Astrophysics of higher order modes

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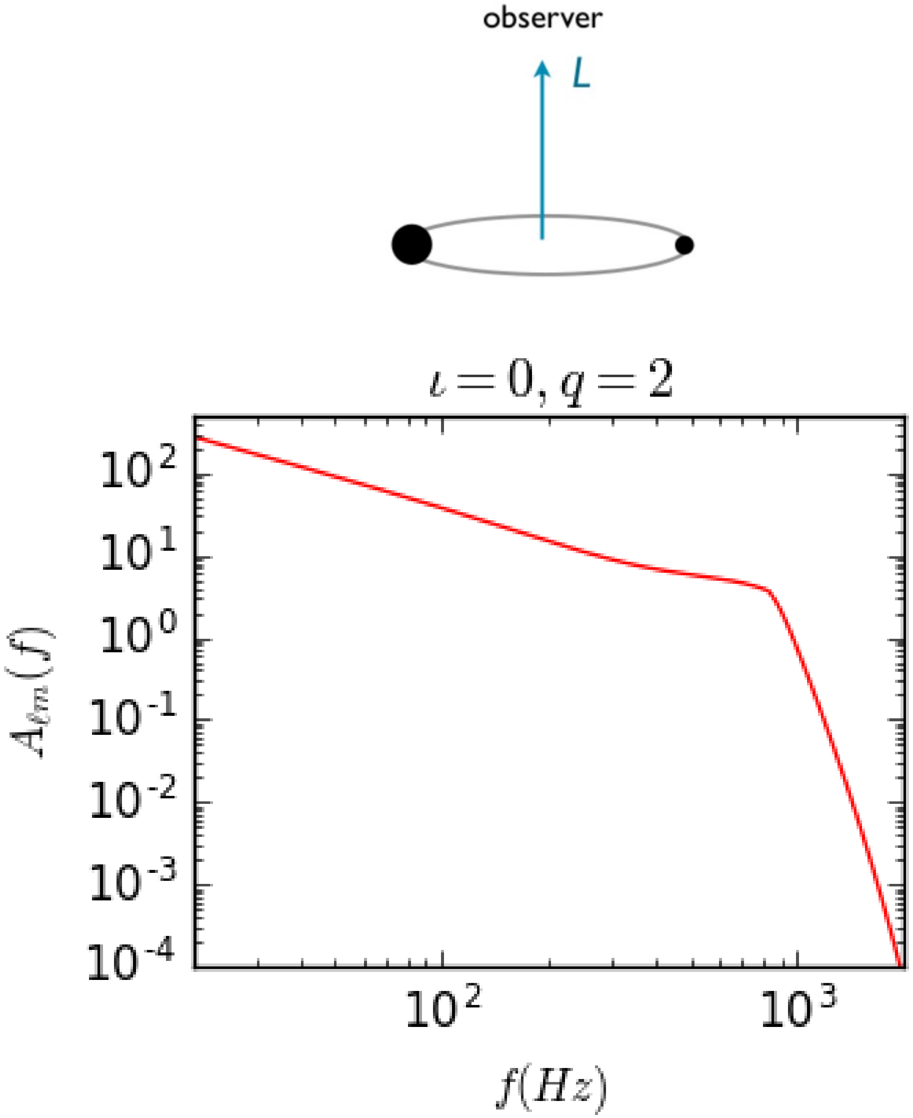
24th January, 2020

