SHADOWS, ECHOES AND MEMORY

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COMMON PHENOMENA, GR TWIST

• Shadows are familiar to all. Eclipses involve shadows.

• Echoes are known for sound waves, as learnt in high school.

• Memory drives us forward in life. In physics, magnetic memory (residual magnetisation) is well-known.

• **All three have been very important in gravitational physics, of late.**

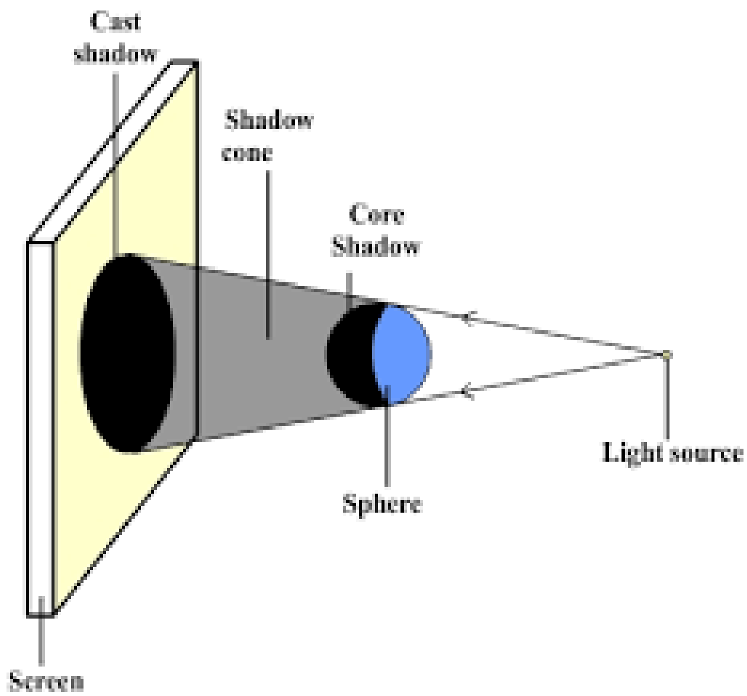
• The black hole shadow (M87); GW echoes from black hole mimickers; GW memory!

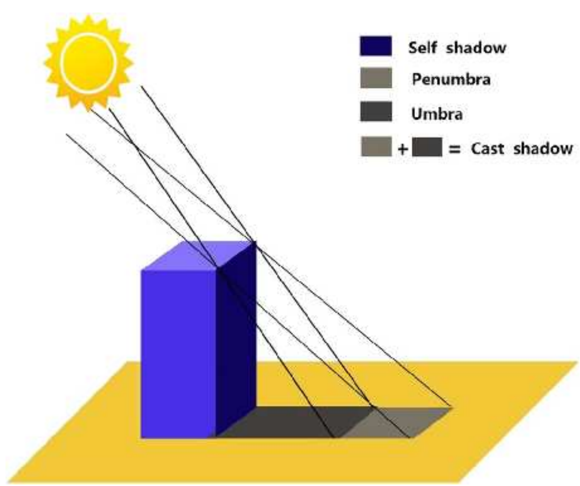
• Thus, a lot of gravitational physics today revolves around the study of:

SHADOWS, ECHOES AND MEMORY.

SHADOWS

SHADOWS EVERYWHERE!

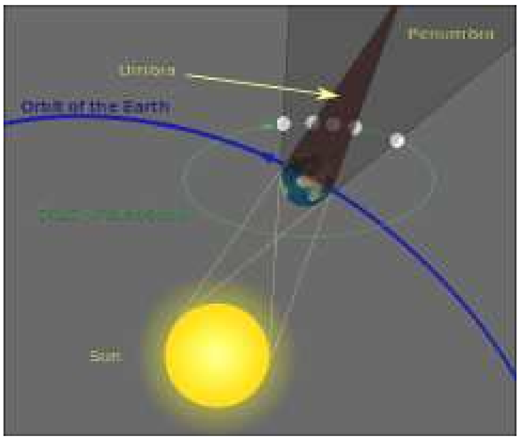




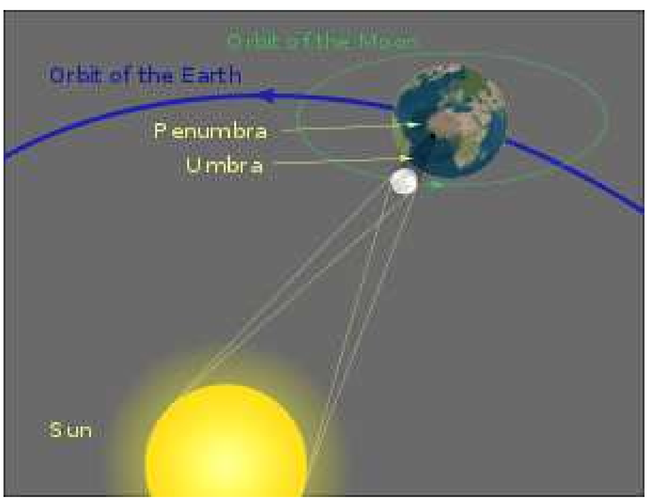
• Understood using ray-tracing (Appel, 1969). Extensively studied in computer graphics with useful programs.

SHADOWS: ECLIPSES

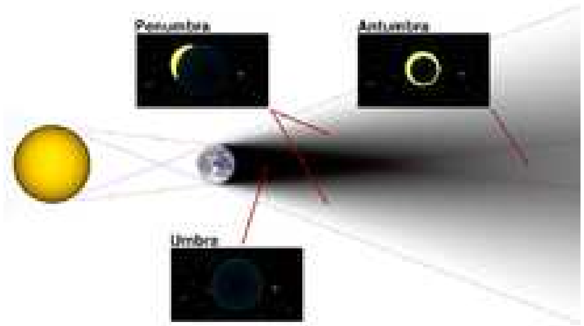
LUNAR: Earth's shadow on moon.



SOLAR: Moon's shadow on Earth.



Umbra, Penumbra, Antumbra:



- **Rectilinear propagation of light.**

Aside: Interesting recent study (2020):

Eclipses of continuous gravitational waves as a probe of stellar structure

Marchant, Breivik, Berry, Mandel, Larson

.... If a gravitational wave source is eclipsed by a star, measuring these perturbations provides a way to directly measure the distribution of mass throughout the stellar interior....We identify continuous gravitational waves from neutron stars as the best candidates to detect this effect. When the Sun eclipses a far-away source, depending on the depth of the eclipse the time-delay can change by up to 0.034 ms, the gravitational-wave strain amplitude can increase by 4 percent.

SOMMERFELD ON SHADOWS

‘We consider the light source as given. From it there emerge rectilinear rays. A screen can be called opaque if it absorbs all rays falling on it and does not itself emit any rays. Then the shadow behind the screen is bounded by straight light ray directions which emerge from the light source.... The rays which do not meet the screen continue unobstructed along straight lines.’

— Sommerfeld (Optics).

- Discusses theory of shadow formation from both the geometrical optics point of view as well as full wave optics.

- We will look at shadows formed due to gravitational deflection of light rays, which is a GR effect.

SHADOWS OF BLACK HOLES

• Synge (1966) first calculated the shadow of a black hole. We will analyse shadows for a general, static, spherically symmetric line element.

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2 d\Omega_2^2$$

• Steps:

(1) Set up the coordinates (observers sky).

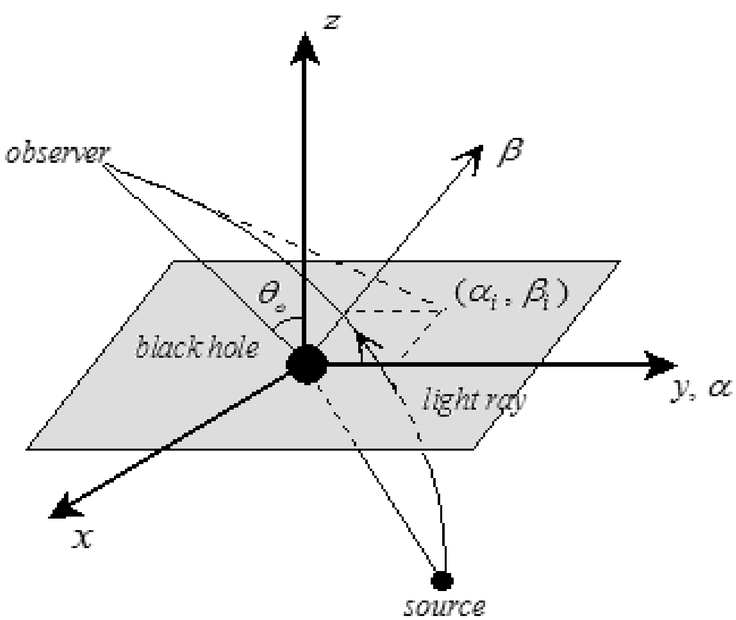
(2) Find photon sphere (not mandatory).

