String Cosmology

Anshuman Maharana

Harish-Chandra Research Institute

Chennai, January 2020

Introduction

String theory is a candidate theory for providing us with quantum theory of gravity. There is growing evidence for this from calculations of perturbative amplitudes and study of blackhole mechanics.

How do string theory and cosmology come together ?

- Big Bang Singularity
- Connect with the Standard Model of Cosmology

Outline

- Dimensional Reduction, Flux Compactifications and Effective Actions.

- Examples:

    - Inflation

    - Moduli dynamics and inflationary predictions

    - Quintessence, Fifth forces

Moduli

$$S_{IIB} \supset \frac{1}{2\kappa_{10}^2} \int d^{10}x \left(\mathcal{R} - \frac{\partial_M \tau \partial^M \tau}{2 \, \mathrm{Im}\tau} + \frac{G_{MNP} G^{MNP}}{12 \mathrm{Im}\tau} + ... \right)$$

simplest compactifications involve backgrounds where all matter fields have vanishing expectation value

$$g^{(10)}_{MN} dX^M dX^N = g_{\mu\nu}(x) dx^\mu dx^\nu + g^{(6)}_{mn} dy^m dy^n$$

$R_{mn} = 0$, $g_{mn} \to (1 + \lambda) g_{mn}$ maps solutions to solutions . Volume $(\mathcal{V})$ of the compactification a free parameter.

- Make λ a dynamical field in four dimensions, EOM

$$\Box \lambda(x) = 0$$

- $\lambda(x)$ a massless scalar which mediates a Planck suppressed fifth force between all matter.

- Moduli continuous deformations of the background solution which map solutions to solutions. Coordinates on the solution space. Thus moduli fields specify the background one is considering.
- Natural to expect that the low energy effective action that gets for the four dimensional observer depends on the background that one is considering.
- Couplings and scales in the four dimensional effective field theory are dependent on the values of the moduli.
- While the geometry of the extra dimensions is cannot be observed directly, it reveals itself to the low energy observer via the moduli fields to the low energy observer.
- e.g. M_s mass of massive string states (scale of string physics) is

$$\frac{M_s}{^{(4)}M_{pl}} \propto \frac{1}{\sqrt{\mathcal{V}}}$$

  $\mathcal{V}$, volume of the compactification in string units
- Similarly values for gauge couplings, soft susy breaking parameters, cosmological constant dependent on values of the moduli.

- To make contact with phenomenology we need to study more complicated solutions; where the values of moduli fields are fixed.

Flux Compactifications

$$S_{IIB} \supset \frac{1}{2\kappa_{10}^2} \int d^{10}x \left(\mathcal{R} - \frac{\partial_M \tau \partial^M \tau}{2\ \mathrm{Im}\tau} + \frac{G_{MNP} G^{MNP}}{12 \mathrm{Im}\tau} - \frac{\tilde{F}_{MNPQR} \tilde{F}^{MNPQR}}{4.5!} \right)$$

Consider solutions with non trivial vevs of the flux fields (generalizations F_{MN}).

- Fluxes are quantised via Dirac quantisation conditions, these obstruct continuous deformation of solutions.
- The flux terms have depend nontrivially on the value of the metric via the inverse metric
- Simplest example only the five form and metric are turned on (Toy example of 5+5 split as opposed to 4+6)
  - $$ds_{10}^2 = L^2 ds_{AdS_5}^2 + R^2 d\Omega_5^2, \qquad \int_{S_5} F = N$$
  - $$V(R) = V_{\mathrm{Curv}} + V_{\mathrm{Flux}} = -\frac{1}{R^{16/3}} + \frac{N^2}{R^{40/3}}$$
  - $R_{\mathrm{min}} \propto N^{1/4}$, $L_{AdS} \propto R_{\mathrm{min}}$

- Value of the modulus now fixed and given in terms of integer fluxes.
- Coming back to the 4+6 split. Calabi Yau manifolds have non-trivial 3-cycles (b_3)
- One can thread three form flux along each of these cycles

$$\int_{\Sigma_a} F_3 = N_{\Sigma_a}, \qquad \int_{\Sigma_a} H_3 = M_{\Sigma_a}$$

- Explicit solutions can be found, metric takes the form of a warped product; internal manifold conformally flat

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2A(y)} g^{\mathrm{CY}}_{mn}(y) dy^m dy^n$$

- D=4, $\mathcal{N} = 1$ Supersymmetry.
- Realisation of the mechanism of Randall and Sundrum; $e^{A_{\min}} = e^{-K/M}$

- IIB Supergravity (follows from the positivity of stress tensor)

$$\nabla_6^2 e^{-4A} = \frac{G_{mnp}G^{mnp}}{48\mathrm{Im}\tau} + \frac{e^{-6A}}{4}\partial_m\alpha\partial^m\alpha$$

no No-go theorem.