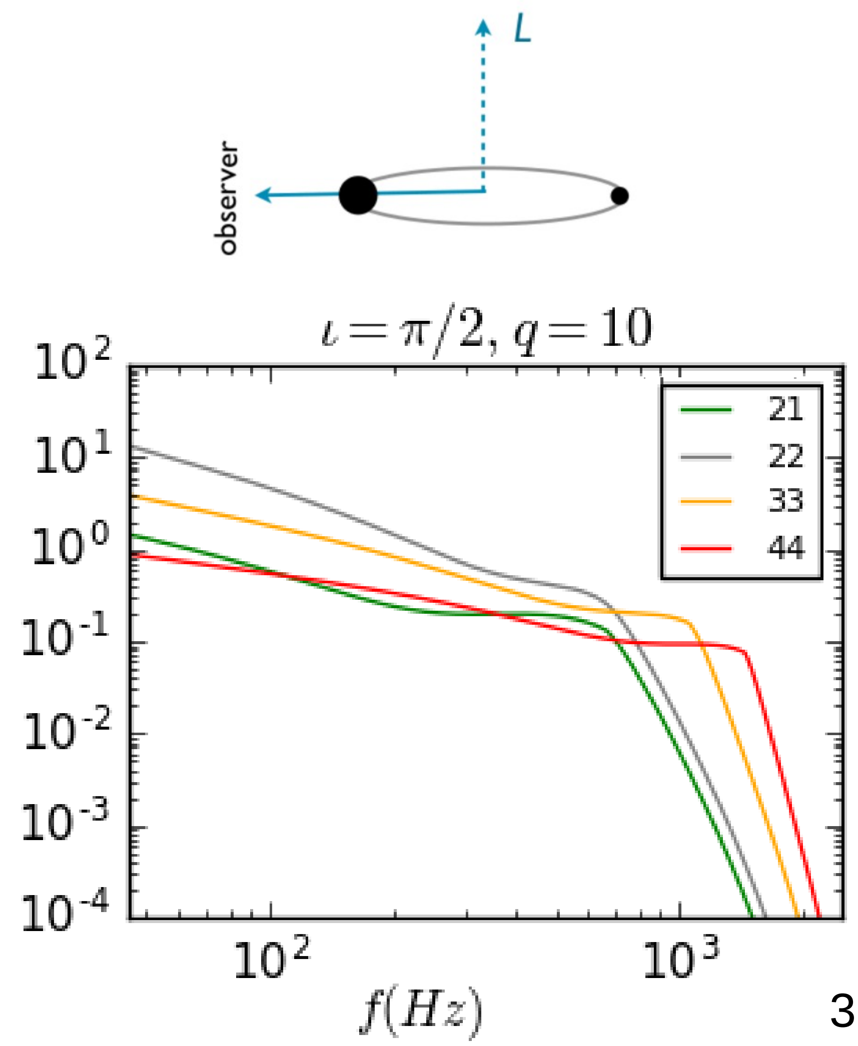
Outline

- Necessity of higher mode waveform
- Parameter Estimation
- Results: 2G detectors
- 3rd generation detectors

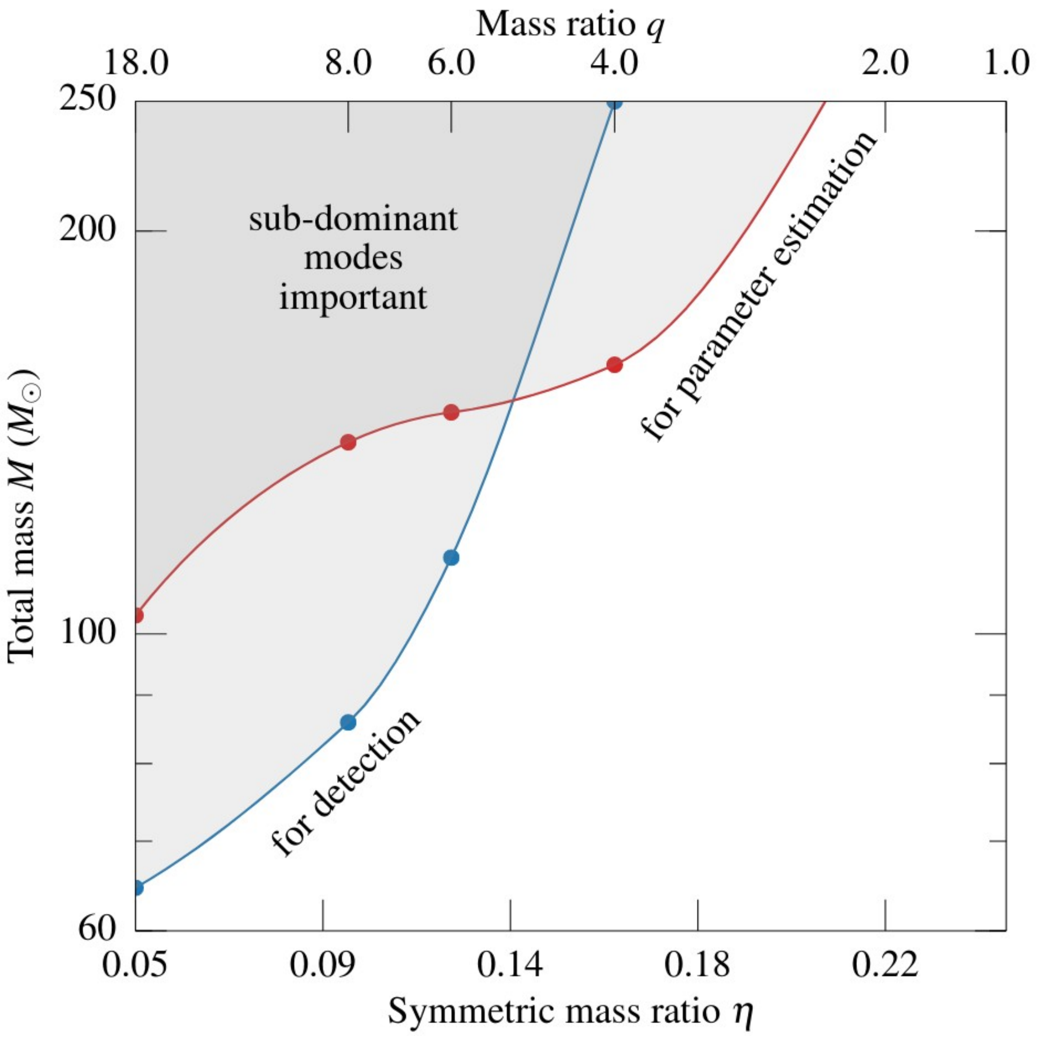
Why higher modes?

- IMR waveform, with non-quadrupole modes: ℓ = 2, m = ±1; ℓ = 3, m = ±3; ℓ = 4, m = ±4





Why higher modes?



