A Canonical Perspective on Holography

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Collaborators and References

- arXiv:1603.02812 with Souvik Banerjee, Jan-Willem Bryan, Kyriakos Papadodimas
- arXiv:1809.10154
- arXiv:1903.11073
- arXiv:200?.???? with Alok Laddha, Siddharth Prabhu, Pushkal Shrivastava

Motivation

- Holography has led to tremendous progress in quantum gravity in the past 20 years.
- However, it is sometimes assumed to be a mysterious feature of gravity that emerges when we consider string-theoretic UV completions.
- We would like to understand whether holography is implicit in canonical gravity.

Explaining old observations

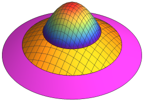
- This analysis yields some useful insights.
- It explains why holography is easy to understand in AdS.
- It explains why gravity is crucial for holography.
- It explains why holography is an intrinsically quantum mechanical phenomenon.

Newer directions

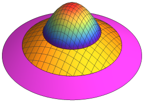
- This approach naturally leads to a proposal for holography in flat space.
- This approach also gives some insight on the information paradox.

Holography is understood in many ways. In this talk, I would like to emphasize the following main point

In quantum gravity, if we specify the wave-function everywhere outside a bounded region, we also specify it completely inside the region.



Canonical Gravity and Localization of Information



- The picture above is a key aspect of holography.
- In this talk, I will emphasize that this picture also follows from semi-classical considerations.

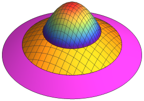
[Marolf, 2008–12]

[Banerjee, Bryan, Papadodimas, S.R, 2016]

[S.R., 2019]

Implication for black holes

This observation has striking implications for quantum information in quantum gravity.



- For black holes, this implies that quantum information never "goes in" and "comes out": information is always outside.

- Many versions of the information paradox are resolved through this simple observation.

Outline

1 Introduction

2 Holography in AdS

3 Holography in Flat Space

4 Information paradox

5 Conclusion

Setup

Consider an asymptotically global AdS space

$$ds^2 \to -(r^2+1)dt^2 + \frac{dr^2}{r^2+1} + r^2 d\Omega_{d-1}^2 + h_{\mu\nu} dx^\mu dx^\nu$$

where

$$h_{r\mu} = 0; \qquad h_{ij} = \mathrm{O}\left(r^{2-d}\right)$$

Physical Observables

- Physical observables are those that are invariant under small diffs

- An interesting class of physical observables are the subleading terms in the metric/other fields near the boundary.

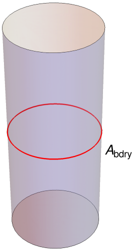
$$t_{ij}(\Omega, t) = \lim_{r \to \infty} r^{d-2} h_{ij}(r, \Omega, t)$$
$$O_{\phi}(\Omega, t) = \lim_{r \to \infty} r^{\Delta} \phi(r, \Omega, t)$$

The Boundary Algebra

We are interested in the algebra of these boundary observables, formed by taking all functions of these operators in an infinitesimal time interval.

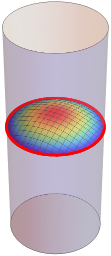
$$\mathcal{A}_{\text{bdry}} = \{t_{ij}(t_1, \Omega_1), t_{ij}(t_1, \Omega_1) t_{ij}(t_2, \Omega_2) \ldots\}$$

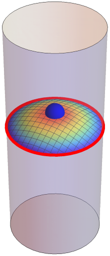
where $t_i \in [0, \epsilon]$.



Holography of Information in AdS

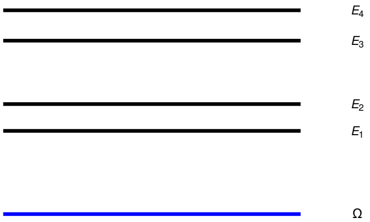
If two bulk states are distinct, they can be distinguished by observables in $\mathcal{A}_{\text{bdry}}$.





Assumptions 1: Low energy structure

- In global AdS,the vacuum is unique and separated by a gap from the lowest excited state.
- We will assume that in the full theory of quantum gravity, the vacuum remains unique.