- String theory has local sources which carry *negative tension* (Orientifold planes)

$$\nabla_6^2 e^{-4A} = \frac{G_{mnp}G^{mnp}}{48\mathrm{Im}\tau} + \frac{e^{-6A}}{4}\partial_m\alpha\partial^m\alpha + T^{loc}$$

- Usually, negative tension objects dangerous due to pair production making the ground state unstable. But O-planes are related to boundary conditions.
  Negative tension (intrinsic to string theory) objects play a crucial role in obtaining these solutions.

Summary of Effects of Fluxes

- All complex structure moduli (z_i) are fixed by fluxes.
- The effect of fluxes is to introduce a superpotential in the 4d effective field theory

$$W = \int G \wedge \Omega(z_i)$$

- Kahler Moduli (T_a) remain flat directions. Can fixed by making use of non-perturbative effects and higher derivative corrections in the effective action.

- As the values of the flux quanta are varied, we get four dimensional solutions with different properties (solution are characterised by the integer quanta)
- Large number of possible choices for the choice of flux quanta.
- The cosmological constant varies by a small amount as the flux quanta are changed.
- This tuning can be used to obtain de Sitter space with a small cosmological constant.
- Many aspects of these constructions remain to be understood.

- In this setting: Phenomenological requirements translated to requirements on the higher dimensional geometry.

- This often leads to constraints and correlations between observables.

Inflation and String Theory

- Simplest models of inflation involve a scalar field rolling down a potential. To get exponential expansion for a sufficiently long epoch, slow roll conditions need to be satisfied.

$$\epsilon \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V'(\varphi)}{V(\varphi)}\right)^2 \ll 1 \qquad \eta \equiv M_{\rm pl}^2 \left(\frac{V''(\varphi)}{V(\varphi)}\right) \ll 1$$

- The potential has to be flat in Planck units !

- The vacuum energy is positive. In general, scalar masses are not stable against loop corrections

$$\Delta\eta = \mathcal{O}(1)$$

- Options: having symmetries or trying to tune the potential. Whether one can actually realise this depends the properties of the UV theory.

- For the symmetries, we need to understand the fate of the symmetries in a UV complete theory, the existence of higher dimensional operators that spoil the symmetry

Moduli as Inflatons

- At tree level Moduli fields are massless.

- Going beyond tree level, moduli fields acquire masses. The potentials generated for them are often flat, moduli are good candidate inflatons.

- Moduli parametrise the shape and size of the extra dimensional geometry. Rolling modulus changing shape of extra dimensions.

- Example: Fibre inflation models, in the regime inflation takes place

$$V \simeq \frac{V_0}{\mathcal{V}^{10/3}}\left(3 - 4e^{-k\hat{\varphi}}\right) \quad \text{with} \quad k = \frac{2}{\sqrt{3}}$$

- $\hat{\varphi}$ rolls from higher to lower values. But form of potential very different for larger values of $\hat{\varphi}$ .Constraint on the number of e-foldings.

- Size of hole in a manifold cannot be bigger than the size of the object.

$$\mathcal{V} = \tau_0^{3/2} - \tau_1^{3/2}$$

Inflation, Moduli and Cosmology

- From the very early days of model building in supergravity models in was realised that

inflation + moduli fields

can lead to cosmological timeline distinct from the standard one.

modular cosmology

Cosmology and Moduli

- Starting point of the analysis moduli dynamics during inflation.

- Analysis of dynamics during inflation gives, for $m_\varphi \lesssim H_{\rm infl}$

At the end of inflation the modulus φ has VEV $\hat{\varphi}$,

$$Y = \frac{\hat{\varphi}}{M_{\rm pl}} \lesssim 1$$

Cosmology and Moduli

Thus just after reheating, energy density has two components

- Radiation: To which the inflaton has dumped its energy density.

- Modulus: Potential energy due to displacement.

- As the universe expands time average of energy density falls off as

$$\rho_{\mathrm{modulus}}(t) \propto \frac{1}{a^3(t)}$$

Cosmological evolution of cold moduli particles.

- Quickly dominates over energy density present in the form of radiation

- Modulus domination continues until decay of modulus at

$$\tau_{\mathrm{mod}} \approx \frac{16\pi M_{\mathrm{pl}}^2}{m_\varphi^3}$$

the characteristic lifetime for decay via their Planck suppressed interactions.

Modular Cosmology

Inflation

↓

Reheating

↓

Radiation Domination

↓

Modulus Domination

↓

Reheating (after modulus decay)