(3) Note first integrals of geodesic equation.

(4) Use (3) in coordinates to get shadow.

(5) Write down shadow radius and angular diameter.

(1) Coordinates (Vasquez, Esteban (2003))



→ xyz **coordinate system with BH at origin.**

→ **Large** r**, Boyer–Lindquist** $\equiv xyz$**.**

→ **Observer coordinates:** $(r_0 \sin\theta_0, 0, r_0 \cos\theta_0)$

→ **Tangent to light ray hits** $\alpha\beta$ **plane at** $(\alpha, \beta) \equiv (-\beta\cos\theta_0, \alpha, \beta\sin\theta_0)$.

→ **Line from BH to Obs.** $\perp$ **to** $\alpha\beta$ **plane.**

- **Eqn. of st.line joining obs. and (α, β):**

$$\frac{x - r_0 \cos\theta_0}{r_0 \sin\theta_0 + \beta \cos\theta_0} = \frac{y}{-\alpha} = \frac{z - r_0 \cos\theta_0}{r_0 \cos\theta_0 - \beta \sin\theta_0}$$

$\rightarrow$ **Yields two equations for α, β. Solve.**

$$\alpha = \frac{r_0 y}{r_0 - x \sin\theta_0 - z \cos\theta_0}, \beta = \alpha \frac{z \sin\theta_0 - x \cos\theta_0}{y}$$

$\rightarrow$ **At the observer, α, β are:**

$$\alpha = -r_0^2 \sin\theta_0 \frac{d\phi}{dr}|_{r_0}$$

$$\beta = r_0^2 \frac{d\theta}{dr}|_{r_0}$$

Note: Used $x = r \sin\theta \cos\phi$ etc. and L'Hospital rule to evaluate at observer point.

$\rightarrow$ **Inputs on $\frac{d\theta}{dr}$ and $\frac{d\phi}{dr}$ come from first integrals of geodesic motion.**

(2) Photon sphere:

→ **Line element:**

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2 d\Omega_2^2$$

→ $u^i = \dot{x}^i; L = \frac{1}{2}g_{ij}\dot{x}^i\dot{x}^j; H(x^i, p_i) = \frac{1}{2}g^{ij}p_i p_j$

→ **Null geodesics,** $H = 0$**;** $p_i = \frac{\partial S}{\partial x^i} = g_{ij}\dot{x}^j$

→ **Constants of motion:**

$$E = -p_t; \ L = p_\phi = r^2 \sin^2\theta\, \dot{\phi}.$$

→ **Action:**

$$S = -Et + L\phi + S^r(r) + S^\theta(\theta)$$

→ $H = 0$ **results in:**

$$- e^{-\nu}E^2 r^2 + e^{-\lambda}\left(\frac{dS^r}{dr}\right)^2 r^2 + L^2$$

$$- \left(\frac{dS^\theta}{d\theta}\right)^2 - L^2 \cot^2\theta = -C$$

→ **Finally, we obtain:**

$$\frac{dS^r}{dr} = \sqrt{e^{\lambda}\left[e^{-\nu}E^2 - \frac{C}{r^2} - \frac{L^2}{r^2}\right]} = E\sqrt{-V(r)} = e^{\nu}\dot{r}$$

$$\frac{dS^{\theta}}{d\theta} = \sqrt{C - L^2\cot^2\theta} = E\sqrt{\Theta(\theta)} = r^2\dot{\theta}$$

→ **Effective potential:**

$$V(r) = e^{\lambda}\left[\frac{\chi + \ell^2}{r^2} - e^{-\nu}\right]$$

where $\chi = \frac{C}{E^2}$**,** $\ell = \frac{L}{E}$**.**

→ **Photon sphere condition:**

$$\dot{r} = 0,\ \dot{p}_r = 0 \rightarrow V(r) = 0,\ V'(r) = 0$$

$$V(r) = 0 \rightarrow \chi + l^2 = r_p^2 e^{-\nu(r_p)}$$

$$V'(r) = 0 \rightarrow \chi + l^2 = \frac{1}{2}\nu'(r_p) r_p^3 e^{-\nu(r_p)}$$

→ **Photon sphere eqn.:** $r_p\nu'(r_p) = 2$.

(3) First integrals:

$$e^{\nu}\dot{r} = E\sqrt{-V(r)}$$

$$r^2\dot{\theta} = E\sqrt{\Theta(\theta)}$$

$$r^2 \sin^2\theta\,\dot{\phi} = L$$

(4) Use (3) in α, β:

$$\alpha = -r_0^2 \sin\theta_0 \frac{d\phi}{dr}|_{r_0} = -\frac{\ell}{\sin\theta_0}$$

$$\beta = r_0^2 \frac{d\theta}{dr}|_{r_0} = \sqrt{\Theta(\theta)}.$$

(5) Shadow radius, angular diameter:

$$\alpha^2 + \beta^2 = \chi + \ell^2 = r_{sh}^2 = r_p^2 e^{-\nu(r_p)}$$

→ **For Schwarzschild: Circular shadow**

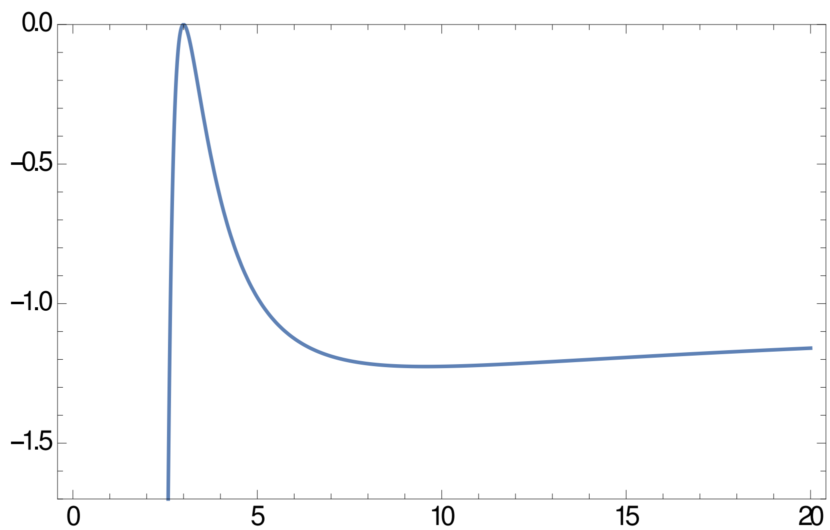
Radius: $r_{sh} = 3\sqrt{3}\frac{GM}{c^2}$

Ang. diam.: $2\alpha_D = \frac{2r_{sh}}{D} = \frac{6\sqrt{3}GM}{c^2 D}$.

Schwarzschild shadow

- **Effective potential:**

$$V_{eff} = \frac{1}{\left(1-\frac{2M}{r}\right)^2}\left[-1+\frac{27M^2}{r^2}\left(1-\frac{2M}{r}\right)\right]$$



Negative, max at $r = 3M$, diverges to negative infinity at $r = 2M$, asymptotically of value -1.

- **Photon sphere: $r_{ph} = 3M$.**

Physical meaning

$\rightarrow$ A finely tuned photon at the photon orbit could, in principle, orbit the black hole an infinite number of times.

$\rightarrow$ However, since the orbit is unstable, any slight perturbation would cause the photon either to fall into the black hole or to escape to infinity.

$\rightarrow$ The photons that escape are seen by a distant observer to have an impact parameter $r_{sh} = 3\sqrt{3}M$ with respect to the mass M.