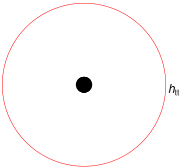


Hamiltonian as a boundary term in gravity

- In canonical quantum gravity, the Hamiltonian is a boundary term; direct result of imposing invariance under small diffeomorphisms on the wave-function.

$$H = (d/16\pi G_N) \lim_{r \to \infty} r^{d-2} \int d^{d-1}\Omega h_{tt}$$

- Provided we regulate the theory respecting diffeomorphism-invariance, H will remain a boundary term.



Assumption 2: unchanged vacuum

We will assume that, the vacuum of H coincides with the vacuum of the full theory and H remains a positive operator.

This is not to assume that H is the full Hamiltonian of the theory.

Projector onto the vacuum

- Since $H \in \mathcal{A}_{\text{bdry}}$, all spectral projectors of H are also in $\mathcal{A}_{\text{bdry}}$. (standard property of von Neumann algebras.)
- We are interested in $P_{\Omega} = |\Omega\rangle\langle\Omega|$ and $\mathcal{P}_{\Omega} \in \mathcal{A}_{\text{bdry}}$.
- $\mathcal{P}_{\Omega}$ is a one-dimensional projector by Assumption 1.

Assumption 3: Hilbert space

- We will assume the theory can be formulated in the space given by exciting the vacuum by boundary operators at arbitrary times.

  $$\mathcal{H} = t_{i_1 j_1}(t_1) \ldots t_{i_n j_n}(t_n)|\Omega\rangle$$

  The operators above are at arbitrary times: i.e. naively the product above does not appear to lie in $\mathcal{A}_{\text{bdry}}$.

- $\mathcal{H}$ is manifestly closed under time-evolution. So unitarity cannot force us to include additional states.

- $\mathcal{H}$ is quite large. Accommodates all black holes formed from collapse.

Squeezing the Hilbert space

Consider

$$\mathcal{H}' = \mathcal{A}_{\text{bdry}} |\Omega\rangle$$

Claim: Any vector in $\mathcal{H}$ can be approximated arbitrarily well by a vector in $\mathcal{H}'$

This means the Hilbert space can be generated through

$$t_{i_1 j_1}(t_1) \ldots t_{i_n j_n}(t_n) |\Omega\rangle$$

with $t_i \in [0, \epsilon]$.

Squeezing the Hilbert space

Proof of Claim: Say $|\Psi\rangle \in \mathcal{H}$ but $\langle\Psi|A|\Omega\rangle = 0, \forall A \in \mathcal{A}_{\text{bdry}}$. Then

$$f(t_i) = \langle\Psi|t_{i_1 j_1}(t_1)\ldots t_{i_n j_n}(t_n)|\Omega\rangle = 0, \quad t_i \in [0, \epsilon]$$

Inserting a complete set of energy eigenstates, $f(t_i)$ is analytic when

$$t_1, \;\; t_2 - t_1, \;\; t_3 - t_2 \ldots$$

are extended in the upper half plane. [positivity of energy.]

So if f vanishes for t_i real in some finite domain, then f must vanish everywhere! [edge-of-wedge theorem]

Therefore $|\Psi\rangle$ cannot exist.

Squeezing the Hilbert space

- We have proved that all states in the Hilbert space are of the form

$$|\Psi\rangle = A|\Omega\rangle$$

for some $A \in \mathcal{A}_{\text{bdry}}$.

- We now want to show that any two distinct states can be distinguished by an element of $\mathcal{A}_{\text{bdry}}$.

Putting the elements together

- Say, there are two states $|\Psi_1\rangle \neq |\Psi_2\rangle$. Then, $\exists X$ s.t $\langle\Psi_1|X|\Psi_1\rangle \neq \langle\Psi_2|X|\Psi_2\rangle$. We will show $X \in \mathcal{A}_{\text{bdry}}$.

- Expand X as
$$X = \sum_{nm} c_{nm}|n\rangle\langle m| = \sum c_{nm} X_n |\Omega\rangle\langle\Omega| X_m^\dagger$$
where $X_n \in \mathcal{A}_{\text{bdry}}$ (Expansion exists by previous proof.)

- Then
$$X = \sum c_{nm} X_n P_\Omega X_m^\dagger$$
by Assumption 2. But $\mathcal{P}_\Omega \in \mathcal{A}_{\text{bdry}} \Rightarrow X \in \mathcal{A}_{\text{bdry}}$.