- Improve the parameter estimation by breaking the degeneracies
- Better measurements of inclination angle and distance can help with the modelling of off-axis GRBs
- Using the improved measurements to put better constraints on cosmological parameters

Figure reference : Varma et al., Phys. Rev. D 90, 124004 (2014)

Error Estimation

For large SNR, errors in the estimation of parameters obey Gaussian probability distribution,

$$p(\Delta\theta^a) = p^{(0)} e^{-\frac{1}{2}\Gamma_{bc}\Delta\theta^b\Delta\theta^c}$$

where the Fisher Information Matrix is given by,

$$\Gamma_{bc} = 2\int_0^\infty \frac{\tilde{h}_a^*(f)\tilde{h}_b(f) + \tilde{h}_b^*(f)\tilde{h}_a(f)}{S_h(f)} df$$

The covariance matrix is defined as,

$$\Sigma^{ab} \equiv \langle \Delta\theta^a \Delta\theta^b \rangle = (\Gamma^{-1})^{ab}$$

Finally, RMS error is given by,

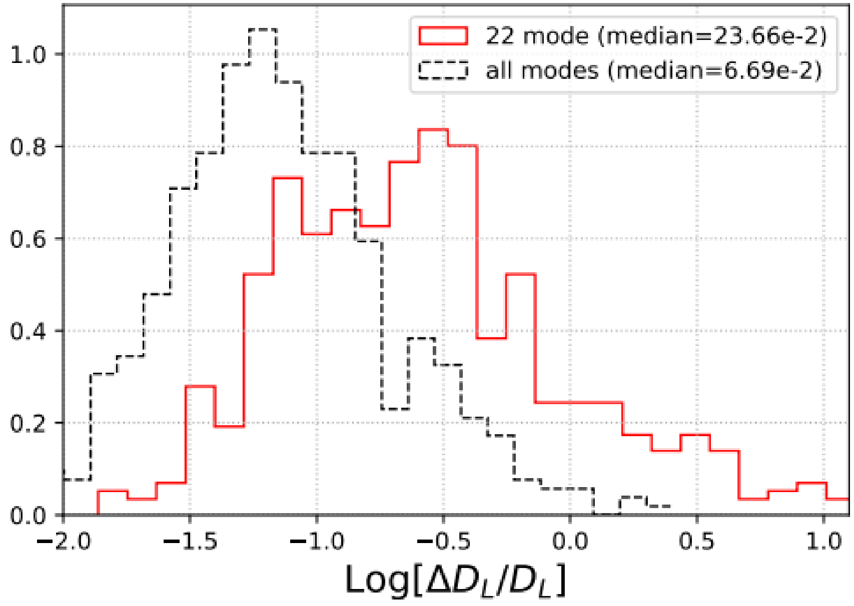
$$\sigma_a = \langle (\Delta\theta^a)^2 \rangle^{1/2} = \sqrt{\Sigma^{aa}}$$

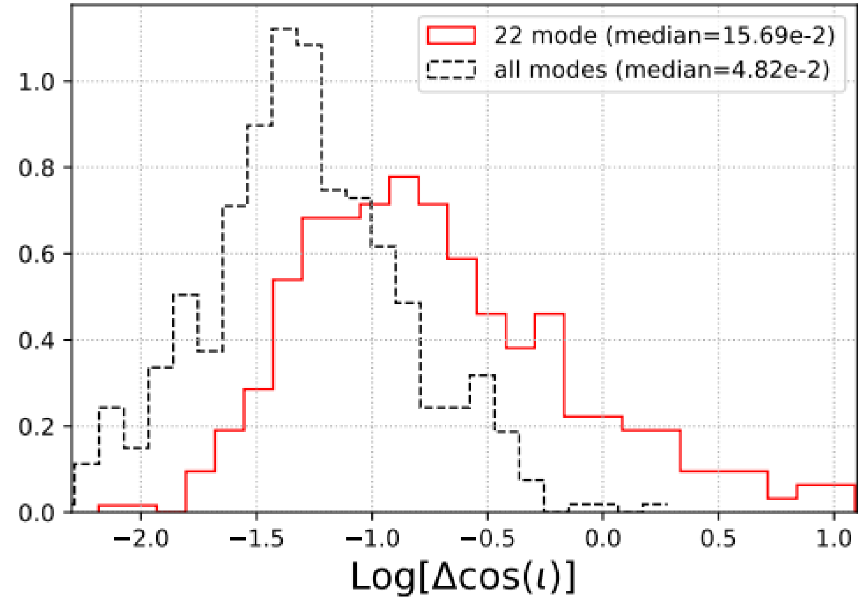
Parameters: D_L , ι , t_c , ϕ_c , M_c , η, θ, ϕ, Ψ

Reference: K.G. Arun, Bala R Iyer, B.S. Sathyaprakash, and Pranesh A Sundararajan Phys.Rev.D71:084008,2005

