Just Enough Inflation

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Overview

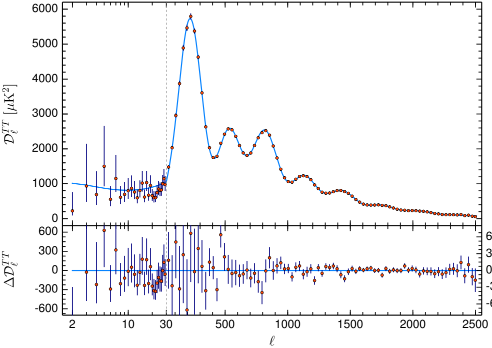
- Motivation for ‘just enough inflation’
- Power spectrum from just enough inflation
- Bispectrum from just enough inflation
- Behaviour of the non-Gaussianity parameter
- Conclusion

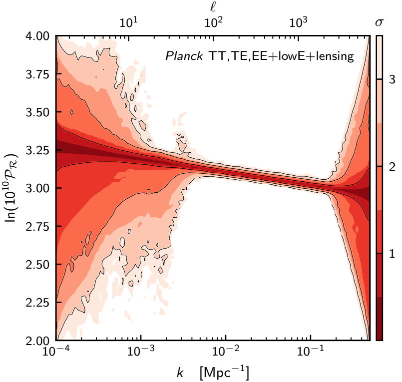
Inflation - just enough

- Inflation is an epoch of accelerated expansion of spacetime before the onset of radiation dominated era.
- About 60 e-folds of this epoch is necessary to resolve the cosmological problems - horizon problem, flatness problem, etc.
- The scalar perturbations evolved from such an epoch explains the origin of tiny anisotropies observed in the Cosmic Microwave Background (CMB).
- If this epoch had lasted only for the required number of e-folds, such a scenario is termed as 'just enough inflation'[1].

[1] Carlo R Contaldi, et.al., JCAP, **2003**, 002, (2003);
E. Ramirez and D. J. Schwarz, Phys. Rev. D., **85**, 103516, (2012);
L.T. Hergt, et.al. Phys. Rev. D., **100**, 023501, (2019).

Motivation



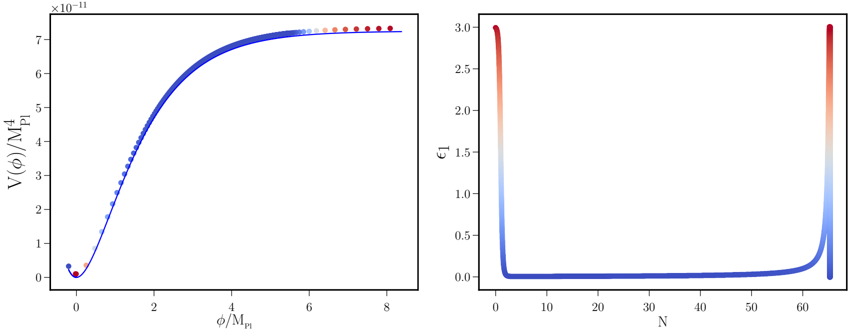


The $\mathcal{C}_\ell$ s of the anisotropies in CMB have consistently exhibited low power over large scales[2].

[2] Planck Collaboration, (2018) [arXiv: 1807.06211v2[astro-ph.CO]].

Dynamics

Scalar field with large initial kinetic energy can effect this scenario.

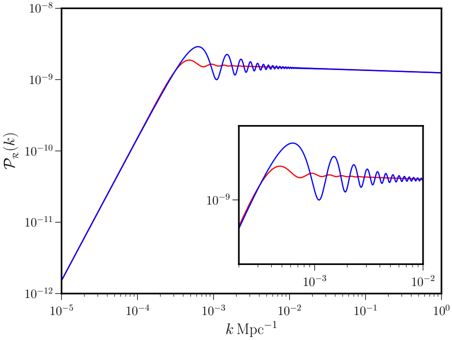


Field excursion in 'just enough inflation'. $\epsilon_1(N) = \phi_N^2/2$

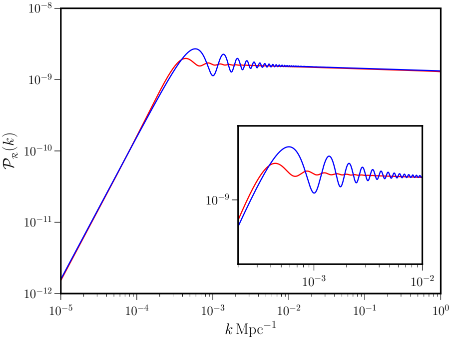
Power spectrum from just enough inflation