- So $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are distinguishable by operators in $\mathcal{A}_{\text{bdry}}$!

Summary

We assumed

1. vacuum remains unique in global AdS.
2. the vacuum can be identified through a boundary Hamiltonian.
3. We can restrict to states produced by exciting this vacuum from the boundary.

We showed

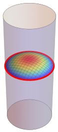
all information about the bulk is contained in an infinitesimal time interval on the asymptotic boundary.

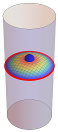
The importance of gravity

- This trick cannot work without gravity.
- Non-gravitational gauge theories contain exactly local gauge-invariant bulk operators which commute with all elements of $\mathcal{A}_{\text{bdry}}$.
- Eg.

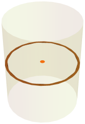
$$|\Psi\rangle \qquad \text{and} \qquad e^{i\mathrm{Tr}(F^2)(0)}|\Psi\rangle$$

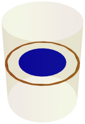
cannot be distinguished by any boundary measurement without gravity.





In the classical theory, one can specify initial data in the interior of a ball independently of data outside.





$$|\Psi_1\rangle = \sum a_i |E_i\rangle \qquad |\Psi_2\rangle = \sum a'_i |E_i\rangle$$

Classically, we can only measure

$$\langle H\rangle = \sum |a_i|^2 E_i$$

But in QM we get access to

$$\langle H^m\rangle = \sum |a_i|^2 E_i^m \neq \langle H\rangle^m$$

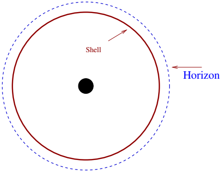
Perturbative verification

- This may seem like an abstract argument but can be verified in perturbation theory.
- For $|\Psi\rangle = e^{i \int \phi(r) f(r) dr} |\Omega\rangle$

$$\int d\Omega \langle \Psi | h_{tt}(\infty, \Omega) \phi(\infty, t) | \Psi \rangle$$
$$\sim G_N \int dr \langle \Omega | \dot{\phi}(r) \phi(\infty, t) f(r) | \Omega \rangle$$

If we know RHS for a small range of t we can infer $f(r)$.

Perturbative verification for black hole interiors



- Also works about black holes! Consider microstate with a very small spread in energy.

- For $|\Psi\rangle = e^{i \int \phi(r) f(r)} |E\rangle$

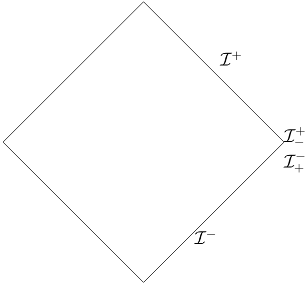
$$\int d\Omega \langle \Psi | \left(h_{tt}(\infty, \Omega) - \langle h_{tt}(\infty, \Omega) \rangle \right) \phi(\infty, t) | \Psi \rangle$$

$$\sim G_N \int dr \langle E | \dot{\phi}(r) \phi(\infty, t) f(r) | E \rangle$$

Can detect excitations inside black holes.

Flat Space

How is quantum information stored in flat space?



[de Boer, Solodukhin, 2003]

[Marolf, 2008–13]

[Bagchi, Grumiller, 2013]

[Pasterski, Shu-Heng Shao, Strominger ..., 2016–19]

Flat Space: Degenerate vacua

- I will describe the generalization of our story for massless particles in four dimensions.
- The main complication in repeating our program in 4D flat space is that the vacuum is infinitely degenerate.
- Different vacua are labelled by their charges under supertranslations.

Asymptotic boundary conditions

We consider four dimensional asymptotically flat spacetimes

$$ds^2 = (-1 + 2\frac{m_b}{r})du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + \ldots$$

The observables at a cut u, $\mathcal{A}(u)$ consists of the algebra generated by $C_{zz}(u), C_{\bar{z}\bar{z}}(u)$ and $m_b(u)$

The Hilbert Space

The Hilbert space can be written as a direct sum

$$\mathcal{H} = \bigoplus_{S} \mathcal{H}_S$$

where

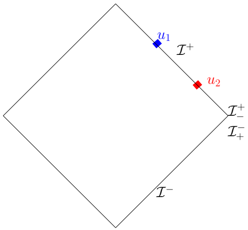
$$\mathcal{H}_S = \partial_u C_{A_1 B_1}(u_1) \ldots \partial_u C_{A_n B_n}(u_n) |S\rangle$$

where $M(-\infty)|S\rangle = 0$ and $|S\rangle$ is also an eigenstate of supertranslations.