LHV Network: Effect of non-quadrupole modes

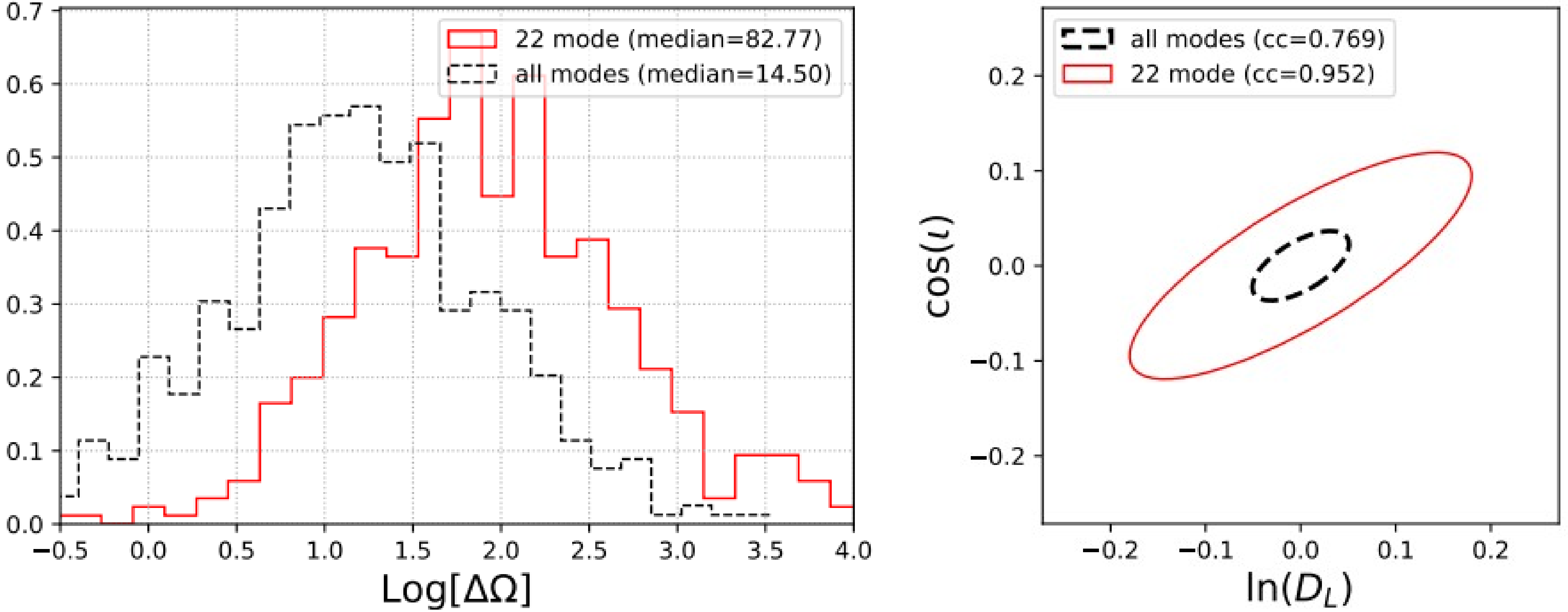
- Comoving number density between **10^3 – 500^3 Mpc^3**,
  Total mass M: **5 – 200 solar mass,**
  Mass ratio q: **1 – 10**





Roughly a factor of **3.5** improvement

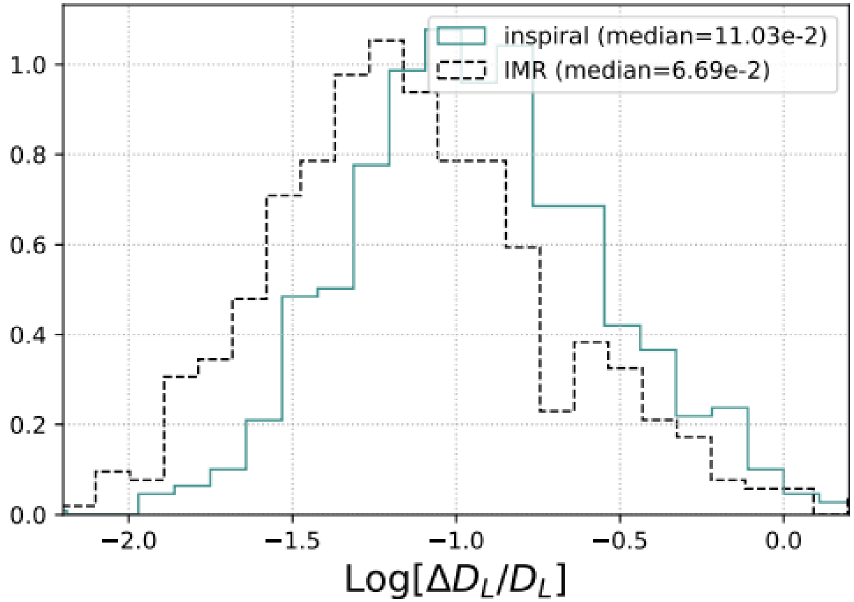
LHV Network: Effect of non-quadrupole modes

