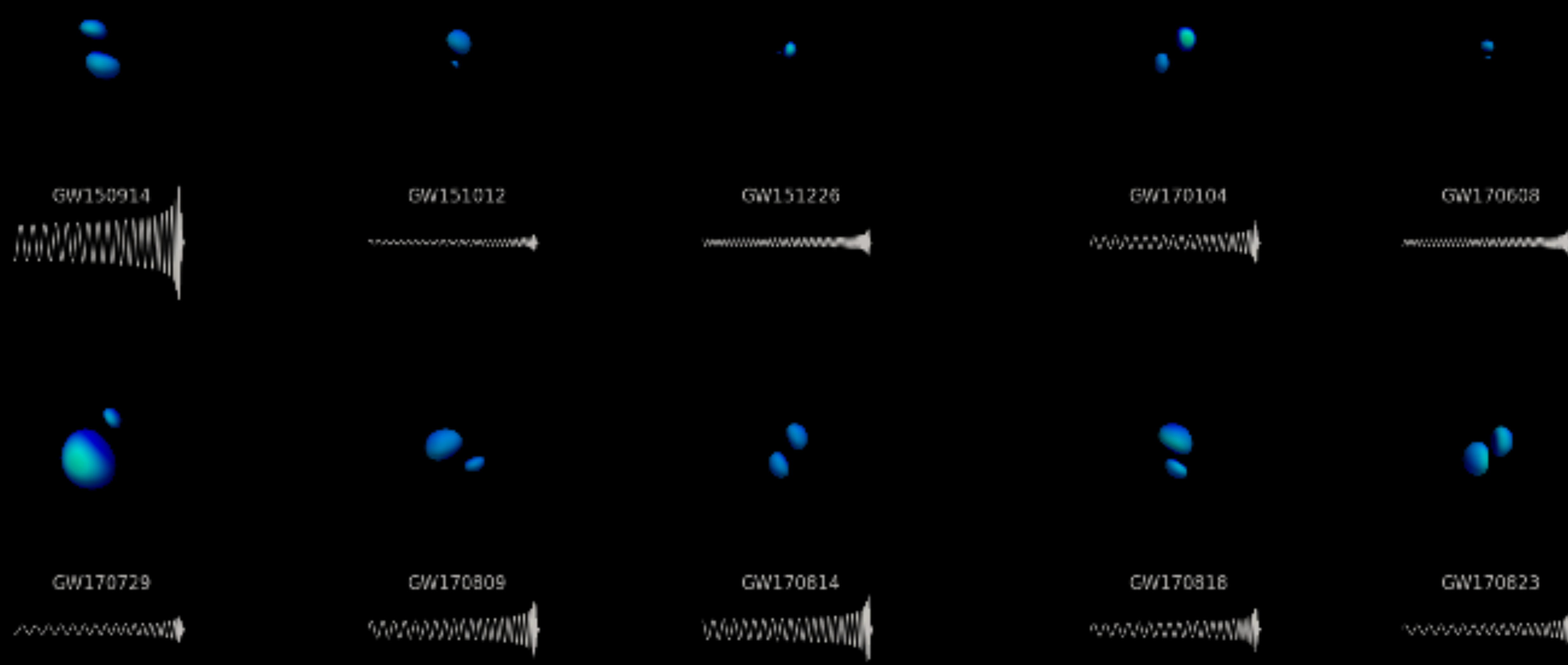
Testing GR using GWs



Chandra Kant Mishra
IIT Madras

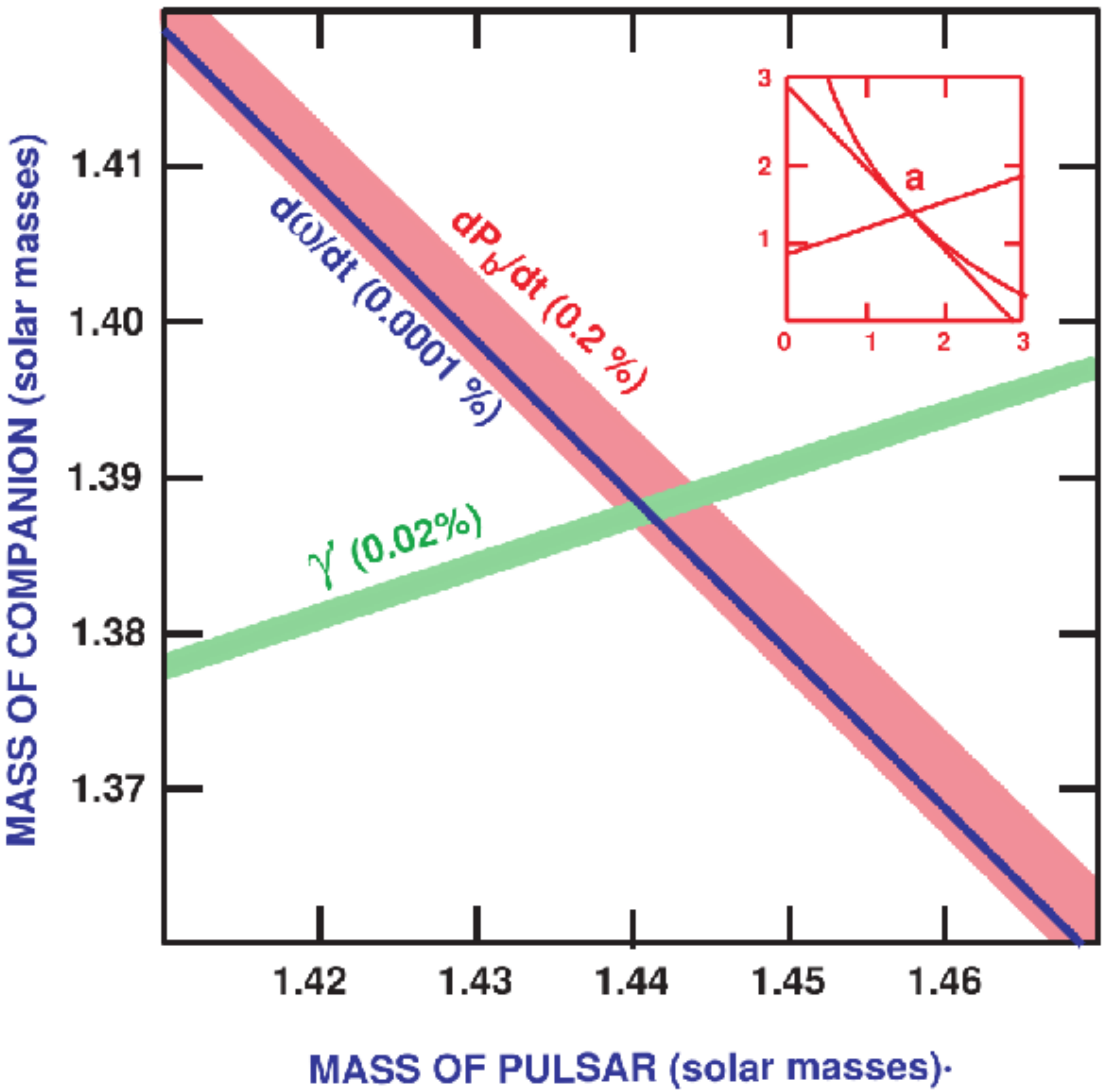
Chennai Symposium, IIT Madras

Overview

- Testing GR — pre-detection proposals
- Firsts tests of GR using GWs — GW150914
- Testing GR with O1/O2 BBHs and GW170817
- GW observations and BH signatures
- “No-hair” tests for inspiralling CBCs — proposal and the test on real data (aLIGO)
- Beyond aLIGO: 3rd generation detectors, LISA

pre-detection proposals

Consistency Test (PSR1913+16)

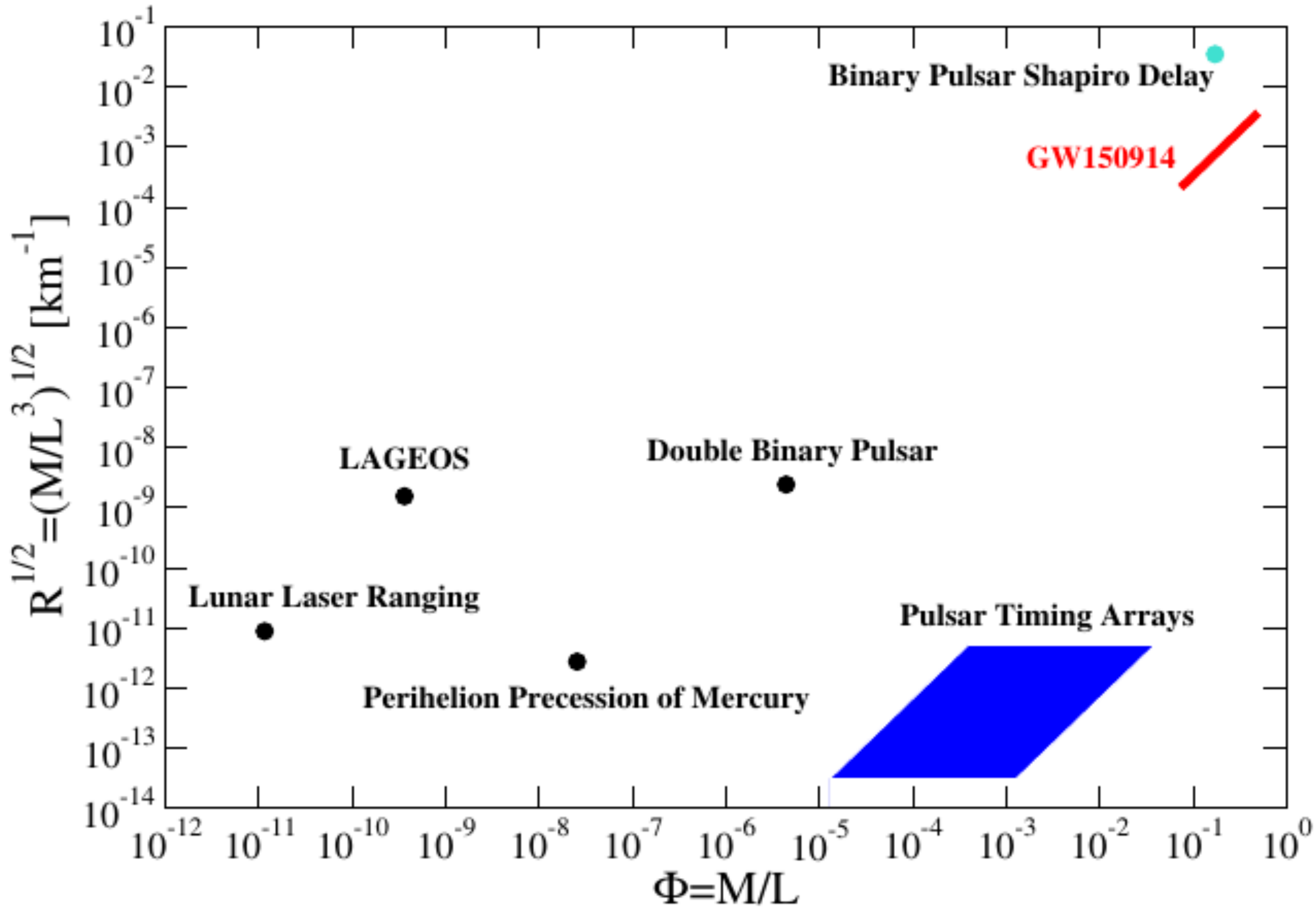


Measurement of two Keplerian (e, P_b) and three post-Keplerian parameters (all functions of e, P_b, m_1, m_2) provides three constraints on the two unknown masses.

GR passes the test if it provides a consistent solution to these constraints, within the measurement errors.

Clifford Will (2014)

Probing highly relativistic, highly dynamical, strong gravity regime



Yunes et al., (2014)

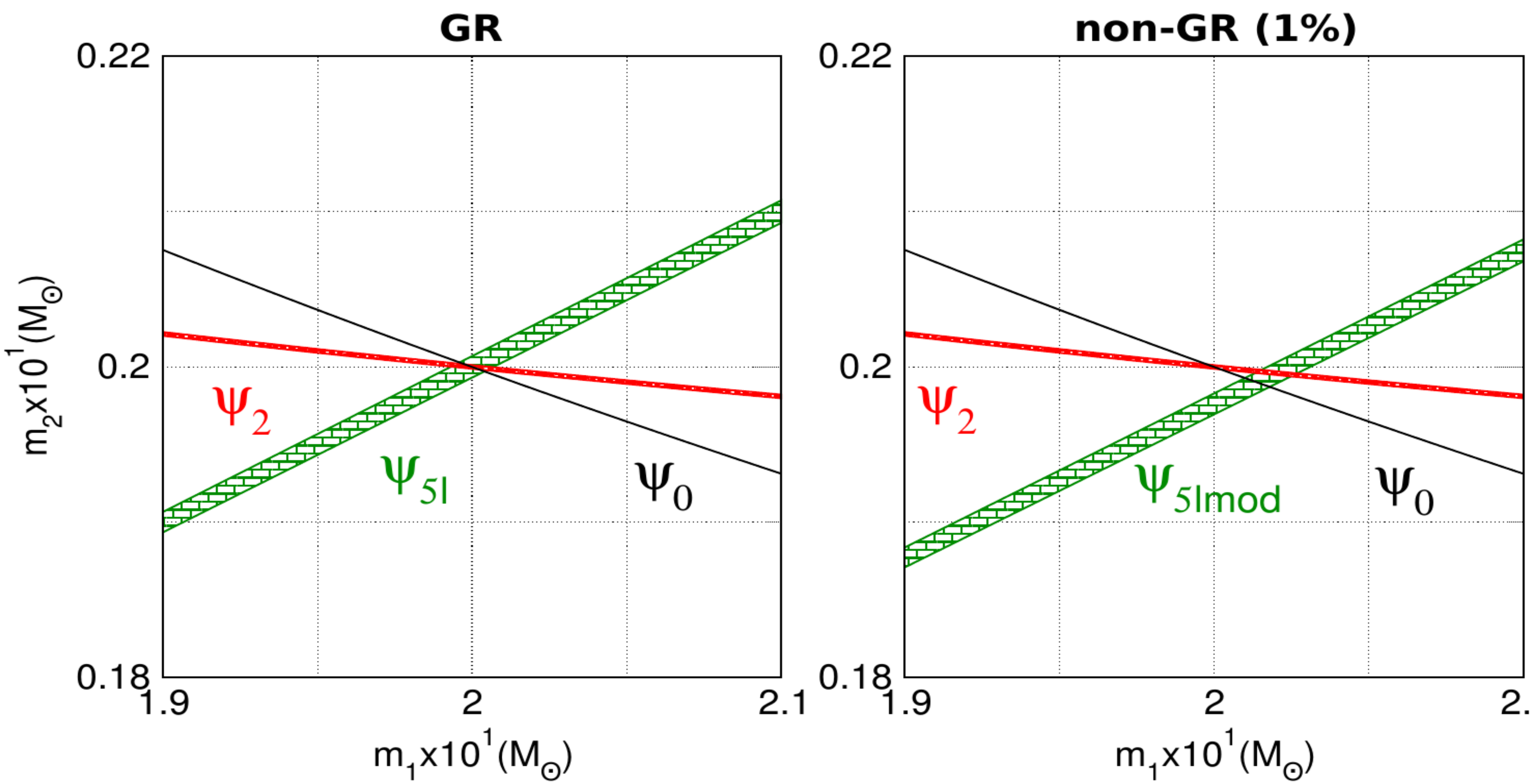
Parametrized tests of the post-Newtonian theory

- In the post-Newtonian (PN) theory, various quantities are expressed as a series expansion in a small parameter $v/c \sim f^{1/3}$.
- The PN inspiral waveform

$$\boxed{\tilde{h}(f) = A(f)\, e^{\mathrm{i}\,\Psi(f)}} \qquad \boxed{\Psi(f) = \sum_{j=0}^{7} \left[\psi_j + \psi_{jl} \ln f\right] f^{(j-5)/3}}$$

- For a binary with nonspinning components, $\psi_i \equiv \psi_i(m_1, m_2)$.
- Measure each of the phasing coefficients (ψ_j, ψ_{jl}), treating them all as independent of one another.
- Measurement of any two of these parameters can be used to infer the binary's component masses (m_1, m_2).
- Measurement of a third parameter would constitute a test of the theory, by requiring a consistency of the component masses in the m_1-m_2 plane, similar to the binary pulsar test.