The power spectra of perturbations in such a scenario exhibit suppression over modes of that are outside the Hubble radius.

Models :$V(\phi) = m^2\phi^2/2, \quad V_0[1 - e^{-\sqrt{2/3}(\phi/M_{\rm Pl})}]^2$.



Quadratic potential



Starobinsky model

Scalar power spectra from just enough inflation.

Red : $\epsilon_{1_i} = 1$, Blue : $\epsilon_{1_i} = 2.99$

Scalar bispectrum - Cubic order action

The cubic order action contributing to the bispectrum of scalar perturbation $\mathcal{R}$ is of the form[3]

$$S_3^{\rm Bulk}[\mathcal{R}] = M_{\rm Pl}^2 \int {\rm d}\eta\, {\rm d}^3\boldsymbol{x} \left[a^2 \epsilon_1^2\, \mathcal{R}{\mathcal{R}'}^2 + a^2 \epsilon_1^2\, \mathcal{R}(\partial\mathcal{R})^2 - 2\, a\, \epsilon_1\, \mathcal{R}'\partial_i\mathcal{R}\partial^i\chi \right.$$
$$+ \frac{a^2}{2}\epsilon_1\epsilon_2'\, \mathcal{R}^2\mathcal{R}' + \frac{\epsilon_1}{2}\partial_i\mathcal{R}\partial^i\chi\partial^2\chi + \frac{\epsilon_1}{4}\partial^2\mathcal{R}(\partial\chi)^2$$
$$\left. + 2\, \mathcal{F}_1(\mathcal{R})\frac{\delta\mathcal{L}_{\mathcal{R}\mathcal{R}}}{\delta\mathcal{R}}\right]$$

[3] Arroja and Tanaka, JCAP, **2011**, 005, (2011).

Scalar bispectrum - Cubic order action

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\begin{aligned}
S_3^{\mathrm{Bulk}}[\mathcal{R}] &= M_{\mathrm{Pl}}^2 \int \mathrm{d}\eta\, \mathrm{d}^3\boldsymbol{x} \Big[a^2\epsilon_1^2\, \mathcal{R}\mathcal{R}'^2 + a^2\epsilon_1^2\, \mathcal{R}(\partial\mathcal{R})^2 - 2\, a\, \epsilon_1\, \mathcal{R}'\partial_i\mathcal{R}\partial^i\chi \\
&+\frac{a^2}{2}\epsilon_1\epsilon_2'\, \mathcal{R}^2\mathcal{R}' + \frac{\epsilon_1}{2}\partial_i\mathcal{R}\partial^i\chi\partial^2\chi + \frac{\epsilon_1}{4}\partial^2\mathcal{R}(\partial\chi)^2 \\
&+2\,\mathcal{F}_1(\mathcal{R})\frac{\delta\mathcal{L}_{\mathcal{R}\mathcal{R}}}{\delta\mathcal{R}}\Big] \\
S_3^{\mathrm{Boundary}}[\mathcal{R}] &= M_{\mathrm{Pl}}^2 \int \mathrm{d}\eta\, \mathrm{d}^3\boldsymbol{x}\, \frac{\mathrm{d}}{\mathrm{d}\eta}\Big[-9\, a^3 H\, \mathcal{R}^3 + \frac{a}{H}\mathcal{R}(\partial\mathcal{R})^2 \\
&-\frac{1}{4\, aH^3}(\partial\mathcal{R})^2\partial^2\mathcal{R} - \frac{a\epsilon_1}{H}\mathcal{R}(\partial\mathcal{R})^2 \\
&-\frac{a\epsilon_1}{H}\mathcal{R}\mathcal{R}'^2 - \frac{a\epsilon_2}{2}\mathcal{R}^2\partial^2\chi + \frac{\mathcal{R}}{2\, aH^2}\left(\partial_i\partial_j\mathcal{R}\partial^i\partial^j\chi - \partial^2\mathcal{R}\partial^2\chi\right) \\
&-\frac{\mathcal{R}}{2\, aH}\left(\partial_i\partial_j\chi\partial^i\partial^j\chi - \partial^2\chi\partial^2\chi\right)\Big]
\end{aligned}
$$