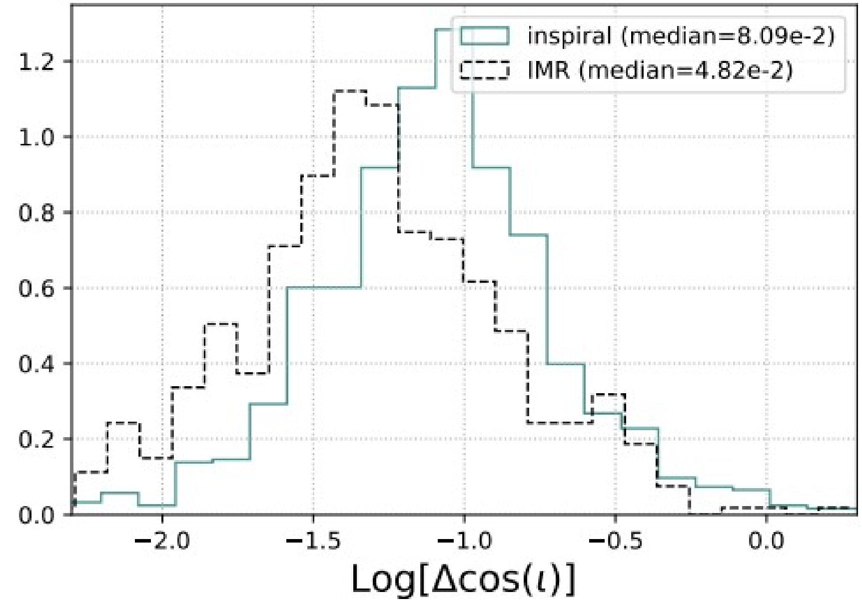


Roughly a factor of **6** improvement

LHV network: Effect of merger and ringdown

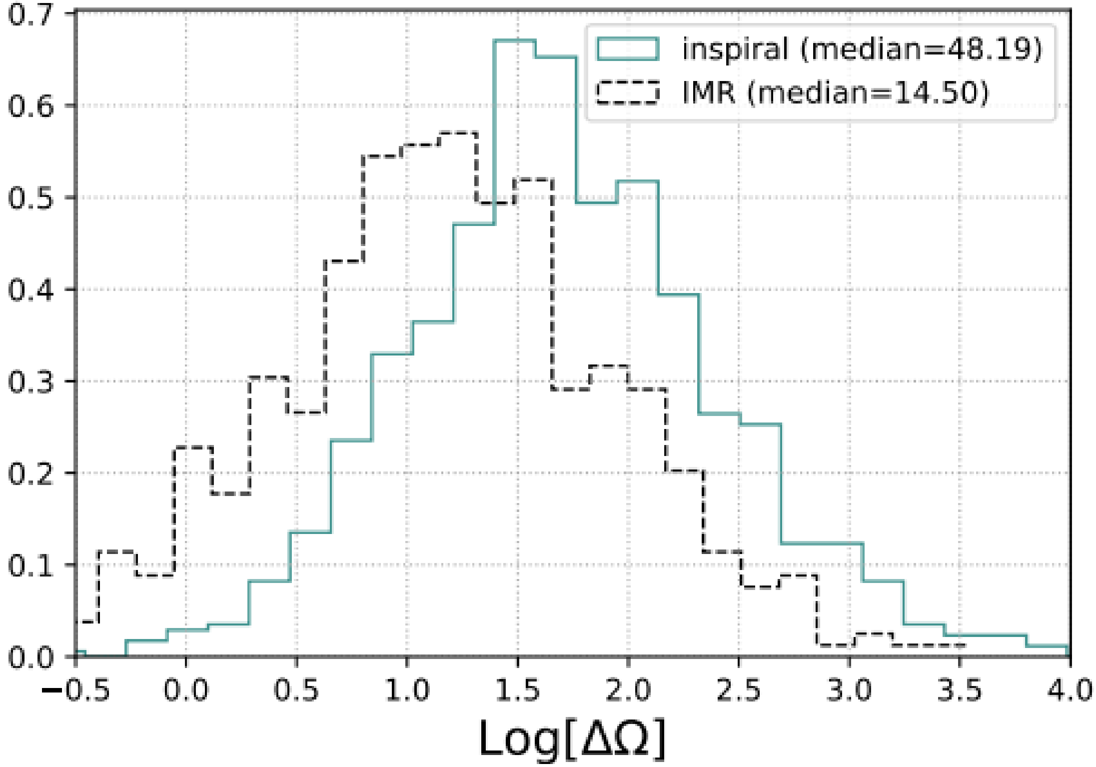
- Comoving number density between **10^3 – 500^3 Mpc^3**,
  Total mass M: **5 – 200 solar mass,**
  Mass ratio q: **1 – 10**
- Inspiral is restricted at 2*f_lso