Parametrised tests of PN theory



GR is correct **GR needs modifications**

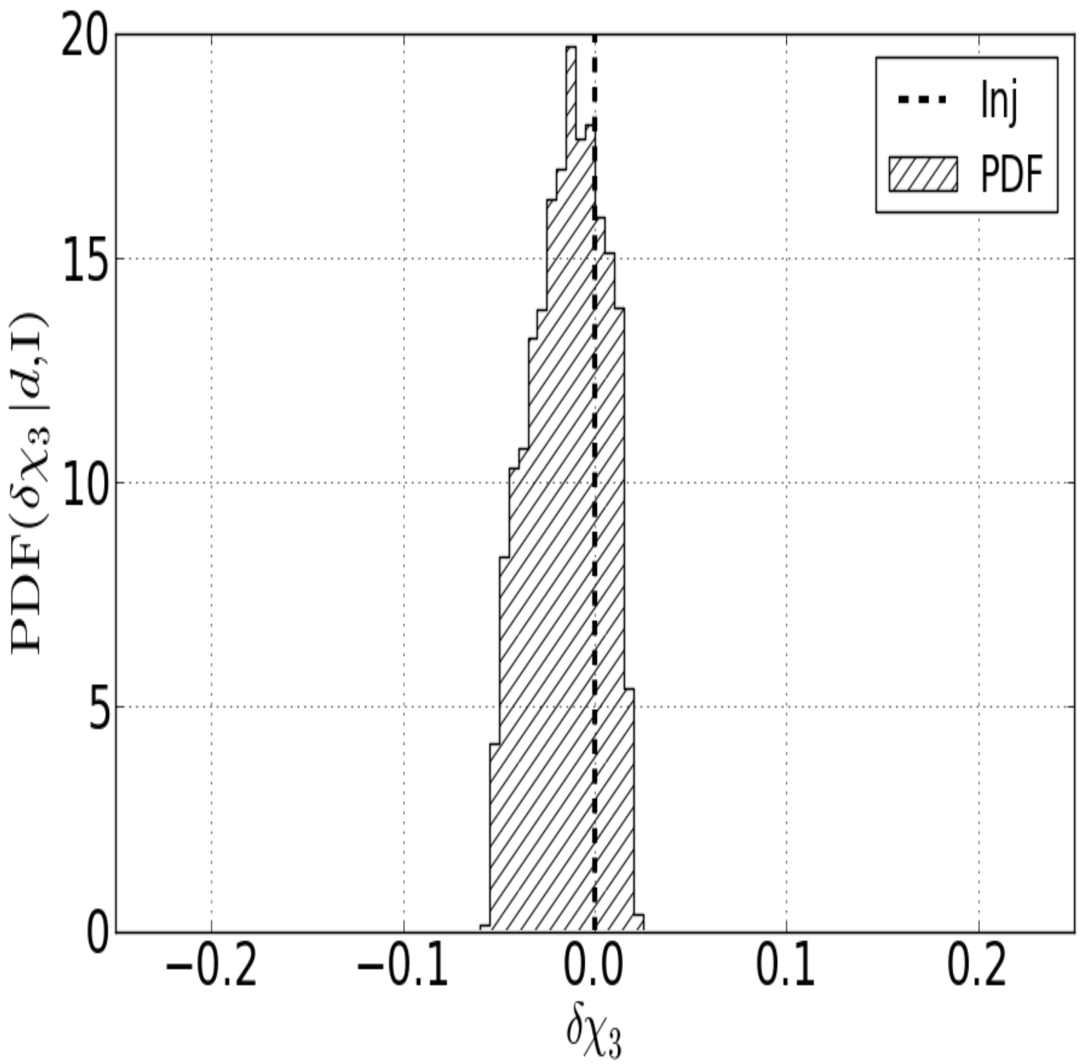
non-GR:

$$\psi_i \to \psi_i(1+\delta\psi_i) \text{ for } i \geq 5$$

Mishra, Arun, Iyer, Sathyaprakash, PRD 82, 064010 (2010) ; Arun+, PRD (2006); Arun+, CQG (2006)

Generalisations — model independent test

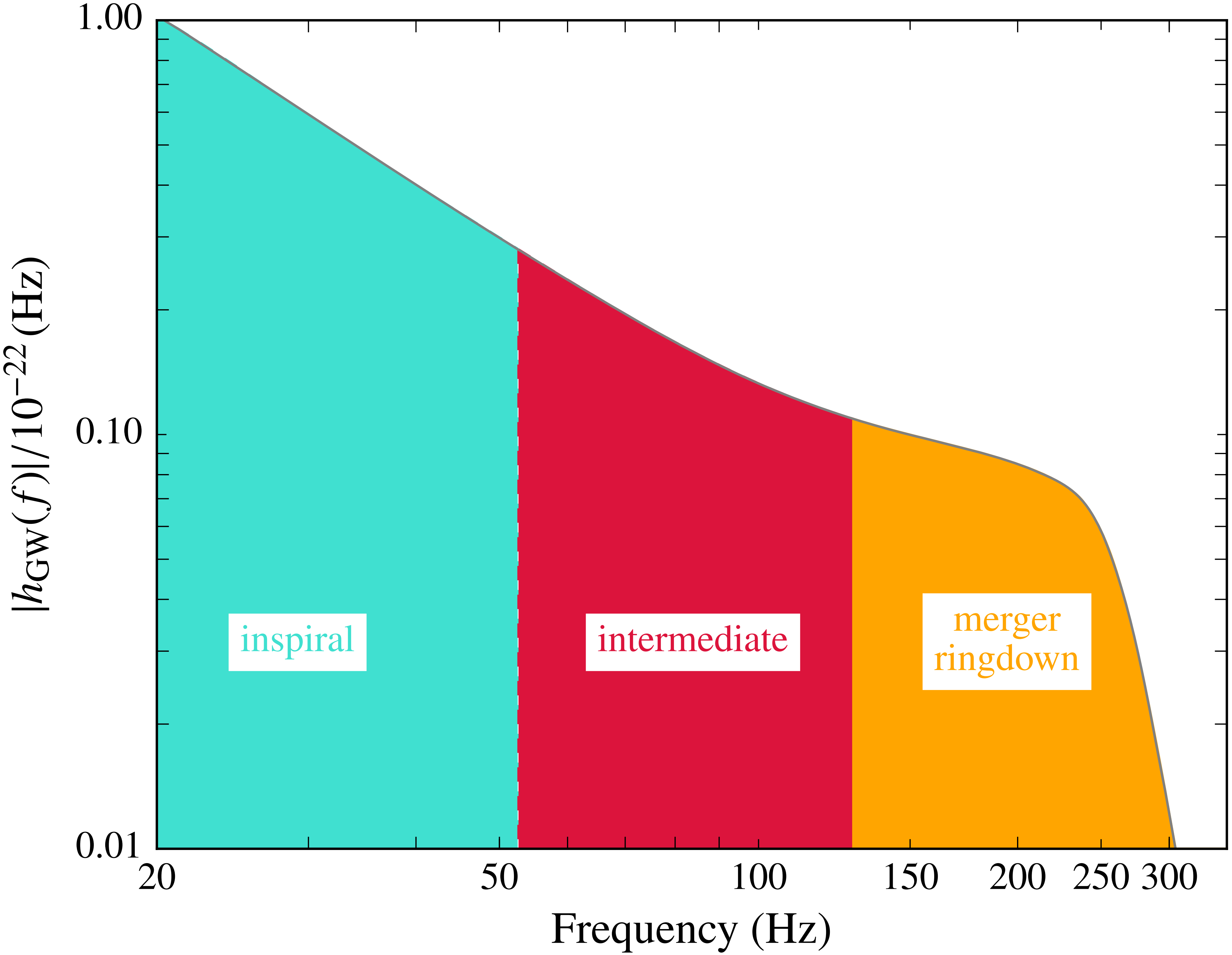
- Generic tests of GR in Bayesian framework.
- $\psi_i \to \psi_i(1+\delta\chi_i); \delta\chi_i^{\mathrm{GR}} = 0$.
  Li *et al.* (2012)



Tests with GW150914

LVC, Phys. Rev. Lett. 116, 061102 (2016) — BBH Discovery Paper
LVC, Phys. Rev. Lett. 116, 221101 (2016) — Testing GR Paper

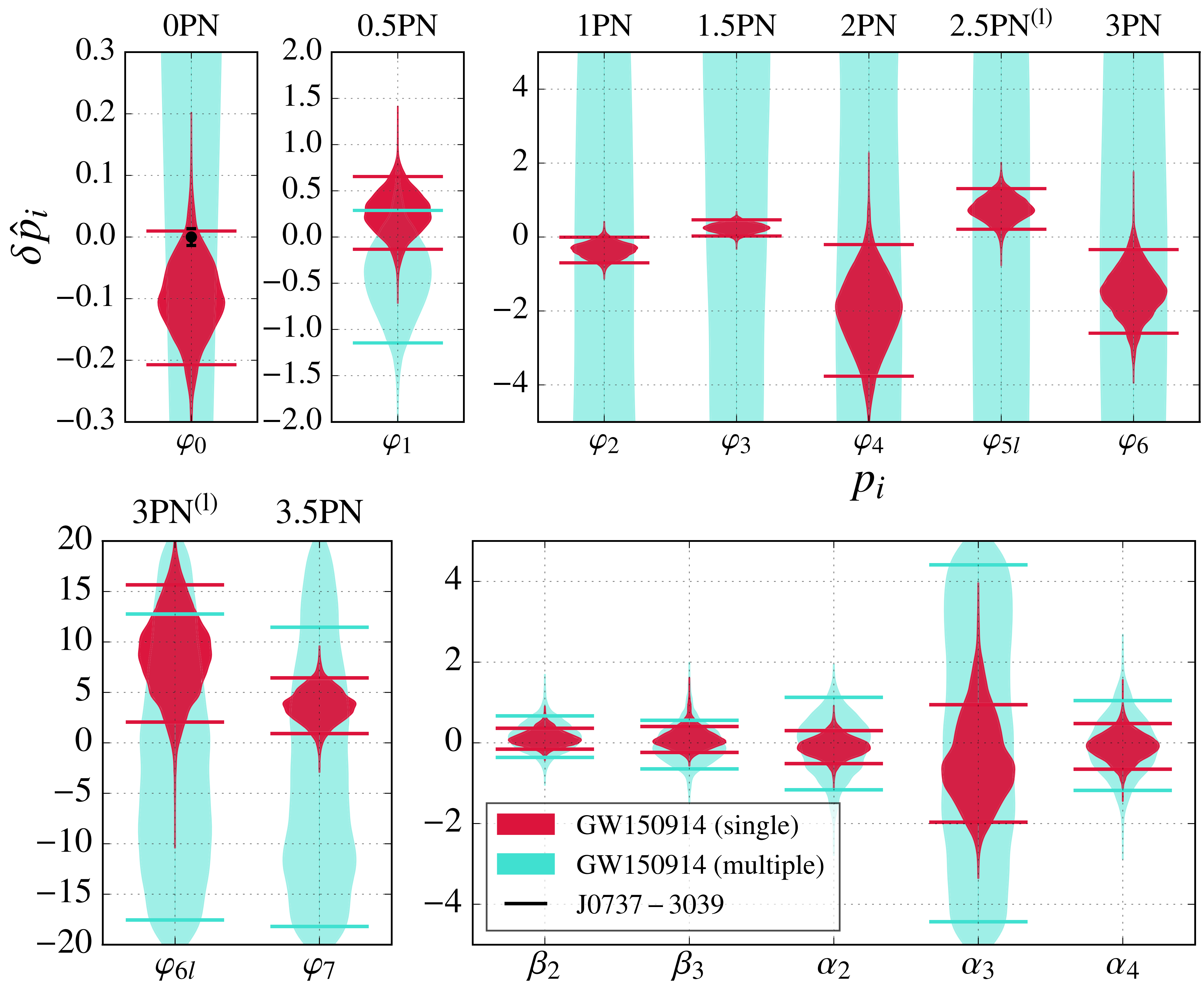
Parameterised tests — IMR signals



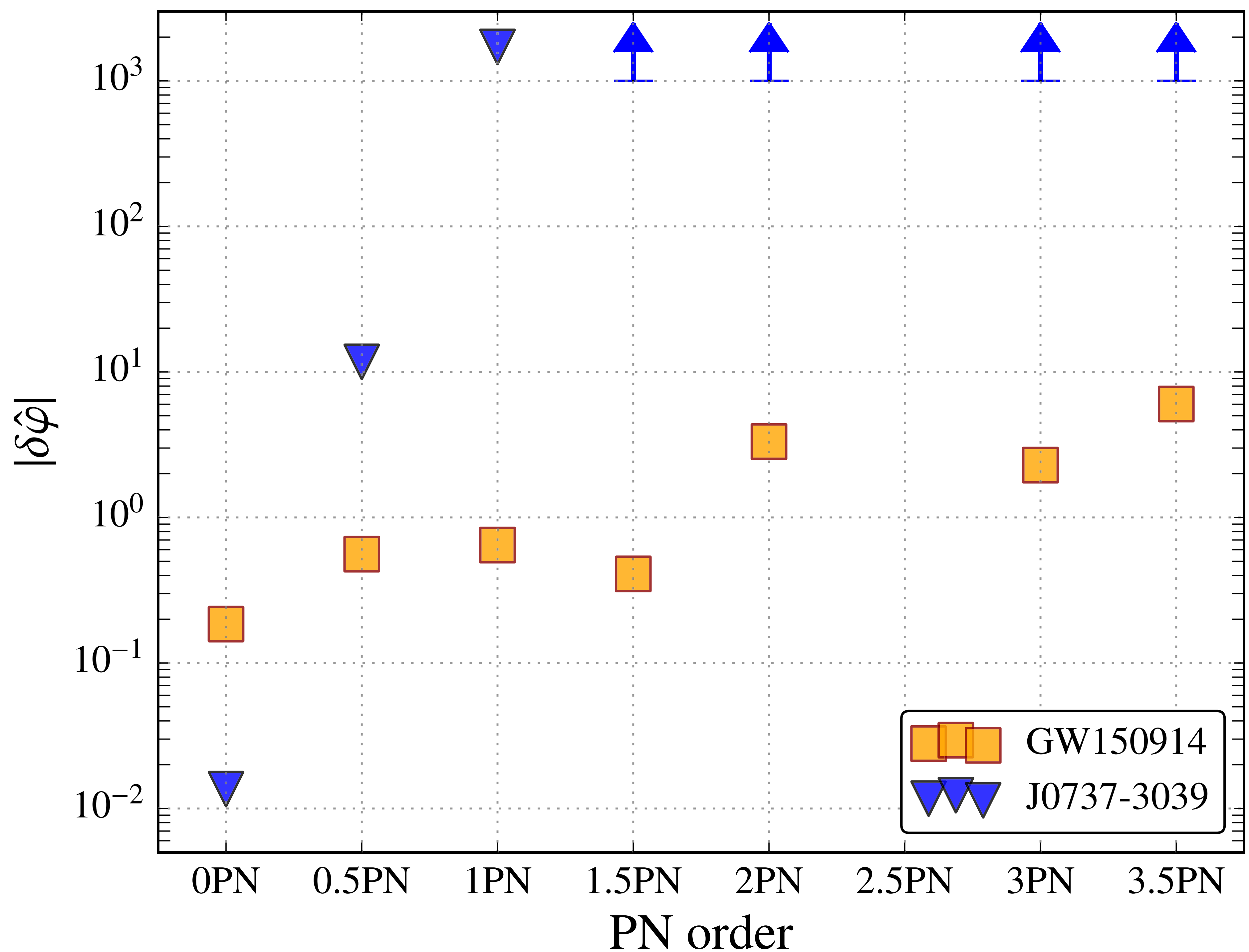
Frequency Evolution

Waveform regime	Parameter	f dependence
Early-inspiral regime	$\delta\hat{\varphi}_0$	$f^{-5/3}$
	$\delta\hat{\varphi}_1$	$f^{-4/3}$
	$\delta\hat{\varphi}_2$	f^{-1}
	$\delta\hat{\varphi}_3$	$f^{-2/3}$
	$\delta\hat{\varphi}_4$	$f^{-1/3}$
	$\delta\hat{\varphi}_{5l}$	$\log(f)$
	$\delta\hat{\varphi}_6$	$f^{1/3}$
	$\delta\hat{\varphi}_{6l}$	$f^{1/3}\log(f)$
	$\delta\hat{\varphi}_7$	$f^{2/3}$
Intermediate regime	$\delta\hat{\beta}_2$	$\log f$
	$\delta\hat{\beta}_3$	f^{-3}
Merger-ringdown regime	$\delta\hat{\alpha}_2$	f^{-1}
	$\delta\hat{\alpha}_3$	$f^{3/4}$
	$\delta\hat{\alpha}_4$	$\tan^{-1}(af + b)$

