Einstein or Jordan:

seeking answers from the reheating constraints

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Introduction

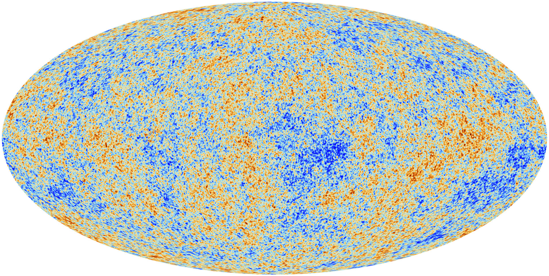

Figure 1: Cosmic Microwave Background Radiation (CMBR)

- CMB observation $\rightarrow$ Temperature fluctuations $\sim 10^{-5}$ with $T_0 \simeq 2.73\ k$ $\rightarrow$ Early Universe was isotropic and homogeneous (imposing spatial symmetry).
- Perturbed spectra $\rightarrow$ nearly scale-invariant.
- Homogeneous and isotropic Universe brings two problems in the early Universe:
  - Casually disconnected region showing similar behavior behavior $\rightarrow$ Horizon problem.
  - $\Omega_k \propto a^{-2}(t)$ and $\Omega_k^0 \sim 10^{-2} \rightarrow \Omega_k^{earlier}$ is extremely small and fine-tuned $\rightarrow$ Flatness problem.

- FLRW (homogeneous and isotropic) line element:

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\mathbf{x}^2 = a^2(\eta)\left(-\,\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2\right)$$

  where t is cosmic time, $\eta \equiv \int \mathrm{d}t/a(t)$ is conformal time (comoving horizon) and a is the scale factor.

- Solving Horizon problem:
  - At very early times $\rightarrow -\infty < \eta \leq 0$,
  - and later, $0 \leq \eta < \infty$.

  i.e., the comoving horizon η shrinks and later expands.

- $\Omega_k \propto (-k_0\,\eta)^2 \rightarrow$ Even if the spatial curvature is present, it will die down quickly at early times.
- Inflation: accelerating universe $\ddot{a}(t)/a(t) > 0 \rightarrow$ solves the issue.
- Scale factor $a(\eta) \propto (-\eta)^{\alpha},\ \alpha \leq -1$.

- Curvature $\zeta(t, \mathbf{x})$ and tensor perturbations $h_{ij}(t, \mathbf{x})$ generate deep inside the horizon (vacuum fluctuations) $\rightarrow$ cross it and then freeze $\rightarrow$ enters again at late times $\rightarrow$ leave imprints on the CMB and also, seed the Large Scale Structure formation.

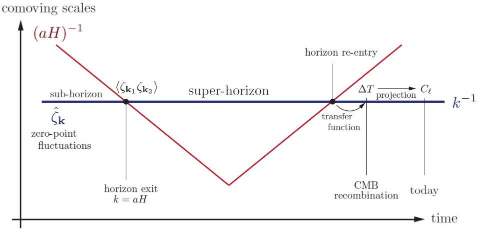

Figure 2: Evolution of Hubble horizon.

- Observations: $\mathcal{P}_\zeta \equiv A_s \ (k/k_\star)^{n_s - 1}, \quad \mathcal{P}_T \equiv A_T \ (k/k_\star)^{n_T}, \quad r \equiv A_T/A_s.$
- $n_s = 0.9649 \pm 0.0042, \quad A_s \simeq 2.1 \times 10^{-9}, \quad r_{0.002} < 0.0056.$
- The simplest inflationary model consistent with the observations $\rightarrow$ slow-roll inflation $\rightarrow$ driven by the simplest field: a scalar field.