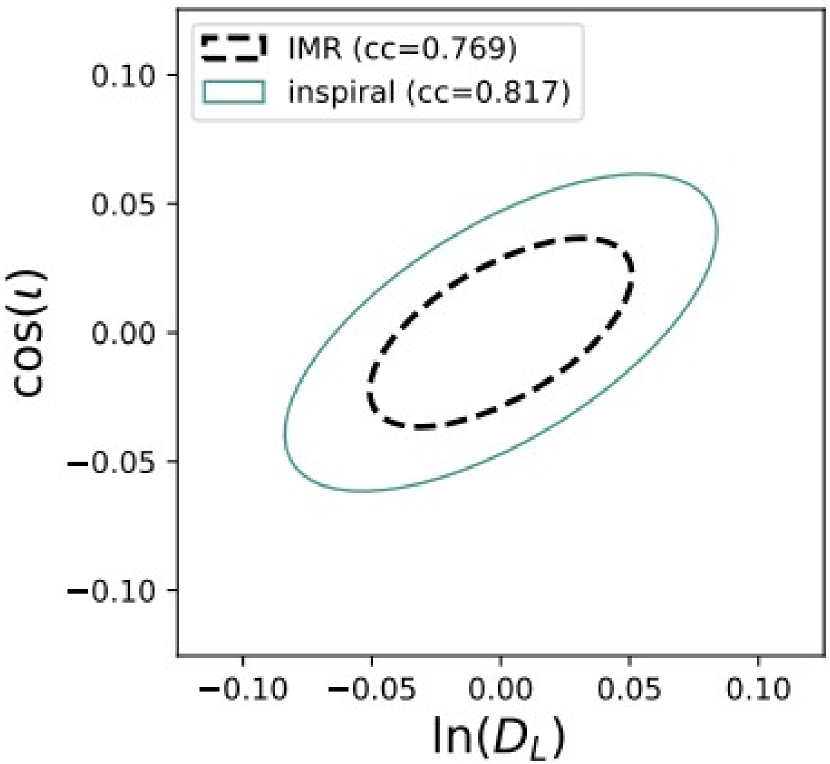




Roughly a factor of **1.5** improvement

LHV network: Effect of merger and ringdown





Roughly a factor of **3** improvement

3^{rd} Generation Detectors

Cosmic Explorer (CE)

- L-shaped, above/under the ground, each arm – 40 km
- Proposed for the LIGO Livingston site in the U.S.
- Other proposals: Australia, China and India

Einstein Telescope (ET)

- In the shape of an equilateral triangle, underground, each arm – 10 km
- 2 detectors in each corner of the triangle
- Proposed by the European Union

Networks: LIE and LCIEA

A Network of 3G Detectors

Locations taken from : Salvatore Vitale and Matthew Evans,Phys. Rev. D 95, 064052 (2017)

