Generalized Schwinger effect and Cosmological particle production

Karthik Rajeev

IUCAA, Pune
IACS, Kolkata (Present)

Collaborators: Dr. Sumanta Chakraborty (IACS, Kolkata) and Prof. Thanu Padmanabhan (IUCAA, Pune)

Outline

- Pair production in homogeneous and time dependent background.
- (Generalized) Schwinger effect.
- Cosmological particle production.
- Connection between the two.

Quantum Simple Harmonic Oscillator (SHO)

Classical EOM: $q'' + \omega_0^2 q = 0$

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Quantization: $q(\eta) = a\xi(\eta) + a^\dagger \xi^*(\eta)$

$$im_0 \left[\xi^*\xi' - \xi(\xi^*)'\right] = 1 \quad \text{(Wronskian condition)}$$

$$[a, a^\dagger] = 1$$

$$a\,|0_a\rangle = 0$$

$$\langle 0_a|\hat{H}|0_a\rangle \to \text{Minimize}$$

$$\xi = \frac{e^{-i\omega\eta}}{\sqrt{2\omega_0 m_0}} \Rightarrow |0_a\rangle = \text{Groundstate (vacuum)}$$

Free (real) scalar field

- Each Fourier (momentum) mode is associated with $\xi_{\mathbf{k}}$ and hence, $a_{\mathbf{k}}$ and $a^{\dagger}_{\mathbf{k}}$.
- Define $|0_a\rangle$ by $a_{\mathbf{k}}\,|0_a\rangle = 0$.
- $|0_a\rangle$ is the vacuum iff $\xi_{\mathbf{k}} \propto e^{-i\omega_{\mathbf{k}}\eta}$.
- Particle number (density) operator $N = \sum_k a^{\dagger}_{\mathbf{k}} a_{\mathbf{k}}$.
- $\langle 0|N|0\rangle = 0$ for all times.
- For complex fields there will be two sets of operators $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$, for particles and antiparticles respectively.

Scalar field interacting with a homogeneous by time dependent background

- Examples: (i) Complex scalar field in a time dependent homogeneous electric field (ii) Scalar field in FRW background.
- Each Fourier modes in these cases are described by a ‘time dependent harmonic oscillator’ (TDHO), instead of SHO.

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$$\langle 0_a|H|0_a\rangle \propto \frac{mf^2}{2(\omega)}\left[\left(-\omega + \frac{1}{2mf^2}\right)^2 + \left(\frac{f'}{f}\right)^2\right] + \frac{1}{2} : \quad \text{Time dependent}$$

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- Define $|in\rangle$ as the vacuum at asymptotic past and $|out\rangle$ as the vacuum at asymptotic future.
- $\langle in|a^\dagger_{out} a_{out}|in\rangle \equiv < n > \neq 0$

Homogeneous Electric field in Minkowski spacetime $\leftrightarrow$ FRW universe [1]

Hom. Elec. Field

$$A_a = (0, 0, 0, A_z(\eta))$$

$$\mathcal{S} = \mathcal{S}_{EM}[A] + \mathcal{S}_{Min}[\psi; m]$$

$$\psi_{\mathbf{p}}'' + \left(m^2 + |\mathbf{p}_\perp|^2 + \{p_z + qA(t)\}^2\right)\psi_{\mathbf{p}} = 0$$

FRW universe

$$ds^2 = a(\eta)^2\left(-d\eta^2 + |d\mathbf{x}|^2\right)$$

$$\mathcal{S} = \mathcal{S}_{EH}[a] + \mathcal{S}_{Min}[\phi; M] - \frac{1}{6}\int \phi^2 R$$

$$\Phi_{\mathbf{k}}'' + \left\{k^2 + M^2 a^2(\eta)\right\}\Phi_{\mathbf{k}} = 0$$

$$m^2 + p_\perp^2 + \{p_z + qA_z(\eta)\}^2 \rightleftarrows k^2 + M^2 a^2(\eta)$$

[1] K R, Chakraborty S. and Padmanabhan T., Phys. Rev. D **100**, no. 04, 045019 (2019), arXiv:1904.03207[gr-qc].

Particle Production

- 'Conformally coupled' scalar field.

$$\mathcal{S} = \frac{1}{2}\int d^4x\sqrt{-g}\phi\left[\Box - \frac{R}{6} - M^2\right]\phi$$
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where $\Phi = a\phi$.

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- 'Trivial' in $M = 0$. For an arbitrary background $a(\eta)$, can we describe small M case perturbatively?
- Our correspondence immediately gives the answer: No!
- Why?: Schwinger effect is known to be a **non-perturbative** result. For eg:

$$n_p = e^{-\frac{\pi(p_\perp^2+m^2)}{|qE_0|}}$$

Constant Electric Field $\leftrightarrow$ Radiation Dominated universe

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  $A(\eta) = E_0\eta \rightleftarrows a(\eta) = a_0 + a_1\eta \rightarrow \text{Radiation Dominated universe}$

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- $k^2 \rightleftarrows p_\perp^2 + m^2; \qquad Ma_1 \rightleftarrows p_z; \qquad \frac{Ma_0}{2} \rightleftarrows qE_0$
- Particle production is indeed non-perturbative

$$n_{\mathbf{k}} = \exp\left(-\frac{\pi k^2}{Ma_1}\right)$$

Homogeneous Electric field in $\mathcal{M} \leftrightarrow$ FRW universe

$a(\eta)$	$A(\eta)$	**Remarks**
$a_0 + a_1 \tanh \mathcal{H}\eta$	$\frac{E_0}{\omega} \tanh \omega\eta + C$	smooth-step-fn uni. $\leftrightarrow$ Sauter
$a_0 + \frac{1}{1-H\eta}$	$\frac{E_0}{\omega(1-\omega\eta)} - \frac{E_0}{\omega}$	
$a(\eta) = a_0 + e^{\mathcal{H}\eta}$	$\frac{E_0}{\omega} e^{\omega\eta}$	

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$a_0 + \frac{1}{1-H\eta}$	$\frac{E_0}{\omega(1-\omega\eta)} - \frac{E_0}{\omega}$	ds$\leftrightarrow$ quadratically-singular E
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$a(\eta) = a_0 + e^{\mathcal{H}\eta}$	$\frac{E_0}{\omega} e^{\omega\eta}$	Milne universe $\leftrightarrow$ exponential E

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$a_0 + \frac{1}{1-H\eta}$	$\frac{E_0}{\omega(1-\omega\eta)} - \frac{E_0}{\omega}$	
$a(\eta) = a_0 + e^{\mathcal{H}\eta}$	$\frac{E_0}{\omega} e^{\omega\eta}$	$\Pi_z \ll \epsilon_p$; $n_p \sim$ Planck; $T = \frac{\omega}{2\pi}$

Summary and Comments

- Pair production in homogeneous and time dependent backgrounds can be studied using quantum TDHOs.
- Two cases of pair production of a scalar field, namely one in a homogeneous electric field and the other in FRW background, can be algebraically mapped to each other.
- This mapping can be used to compare pair creation processes in different interesting scenarios in both sides.
- For example, spectrum of created particles in the electric field corresponding to Milne universe is almost Planckian.
- It is hoped that next-gen high energy laser facilities can probe (gen)-Schwinger effect.
- What important physical questions can be translated from cosmology side to electric field side? Can we set up 'QFT in FRW' in the lab?