Parameterised tests

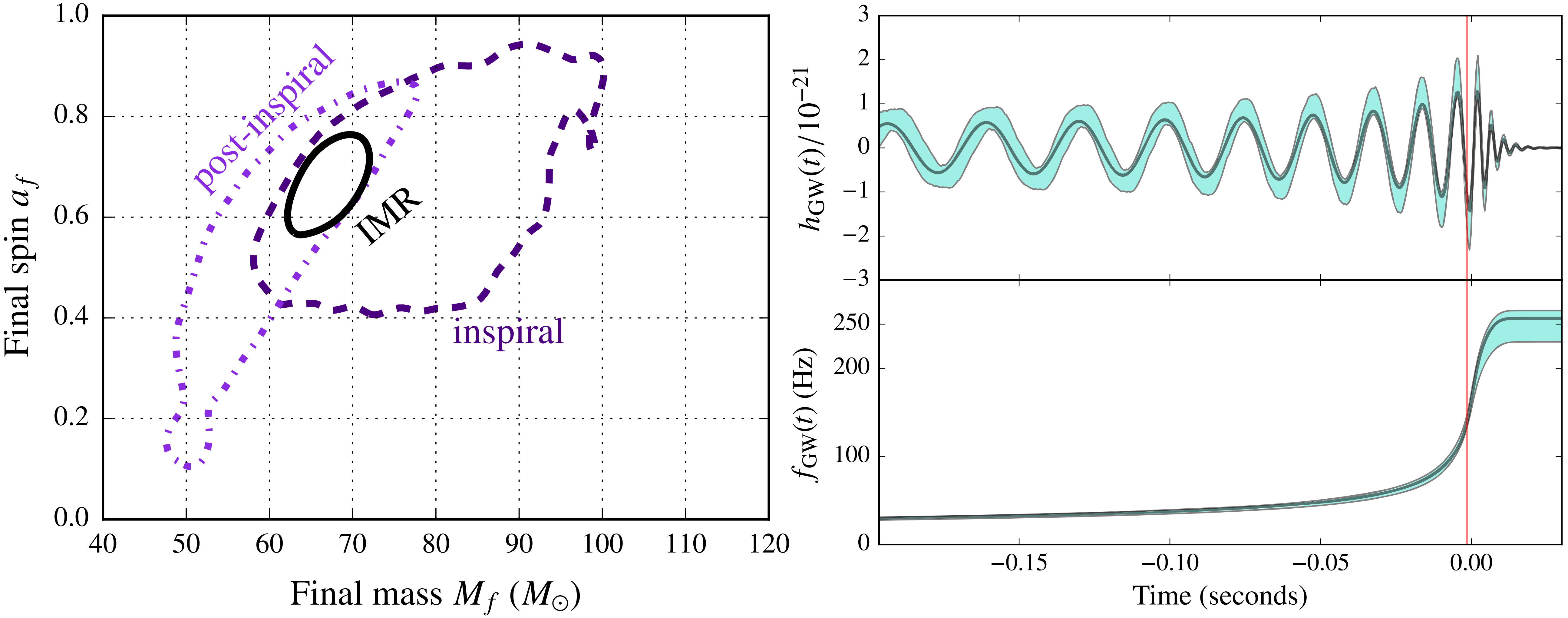


Bounds (90%)

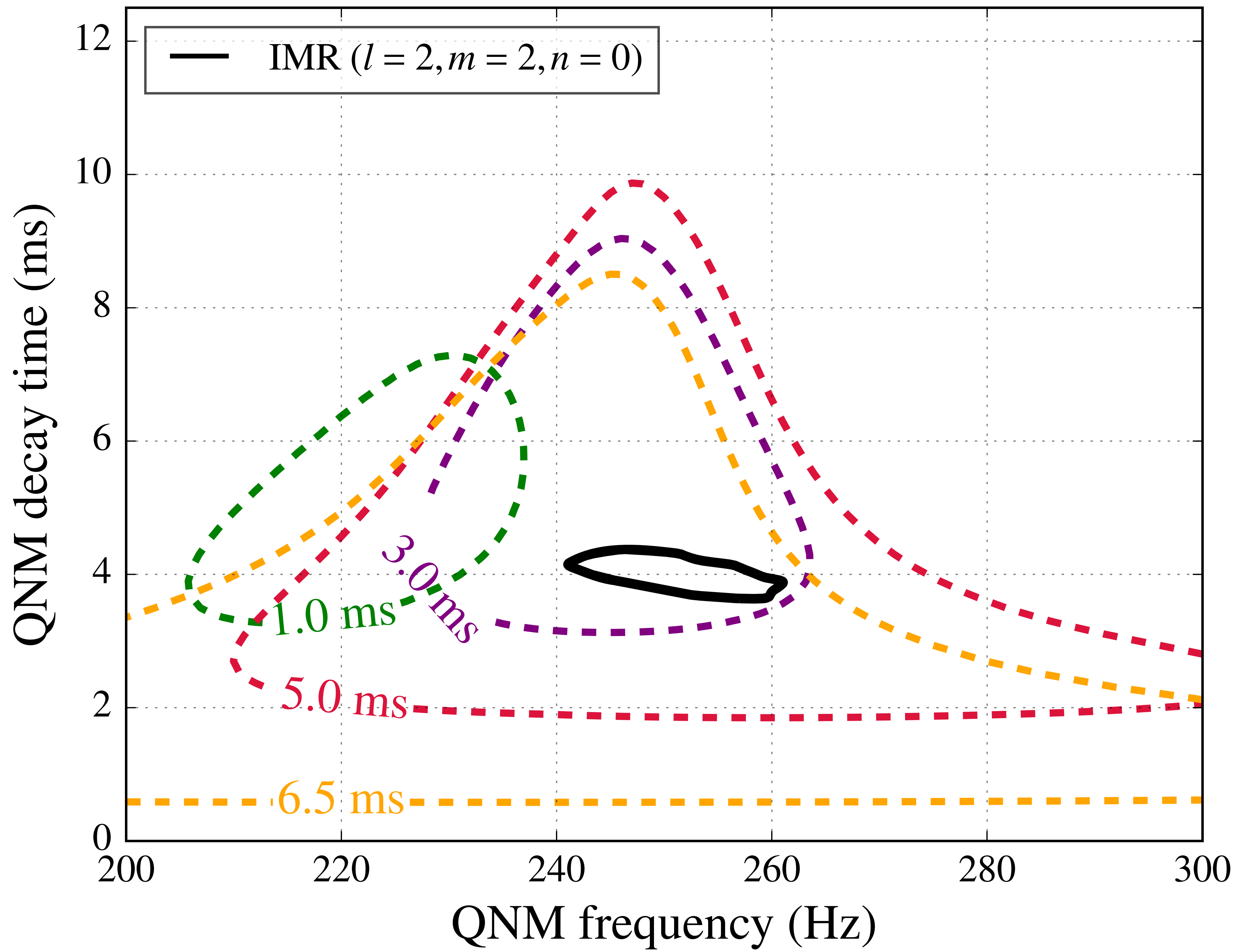


10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
$|\delta\hat{\varphi}|$
0PN
0.5PN
1PN
1.5PN
2PN
2.5PN
3PN
3.5PN
PN order
GW150914
J0737-3039

IMR consistency test



Test of the least damped mode



Tests with O1-O2 data

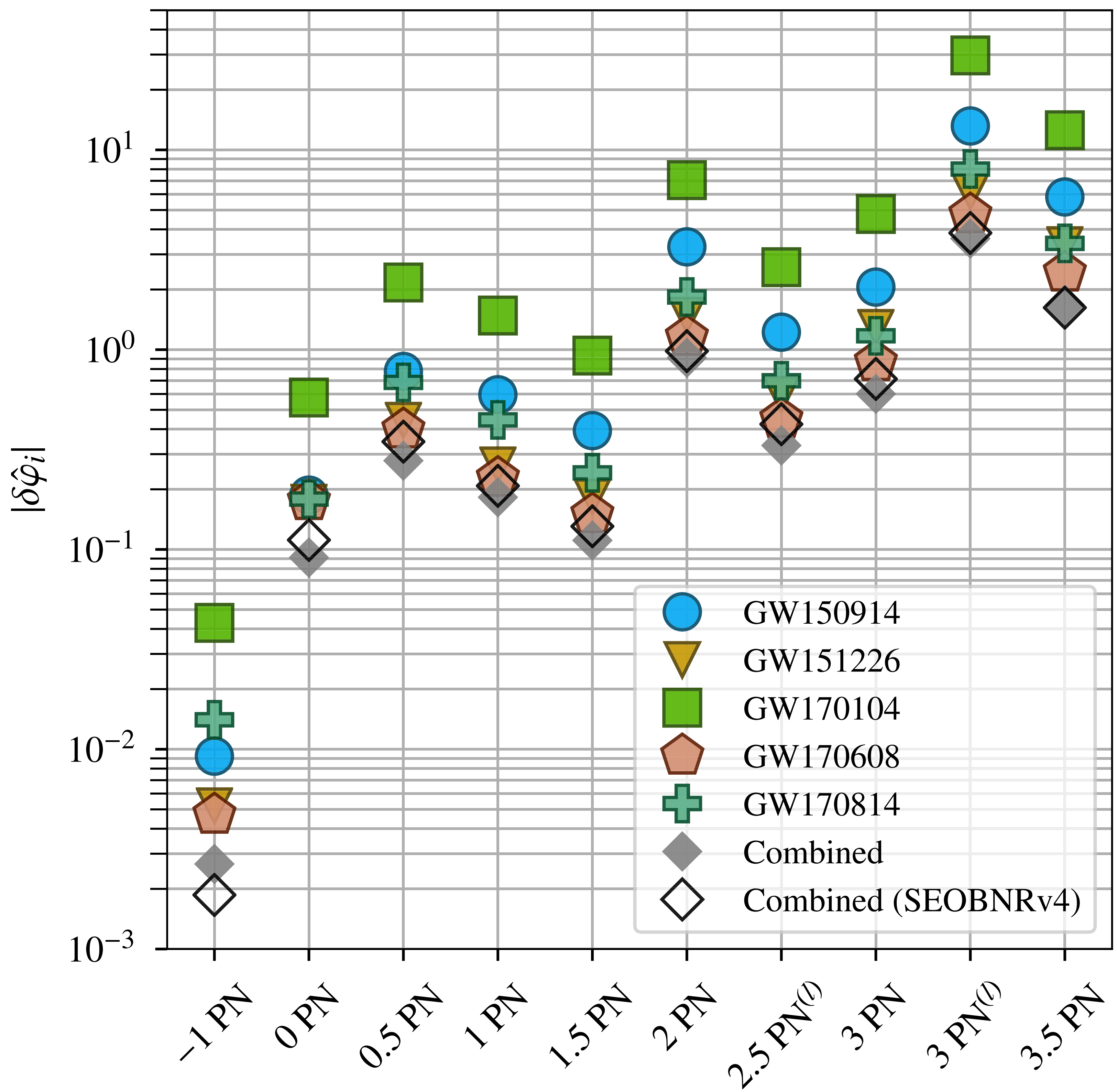
LVC, PRD, 100, 104036 (2019)

Events (O1-O2) and tests of GR

Event	Properties				GR tests performed				
	D_L [Mpc]	M_{tot} [$M_\odot$]	M_f [$M_\odot$]	a_f	RT	IMR	PI	PPI	MDR
GW150914[a]	440^{+150}_{-170}	$66.1^{+3.8}_{-3.3}$	$63.1^{+3.4}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	✓	✓	✓	✓	✓
GW151012[a]	1080^{+550}_{-490}	$37.2^{+10.6}_{-3.9}$	$35.6^{+10.8}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	✓	...	...	✓	✓
GW151226[a,b]	450^{+180}_{-190}	$21.5^{+6.2}_{-1.5}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	✓	...	✓	...	✓
GW170104	990^{+440}_{-430}	$51.0^{+5.3}_{-4.1}$	$48.9^{+5.1}_{-4.0}$	$0.66^{+0.08}_{-0.11}$	✓	✓	✓	✓	✓
GW170608	320^{+120}_{-110}	$18.6^{+3.2}_{-0.7}$	$17.8^{+3.4}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	✓	...	✓	✓	✓
GW170729[c]	2840^{+1400}_{-1360}	$84.4^{+15.8}_{-11.1}$	$79.5^{+14.7}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	✓	✓	...	✓	✓
GW170809	1030^{+320}_{-390}	$59.0^{+5.4}_{-4.1}$	$56.3^{+5.2}_{-3.8}$	$0.70^{+0.08}_{-0.09}$	✓	✓	...	✓	✓
GW170814	600^{+150}_{-220}	$55.9^{+3.4}_{-2.6}$	$53.2^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	✓	✓	✓	✓	✓
GW170818	1060^{+420}_{-380}	$62.2^{+5.2}_{-4.1}$	$59.4^{+4.9}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	✓	✓	...	✓	✓
GW170823	1940^{+970}_{-900}	$68.7^{+10.8}_{-8.1}$	$65.4^{+10.1}_{-7.4}$	$0.72^{+0.09}_{-0.12}$	✓	✓	...	✓	✓