↓

Radiation Domination

⋮

Today

Conventional Cosmology

Inflation

↓

Reheating

↓

Radiation Domination

⋮

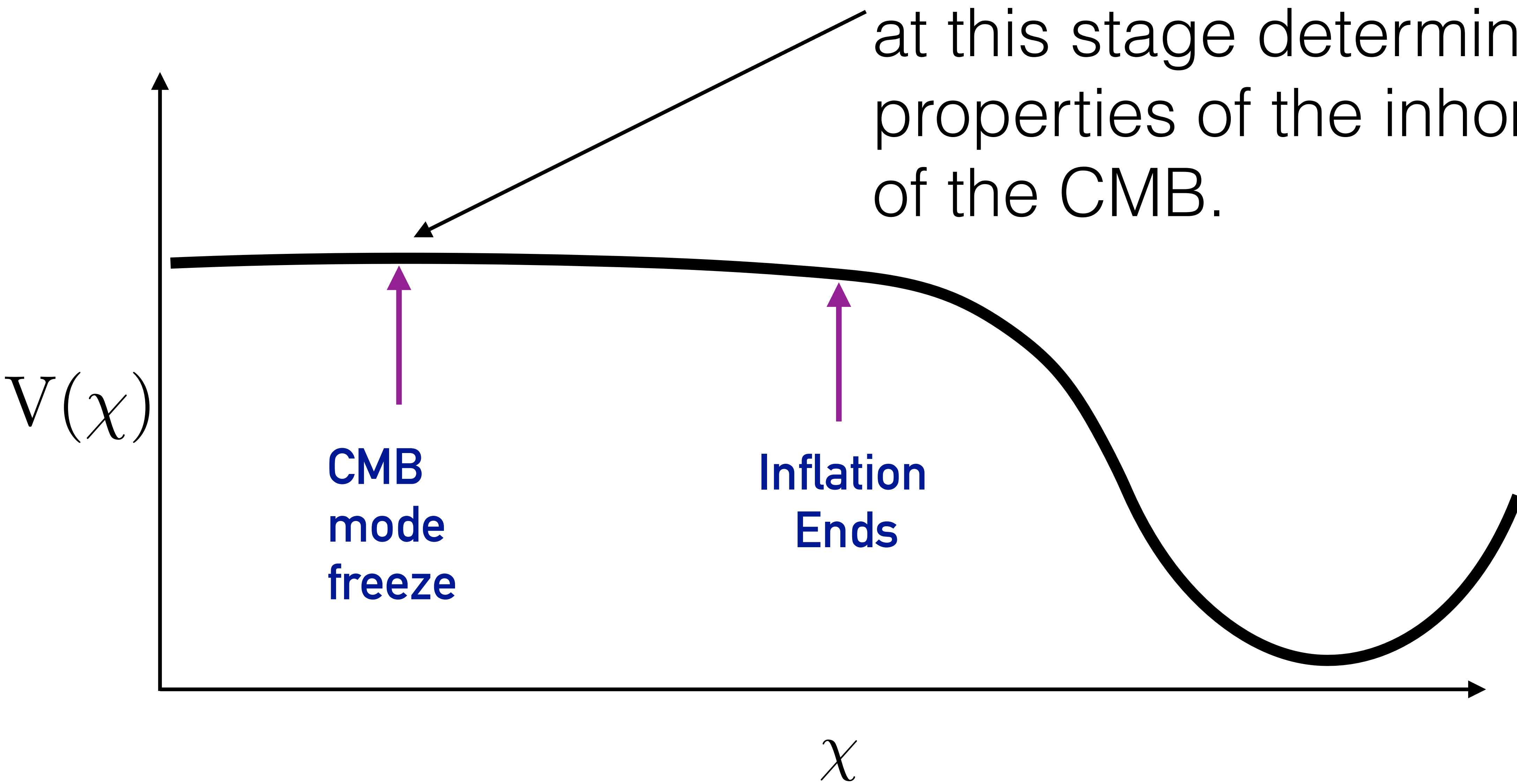
Today

Inflation and Inhomogeneities

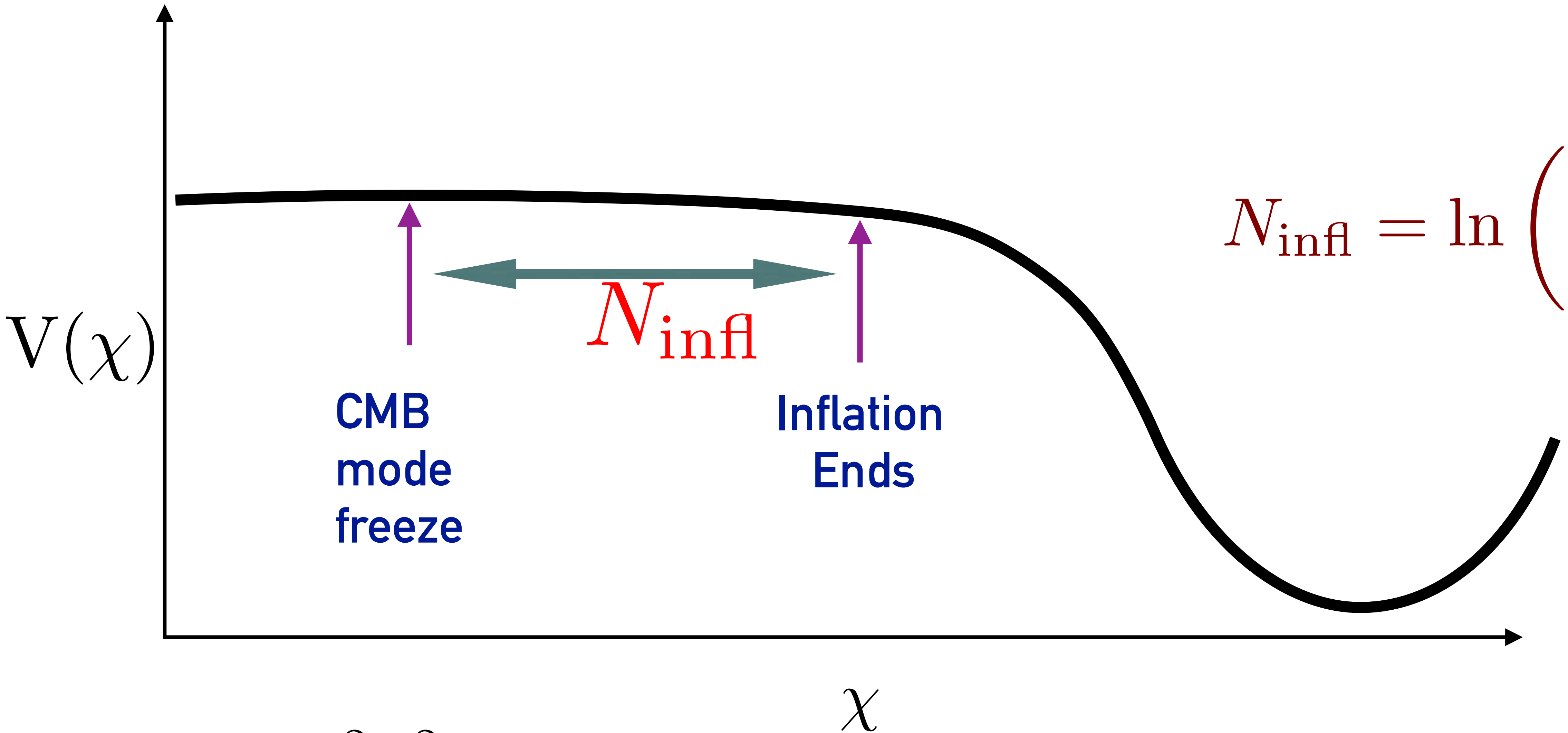
- Inhomogeneities are a result of freezing of quantum fluctuations at the time of horizon exit; $k/a \approx H$.

$k \approx 0.05\ \ \mathrm{Mpc}^{-1}$ for CMB observations by the PLANCK satellite.

Properties of the universe at this stage determine the properties of the inhomogeneities of the CMB.



It is conventional to keep track of the point of freezing by the number of e-folding between freezing and end of inflation.



$$N_{\rm infl} = \ln\left(\frac{a(t_{\rm end})}{a(t_{\rm freeze})}\right)$$

For e.g. $m^2\chi^2$ potential (similar expressions for all models)

$$n_s = 1 - 2/N \qquad r = 8/N$$

Given a potential we need the value of $N_{\rm infl}$ to extract predictions

Inflation and Inhomogeneities

- How is $N_{\rm infl}$ determined ?

Observed Inhomogeneties ⟷ INFLATION ⟷ Energy Density at an early time

- More precisely,

$$A_s = \frac{2}{3\pi^2 r}\left(\frac{\rho}{M_{\rm pl}^4}\right)$$

- ρ - Energy density of universe at the time of horizon exit of mode relevant for CMB observations.

- r - Strength of gravity waves.

Inflation, Inhomogeneities and Energy Densities

- An early time and today's energy densities known. This implies a consistency condition

Any history we ascribe must be such that the early time energy density evolves to the energy density today.