$\rightarrow$ The circle in the observer's sky with radius r_{sh}, centered on M, is the boundary of the shadow. Shadow boundary is the critical curve in lensing.

M87 (EHT Results 2019)

For the supermassive BH in M87

$\rightarrow$ **Use GR-Schwarzschild result for shadow.**

$\rightarrow$ $M = 6.5 \times 10^9 M_{Sun}$, $D = 16.8 MPc$

$\rightarrow$ $2\alpha_D = 39.6 \mu as$ **by the Synge formula!**

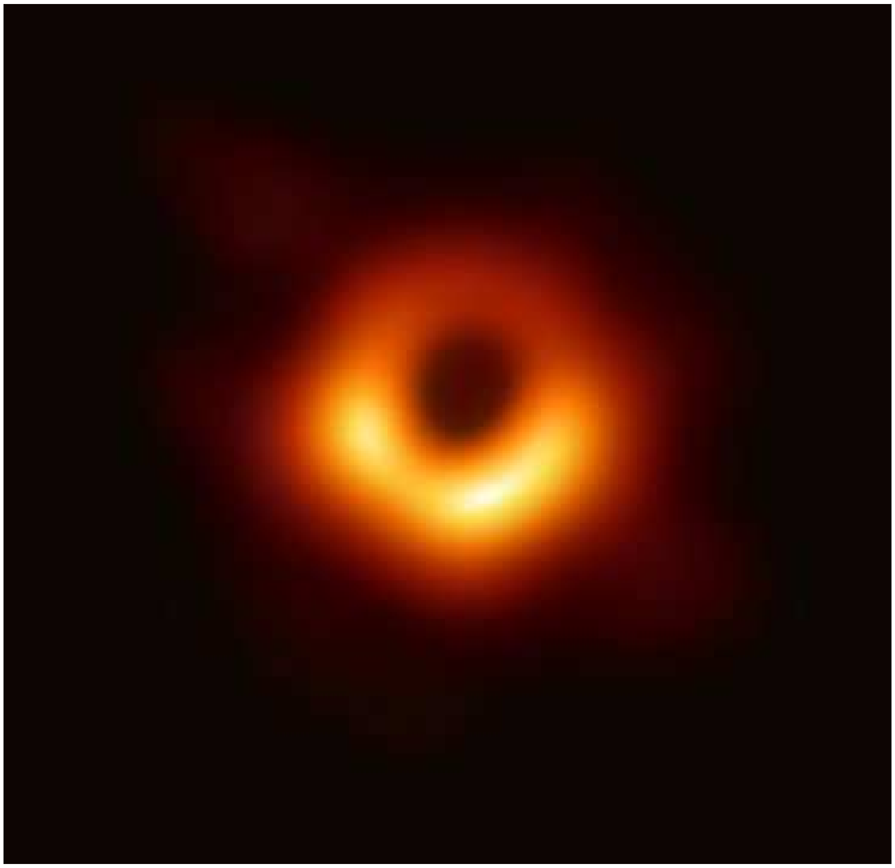
Observed result

$\rightarrow$ **For ring diameter: 42 $\pm$ 3 μ-arcseconds**

$\rightarrow$ **Deviation from circularity: 10 percent.**

- **See details in:**

First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole, also paper VI.



Brightness temperature profile of emissions at $\lambda = 1.3mm$.

Inner dark region (no emissions)

Outer emission ring: photon sphere and beyond–shadow radius.

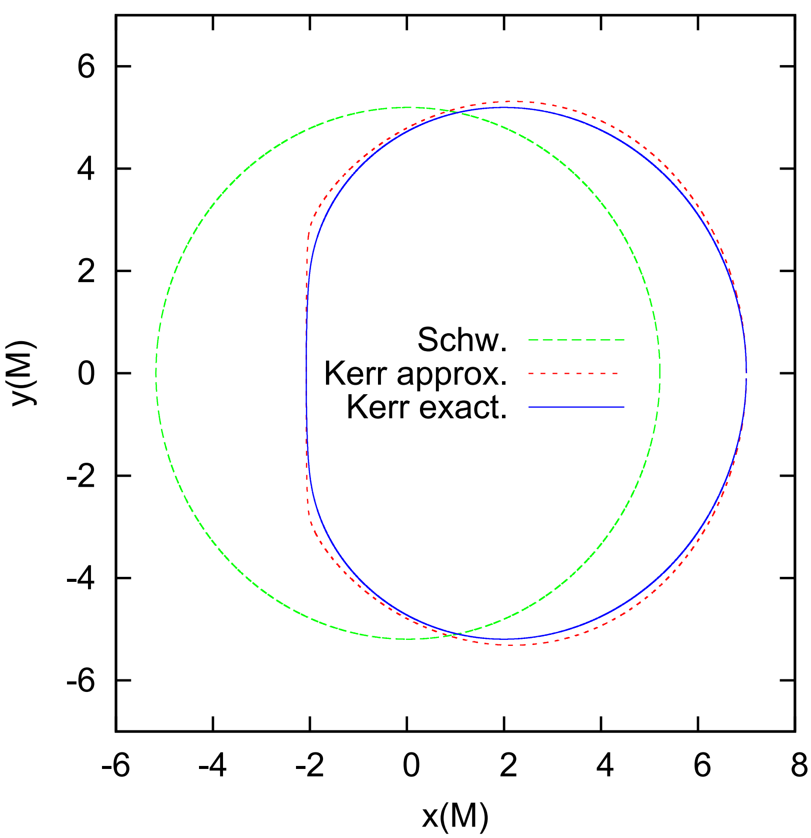
Expected shadow radius matches well with GR expectations.

Rotating black holes

What happens when we include rotation? Kerr BH.

$\longrightarrow$ **Shadow no longer circular.**

Possible profiles:



What can shadows tell us?

• Shadows can help us compare different theories of gravity, apart from further confirming GR. But, one must understand shadows better (see eg. Gralla, Holz and Wald (2019) for critique)

• Are shadows different in the presence of dark matter or plasma or a cosmological background? Several papers (Jusufi et. al, Bisnovatyi-Kogan, Perlick....) which discuss these issues.

• Shadows not necessarily for black holes alone. Wormholes and other BH mimickers can have shadows too. A lot of work on this is ongoing (Shaikh, Konoplya...).

• Accretion disc modeling and explanations using GRMHD have gone a long way in understanding the results for M87. Further work (simulations) surely required.

ECHOES

ECHOES IN SOUND

- When you get to the top of a mountain, absorb the beautiful scenery, and shout, ”Hello!” A second later, your echo replies in a quieter voice: ”Hello!”. Another second later, you might hear a second reply, even quieter than the first.

- Audio signal processing and acoustics: Echo is a reflection of sound that arrives at the listener with a delay after the direct sound.

- The delay is directly proportional to the distance of the reflecting surface from the source and the listener.

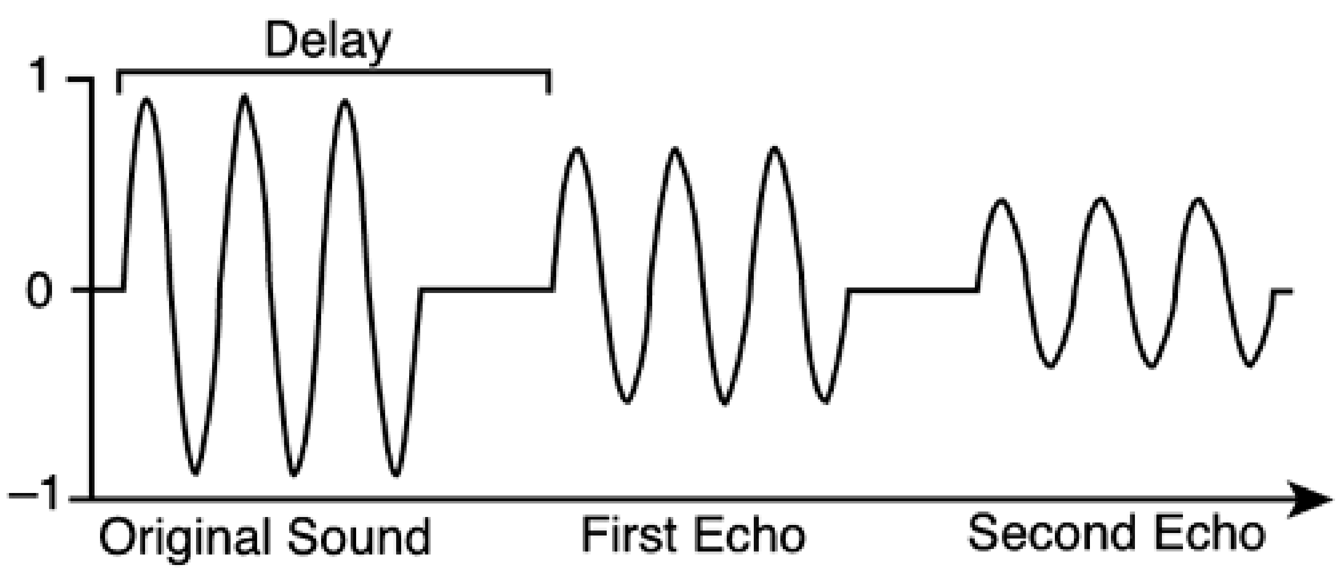
• Two elements of an echo:

delay and decay.