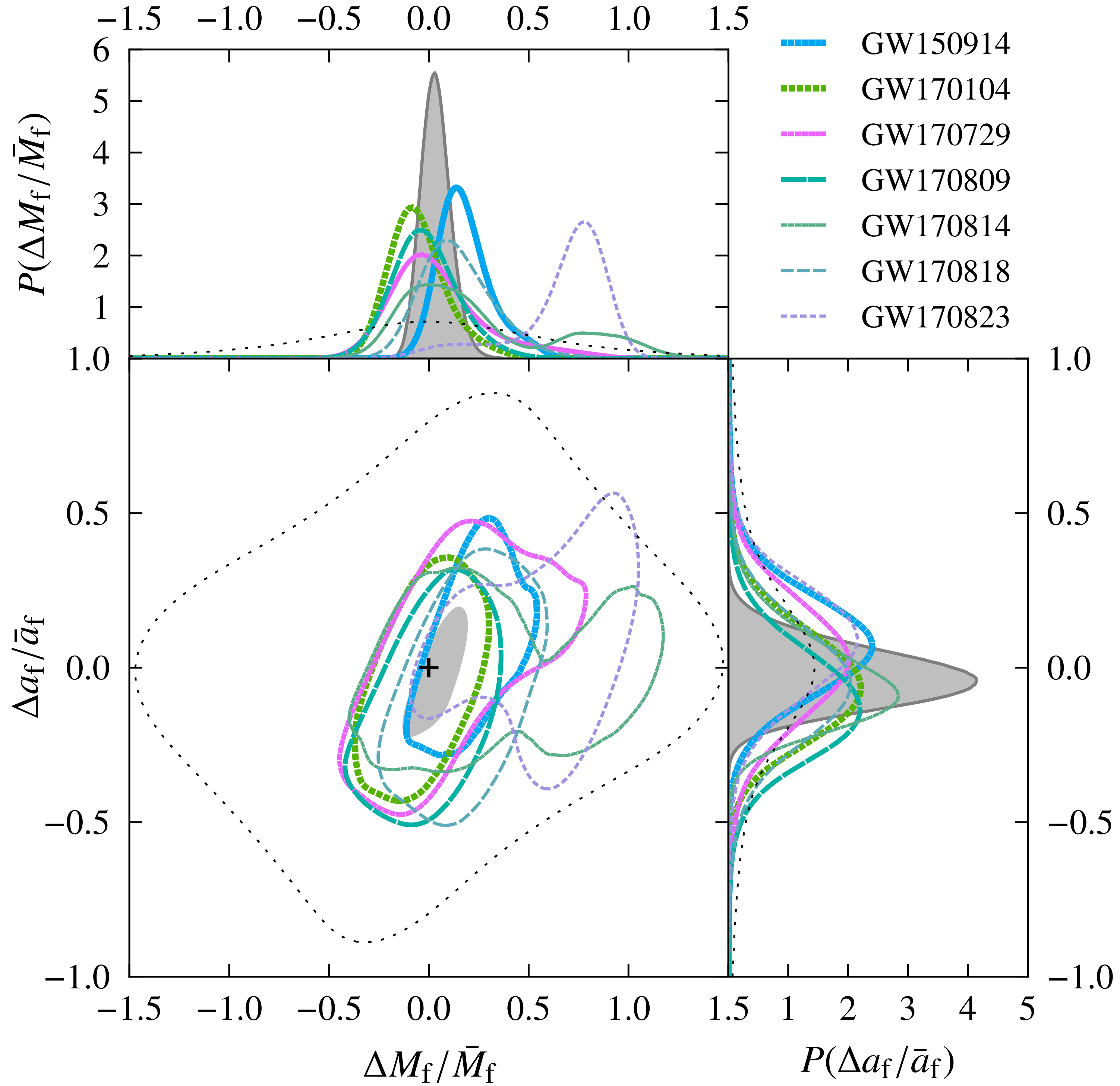
LVC, PRD. 100, 104036 (2016)

Bounds (90%)

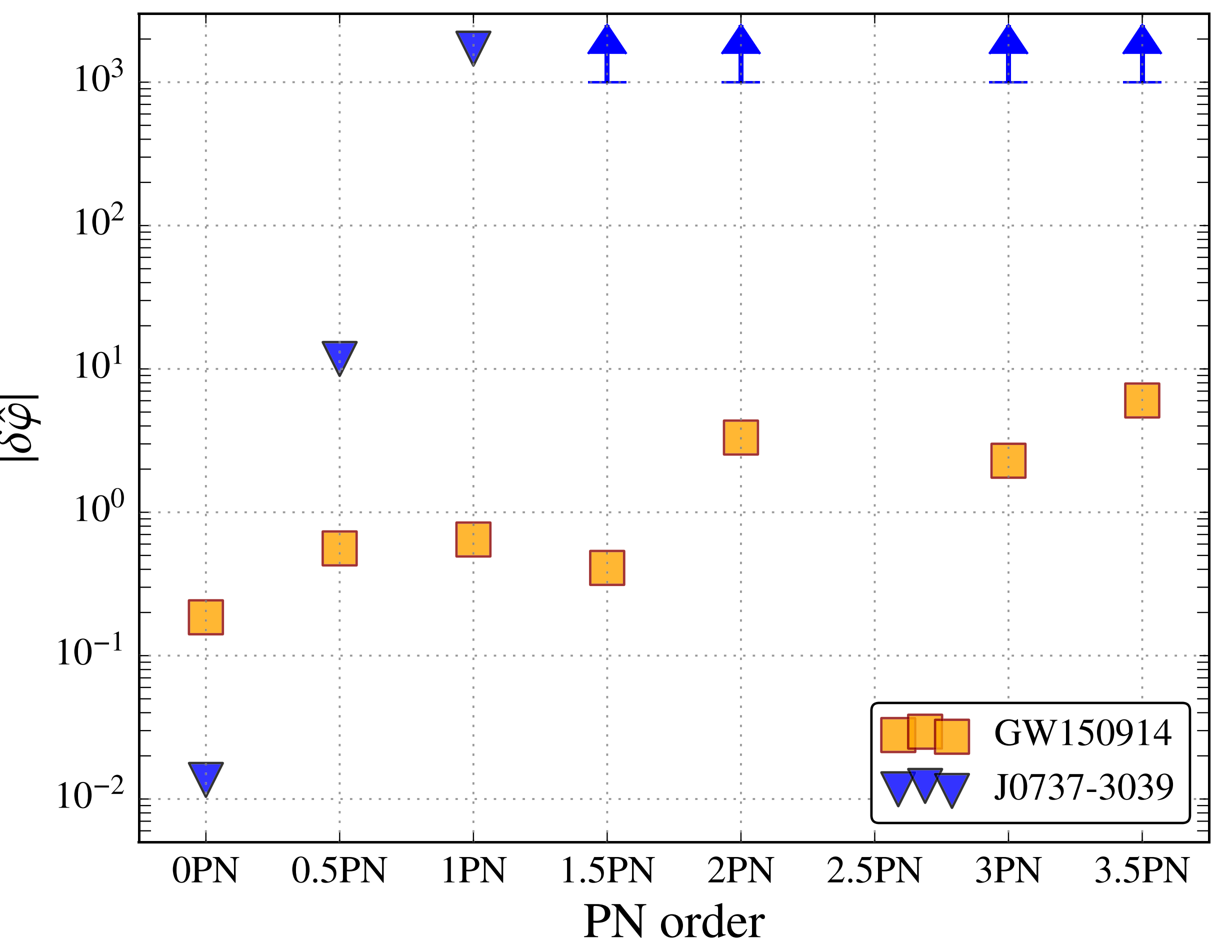


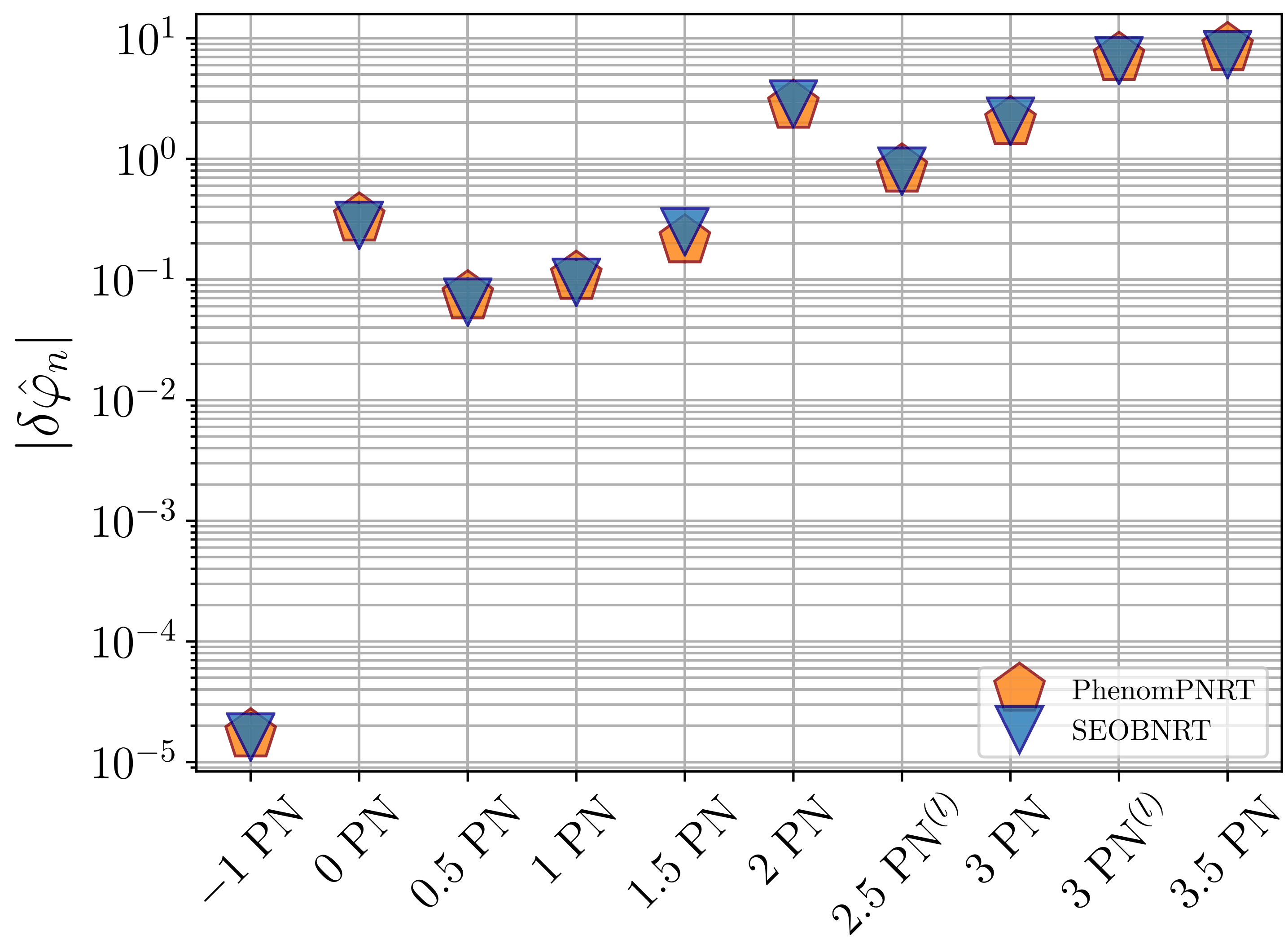
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
$|\delta\hat{\varphi}_i|$
GW150914
GW151226
GW170104
GW170608
GW170814
Combined
Combined (SEOBNRv4)
-1 PN
0 PN
0.5 PN
1 PN
1.5 PN
2 PN
2.5 PN$^{(l)}$
3 PN
3 PN$^{(l)}$
3.5 PN

Combined posteriors (IMR consistency) O1-O2



Parameterised tests with GW170817





LVC, PRL 116, 061102 (2016)

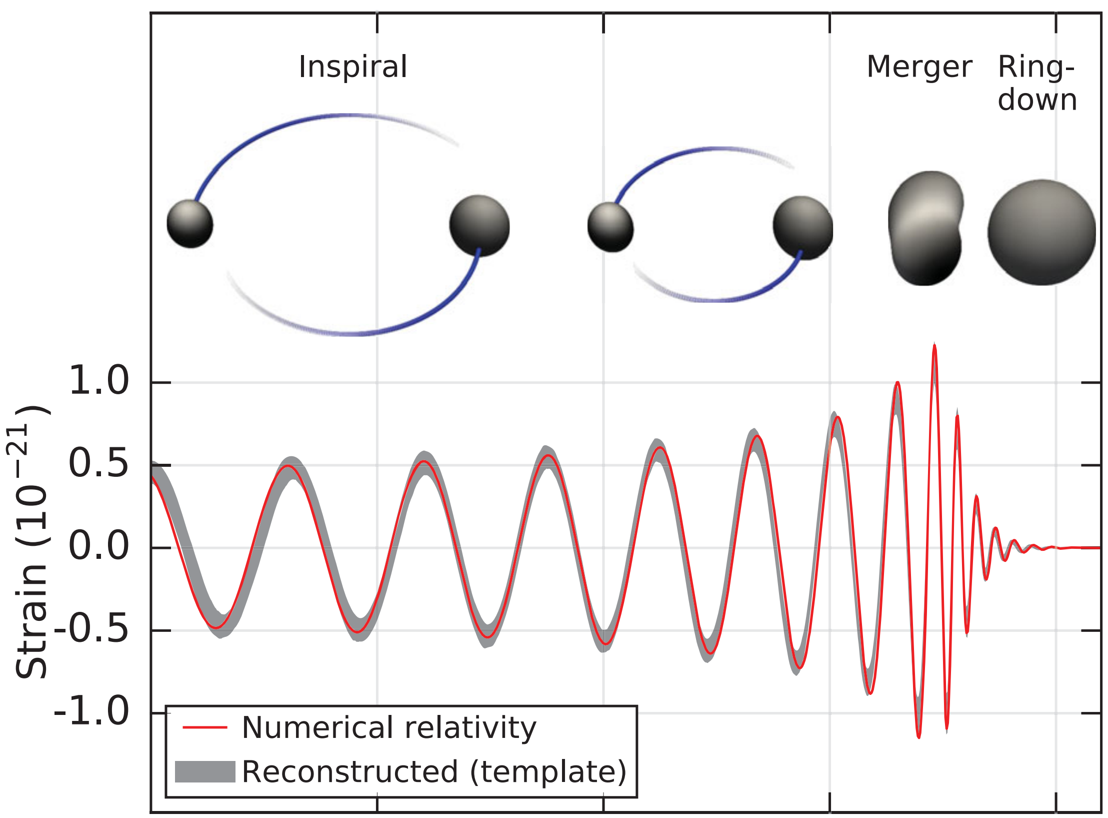
LVC, PRL, 123 011102 (2019)

“No-hair” tests for inspiralling CBCs

GW observations and BH signatures

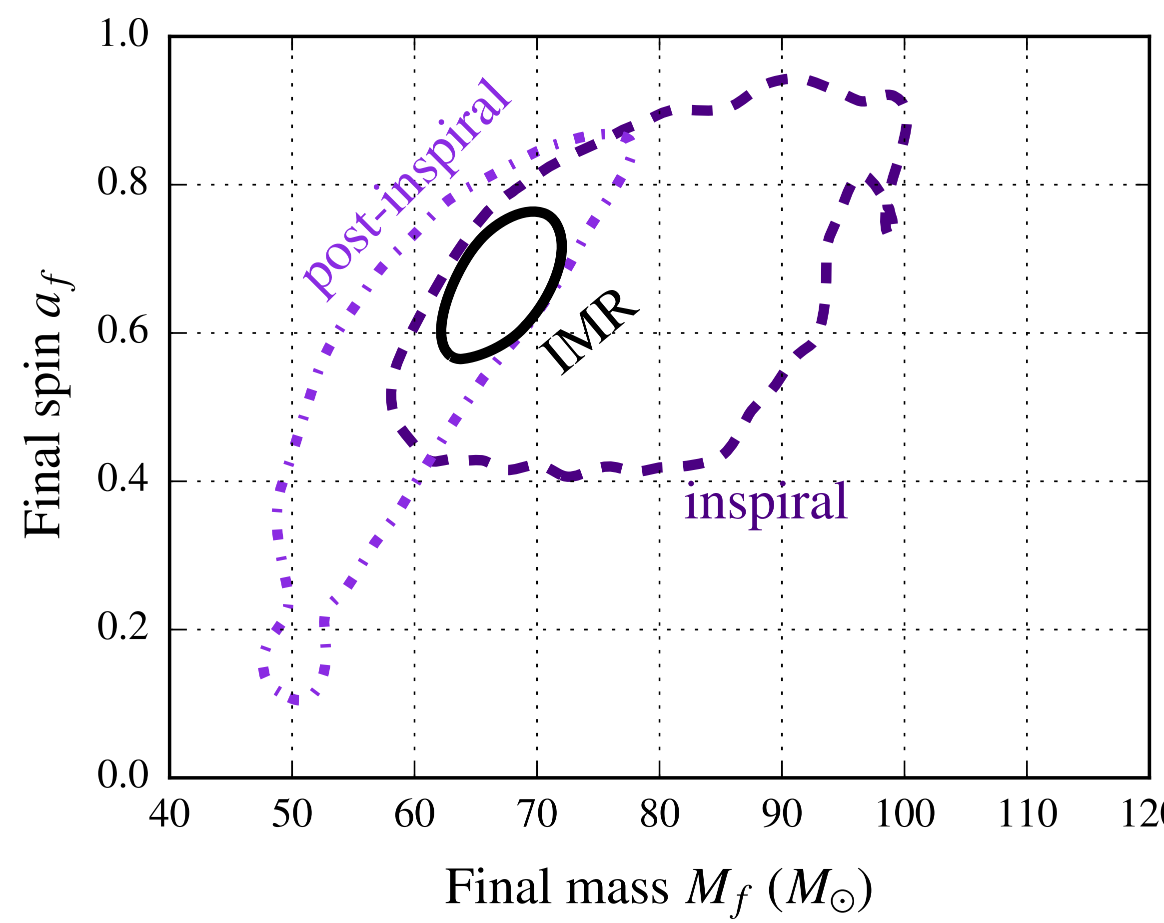
BBH nature

Waveform Reconstructions



LVC, Phys. Rev. Lett. 116, 061102 (2016)