Minimal Einstein Gravity: Evolution of the perturbations

- Minimal → Gravity minimally coupled (Einsteinian GR) with single canonical scalar field:

$$\mathcal{S} = \frac{1}{2}\int \mathrm{d}^4\mathbf{x}\sqrt{-g}\left(\frac{R}{M_{\rm pl}^2} - g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 2V(\varphi)\right).$$

- Background equations of motion:

$$\mathcal{H}^2 = \frac{1}{3\,M_{\rm pl}^2}\left(\frac{1}{2}\varphi'^2 + V(\varphi)\right),$$

$$\varphi'' + 2\mathcal{H}\varphi' + a^2\,V_{,\varphi}(\varphi) = 0, \quad \mathcal{H} \equiv \frac{a'(\eta)}{a(\eta)}$$

- Perturbed equations of motion: $g^s_{ij} = a^2(\eta)\,e^{2\zeta}\,\delta_{ij},\ g^T_{ij} = a^2(\eta)\,e^{2h}|_{ij}.$

$$\zeta''_{\mathbf{k}} + 2\frac{z'}{z}\zeta'_{\mathbf{k}} + k^2\zeta_{\mathbf{k}} = 0, \quad z(\eta) = a(\eta)\sqrt{\epsilon(\eta)}$$

$$h''_{\mathbf{k}} + 2\frac{a'}{a}h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = 0$$

- Scalar and tensor power-spectra: $\mathcal{P}_\zeta = k^3/(2\pi^2)|\zeta_{\mathbf{k}}|^2, \quad \mathcal{P}_T = 2k^3/\pi^2|h_{\mathbf{k}}|^2.$

Slow-roll dynamics and the perturbed observables

- Slow-roll parameters:

$$\epsilon_V \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\varphi}}{V}\right)^2, \quad \eta_V \equiv M_{\rm pl}^2\left(\frac{V_{,\varphi\varphi}}{V}\right).$$

- Slow-roll inflation condition:
  $\{\epsilon_V, \eta_V\} \ll 1 \ \rightarrow a(\eta) \simeq -1/(H\eta)$.
- Perturbed observables:

$$n_s \simeq 1 - 6\epsilon_V + 2\eta_V, \ \ r \simeq \ 16\,\epsilon_V$$

with

$$N_\star \simeq \int_\star^{\rm end} \frac{1}{\sqrt{2\epsilon_V}}\,{\rm d}\varphi/M_{\rm pl}, \ \ N_\star \simeq 50\text{–}60.$$

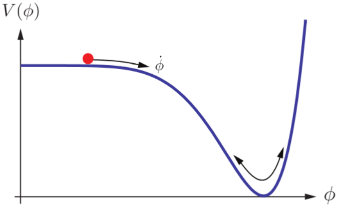

Figure 3: Slow-roll inflationary potential.

- Inflation ends → field oscillating around the minima → Reheating.
- So far the simplest yet the 'best' model of inflation → Starobinsky model of inflation with $V(\varphi) = \frac{3}{4}\,m^2 M_{\rm pl}^2\left(1 - \exp\left(-\sqrt{2/3}\,\varphi/M_{\rm pl}\right)\right)^2$.

Non-minimal Gravity

- However, the non-minimal theory where the matter (scalar field) is coupled with the gravity can also drive slow-roll inflation.
- Simplest non-minimal theory:

$$\mathcal{S}_{\rm nm} = \frac{1}{2}\int \mathrm{d}^4\mathbf{x}\sqrt{-g}\left(\frac{\varphi R}{M_{\rm pl}^2} - \omega(\varphi)\, g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 2V(\varphi)\right).$$

- Original Starobinsky model with near de-Sitter solution $\rightarrow$ $f(R)$ theory $\rightarrow$ can be written with $\omega(\phi) = 0$ with the help of an auxiliary field as:

$$\mathcal{S}_{\rm nm} = \frac{1}{2}\int \mathrm{d}^4\mathbf{x}\sqrt{-g}\left(\frac{\varphi R}{M_{\rm pl}^2} - 2V(\varphi)\right)$$

- Starobinsky model: $f(R) = R + 1/6m^2 R^2 \;\rightarrow V(\varphi) = 3/4 m^2 M_{\rm pl}^2(\varphi - 1)^2$.