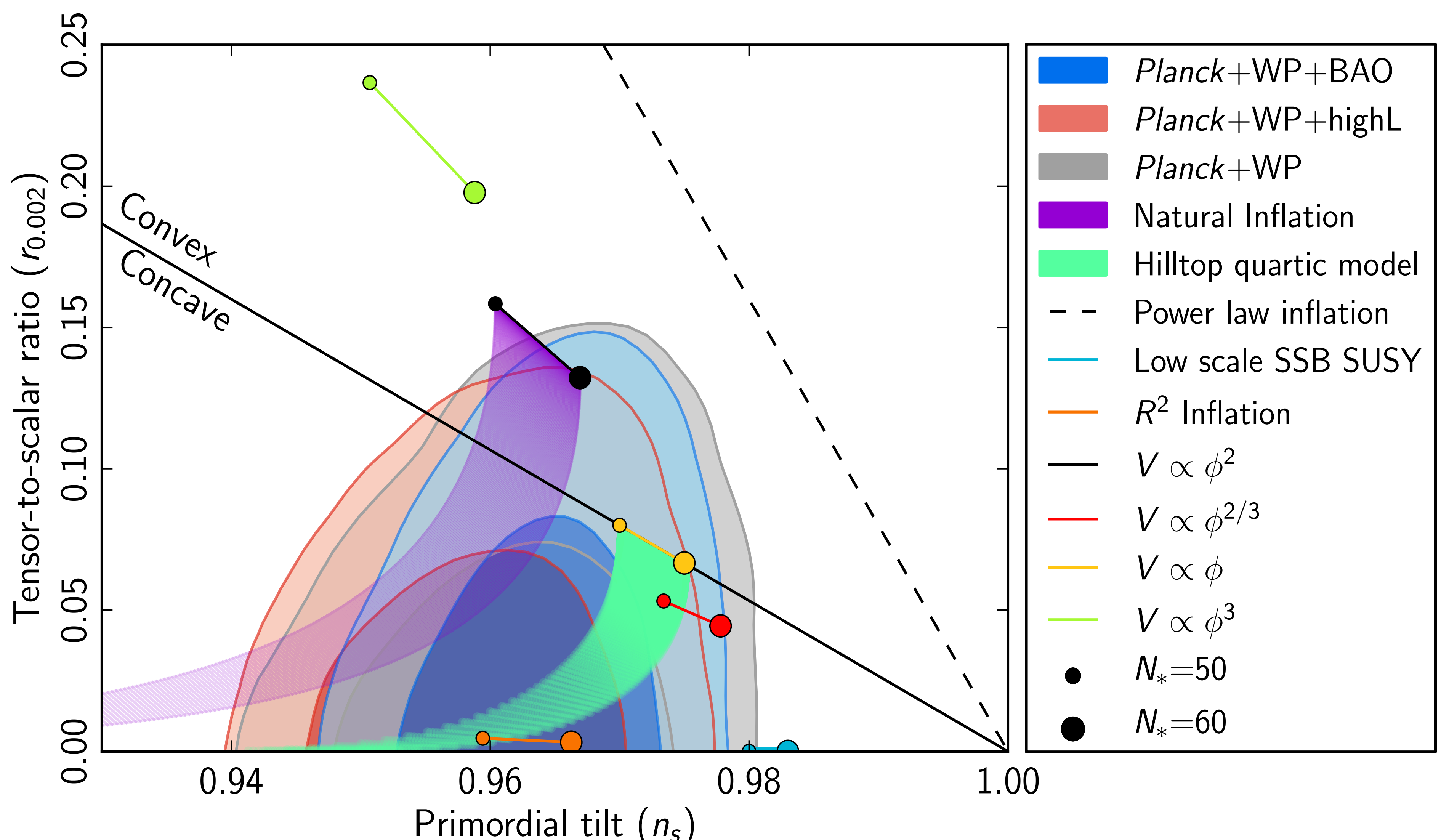
Energy density at horizon exit, ρ_k (from inhomogeneities).

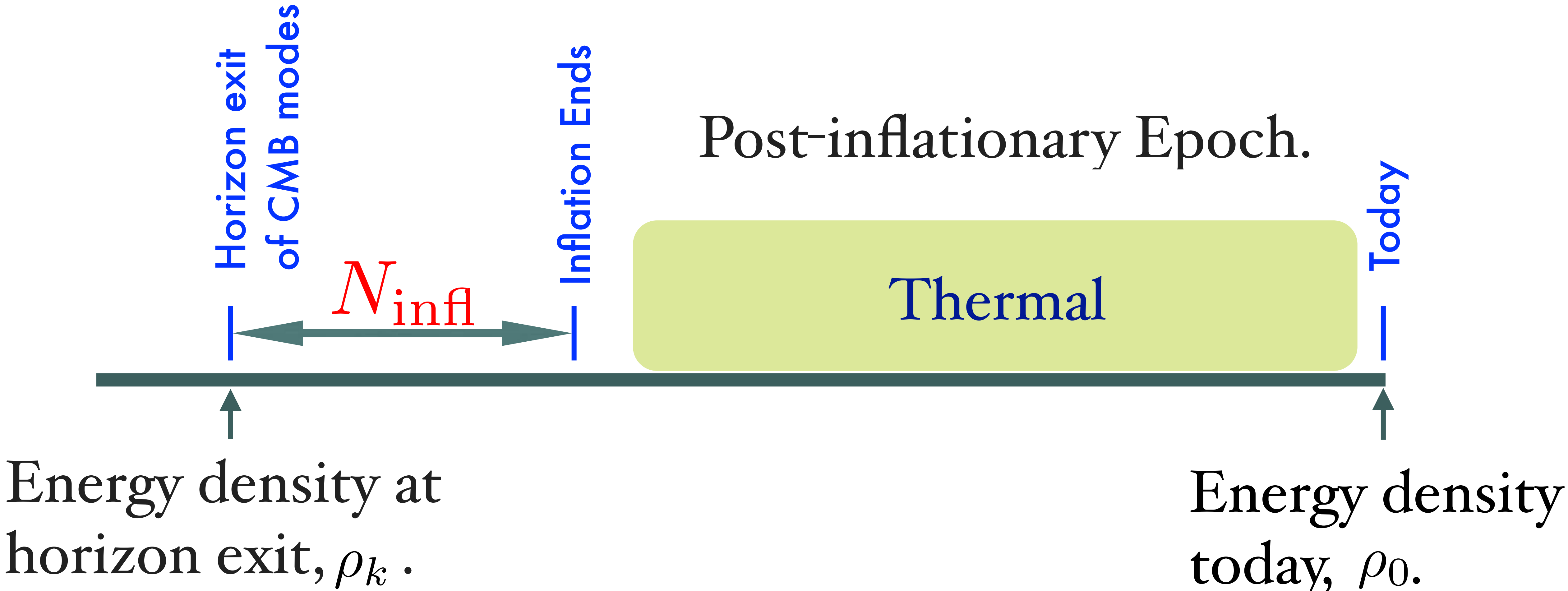
Energy density today, ρ_0

Post-inflationary Epoch consists of reheating followed by thermal history in conventional cosmologies.