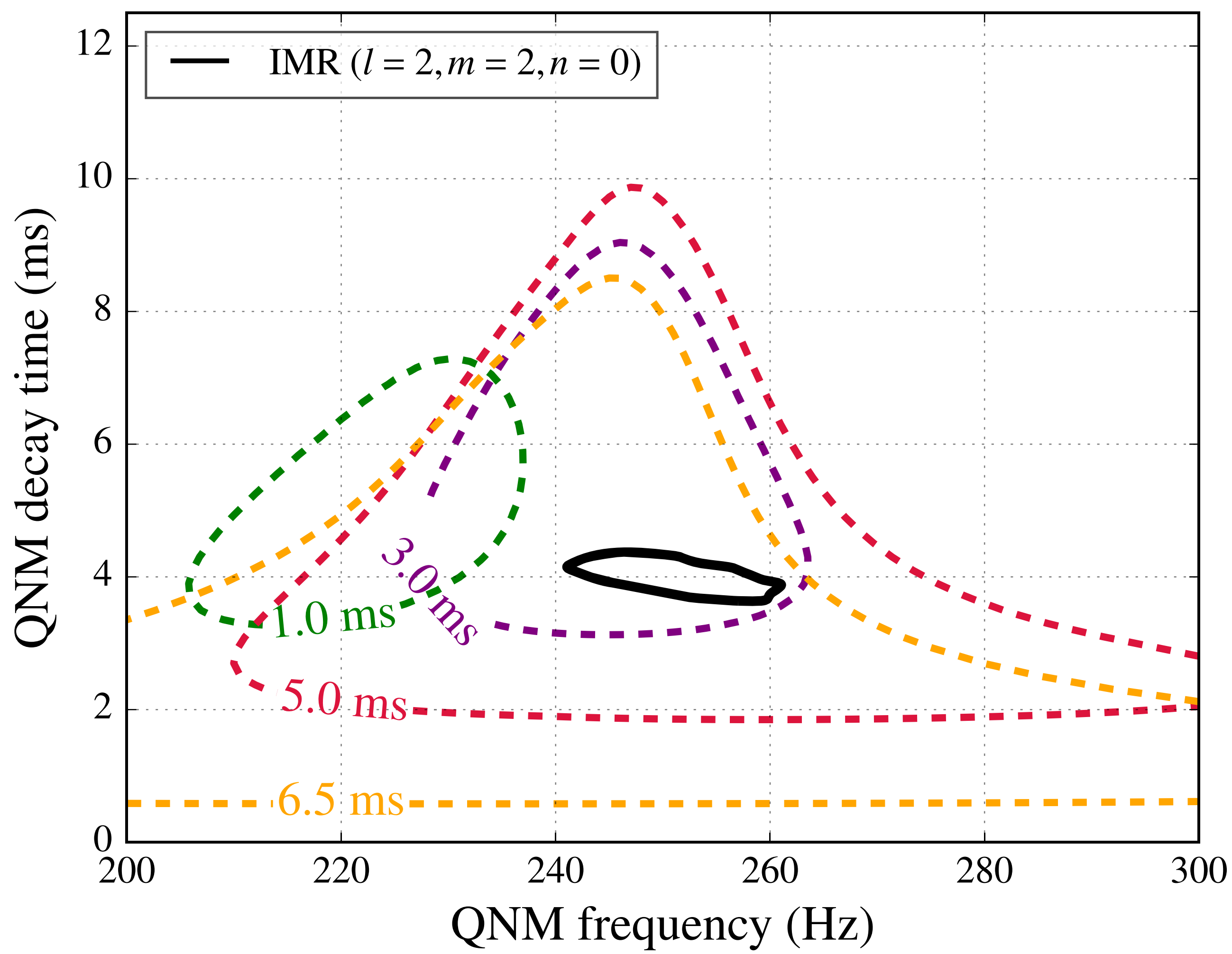
Waveform Consistency Tests



LVC, Phys. Rev. Lett. 116, 221101 (2016)

Is the final object a BH?

Observed quasi-normal mode spectrum of the remnant



LVC, Phys. Rev. Lett. 116, 221101 (2016)

- **Power in the ringdown signal.**
- **Uncertainties in predicting the start of the ringdown.**

What about the binary constituents?

Tidal deformability of the compact object

Induced quadrupole moment

External tidal field

$$Q_{ij} = -\lambda\, \epsilon_{ij}$$

Tidal love number

$$k_2 = \frac{3}{2} G \lambda R^{-5}$$

Dimensionless tidal love number: **0** for BHs

Damour & Nagar, 2009
Bennington & Poisson, 2009

		k_2^E
NSs		210
ECOs	Boson star	41.4
	Wormhole	$\frac{4}{5(8+3\log\xi)}$
	Perfect mirror	$\frac{8}{5(7+3\log\xi)}$
	Gravastar	$\frac{16}{5(23-6\log 2+9\log\xi)}$

Cardoso+ (2017)

What about the binary constituents?

Tidal deformability of the compact object

Induced quadrupole moment

External tidal field

$$Q_{ij} = -\lambda\,\epsilon_{ij}$$

Tidal love number

$$k_2 = \frac{3}{2} G \lambda R^{-5}$$

Dimensionless tidal love number: **0** for BHs

Higher order effects and hence difficult to measure.

Damour & Nagar, 2009
Bennington & Poisson, 2009

Cardoso+, arXiv: 1701.01116 (2017)
Maselli+, arXiv:1703.10612 (2017)

A "No-hair" Test for Compact Binaries

Testing the BH nature
by measuring the spin-induced
multipole moments of
a compact binary system.

Krishnendu, Arun, Mishra, PRL119, 091101,2017
Krishnendu, Mishra, Arun, PRD 99, 064008, 2019
Krishnendu, Saleem, Samajdar, Arun, Del Pozzo, Mishra, PRD 100, 104019 (2019)

The proposal

Measure spin-induced deformations of binary components

$$Q_2 = -\kappa \, M^3 \chi^2 \qquad\qquad S_3 = -\lambda \, M^4 \chi^3$$

$\kappa = 1 \text{ for BHs}$

$\simeq 2 - 14 \text{ for NS}$

$\simeq 10 - 150 \text{ for Boson Stars}$

$- / + \;\text{ for Gravstars}$

$\lambda = 1 \text{ for BHs}$

$\sim 4 - 30 \;\text{ for NSs}$

$\sim 10 - 200 \;\text{ for BSs}$

Poisson, PRD 57, 5287 (1998);
Laarakkers+, ApJ 512, 282 (1999);
Pappas+, PRL 108, 231103 (2012);
F. D. Ryan, PRD 55, 6081 (1997)
Uchikata+, CQG 32,085008(2015)

Waveforms and spin deformations

Inspiral waveform (SPA)

$$\tilde{h}(f) = \frac{M^2}{D_L}\sqrt{\frac{5\pi}{48\eta}}\sum_{n=0}^{4}\sum_{k=1}^{6} V_k^{n-7/2}\, C_k^{(n)}\, e^{\mathrm{i}(k\,\Psi_{\mathrm{SPA}}(f/k)-\pi/4)}$$

Amplitude corrections from k^th harmonic at n^th PN order

λ dependence

$$\Psi_{\mathrm{SPA}}(f) = 2\pi f t_{\mathrm{c}} - \phi_{\mathrm{c}} + \left\{\frac{3}{128\eta\, v^5}\left[\psi_{\mathrm{NS}} + \psi_{\mathrm{SO}} + \psi_{\mathrm{SS}} + \psi_{\mathrm{SSS}}\right]\right\}_{v=V_1(f)}$$

κ dependence

Analysis Setup

Fisher Matrix approach to parameter estimation

— assumes Gaussian noise and high SNR

— gives a lower bound on errors / highly inexpensive

Parameter Space: $\{t_c, \phi_c, D_L, \iota, \mathcal{M}, \delta, \chi_1, \chi_2, \kappa_1, \kappa_2, \lambda_1, \lambda_2\}$

— alternatively, $\{t_c, \phi_c, D_L, \iota, \mathcal{M}, \delta, \chi_1, \chi_2, \kappa_s, \kappa_a, \lambda_s, \lambda_a\}$

Compact Binary

$$\kappa_s = (\kappa_1 + \kappa_2)/2; \kappa_a = (\kappa_1 - \kappa_2)/2$$

$$\lambda_s = (\lambda_1 + \lambda_2)/2; \lambda_a = (\lambda_1 - \lambda_2)/2$$

BBH

$$\kappa_s = 1; \kappa_a = 0$$

$$\lambda_s = 1; \lambda_a = 0$$

Large dimensionality and unreliable measurements

— We set κ_a, λ_s, λ_a to their BBH values

Analysis Setup

New parameter space: $\{t_c, \phi_c, D_L, \iota, \mathcal{M}, \delta, \chi_1, \chi_2, \kappa_s\}$