• The delay is how long it takes for the echo to occur.

• The decay is how much quieter the echo was compared to the original sound.

• In the case of you shouting on top of the mountain, the delay is about one second, and the decay is probably less than 50 percent.



ECHO TEMPLATES

Echoing GW Signal (CIE Template)

$$\psi(t) = \psi_{BH}(t) + \psi_{echo}(t)$$

$$\psi(t) = A\, e^{-\frac{t}{\tau}} \cos\left(2\pi f t + \phi\right) +$$

$$\sum_{n=1}^{\tilde{N}} (-1)^n A_n e^{-\frac{x_n^2}{2\beta_n^2}} \cos\left(2\pi f_n x_n\right)$$

$$x_n = t - t_{echo} - n\Delta t_{echo}$$

$\tilde{N}$ is number of echoes.

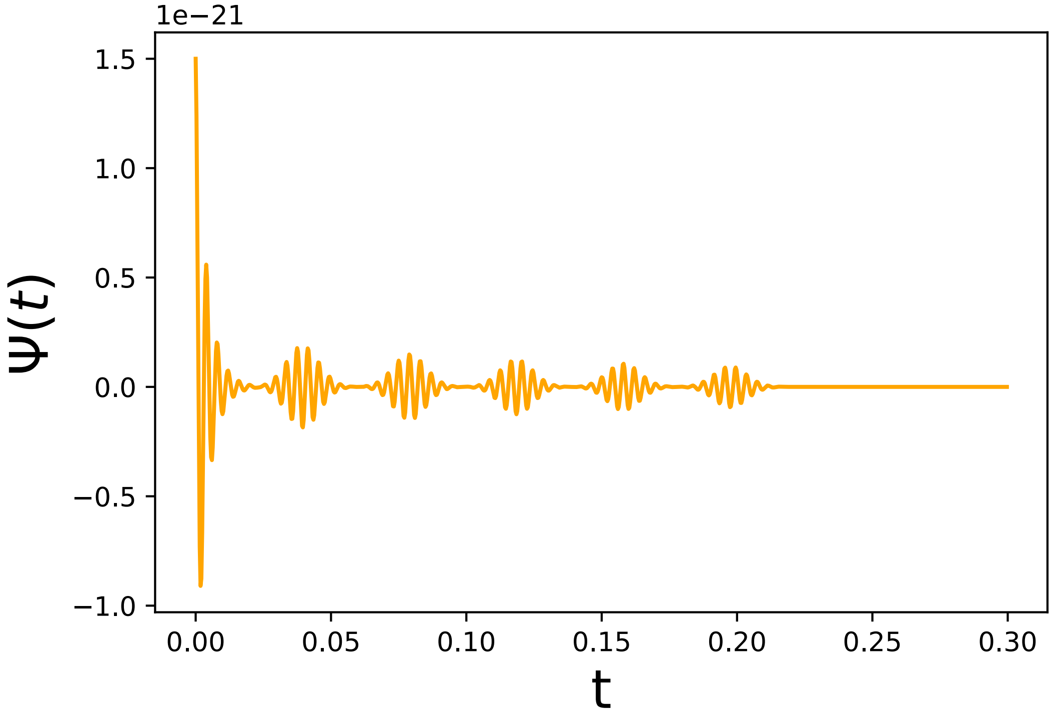
Several templates exist:

→ CIE (constant interval echoes)

→ UIE (unequal interval echoes).

For discussion on templates see Wang et. al (2019).

Example: **Choose** $\beta_n = \beta$**,** $A_n = \mathcal{A}\frac{A}{3+n}$**,** $f_n = f$ $\tau = 4 \times 10^{-3}$**s,** $f = 250Hz$**,** $\phi = 0$**,** $A = 1.5 \times 10^{-21}$**,** $\mathcal{A} = 0.5$**,** $\beta = 0.006$**s,** $\Delta t_{echo} = t_{echo} = 0.0295$



→ **Distinct demo of CI echoes above.**

→ **Other templates: ADA (see Abedi and Afshordi (2020), Review.)**

→ **Forced oscillator with specific forcing produces echoes.**

MODELS WITH ECHOES

• When can echoes occur in GW physics? (Cardoso, Franzin, Pani 2016)

→ Scalar wave eqn, $\phi = \sum_{l,m} Y_{lm} \frac{\psi_{lm}}{r}$

→ Equation for $\psi_{lm}(r)$:

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_l(r) \right] \psi_{lm} = 0$$

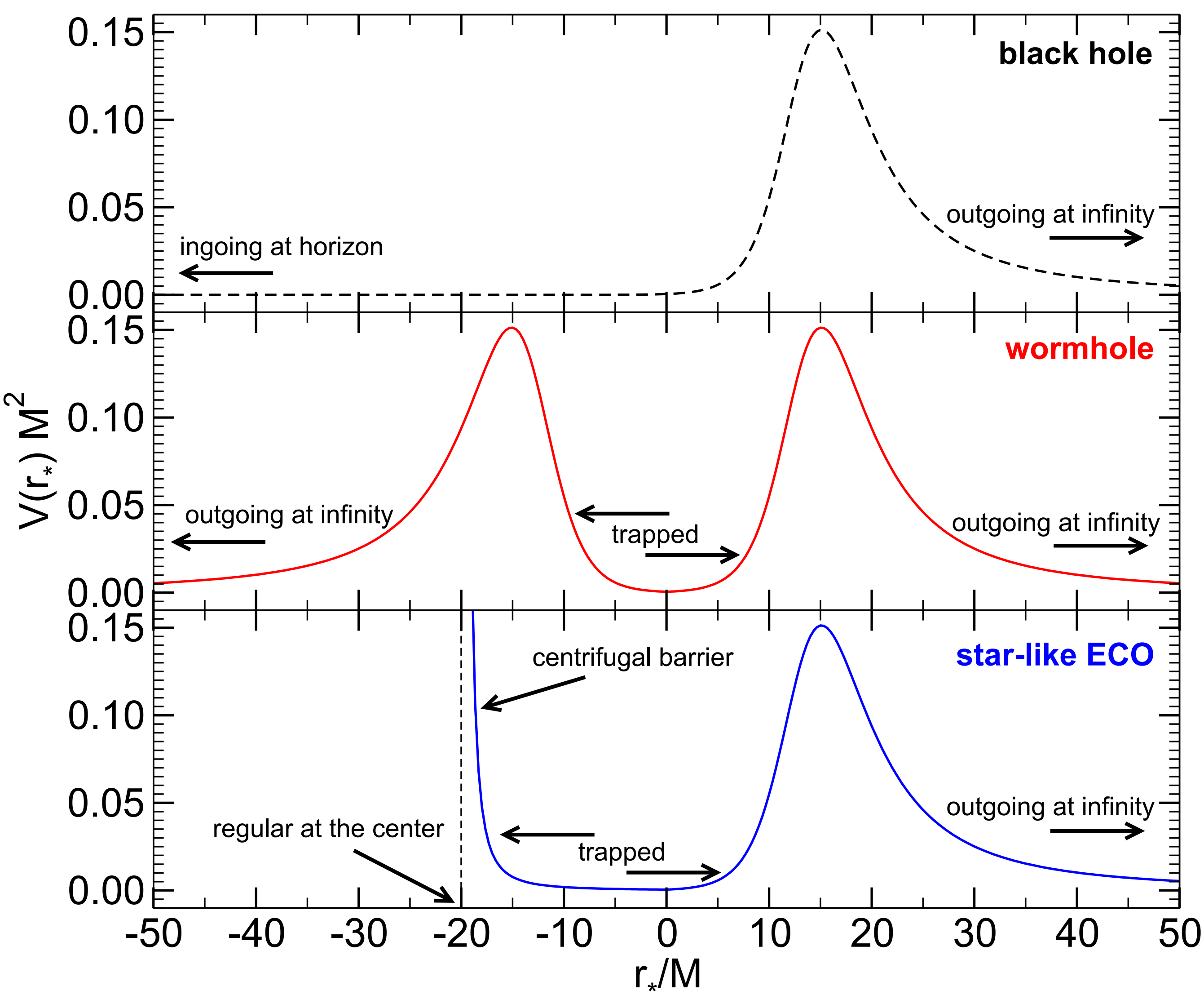
where $\frac{dr}{dr_*} = \sqrt{FB}$, $-g_{00} = F, g_{rr} = B^{-1}$