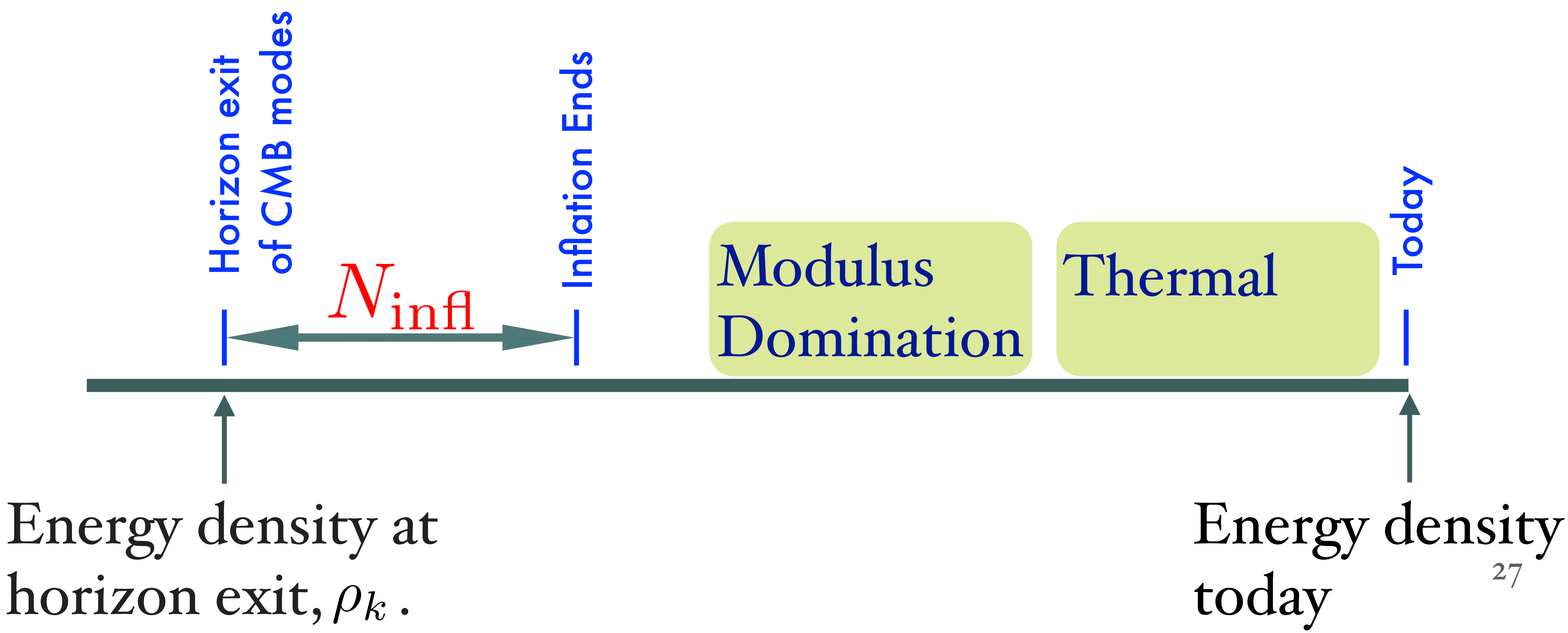
$$N_{\rm infl} + \frac{1}{4}(1 - 3w_{\rm rh})N_{\rm rh} \approx \mathbf{57} + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_{\bf k}}{\rho_{\rm end}}\right)$$

This motivates the usual range of 50-60 for $N_{\rm infl}$





Modular Cosmology



We obtain

$$N_{\rm infl} + \frac{1}{4}N_{\rm modulus} + \frac{1}{4}(1-3w_{\rm rh1})N_{\rm rh1} + \frac{1}{4}(1-3w_{\rm rh2})N_{\rm rh2} \approx \mathbf{57} + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_k}{\rho_{\rm end}}\right)$$