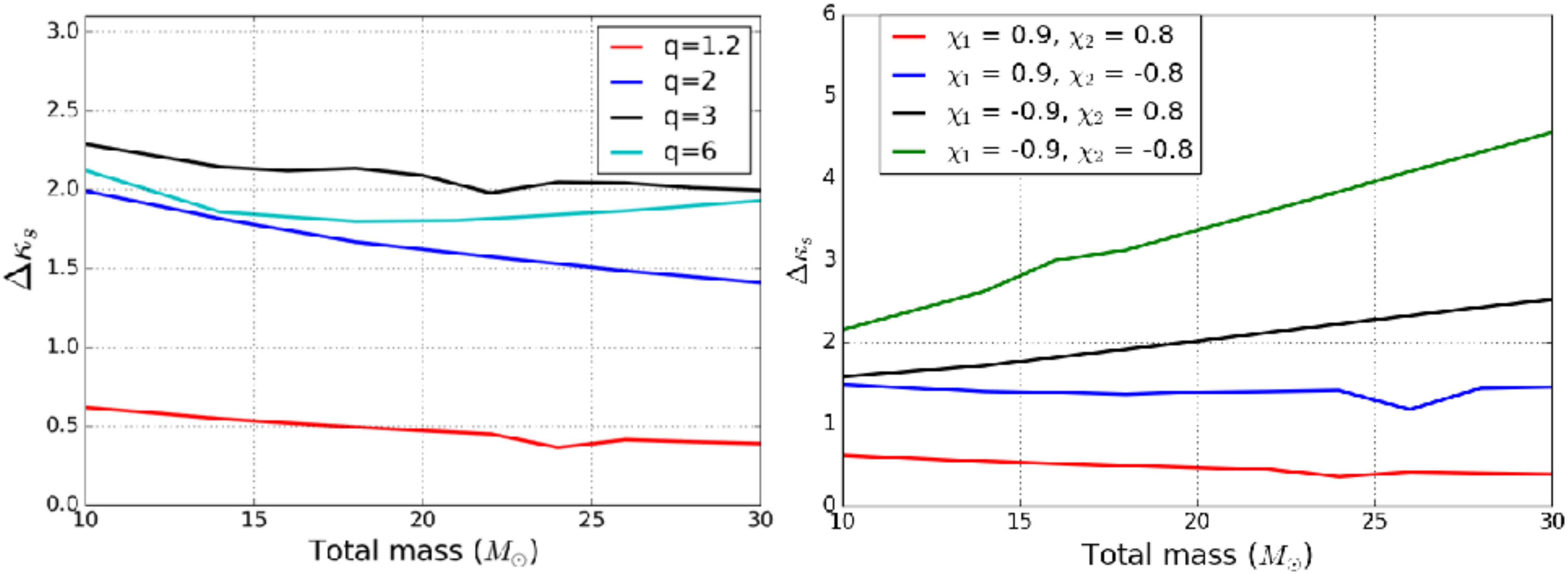
Note that $\kappa_s = 1$ for BBH

— accuracies with which it can be measured gives constrain on BBH nature

The choice $\kappa_a = 0$

— The test may be viewed as a Null test of the BBH nature of a compact binary

Effect of binary params

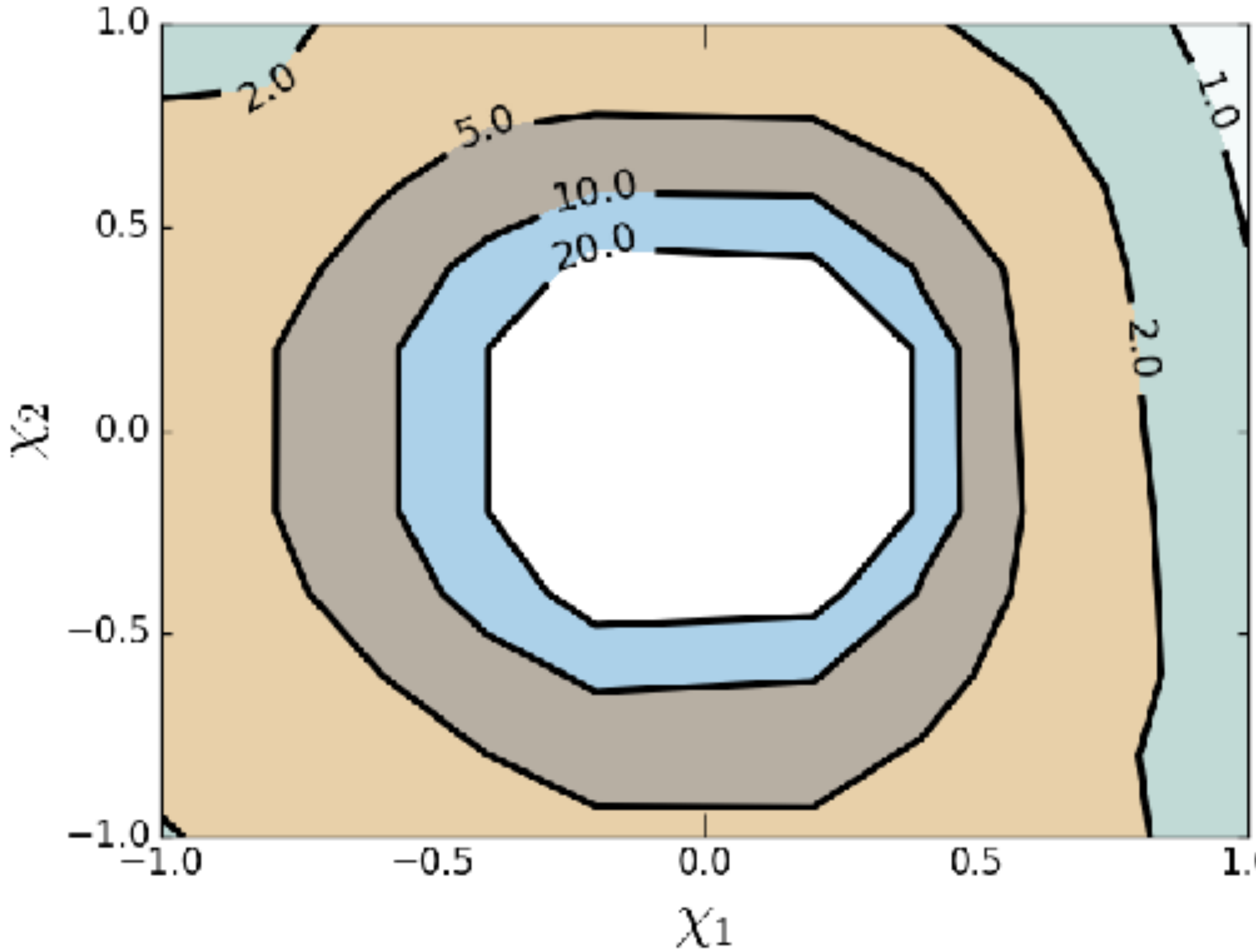


aLIGO, SNR=10

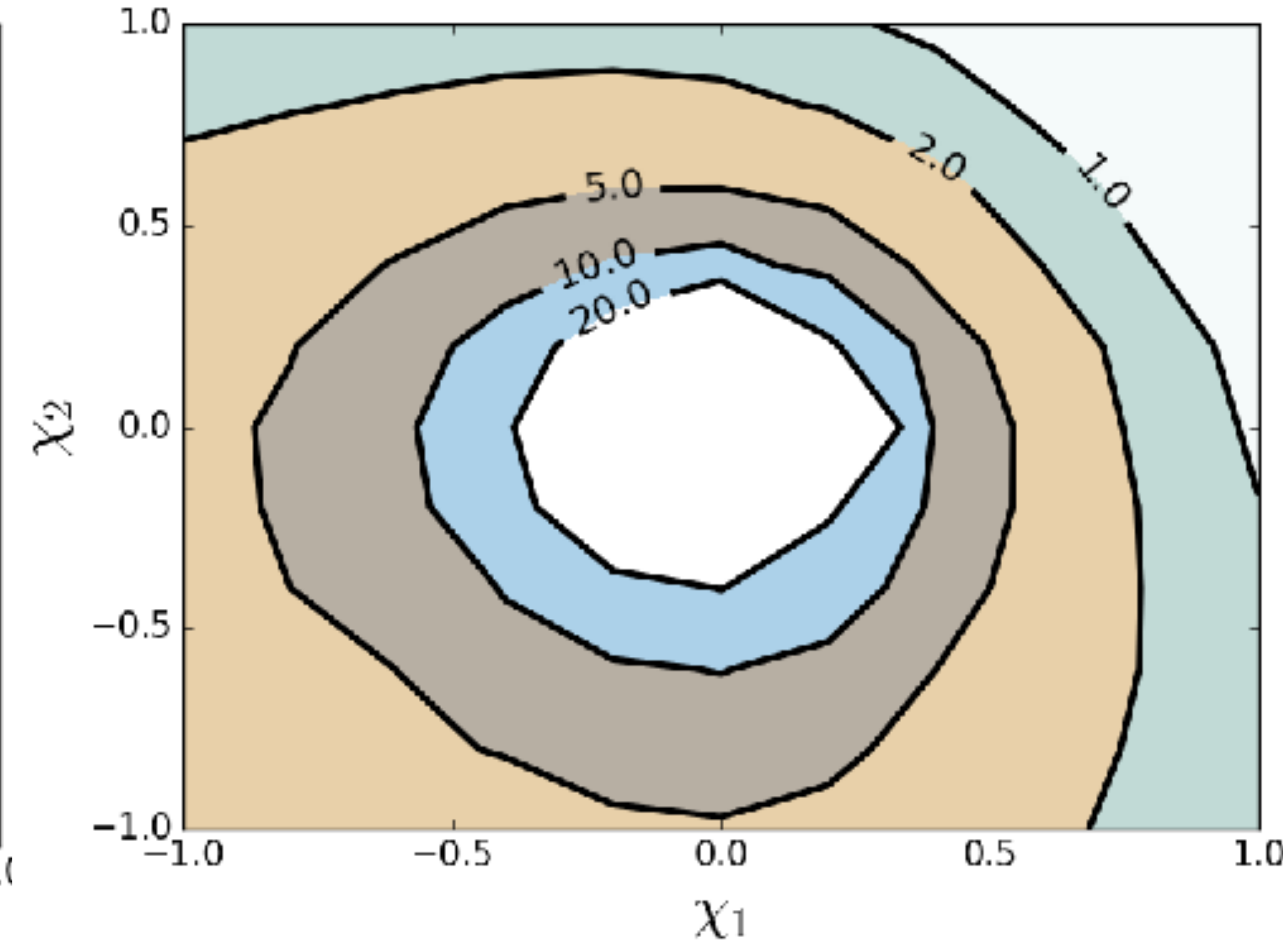
Krishnendu, Arun, Mishra, PRL119, 091101,2017

$1-\sigma$ error contours for κ_s

$(5,4)M_\odot$



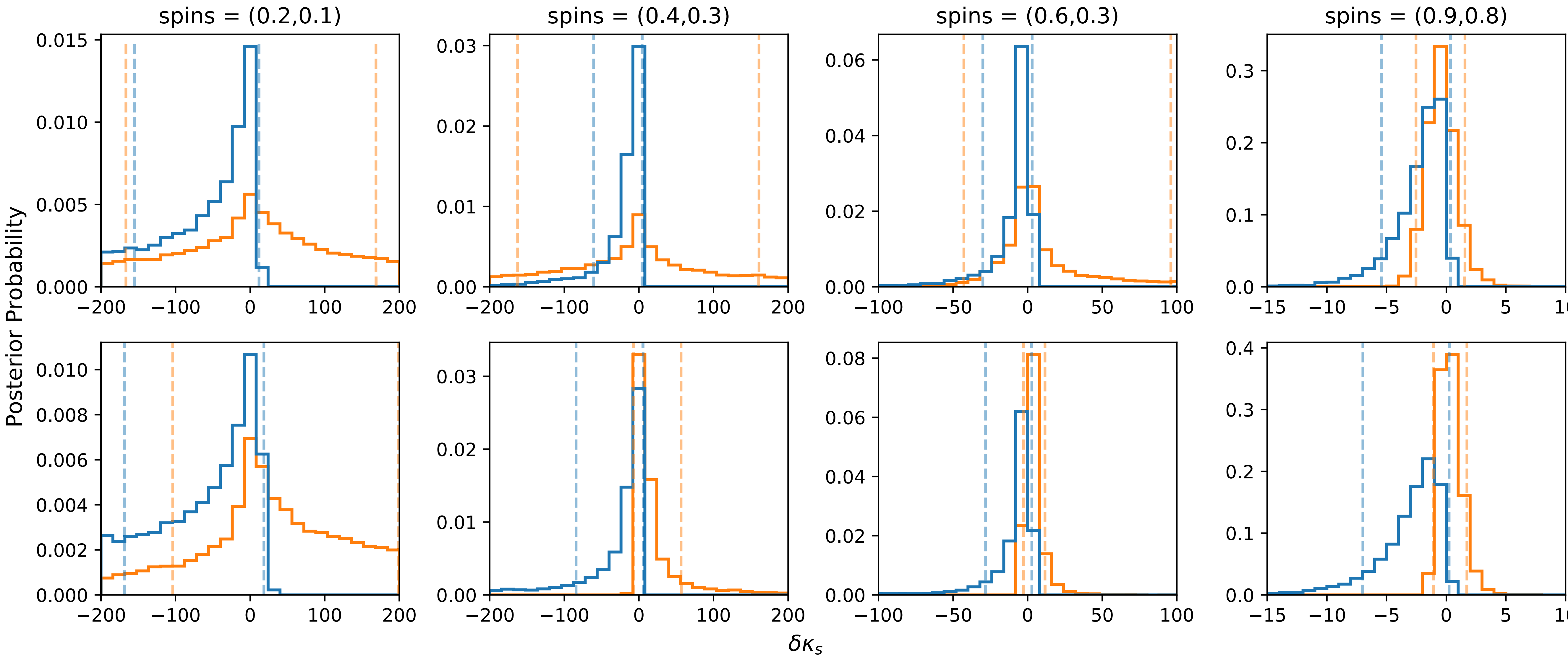
$(10,9)M_\odot$



aLIGO, SNR=10

Krishnendu, Arun, Mishra, PRL119, 091101,2017

Injection study (Bayesian)

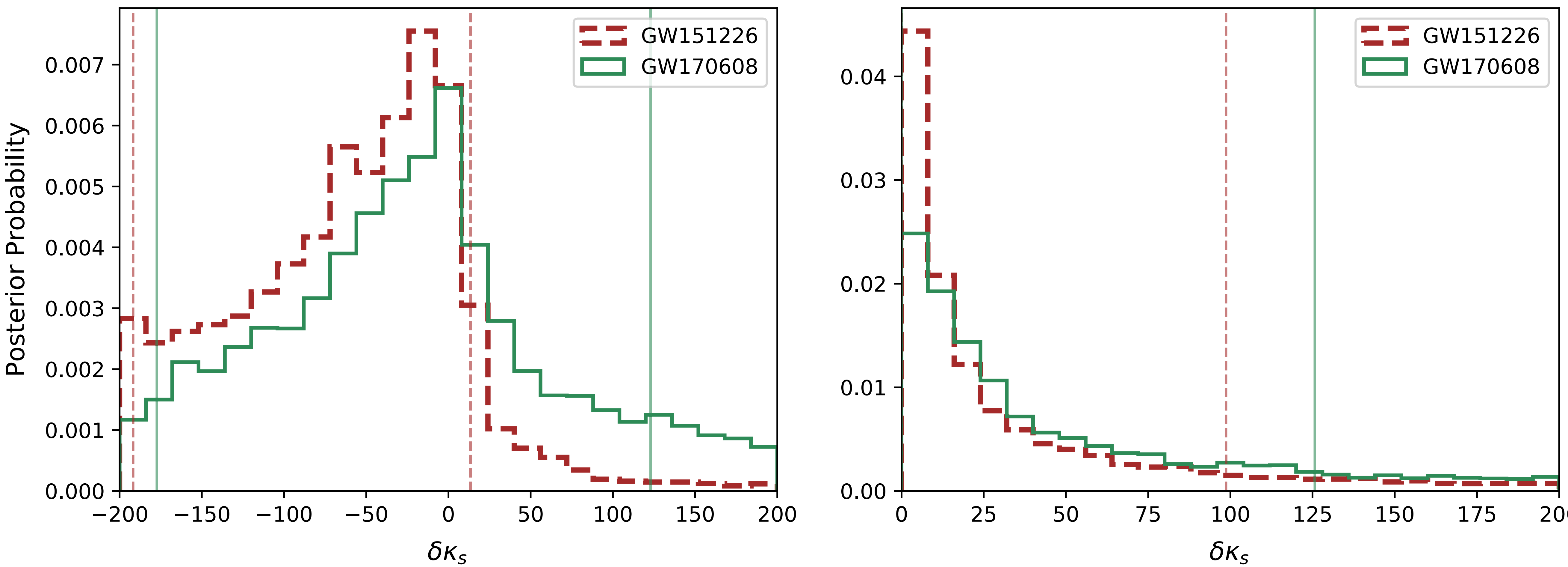


Trends in-line with expectations— 'high component spins lead to better constraints'

$$M = 15M_{\odot}, q = 1(\mathrm{top}),$$
$$q = 2(\mathrm{bottom}), D_L = 400\,\mathrm{Mpc}$$

Krishnendu, Saleem, Samajdar, Arun, Del Pozzo, Mishra, PRD 100, 104019 (2019)

Constraints from observed events



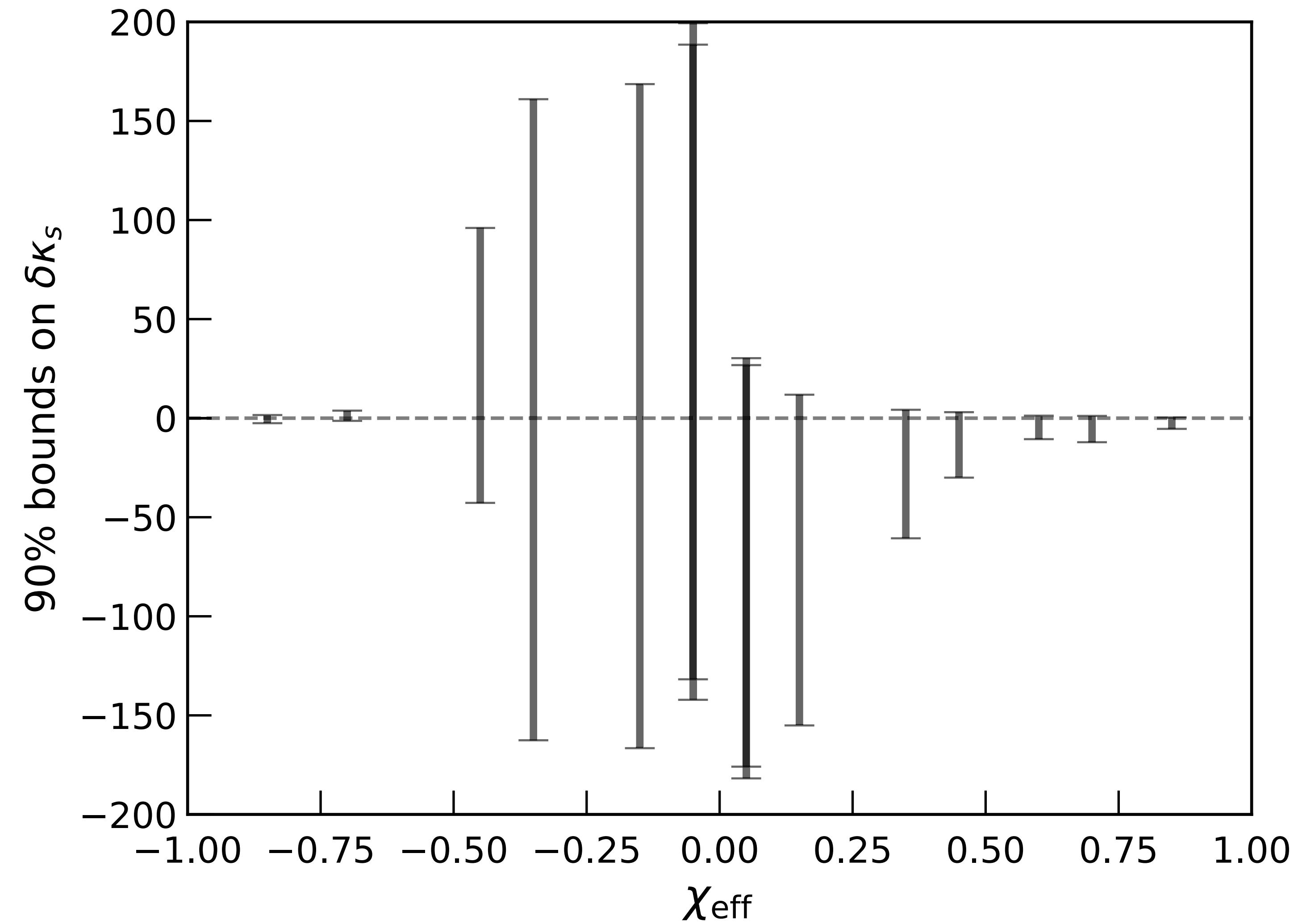
Event	Prior on $\delta\kappa_s$	90% bounds on $\delta\kappa_s$	Bayes factor ($\log \mathcal{B}_{\rm BH}^{\rm NBH}$)
GW151226	$[-200, 200]$	$[-191.78, 13.45]$	-0.94
	$[0, 200]$	≤ 98.67	-2.26
GW170608	$[-200, 200]$	$[-177.36, 122.98]$	-0.15
	$[0, 200]$	≤ 125.69	-1.15

- **No evidence for non-BBH nature though weaker constraints**

Krishnendu, Saleem, Samajdar, Arun, Del Pozzo, Mishra, PRD 100, 104019 (2019)