• Potentials of three types:

→ Black holes, wormholes, star like ECOs

→ Note double barrier nature for wormholes, ECOs

- **Potential features:**

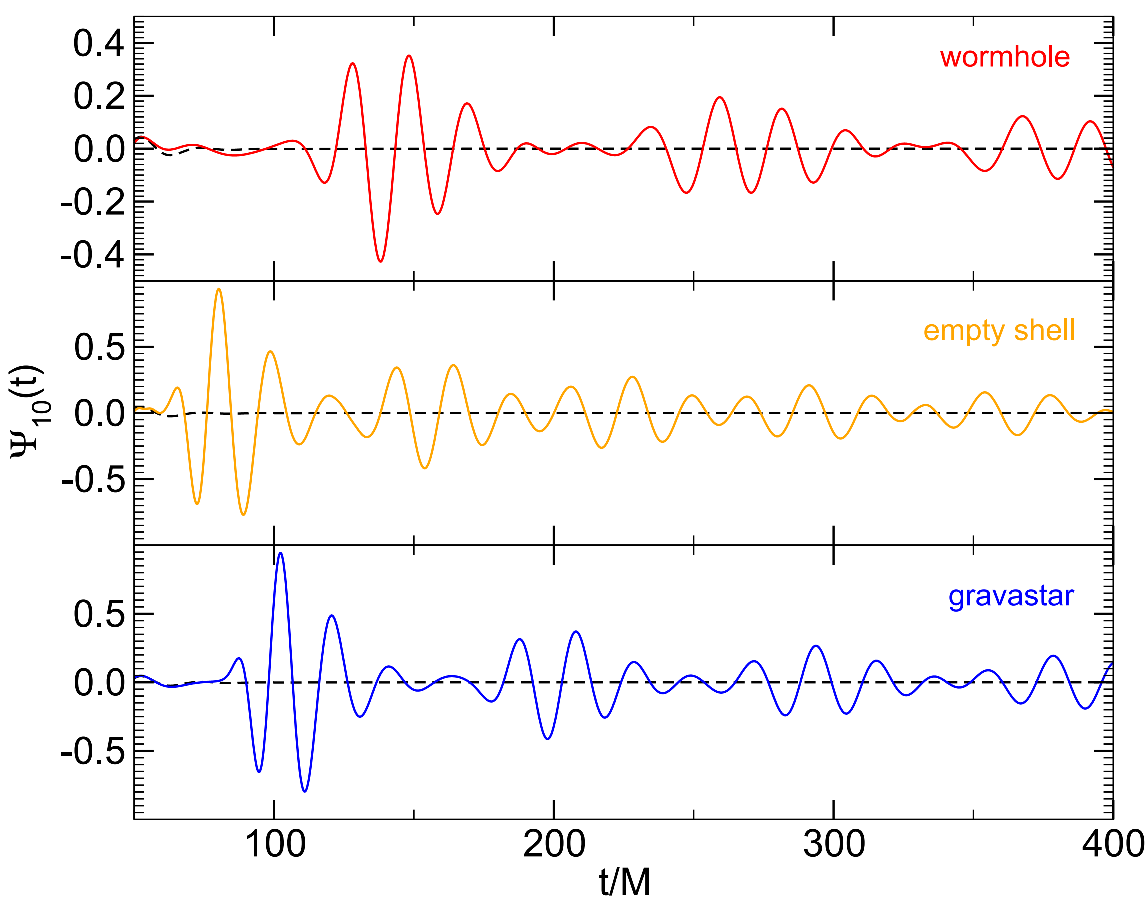


- **Typical solutions:**

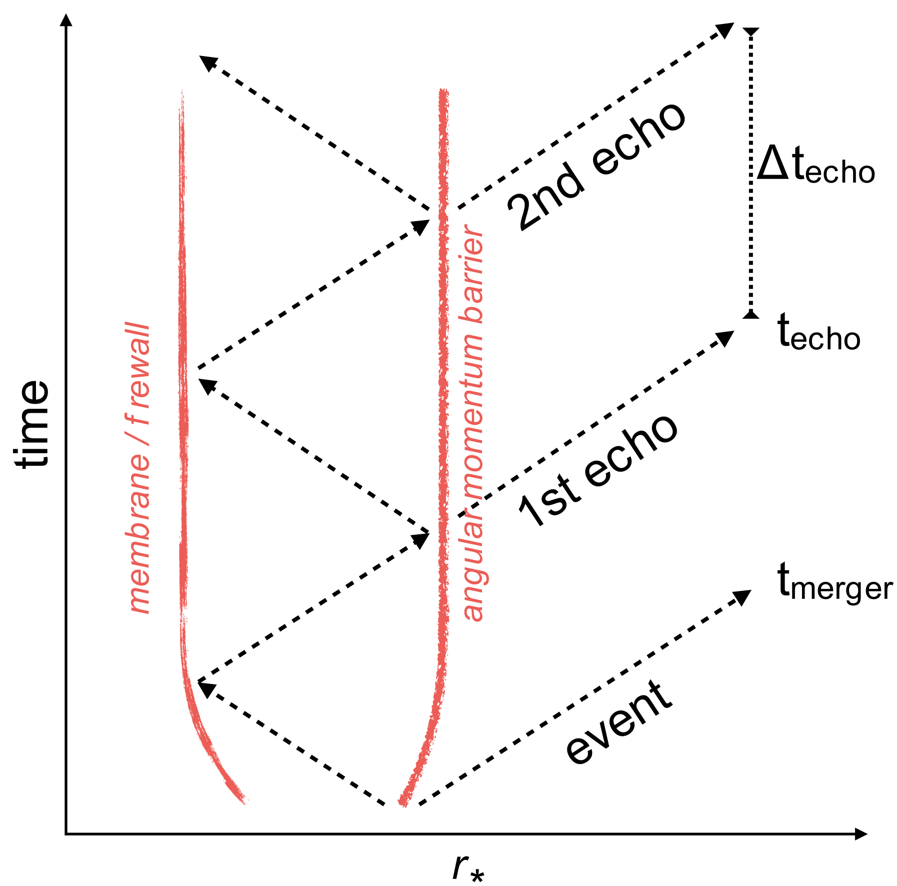
$\rightarrow$ **Solve using initial conditions:**

$\rightarrow \frac{\partial \psi_{lm}}{\partial t}(0, r) = e^{-\frac{(r_* - r_g)^2}{\sigma^2}}$, $\psi_{lm}(0, r) = 0$.

$\rightarrow$ **Waveform profile (in time) using** $r_g = 10M$, $\sigma = 6M$



Multiple reflections in a lossy cavity.



t_{echo}, Δt_{echo} is measured from time-domain profiles.

Double barrier nature of potential crucial.

BETTER UNDERSTANDING?

• How to understand the origin of echoes?

→ The basic eqn is:

$$\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial x^2}-V(x)\right]\psi=0$$

→ Laplace transform to $\bar{\psi}(\omega,x)$ to get:

$$\frac{d^2\bar{\psi}}{dx^2}+(\omega^2-V(x))\bar{\psi}=I(\omega,x)$$

$I=i\omega\psi_0-\dot{\psi}_0$, $\psi_0=\psi(0,x)$, $\dot{\psi}_0=\partial_t\psi(0,x)$.