Here, $\chi = a\epsilon_1\partial^{-2}\mathcal{R}'$ and $\mathcal{F}_1(\mathcal{R}) = \frac{1}{4}\epsilon_2\mathcal{R}^2 + \frac{\mathcal{R}\mathcal{R}'}{aH} + \frac{1}{4a^2H^2}\Big[-(\partial\mathcal{R})(\partial\mathcal{R}) + \partial^{-2}(\partial_i\partial_j(\partial_i\mathcal{R}\partial_j\mathcal{R}))\Big] + \frac{1}{2a^2H}\Big[(\partial\mathcal{R})(\partial\chi) - \partial^{-2}(\partial_i\partial_j(\partial_i\mathcal{R}\partial_j\chi))\Big]$.

[3] Arroja and Tanaka, JCAP, **2011**, 005, (2011).

Scalar bispectrum

The scalar bispectrum, evaluated at the end of inflation, η_e, is defined as

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_e)\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_e)\hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_e)\rangle \equiv (2\pi)^{3/2}G(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3).$$

This is evaluated using the relation

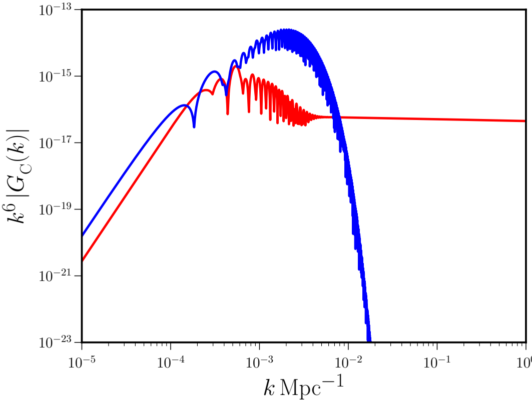
$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_e)\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_e)\hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_e)\rangle = -i\int_{\eta_i}^{\eta_e}\mathrm{d}\eta\langle\left[\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_e)\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_e)\hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_e),\hat{H}^{\mathrm{int}}_{\mathcal{RRR}}(\eta)\right]\rangle$$

where, $\hat{H}^{\mathrm{int}}_{\mathcal{RRR}} = -\hat{L}^{\mathrm{int}}_{\mathcal{RRR}}$. Hence,

$$G(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = G_{\mathrm{Bulk}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + G_{\mathrm{Boundary}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3).$$

Bispectrum - Bulk and Boundary

$$G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = G_{\rm Bulk}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + G_{\rm Boundary}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3).$$



Scalar bispectrum in equilateral limit from just enough inflation.
Red : $G_{\rm Bulk}$, Blue : $G_{\rm Boundary}$

Behaviour of the non-Gaussianity parameter

The scalar non-Gaussianity is quantified as

$$f_{_{\rm NL}} = \frac{-\left(\frac{10}{3}\right)\frac{1}{16\pi^4}(k_1k_2k_3)^3\, G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)}{k_1^3\mathcal{P}_{_{\mathcal{R}}}(k_2)\mathcal{P}_{_{\mathcal{R}}}(k_3) + k_2^3\mathcal{P}_{_{\mathcal{R}}}(k_1)\mathcal{P}_{_{\mathcal{R}}}(k_3) + k_3^3\mathcal{P}_{_{\mathcal{R}}}(k_1)\mathcal{P}_{_{\mathcal{R}}}(k_2)}$$