Effective spin

$$\chi_{\text{eff}} = \frac{m_1\chi_{1z} + m_2\chi_{2z}}{(m_1 + m_2)}$$

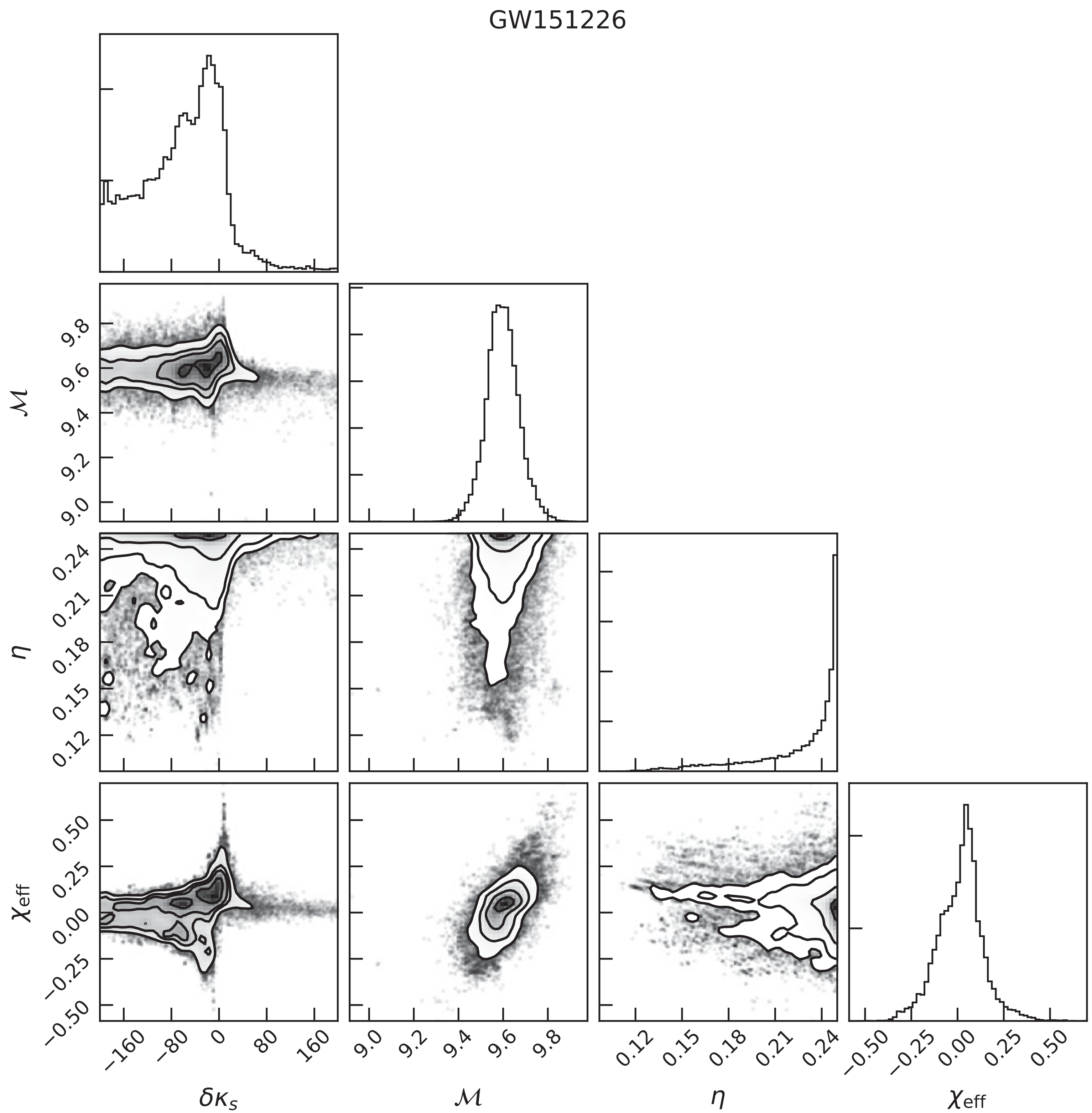


$$M = 15M_\odot, D_L = 400\,\text{Mpc}$$

Single most important parameter indicating the ability to constrain departure from BBH nature.

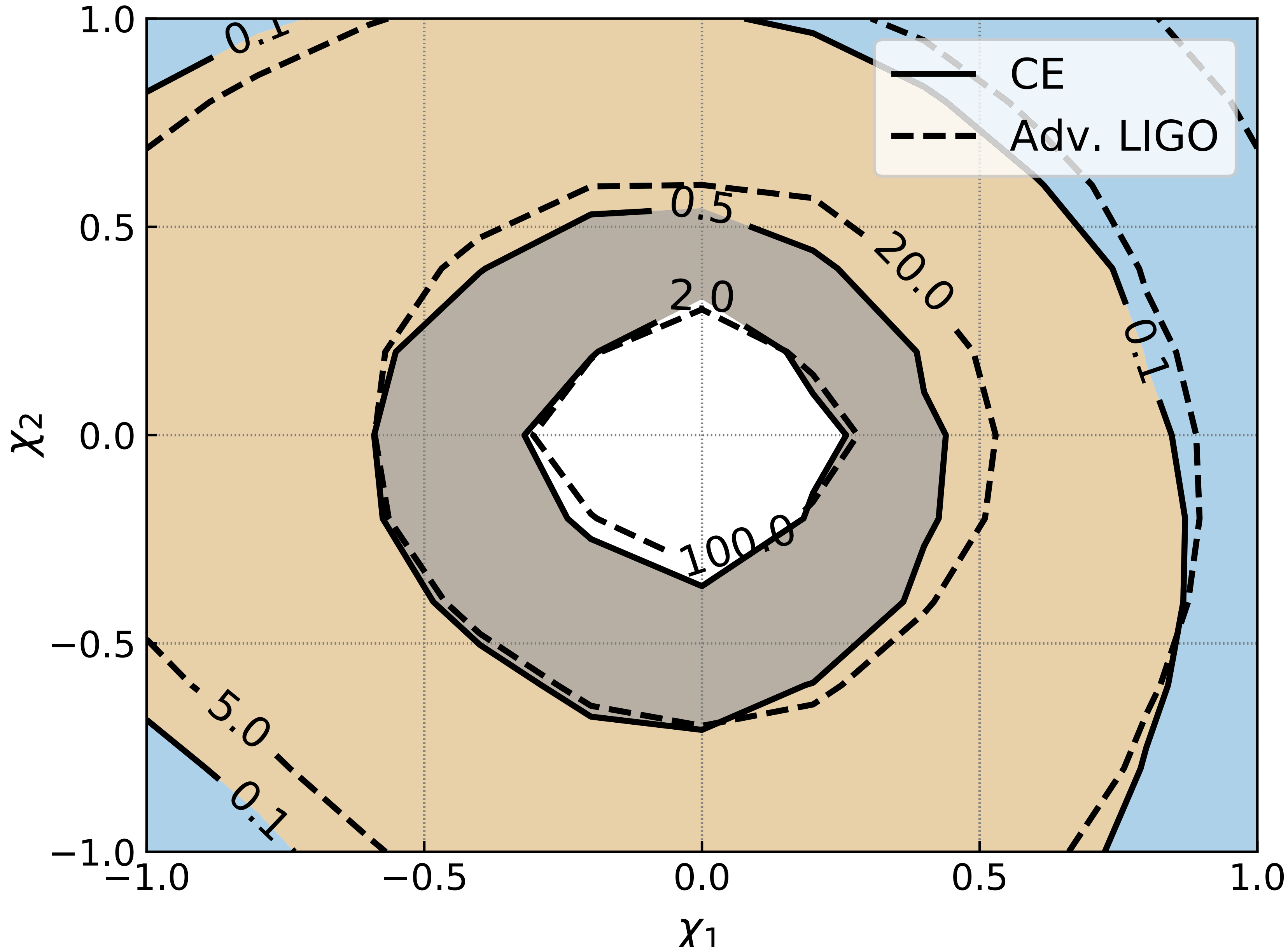
Krishnendu, Saleem, Samajdar, Arun, Del Pozzo, Mishra, PRD 100, 104019 (2019)

Corner plots



Beyond aLIGO
Cosmic Explorer, ET &
LISA

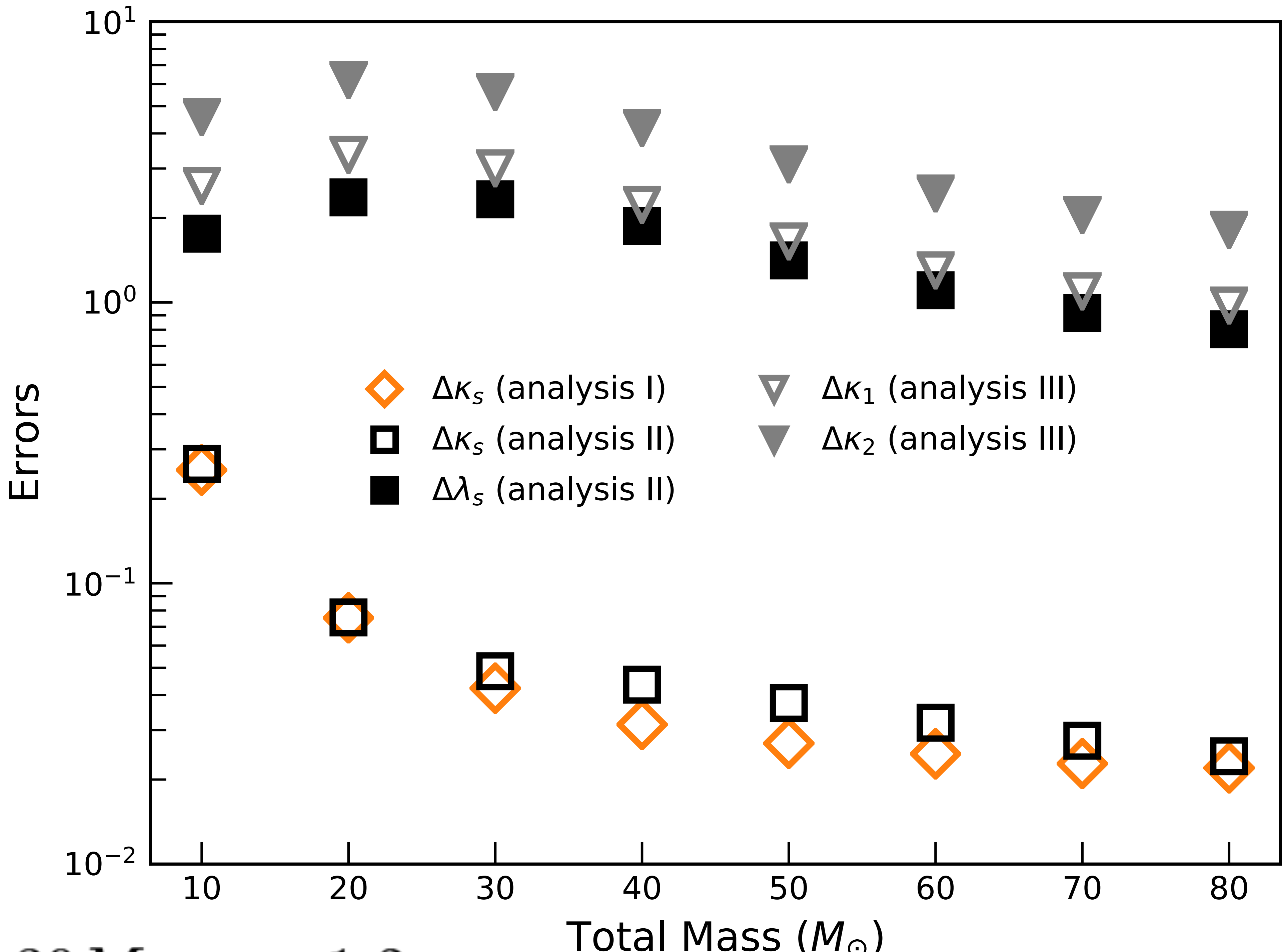
$1 - \sigma$ error contours for κ_s



$$M = 30M_{\odot}, q = 1.2,$$
$$D_L = 400\,\mathrm{Mpc}$$

Krishnendu, Mishra, Arun, PRD 99, 064008 (2019)

Simultaneous bounds (CE)