→ One can solve this using a purely Green's function approach or employ a Dyson series. Final series solution inverted back using the inverse transform.

→ The crucial point is to implement the right boundary conditions: pure outgoing (two sides of a double barrier) or reflection at a mirror say at $x=-L$ and a membrane at $x=0$ (Delta fn.).

→ Dyson series approach

Analytically individualize each echo waveform and show that it can be written as a Dyson series, for arbitrary effective potential and boundary conditions. Apply the formalism to explicitly determine the echoes of a simple toy model: the Dirac delta potential. Correia, Cardoso (2018)

→ Green's function approach

Over short time scales evolution is governed by the quasinormal frequencies of the individual potentials, while the sensitivity to global structure can be understood in terms of echoes. An echo expansion of the Green's function shows, as expected on general grounds, at any finite time, causality limits the number of echoes that can contribute. Hui, Kabat, Wong (2019)

Mostly toy models worked out in some detail

OBSERVATIONS?

• Echoes will play a decisive role in finding out is BH mimickers exist.

• Membrane near horizon

$\longrightarrow$ The ingoing modes of the ringdownreflect back from the membrane (e.g., fuzzball or firewall) near horizon and pass back through the potential barrier.

$\longrightarrow$ Part of the wave goes to infinity with a time delay. We call this the 1st echo.

$\longrightarrow$ This time delay corresponds to twice the tortoise coordinate distance between the peak of the angular momentum barrier (r_{max}) and the membrane (which diverges logarithmically if the membrane approaches the horizon).

$\longrightarrow$ The remaining part of the 1st echo returns back towards the membrane and the process repeats itself. ($\Delta t_{echo} \sim 8M \ln \frac{M}{l_p}$).

Abedi et. al (2017): Predicted

$$\Delta t_{echo}(sec) = 0.2925 \pm 0.00916 (GW150914)$$
$$\Delta t_{echo}(sec) = 0.1013 \pm 0.01152 (GW151226)$$
$$\Delta t_{echo}(sec) = 0.1778 \pm 0.02789 (LVT151012)$$

Abedi et. al (2017): Best fit (1σ)

$$\Delta t_{echo}(sec) = 0.30068 (GW150914)$$

$$\Delta t_{echo}(sec) = 0.09758 (GW151226)$$

$$\Delta t_{echo}(sec) = 0.19043 (LVT151012)$$

Criticism by Ashton et.al (2017)

Nothing confirmed yet.

Update: Abedi et al (2020)

Echoes in GW190425??

MEMORY

EARLY WORK

• Zel'dovich, Polnarev (1974): Radiation of gravitational waves by a cluster of superdense stars.

.... the value of h_{ik} after the encounter of two objects differs from the value before the encounter. As a result the distance between a pair of free bodies should change, and in principle this effect might possible serve as a non-resonance detector.

• Braginskii, Grishchuk (1985): Kinematic resonance and memory effect in free-mass gravitational antenna.

First coined the term memory effect in this paper. Provided a very simple analysis.

Braginskii, Grishchuk (continued)

- **Weak GW (along x):**

$$ds^2 = -dt^2 + dx^2 + (1 + a(u))\, dy^2 + (1 - a(u))\, dz^2 - 2b(u)dydz$$

Particle 1 at origin, proper time w.r.t. 1, $(x, y, z) = (l_1, l_2, l_3)$ initial position of Particle 2. Deviation measured is via (x,y,z).

- **Deviation eqns.:**

$$\ddot{x} = 0$$
$$\ddot{y} = -\frac{1}{2}\left(\ddot{a}y + \ddot{b}z\right)$$
$$\ddot{z} = -\frac{1}{2}\left(\ddot{b}y - \ddot{a}z\right)$$

Total displacement $\Delta l << l$.

$\ddot{a}$, $\ddot{b}$ **and ξ all small. Also $y = l_2(1 + \xi(u))$, $z = l_3(1 + \xi(u))$.**

y, z deviation eqns. reduce to eqns like:

$\ddot{\xi} = F(u)$ (Forced eqn.).

- Memory effect:

$\longrightarrow$ Forcing over a small interval only. Pulse.

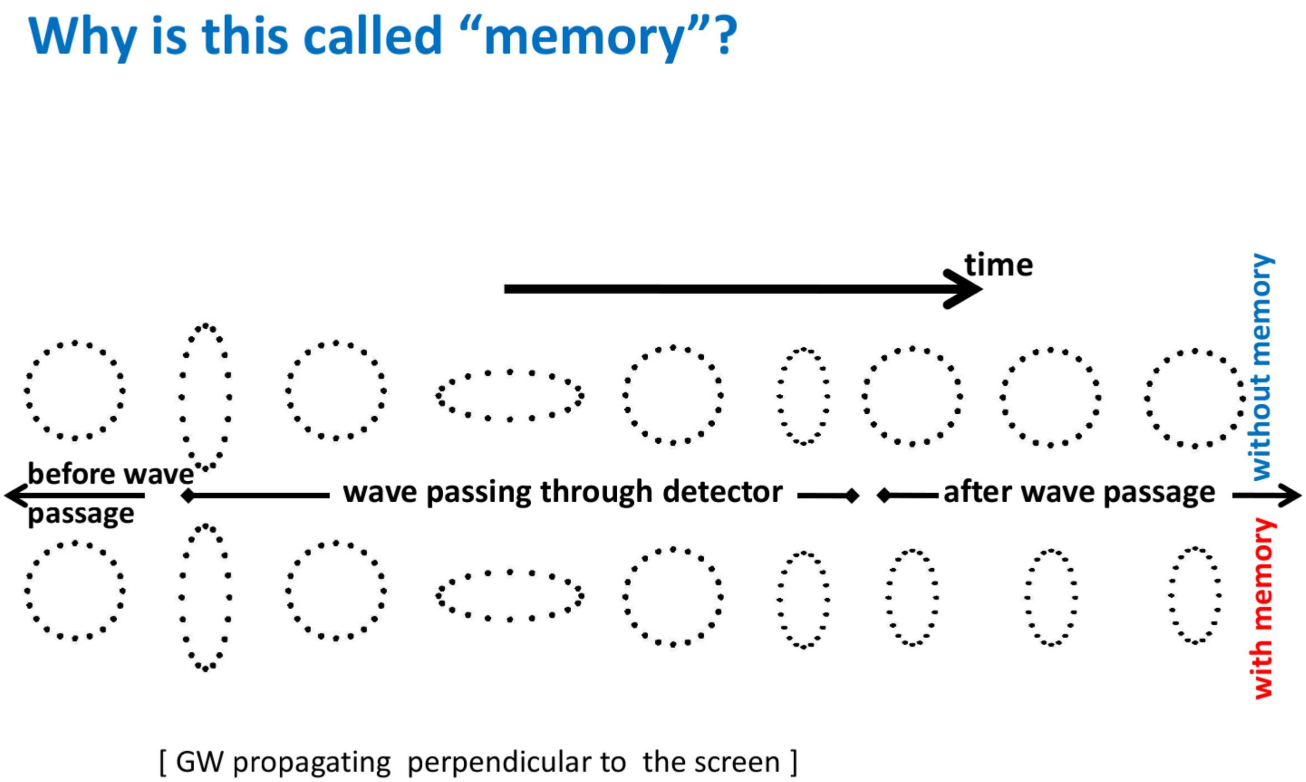
Integrate forced eqn. to get:

$$\xi_2 - \xi_1 = \int_{u_1}^{u_2} \left(\int_{u_1}^{T} F(\tau) d\tau \right) dT + v_1(u_2 - u_1)$$

$\xi_2 - \xi_1 \neq 0$ for $u > \tau$ (beyond the u value where the forcing stops). Separation between geodesics changes in region where there is no pulse.

Change in separation due to a pulse, when initial separation is, say, zero, is memory.

SHEAR AND GW MEMORY



Permanent shear! Memory effect for a ring of particles. Figure from a talk by Marc Favata.

- May actually be seen in future.