Perturbed observables in the non-minimal theory

- Consider the simplest with $\omega(\varphi) = 0$, *i.e.*, $f(R)$ theory of gravity.
- Similar to minimal Einstein theory, slow-roll parameters:

$$\begin{aligned}\epsilon_1 &\equiv -\frac{\dot{H}}{H^2} \simeq \frac{(\varphi V_\varphi - 2V)(\varphi V_\varphi - V)}{3V^2} \\ \epsilon_2 &\equiv \frac{\dot{\varphi}}{2H\varphi} \simeq \epsilon_1 - \frac{2\varphi(\varphi V_{\varphi\varphi} - V_\varphi)}{3V} \\ \epsilon_3 &\equiv \frac{\ddot{\varphi}}{H\dot{\varphi}} \simeq -\frac{(\varphi V_\varphi - 2V)}{3V},\end{aligned}$$

- Inflationary observables:

$$\begin{aligned}n_s &\simeq 1 - 4\,\epsilon_1 + 2\,\epsilon_2 - 2\,\epsilon_3 \\ A_s &\simeq \frac{V^3}{8\pi^2\varphi^2 M_{\rm Pl}^4\,(\varphi V_\varphi - 2V)^2} \\ r &\simeq 48\,\epsilon_3^2\end{aligned}$$

- Perturbed mode leaving the horizon before the end of inflation:

$$N_\star = \int_\star^{\rm end} \frac{1}{2\,\varphi\,\epsilon_2}\,{\rm d}\varphi, \quad N_\star \simeq 50 - 60.$$

Conformal connection and the “equivalence”

- The above non-minimal theory can easily be transformed into a minimal one by using the conformal transformation:

$$g_{\mu\nu}^{\rm m} = \varphi\, g_{\mu\nu}^{\rm nm}.$$

- To make the field canonical, we need the transformation: $\varphi \to \exp(\sqrt{2/3}\,\varphi/M_{\rm Pl})$ and $V(\varphi) \to V(\varphi)/\varphi^2$.
- Example: One can easily check that, under such transformation, the non-minimal Starobinsky potential $3/4m^2 M_{\rm pl}^2(\varphi-1)^2$ reduces to the minimal Starobinsky potential $3/4\, m^2 M_{\rm pl}^2 \left(1 - \exp\left(-\sqrt{2/3}\,\varphi/M_{\rm pl}\right)\right)^2$.
- Under conformal transformation:
  - Background: scalar factor $a_{\rm m}(\eta) = \sqrt{\varphi(\eta)}\, a_{\rm nm}(\eta)$.
  - Perturbations: $\zeta^{\rm m} = \zeta^{\rm nm},\ h_{ij}^{\rm m} = h_{ij}^{\rm nm}$.
- If two theories are conformally connected, the perturbed observables remain the same.
- Example: Perturbed observables in Starobinsky $f(R)$ theory is same as the minimal Starobinsky model.
- In fact, it can be shown that, even the background observables changes accordingly in such a way that the conformal frames become indistinguishable. However, it is still an open problem to show, in general, these frames are indistinguishable.
- What about Reheating as the physics is a complex.

Reheating

- Scalar field φ decays into other particles (eventually standard model particles).
- $\varphi \to \chi + \chi \ \to$ Decay rate $\Gamma_{\phi\to\chi\chi}$.
- Equation: $\ddot{\varphi} + 3H\dot{\varphi} + \Gamma\,\dot{\varphi} + V_{\varphi} = 0$.
- The expansion rate H decreases with time, and reheating completes when $\Gamma = H$.
- $V(\varphi) = 1/2\, m^2\varphi^2 \quad \Rightarrow \varphi \simeq M_{\rm Pl}/(mt)\sin(mt), \quad \rho_{\varphi} \propto a(t)^{-3}$. This implies the effective equation of state is $w_{\rm re} = 0$.
- $V(\varphi) \propto \varphi^p \ \to w_{\rm re} \simeq (p-2)/(p+2)$.
- Too simplistic $\to$ Needs simulations like Lattice $\to$ $w_{\rm re}$ varies and the duration of reheating is small.