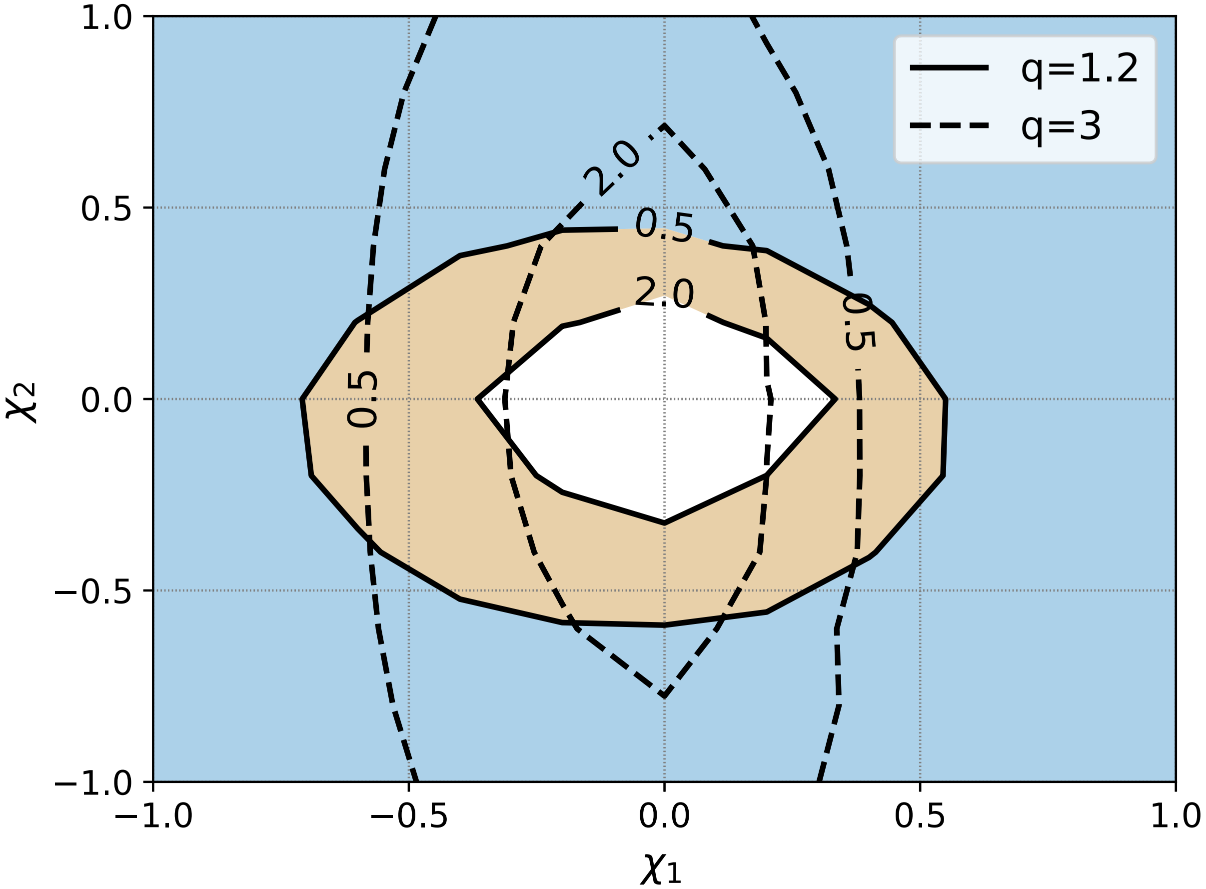


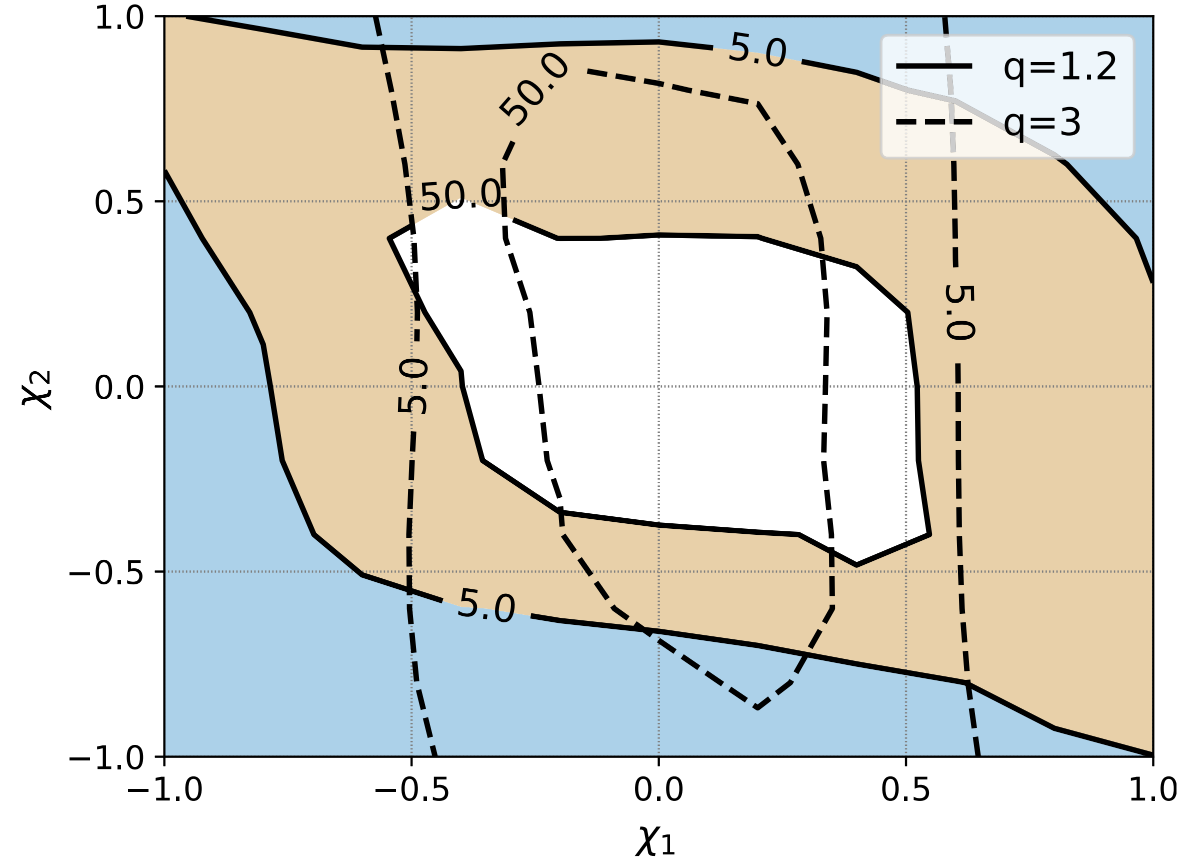
$$M = 30 M_\odot, q = 1.2,$$
$$D_L = 400\,\mathrm{Mpc}$$

Krishnendu, Mishra, Arun, PRD 99, 064008 (2019)

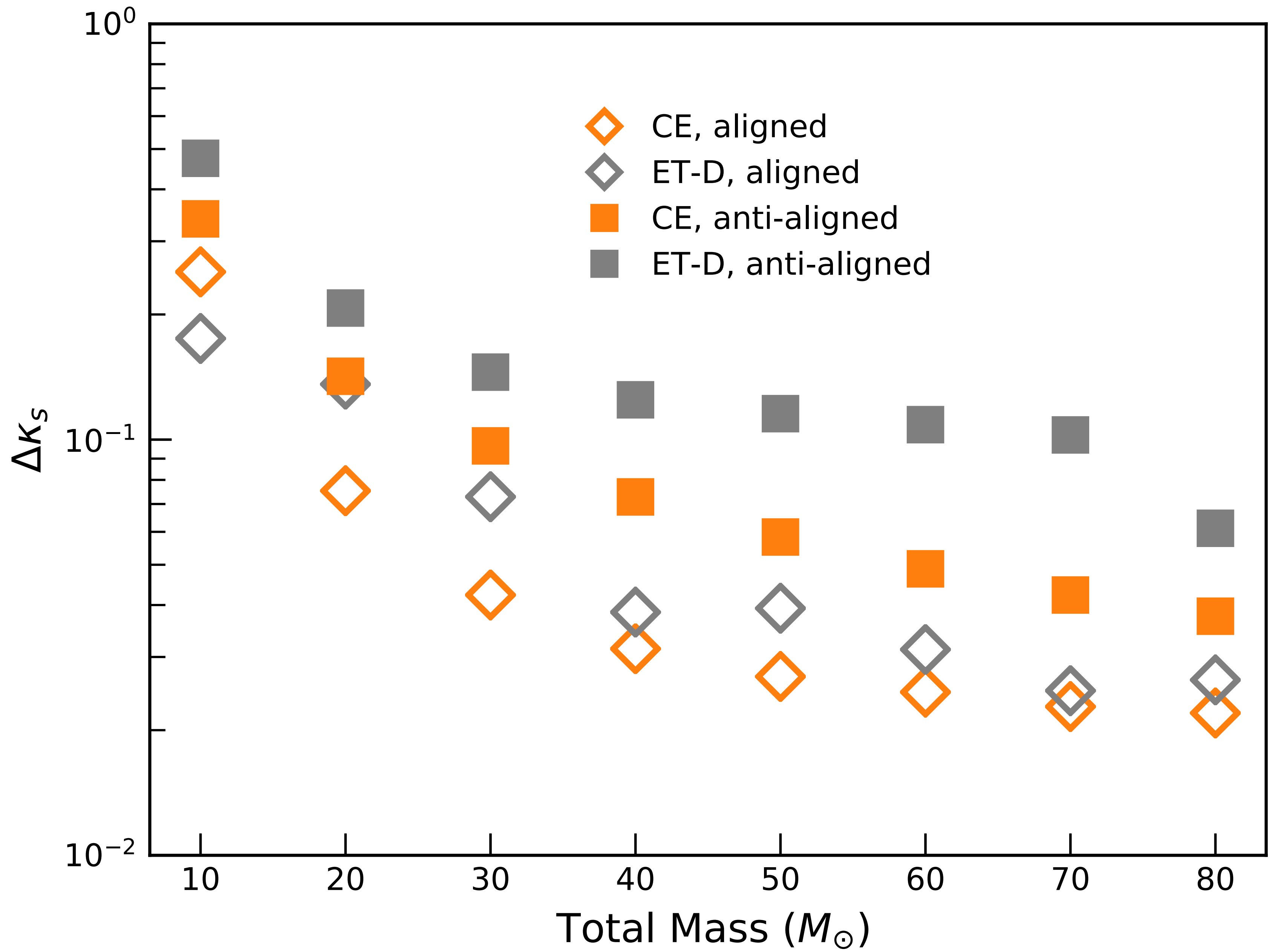
Cosmic Explorer

$$M = 30M_{\odot}, q = 1.2$$





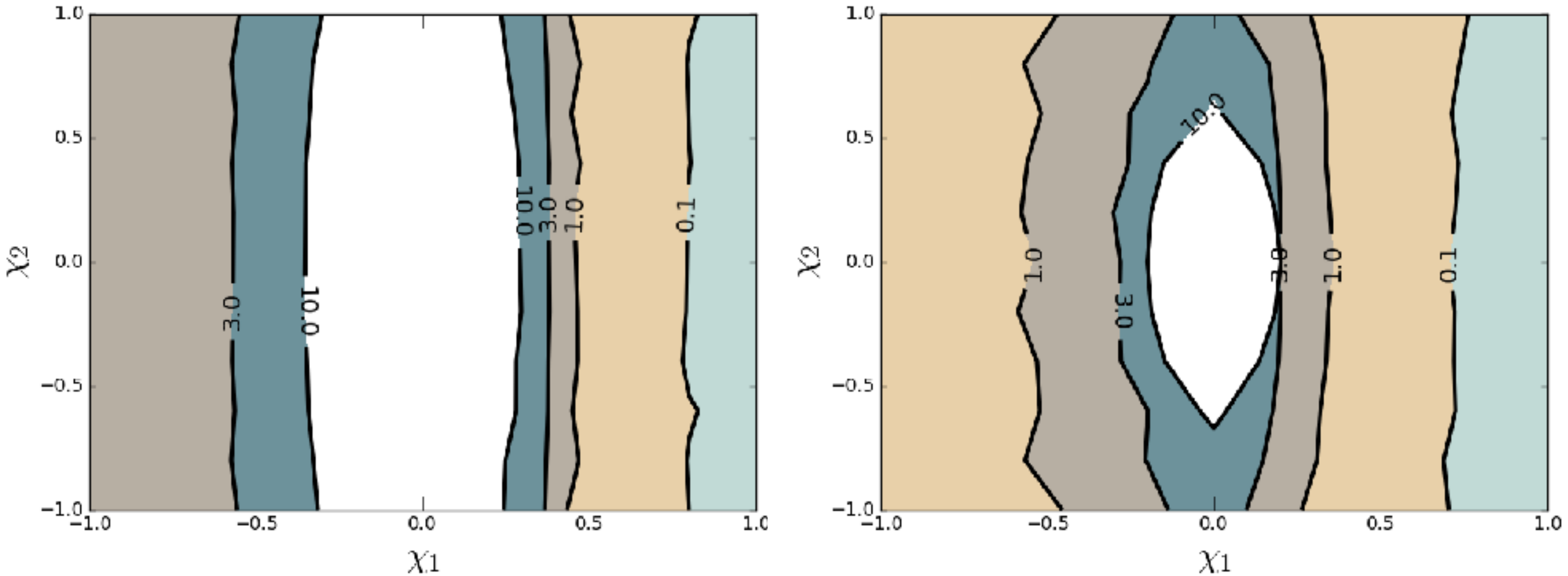
3G cases



$$M = 30M_\odot, q = 1.2,$$
$$D_L = 400\,\mathrm{Mpc}$$

LISA

$(10, 1) \times 10^6 M_\odot$ $\quad$ $(5, 1) \times 10^6 M_\odot$



$1 - \sigma$ error contours for κ_s

Summary and outlook

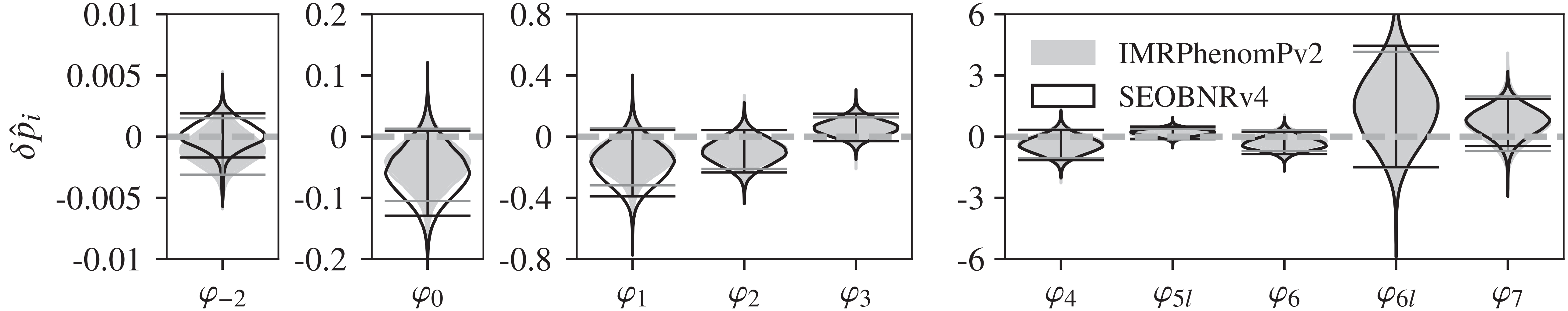
- First tests of GR using GWs probing the highly dynamical, highly relativistic and strong gravity regime performed — No evidence for departures from ‘the theory’.
- Tests further improved by combining the data collected during the first two observations runs —- to be aided by near future observations which are expected to be routine.

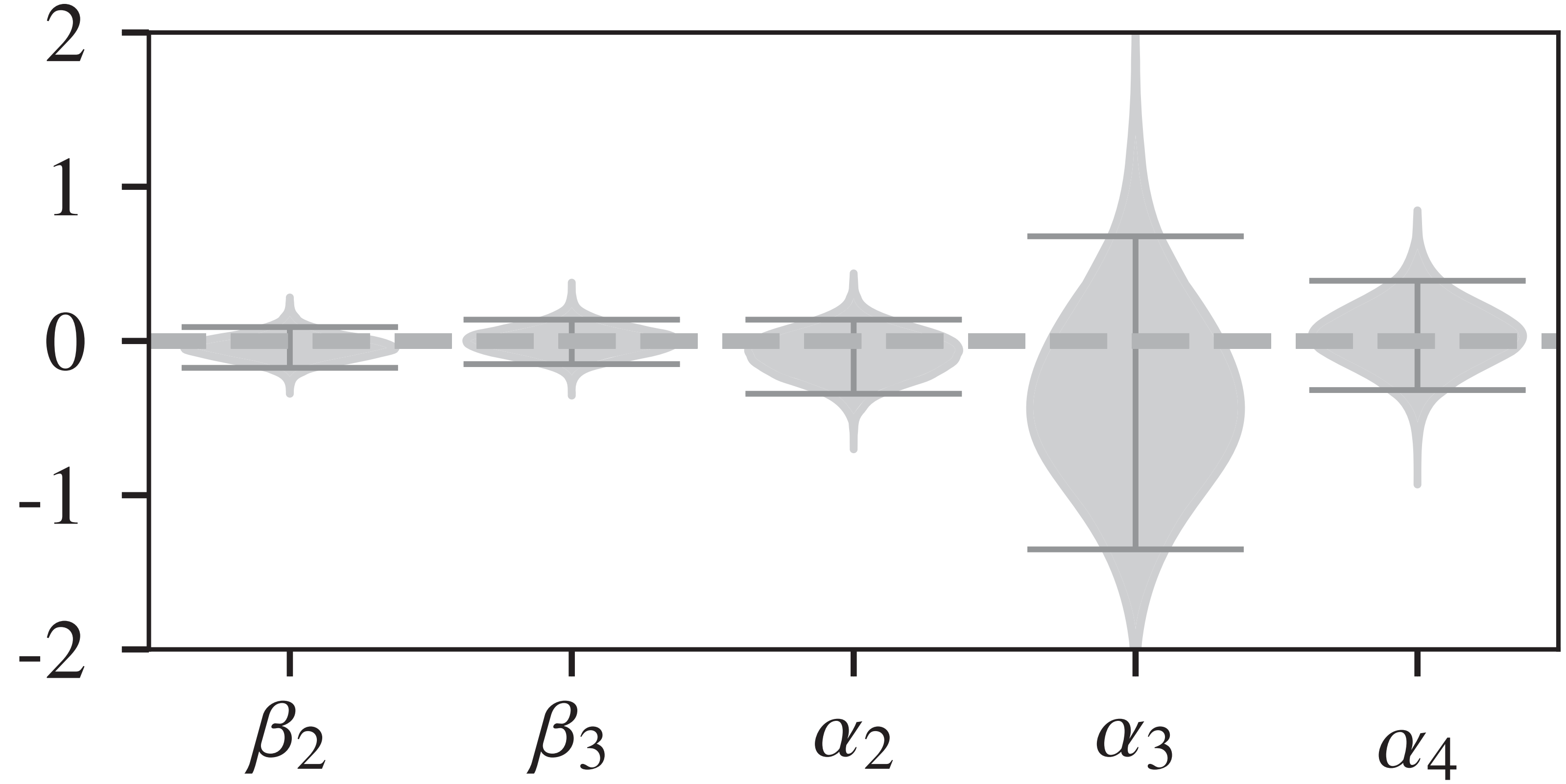
Summary and outlook

- Diverse spinning configurations, inclusion of contributions from the late stages of binary evolution to improve tests such as “No-hair” tests for BBHs.
- Synergy of tidal, spin-induced constraints in case of mixed, strong constraints compact components involving ECOs.
- Tests of General relativity involving observations of spinning-eccentric binaries in context of 2G/3G and LISA observations — probing eccentric space-times.

Additional slides

Combined posteriors (PI/PPI) O1-O2





Parameterised tests with GW170817