BRAGINSKII-THORNE, CHRISTODOULOU

- Change in metric perturbation related to change in separation between geodesics.

$$\Delta\xi^i = \frac{1}{2}\Delta h_{ij}^{TT}\xi^j$$

$$\Delta h_{ij} = \lim_{t\to\infty} h_{ij} - \lim_{t\to-\infty} h_{ij}$$

$$\bar{h}_{ij} \propto \ddot{D}_{ij}$$
$$\Rightarrow \Delta\xi^i \propto \Delta\ddot{D}^i_j$$

- For energy-momentum of a system of n particles one can find h_{ij}^{TT} and Δh_{ij}^{TT}. This gives the → Braginskii–Thorne formula.

- Christodoulou generalised this to full non-linear GR. Memory carried by GWs to null infinity. Effect of gravitons.

EXACT PLANE WAVES (Zhang et al (2017)

- Toy model, exact plane gravitational wave metric:

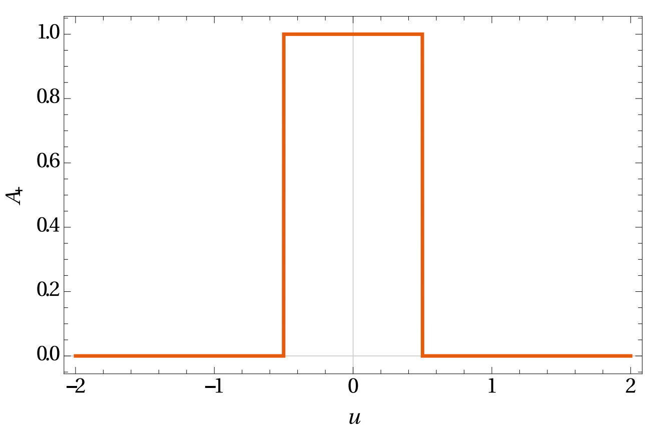
$$ds^2 = dx^2 + dy^2 + 2dudv + \left(\frac{1}{2}A_+(u)(x^2 - y^2)\right) du^2$$

- Vacuum solution of Einstein equations.

- u =constant surfaces are planes which propagate as the waves.

- Riemann tensor not zero everywhere.

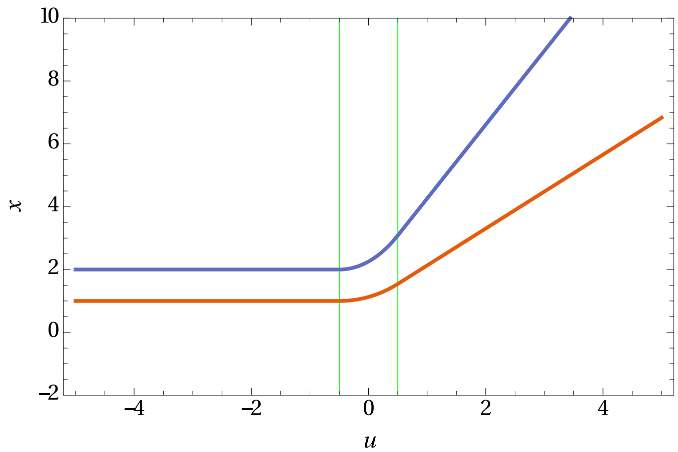
- Choice of $A_+(u)$ is arbitrary. Choose $A_+(u)$ as a square pulse. Write down geodesic equations and solve.



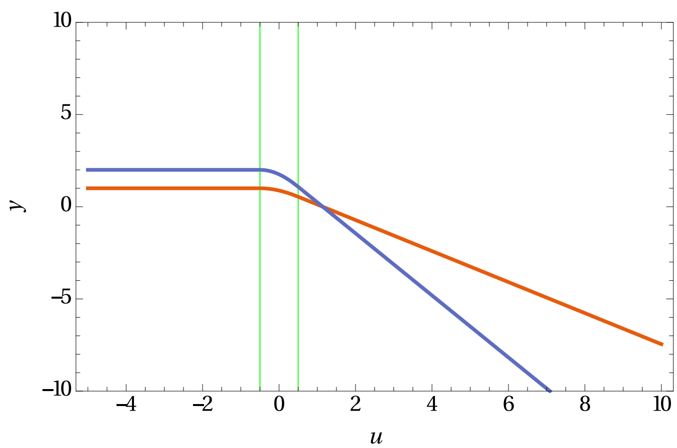
Square pulse

DISPLACEMENT MEMORY

• Nature of separation between trajectories, before the pulse arrives and after it departs.



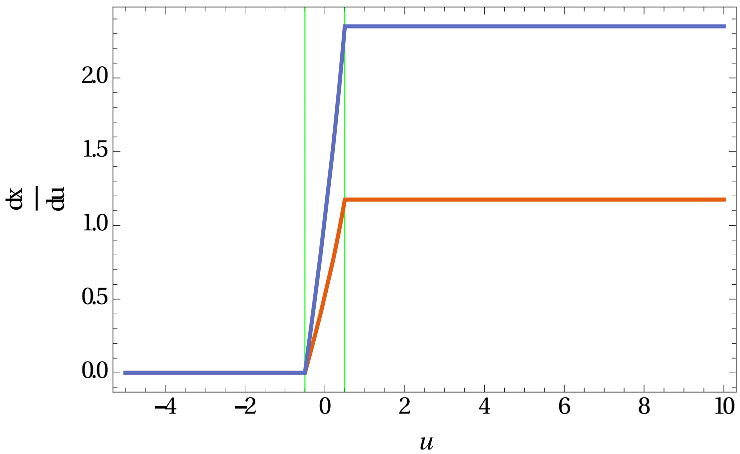
x-coordinate



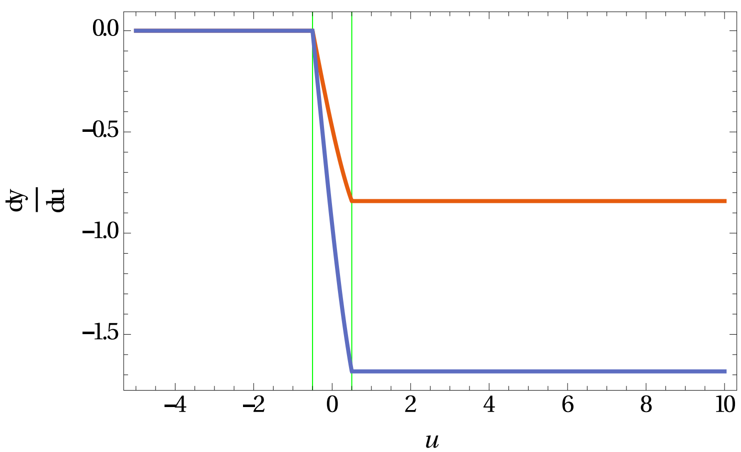
y-coordinate

• Note change in relative separation, caused by pulse. Memory.

VELOCITY MEMORY



Velocity along x.



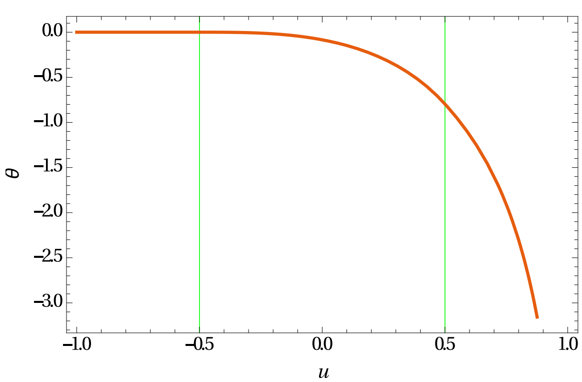
Velocity along y.

• Note jump in velocity, caused by a pulse. Memory!

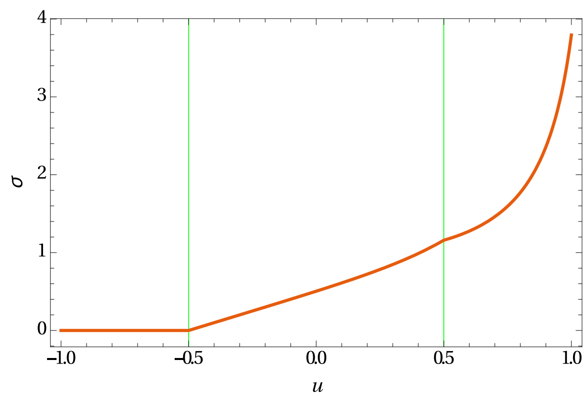
$\mathcal{B}$-MEMORY (O'Loughlin, Demirchian (2019))

• Look at a geodesic congruence. Find how $\nabla_i v_j = B_{ji}$ evolves. Covariant defn. of memory?