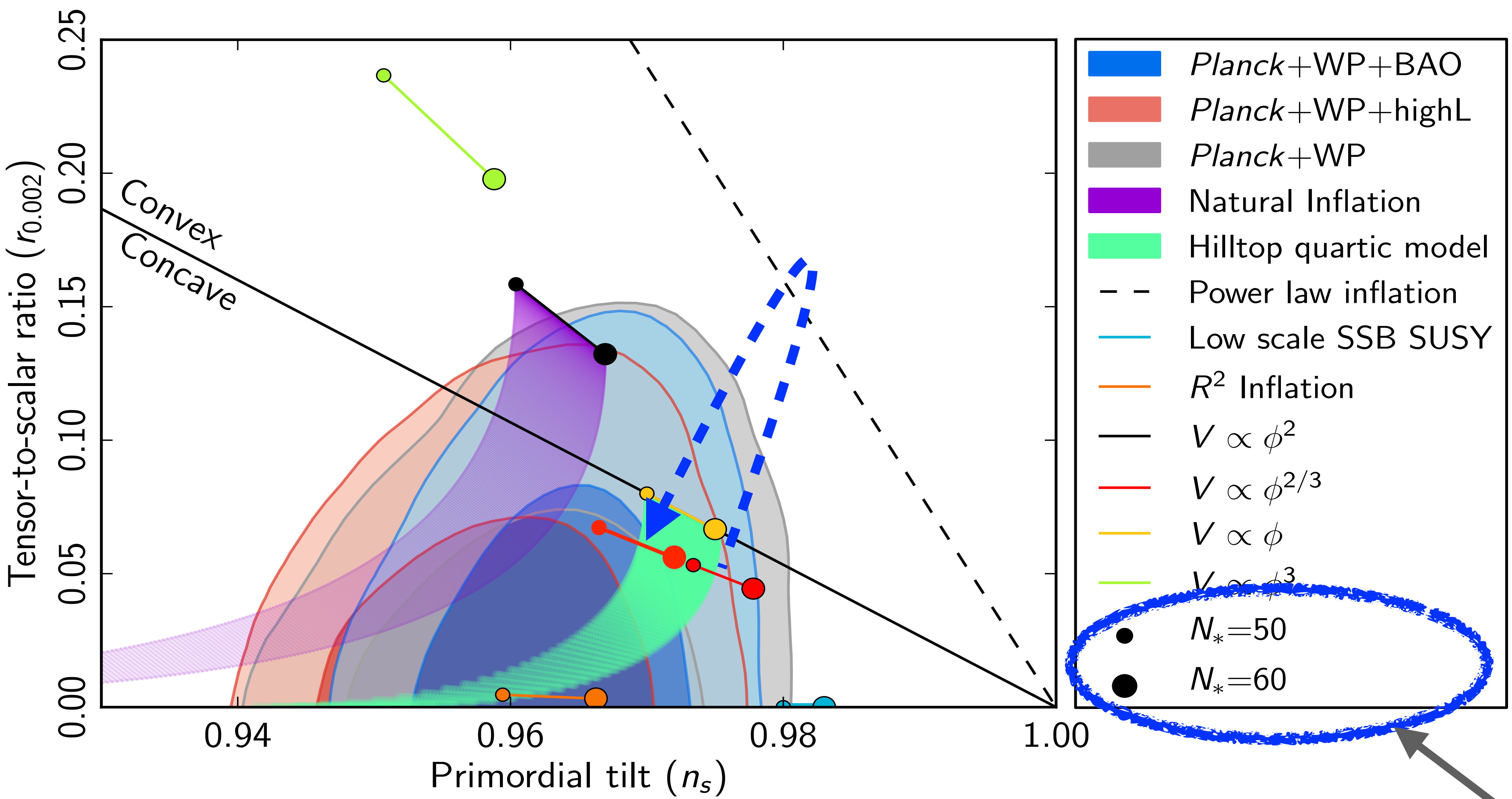
The number of e-folding during modulus domination.

$$N_{\rm modulus} \approx \frac{4}{3}\ln\left(\frac{\sqrt{16\pi}M_{\rm pl}Y^2}{m_\varphi}\right)$$

$Y = \frac{\hat{\varphi}}{M_{\rm pl}}$ The initial displacement in Planck Units (generic estimate from EFT $Y \simeq \mathcal{O}(1)$)

m_φ The post-inflationary mass of the modulus

Since the dependence is on $\ln(M_{\rm pl}/m_\varphi)$ this can significantly bring down the value of $N_{\rm infl}$.



Modulus mass input for inflationary predictions

Change the 50 - 60 range

N_{infl}

$$N_{\text{infl}} + \frac{1}{4}N_{\text{modulus}} + \frac{1}{4}(1-3w_{\text{rh1}})N_{\text{rh1}} + \frac{1}{4}(1-3w_{\text{rh2}})N_{\text{rh2}} \approx \mathbf{57} + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_k}{\rho_{\text{end}}}\right)$$

In a string compactification, one can compute

- The initial displacement of the modulus.
- The inflaton width.

Carrying this out for Kahler Moduli Inflation.

$$N_{\rm infl} \approx 45$$

Exhibits the importance of moduli dynamics for making inflationary predictions. To confront the next generation experiments we need to know $N_{\rm infl}$ with accuracy:

$$\Delta N \approx 5$$

Having an era of early matter domination also

- The initial displacement of the modulus.
- The inflaton width.

Carrying this out for Kahler Moduli Inflation.

$$N_{\rm infl} \approx 45$$

Exhibits the importance of moduli dynamics for making inflationary predictions. To confront the next generation experiments we need to know $N_{\rm infl}$ with accuracy:

$$\Delta N \approx 5$$

Modular cosmology also has implications for dark matter.