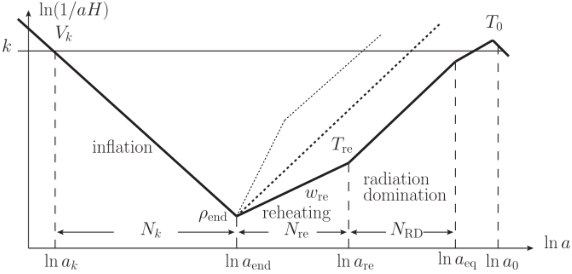

Figure 4: The evolution of comoving Hubble scale with reheating.

- In our case, Quantitative analysis.
- Assumption $\rightarrow$ $w_{\rm re}$ nearly constant: $\rho \propto a(t)^{-3(1+w_{\rm re})}$.
- Constraint on the total amount of the expansion:

$$\frac{k}{a_0 H_0} = \frac{a_k}{a_{\rm end}} \frac{a_{\rm end}}{a_{\rm re}} \frac{a_{\rm re}}{a_{\rm eq}} \frac{a_{\rm eq} H_{\rm eq}}{a_0 H_0} \frac{H_k}{H_{\rm eq}}$$

$$\Rightarrow \ln\left(\frac{k}{a_0 H_0}\right) = -N_k - N_{\rm re} - N_{\rm RD} + \ln\left(\frac{a_{\rm eq} H_{\rm eq}}{a_0 H_0}\right) + \ln\left(\frac{H_k}{H_{\rm eq}}\right).$$

- Reheating e-folding number:

$$N_{\rm re} = \frac{1}{3(1+w_{\rm re})} \ln\left(\frac{\rho_{\rm end}}{\rho_{\rm re}}\right).$$

- Also, entropy conservation provides another similar equation:

$$\frac{3}{4(1+w_{\rm re})} N_{\rm re} = \frac{1}{4}\ln\left(\frac{30}{g_{\rm re}\pi^2}\right) + \frac{1}{4}\ln\left(\frac{\rho_{\rm end}}{T_0^4}\right) + \frac{1}{3}\ln\left(\frac{11 g_{\rm s,re}}{43}\right) + \ln\left(\frac{a_{\rm eq}}{a_0}\right) - N_{\rm RD}.$$

- $\{g_{\rm re}, g_{\rm s,re}\} \rightarrow$ effective number of relativistic species upon thermalization and effective number of light species for entropy at reheating, respectively ~ 100.
- The final expression:

$$N_{\rm re} = \frac{4}{3w_{\rm re}-1}\left[N_k - \ln(H_k) + \frac{1}{4}\ln(\rho_{\rm end}) + \ln\left(\frac{k}{a_0 T_0}\right) + \frac{1}{4}\ln\left(\frac{30}{\pi^2 g_{\rm re}}\right) + \frac{1}{3}\ln\left(\frac{11 g_{\rm s,re}}{43}\right)\right]$$

- Reheating temperature:

$$T_{\rm re} = \left(\frac{43}{11 g_{\rm s,re}}\right)^{1/3}\left(\frac{a_0 T_0}{k}\right) H_k\, e^{-N_k - N_{\rm re}}.$$

- N_k and H_K can be expressed in terms of φ, which, in turn, can be written in terms of the scalar spectral index n_s :

$$N_k = N_k(n_s), \quad H_k = H_k(n_s).$$

- Also, $\rho_{\rm end}$ is different in different conformal frames as $\epsilon_1 = 1$ in one frame does not imply $\tilde{\epsilon}_1 = 1$ in another frame.
- Different equations for different frames, which has been shown before.
- We plot the $N_{\rm re}$ and $T_{\rm re}$ in terms of n_s only.
- This expression provides extra constraint equation for the spectral index n_s.
- Assumption: $w_{\rm re}$ same in both frames.
- The difference:
  - ΔN_k is negligible and does not contribute much.
  - $\Delta\rho_{\rm end}$ is non-zero and greater than ΔN_k, but the contribution is still small.
  - ΔH_k is significant because $H_k^m \simeq \frac{1}{\sqrt{\varphi_k}} H_k^{nm}$.
- We find a difference.