• Initial congruence with zero θ, σ, ω (before the pulse arrives). What is θ, σ after the pulse leaves?



Expansion(square pulse)



Shear(square pulse)

•Focusing, plus a large, non-constant growing shear! (I. Chakraborty, SK (2019)).

Fourier mode of square pulse centred around $\omega \sim 0$(soft graviton??)

MEMORY IN KUNDT SPACETIMES

- **Special case of Kundt spacetimes:**

$$ds^2 = P^2(x, y)\left[dx^2 + dy^2\right] + 2dudv + \left(\frac{1}{2}A_+(u)(x^2 - y^2)\right)du^2$$

- **Result: For a constant negative curvature wavefront ($P(x, y) = \frac{1}{y}$), we get a distinct constant shift displacement memory effect.**

- **Note that before arrival and after departure of the pulse, the spacetimes are not asymptotically flat.**

Largely of theoretical interest.

Ongoing work IC, SK (2020)

OTHER WORK

• Relation between memory, soft theorems and BMS group symmetry at null infinity. (Strominger, Sen,Laddha and others)

• Electromagnetic memory effects, memory for other particles as as well. Transverse kick. (Bieri and Garfinkle).

• Recently discovered logarithmic terms in the soft graviton theorem induce a late time component in the gravitational waveform that falls off as inverse power of time, producing a tail term to the linear memory effect. (Laddha, Sen (2019)

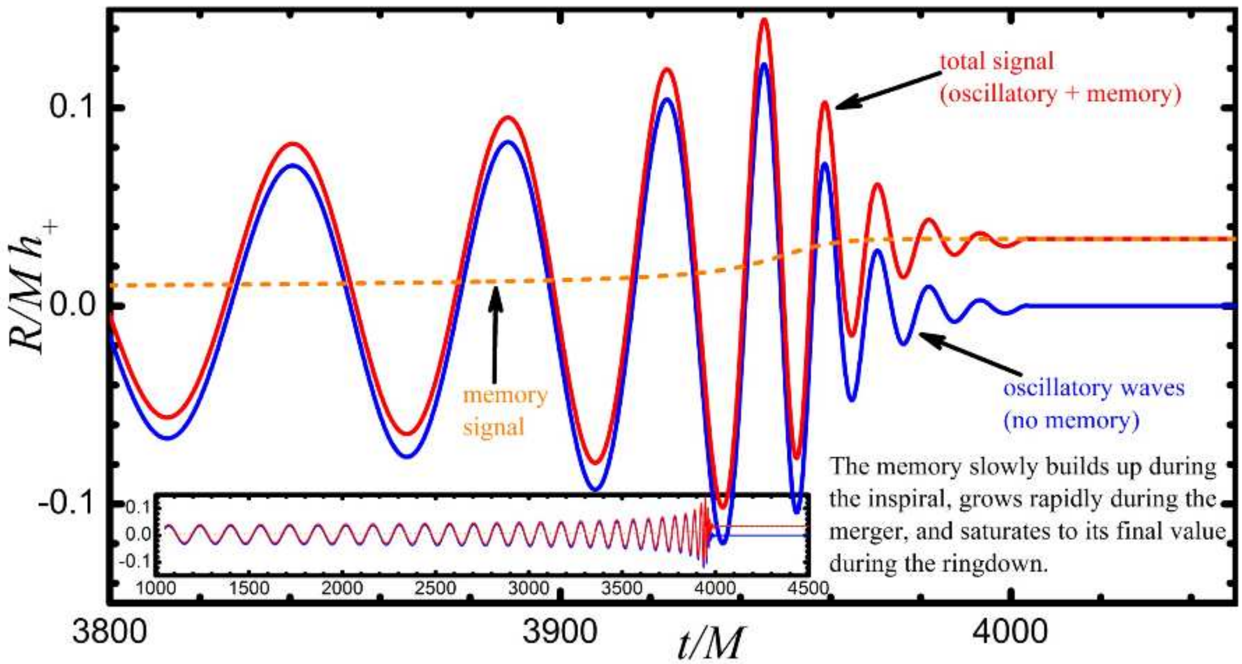
• Persistent observables for memory, work by Flanagan, Grant, Harte, Nichols (2019).

• Black Hole memory (Rahman and Wald (2019)).

.... and many more....

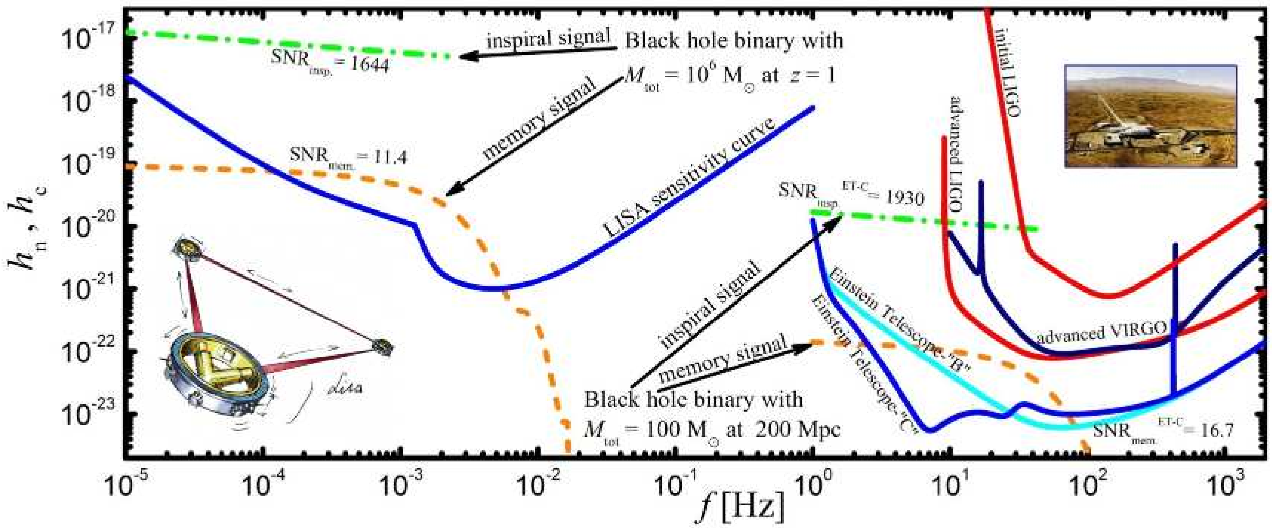
DETECTIONS

- **The expected signal:**



- **Overall tiny DC effect.**

• **Possible detections:**



• **Two cases: (1) BH Binary with** $M_{tot} = 10^6 M_{sun}$ **at** $z = 1$**; (2) BH Binary with** $M_{tot} = 100 M_{Sun}$ **at 200 Mpc.**

• **For (2), Strain Amplitude is** 10^{-23} **(detector arm** 10^3 **to** 10^4 **m) and change in length due to memory signal is** 10^{-19} **m.**

CONCLUSIONS

- Shadows may provide a direct image of an astrophysical object: ‘seeing’. Eg. supermassive BH in M87. Proof of existence very direct. But what we are really seeing must be understood well.

- Echoes specific to exotic objects like wormholes, gravastars. Quantum effects at horizons can also yield echoes. However, this is indirect evidence through echoing time domain profiles and their properties.

- Memory a crucial new phenomenon. Detecting memory amounts to detecting a soft graviton. Inequivalent Minkoswki spacetimes related through BMS transformations at null infinity.

Hopefully many new results forthcoming, maybe this year??

A LIGHTER(?) SIDE!

Shadow, Echo, Memory is a collection of nineteenth, twentieth, and twenty-first century music arranged and written for cello ensemble. ... All together, this varied group of compositions explores the cello's power to sing, to express textures of light and dark, to bring to life sounds and images from another time, and to aid a listener in revisiting their own history.