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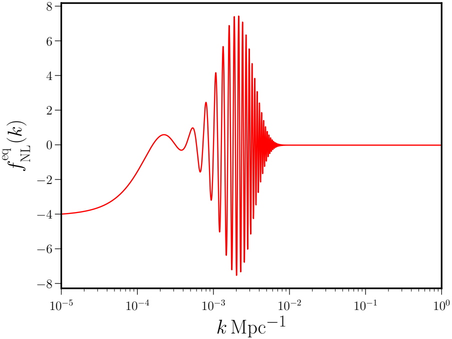
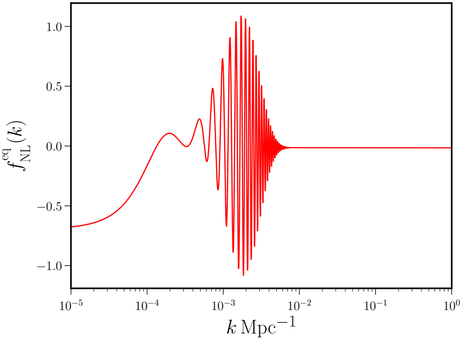
Equilateral limit $(k_1 = k_2 = k_3)$; $\epsilon_{1_i} = 1$



$f_{\rm NL}^{\rm eq}$ from quadratic potential



$f_{\rm NL}^{\rm eq}$ from Starobinsky model

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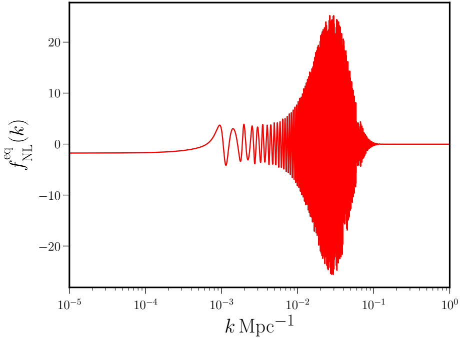
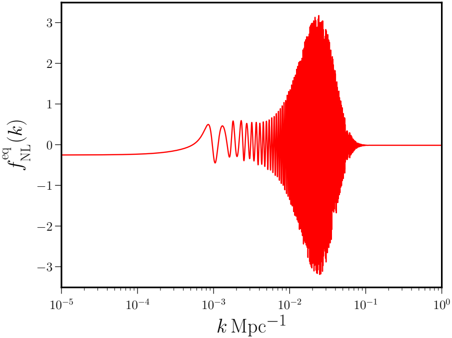
Equilateral limit $(k_1 = k_2 = k_3)$; $\epsilon_{1_i} = 2.99$



$f_{\rm NL}^{\rm eq}$ from quadratic potential



$f_{\rm NL}^{\rm eq}$ from Starobinsky model

Behaviour of $f_{_{\rm NL}}$ in squeezed limit

The non-Gaussianity parameter in the squeezed limit, is expected to satisfy the consistency condition, which is given by

$$f_{_{\rm NL}}^{\rm sq} = \frac{5}{12}(n_{_{\rm S}} - 1),$$

where, $n_{_{\rm S}} - 1 = {\rm d}ln\mathcal{P}_{_{\mathcal{R}}}/{\rm d}lnk$.

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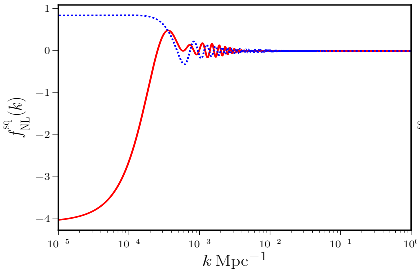
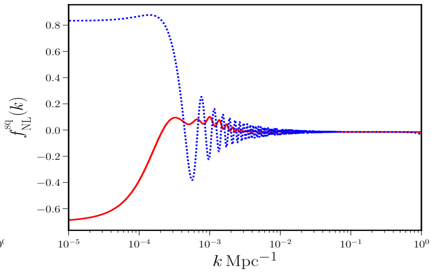
Squeezed limit $(\boldsymbol{k}_1 \simeq -\boldsymbol{k}_2; \boldsymbol{k}_3 \simeq 0)$; $\epsilon_{1_i} = 1$