Results: Starobinsky and chaotic inflation

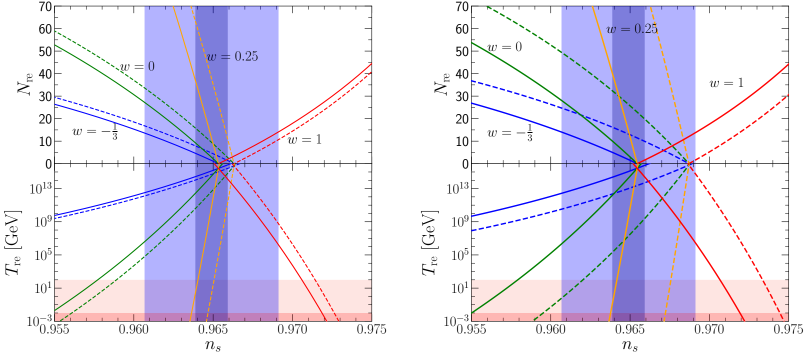

Figure 5: We plot $N_{\rm re}$ and $T_{\rm re}$ with n_s for the Starobinsky model (left) and the chaotic $m^2\phi^2$ model (right). The solid lines are for the Einstein frame while the dashed lines are for the Jordan frame. Different colors represent different effective equation of the state $w_{\rm re}$. The blue shaded region is the Planck 1σ region with $n_s = 0.9649 \pm 0.0042$. The dark blue region indicates the future projected bound on $n_{\rm s}$ with a sensitivity of 10^{-3} assuming the central value of it will remain unchanged. The temperature below the deep red region is excluded due the constraints from BBN while the lighter red region is the electroweak scale taken to be 100 GeV.

Results: Different α-attractor models

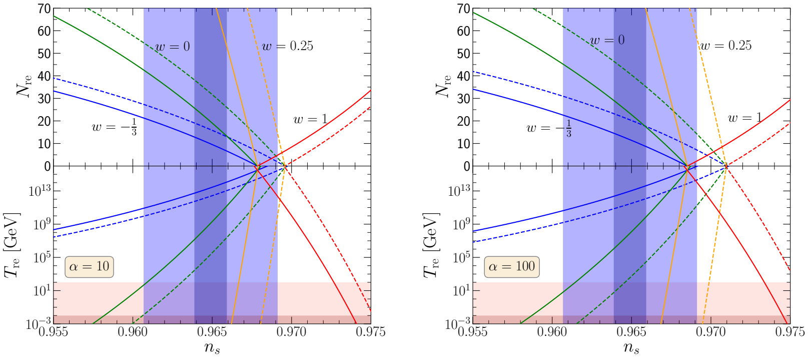

Figure 6: We plot the variation of reheating e-folding number $N_{\rm re}$ and reheating temperature $T_{\rm re}$ with the scalar spectral index n_s for the α-attractor model with $\alpha = 10$ (left) and $\alpha = 100$ (right). Note that the Starobinsky model is a special case of $\alpha = 1$.

Results in a nutshell

- If the perturbations remain the same $\rightarrow$ $N_{\rm re}$ and $T_{\rm re}$ are different and hence the thermal history.
- Difference depends on the field value φ.
  - Small field models $\rightarrow$ small difference.
  - Large field models $\rightarrow$ difference is big.
- Reheating constraints are also difference in different frames.
  - From the future experiment like EUCLID [*arXiv* : 1206.1225] and PRISM [*arXiv* : 1306.2259], cosmic 21-cm surveys [*arXiv* : 0802.1710] and CORE [*arXiv* : 1612.08270] experiments with 10^{-3} sensitivity in the scalar spectral index n_s, it may (can) actually rule out frames.
- There is another way to look at: $N_{\rm re}$ and $T_{\rm re}$ are the same and so the perturbations are different in different frames. Then the thermal history remains the same.
- Always, there is a difference either in the thermal history or in the perturbations.

Summary and conclusions

- We found a difference in conformal frames which indicates that the conformal frames may not actually be 'equivalent" and the future experiments may distinguish minimal and non-minimal theories.
- It needs further investigation:
  - Qualitative reheating analysis.
  - Full numerical evolution of the background till now.
- We should look for other signatures in different cosmic ages.
- A new outlook to look at the conformal theories.