Sensitivity of 3G Detectors

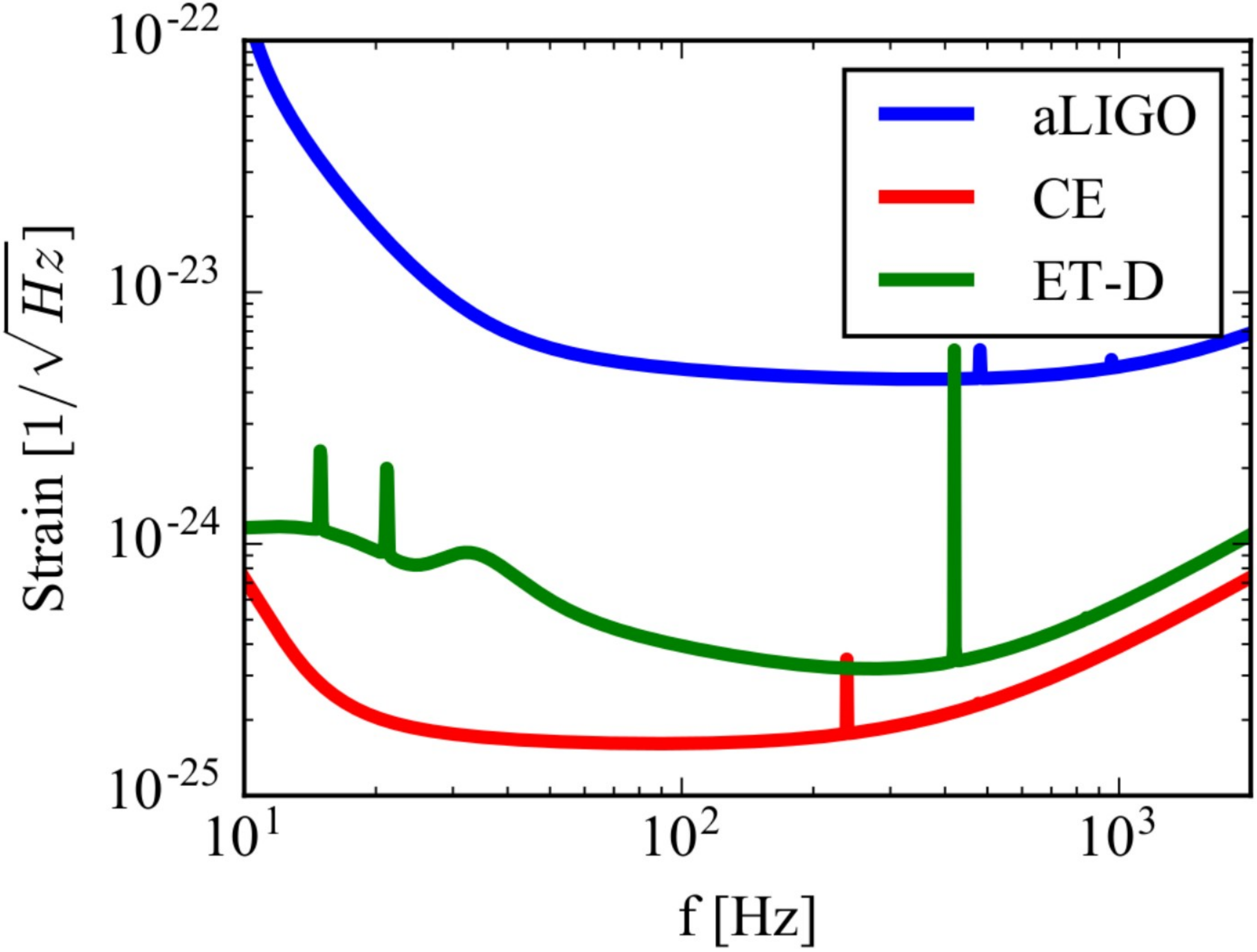
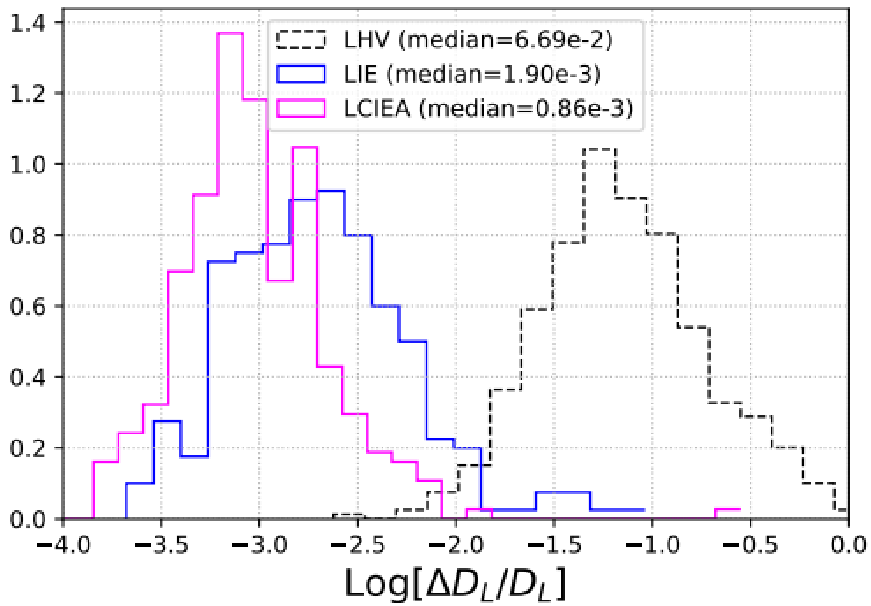
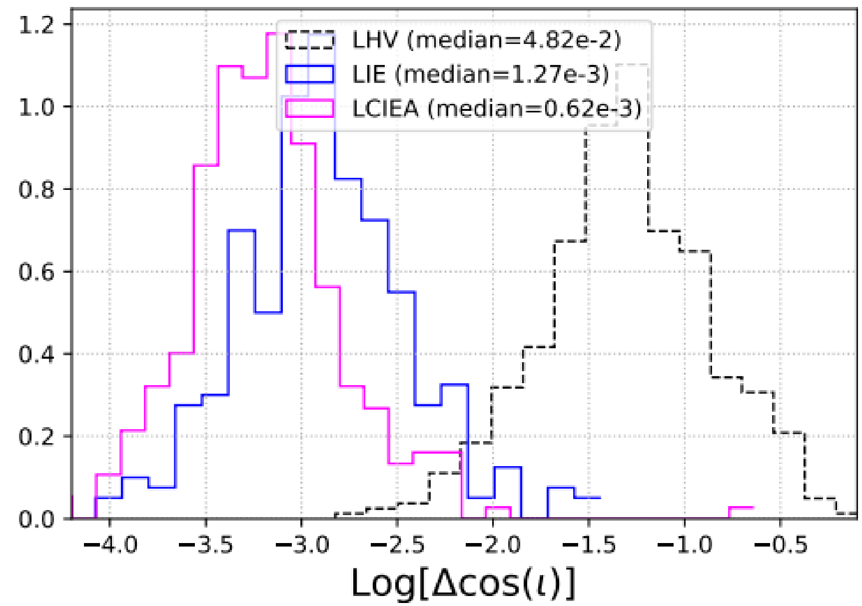


Figure reference : Abbott et al., LIGO-P1600143 (2016)