$f_{_{\rm NL}}^{\rm sq}$ from quadratic potential



$f_{_{\rm NL}}^{\rm sq}$ from Starobinsky model

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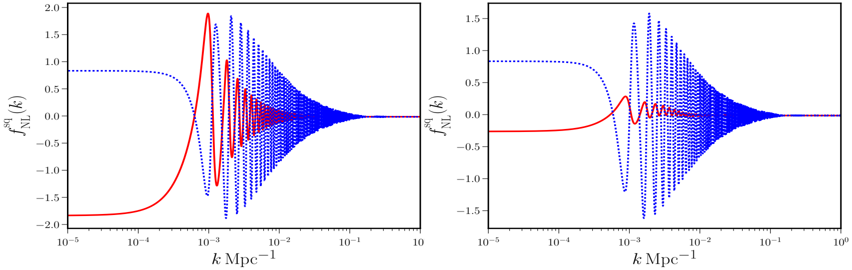

$f_{\rm NL}^{\rm sq}$ from quadratic potential $f_{\rm NL}^{\rm sq}$ from Starobinsky model

Conclusion

- Just enough inflation effects a model independent feature in the power spectrum.
- The onset of inflation gives a unique shape to the non-Gaussianity parameter across models.
- However, one can discriminate models based on the magnitude of the non-Gaussianity parameter.
- It also violates the consistency condition relating the bispectrum in the squeezed limit to the power spectrum.

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Thank You.

This presentation is based on the work arXiv:1906.03942 [astro-ph.CO] by HVR, Debika Chowdhury, and L. Sriramkumar.

Appendix - I

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$$\begin{aligned} G(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= \sum_{C=1}^{7} G_{_C}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \\ &= M_{_{\rm Pl}}^2 \sum_{C=1}^{6} \Big[f_{k_1}(\eta_{\rm e})\, f_{k_2}(\eta_{\rm e})\, f_{k_3}(\eta_{\rm e}) \\ &\quad \times \mathcal{G}_{_C}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \text{complex conjugate}\Big] \\ &\quad + G_7(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \\ &\quad + G_{\rm B_1}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + G_{\rm B_2}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \end{aligned}$$

Appendix - I

The different contributions $\mathcal{G}$'s are given by,

$$\mathcal{G}_1(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = 2\,i \int_{\eta_\mathrm{i}}^{\eta_\mathrm{e}} \mathrm{d}\eta\; a^2\, \epsilon_1^2 \left(f_{k_1}^{*}\, f_{k_2}^{\prime *}\, f_{k_3}^{\prime *} + \text{two permutations}\right),$$

$$\mathcal{G}_2(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -2\,i\; (\boldsymbol{k}_1 \cdot \boldsymbol{k}_2 + \text{two permutations}) \times \int_{\eta_\mathrm{i}}^{\eta_\mathrm{e}} \mathrm{d}\eta\; a^2\, \epsilon_1^2\, f_{k_1}^{*}\, f_{k_2}^{*}\, f_{k_3}^{*},$$

$$\mathcal{G}_3(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -2\,i \int_{\eta_\mathrm{i}}^{\eta_\mathrm{e}} \mathrm{d}\eta\; a^2\, \epsilon_1^2 \left(\frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{k_2^2}\, f_{k_1}^{*}\, f_{k_2}^{\prime *}\, f_{k_3}^{\prime *} + \text{five permutations}\right),$$

$$\mathcal{G}_4(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = i \int_{\eta_\mathrm{i}}^{\eta_\mathrm{e}} \mathrm{d}\eta\; a^2\, \epsilon_1\, \epsilon_2^{\prime} \left(f_{k_1}^{*}\, f_{k_2}^{*}\, f_{k_3}^{\prime *} + \text{two permutations}\right),$$