- Thermal overproduction before the epoch of early matter domination.

- Dilution upon reheating

The dilution factor is given by $\left(\frac{H(t_{\rm r})}{H(t_{\rm r})}\right)^{1/2}$ thus is directly related to the shift in $N_{\rm infl}$

- Dark radiation production from the decay of the Modulus.

Quintessence and Fifth Forces

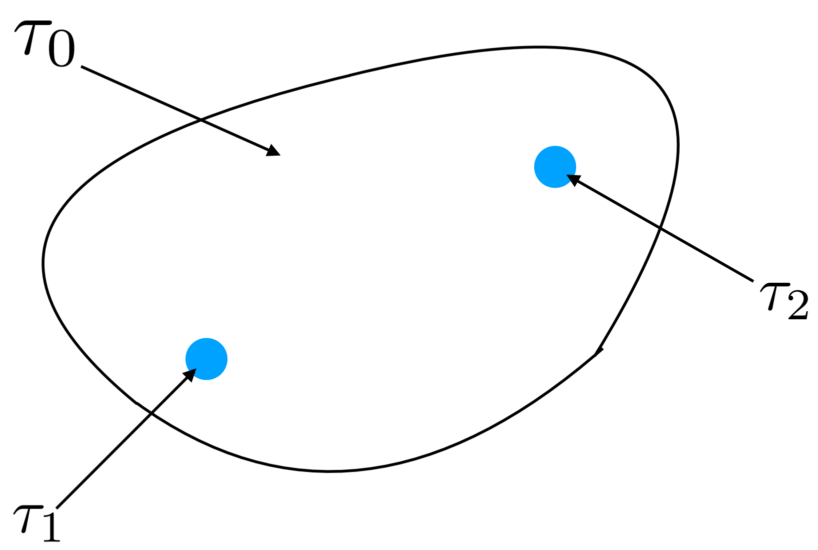
- The acceleration of the present universe is due to a slowly rolling scalar field.

- A scalar field driving quintessence has to have Compton wavelength of the order of the cosmological horizon.

- Natural candidate for this is modulus field in a hidden sector.

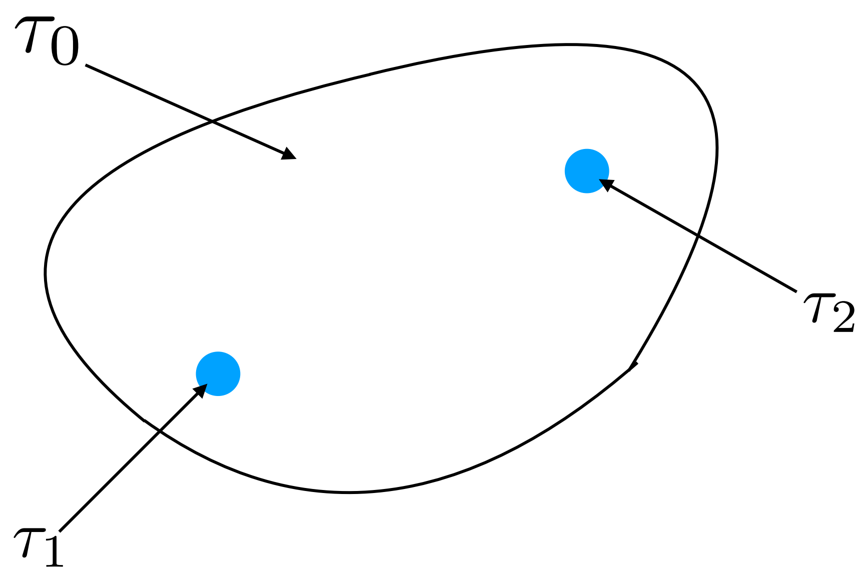
Quintessence in String Theory ?

- Natural candidate for this is modulus field in a hidden sector.
- In string compactications, this would arise from a sector that is geometrically separated from the Standard Model.
- One of the localised moduli corresponds to the quintessence field.



Quintessence in String Theory ?

- One of the localised moduli corresponds to the quintessence field.



- Interactions that arise from kinetic mixings between the sectors can be computed from the Kahler potential which is closely related to the geometry of the internal manifold.

Quintessence in String Theory ?

- Interactions that arise from kinetic mixings between the sectors can be computed from the Kahler potential which is closely related to the geometry of the internal manifold.

- For e.g. internal manifold with two point like singularities

$$K = -2\log\left(\tau_0^{3/2} - \tau_1^{3/2} - \tau_2^{3/2}\right)$$

- Fifth force bounds give

$$\mathcal{V} > 10^{12}$$

A very low string scale.

Conclusions

- In order to make contact with cosmology we require to work with solutions of string theory in which the moduli field are stabilised.

- Flux compactifications provide such as setting. Developing a detailed understanding of the effective actions associated with flux compactifications remains an area of active study.

- Moduli dynamics and inflation. Quintessence

- Properties of extra dimensional geometry, correlation between observables.