stellar mass binary blackhole systems that would be jointly detected in LISA and CE.bands.

- We demonstrate that such multi-band observations would facilitate multi-parameter test of GR that is more rigorous when it comes to ruling out/in alternative theories of gravity.

- All the post-Newtonian phasing coefficients can be tested simultaneously to less than 10% accuracy, with the lower order PN coefficients being constrained to 0.01% accuracy.

Thank You!