Appendix - I

$$
\begin{aligned}
\mathcal{G}_5(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= \frac{i}{2} \int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\, a^2\, \epsilon_1^3 \left(\frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{k_2^2}\, f_{k_1}^{*}\, f_{k_2}'^{*}\, f_{k_3}'^{*} \right. \\
&\qquad \left. + \text{five permutations} \right), \\
\mathcal{G}_6(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= \frac{i}{2} \int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\, a^2\, \epsilon_1^3 \left(\frac{k_1^2\, (\boldsymbol{k}_2 \cdot \boldsymbol{k}_3)}{k_2^2\, k_3^2}\, f_{k_1}^{*}\, f_{k_2}'^{*}\, f_{k_3}'^{*} \right. \\
&\qquad \left. + \text{two permutations} \right),
\end{aligned}
$$

$$
\begin{aligned}
G_7(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= -i\, M_{_{\rm Pl}}^2\, \left(f_{k_1}(\eta_{\rm e})\, f_{k_2}(\eta_{\rm e})\, f_{k_3}(\eta_{\rm e})\right) \\
&\quad \left[a^2 \epsilon_1 \epsilon_2\, f_{k_1}^{*}(\eta)\, f_{k_2}^{*}(\eta)\, f_{k_3}'^{*}(\eta) \right. \\
&\quad \left. + \text{two permutations}\right]_{\eta_i}^{\eta_{\rm e}} + \text{ complex conjugate.}
\end{aligned}
$$

Appendix - I

$$
\begin{aligned}
G_{\mathrm{B}_1} &= i\left(f_{k_1}(\eta_{\mathrm{e}})\, f_{k_2}(\eta_{\mathrm{e}})\, f_{k_3}(\eta_{\mathrm{e}})\right) \times \left[\frac{a(\eta)}{H} f_{k_1}^{*}(\eta)\, f_{k_2}^{*}(\eta)\, f_{k_3}^{*}(\eta)\right]_{\eta_i} \\
&\times \Bigg[54\left(a(\eta_i)H(\eta_i)\right)^2 + 2\,(1-\epsilon_1)\,(\boldsymbol{k}_1\cdot\boldsymbol{k}_2 + \boldsymbol{k}_1\cdot\boldsymbol{k}_2 + \boldsymbol{k}_1\cdot\boldsymbol{k}_2) \\
&+ \frac{((\boldsymbol{k}_1\cdot\boldsymbol{k}_2)\boldsymbol{k}_3^2 + (\boldsymbol{k}_2\cdot\boldsymbol{k}_3)\boldsymbol{k}_3^2 + (\boldsymbol{k}_3\cdot\boldsymbol{k}_1)\boldsymbol{k}_2^2)}{2\left(a(\eta_i)H(\eta_i)\right)^2}\Bigg] + \text{complex conjugate} \\
G_{\mathrm{B}_2} &= i\left(f_{k_1}(\eta_{\mathrm{e}})\, f_{k_2}(\eta_{\mathrm{e}})\, f_{k_3}(\eta_{\mathrm{e}})\right) \\
&\times \Bigg\{\left[\frac{\epsilon_1}{2H^2} f_{k_1}^{*}(\eta)\, f_{k_2}^{*}(\eta)\, f_{k_3}^{\prime *}(\eta)\right]\left[k_1^2 + k_2^2 - \left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_3}{k_3}\right)^2 - \left(\frac{\boldsymbol{k}_2\cdot\boldsymbol{k}_3}{k_3}\right)^2\right] \\
&- \left[\frac{a(\eta)\epsilon_1}{H} f_{k_1}^{*}(\eta)\, f_{k_2}^{\prime *}(\eta)\, f_{k_3}^{\prime *}(\eta)\right]\left[2 - \epsilon_1 + \epsilon_1\left(\frac{\boldsymbol{k}_2\cdot\boldsymbol{k}_3}{k_2 k_3}\right)^2\right]\Bigg\}_{\eta_i}^{\eta_e} \\
&+ \text{two permutations} + \text{complex conjugate}
\end{aligned}
$$