Results: Comparison between 2G-3G detectors

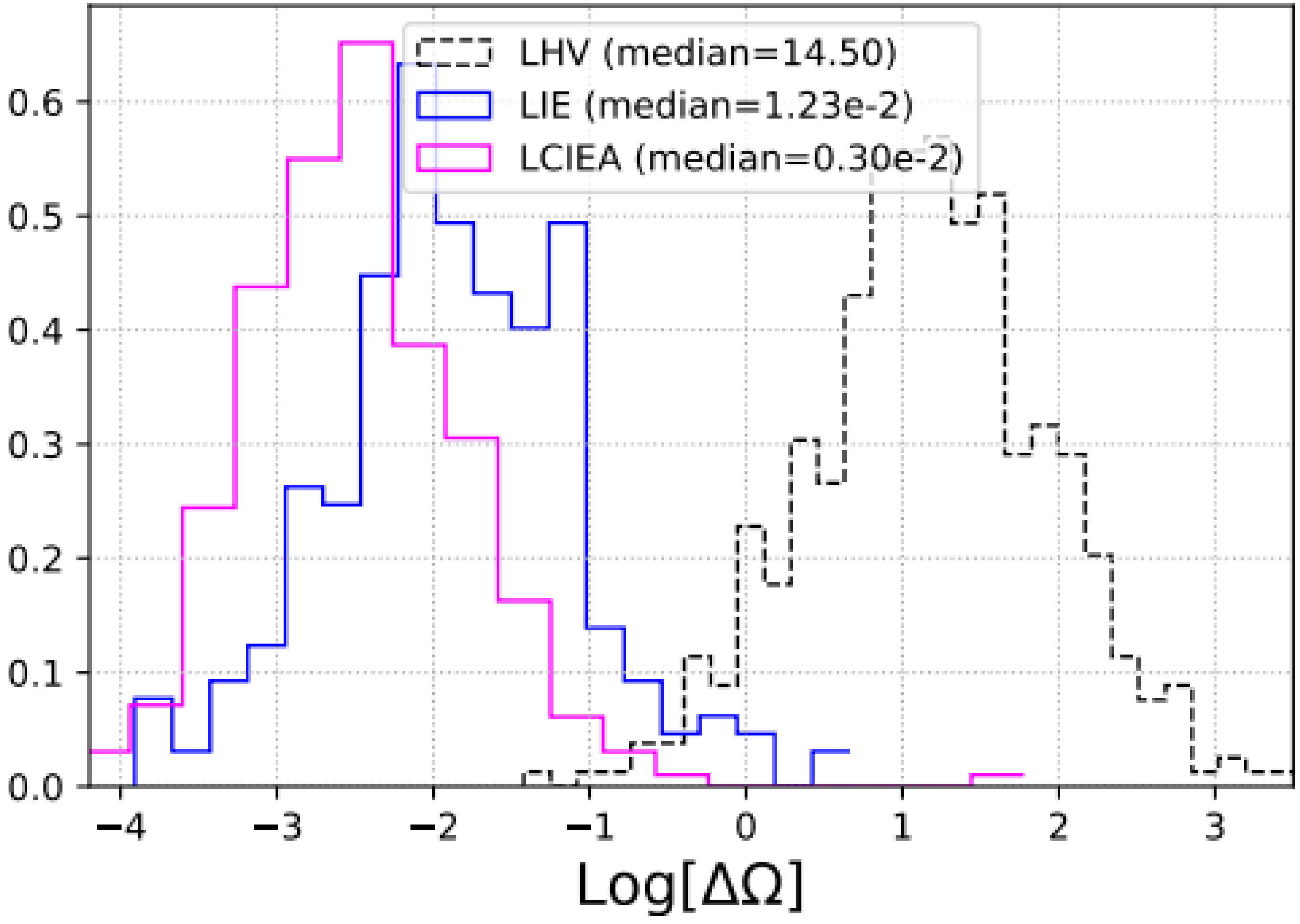
- Comoving number density between **10^3 – 500^3 Mpc^3**, Total mass M: **5 – 200 solar mass,** Mass ratio q: **1 – 10**





Roughly a factor of **35-38** improvement between LHV and LIE, and **2** between LIE and LCIEA

Results: Comparison between 2G-3G detectors



Roughly by **3** orders of magnitude improvement between 2G and 3G detectors

Concluding Remarks

- Contribution from higher modes becomes relatively significant at higher inclination angles, and in highly asymmetric systems (high mass ratio)
- It can contribute to more detections and better parameter estimation
- Inclusion of higher modes gives better constraints on distance and inclination angle measurements, and increases the angular resolution
- Switching from 2G to 3G detectors can improve the parameter estimation significantly, especially the angular resolution
- Improvement in the estimates of distance, inclination angle and angular resolution can further help in putting better constraints on cosmological constants.

Thank You

Structure of waveform

- Analytical IMR waveform having inspiral upto 3.5 post-Newtonian orders

- The strain amplitude of gravitational radiation for different modes, in Fourier domain is given by,

$$h_{lm}(f) = A_{lm}(f)e^{i\Psi_{lm}(f)}$$

$$A_{lm}(f) = \begin{cases} A_{lm}^{IM}(f), & f < f_{lm}^{A} \\ A_{lm}^{RD}(f), & f \geq f_{lm}^{A} \end{cases}$$

$$\Psi_{lm}(f) = \begin{cases} \Psi_{lm}^{IM}(f), & f \leq f_{lm}^{P} \\ \Psi_{lm}^{RD}(f), & f \geq f_{lm}^{P} \end{cases}$$

Reference: Ajit Kumar Mehta, Chandra Kant Mishra, Vijay Varma, and Parameswaran Ajith 10.1103/PhysRevD.96.124010