Two Conjectures

Two states distinguishable anywhere on null infinity can also be distinguished in an infinitesimal neighbourhood of null infinity near spatial infinity.

If two states can be distinguished by observables on any cut of future null infinity, they can also be distinguished by observables on any cut to its past.



Sketch of Argument: information at spatial infinity

- In the canonical theory, the *vacuum* is completely specified by charges supported near spatial infinity.

$$Q[f] = \frac{1}{4\pi G} \lim_{u \to -\infty} \int \gamma_{z\bar{z}} f(z, \bar{z}) m_b(u, z, \bar{z}) d^2 z$$

- We need to assume that in the full theory, the vacua can be completely identified through operators near $\mathcal{I}^+_-$.

- Allows us to project onto a specific vacuum,

$$P_{\Omega,S} = |S\rangle\langle S|$$

Sketch of Argument: information at spatial infinity

- The rest of the argument is very similar to the argument in AdS.

- Once we have
$$P_{\Omega,S} = |S\rangle\langle S|$$
We then "lift" the vacuum to excited states using asymptotic operators near $\mathcal{I}^+_-$, and perform an AdS-like construction to write any operator as
$$O = \sum_{n,m,i,j} X_n |S_i\rangle\langle S_j| X_m^\dagger$$
with $X_m \in A_{\mathcal{I}^+_-}$.

Two Conjectures

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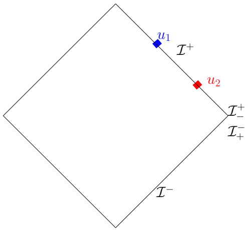


Sketch of argument: nested quantum information

In the canonical theory, one can derive some simple commutators in the algebra on $\mathcal{I}^+$.

[Ashtekar, 1981]

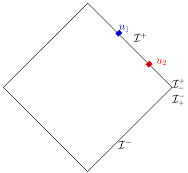
$$[M(u_1), O(u_2, z, z')] = 4\pi G \partial_{u_2} N_{AB}(u_2, z, z') \theta(u_2 - u_1)$$

and also the constraints

$$\partial_u M(u) = -T_{uu}$$
$$T_{uu} = \frac{1}{4} N_{AB} N^{AB} + 4\pi G t_{uu}$$

These relations depend only on the quadratic part of the action; unaffected by arbitrary nonlinear terms in the effective action.

Moving along $\mathcal{I}^+$



- If we assume that constraints and commutators are exact in the full quantum theory we have

$$O(u + U + \epsilon) = e^{\frac{-i}{4\pi G} M(u) U} O(u + \epsilon) e^{\frac{-iM}{4\pi G} U}, \quad U > 0.$$

- So information at a cut can be translated to a later cut

- Unlike AdS, we seem to lose information as we move away from $\mathcal{I}^+_-$.

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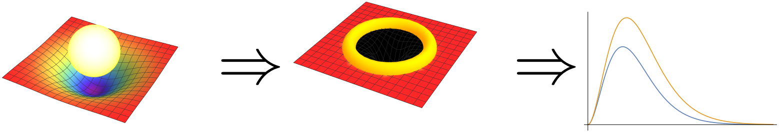
Old Information Paradox



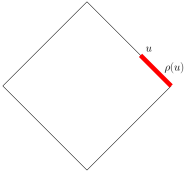
- Original paradox (still used for popular description): black hole forms and evaporates: star $\longrightarrow$ thermal radiation.

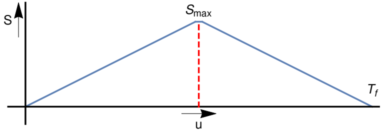
  [Hawking, 1975]

- But this is not a paradox; as in standard description of thermalization, information stored in exponentially small corrections.

The Page Curve

- A more refined demand is that one must derive the "Page curve" of the radiation.
- Page analyzed the average entropy of one subsystem (for typical pure states) on splitting a system into two. [Page, 1993]
- It is sometimes believed that the von Neumann entropy at null infinity will obey the Page curve.

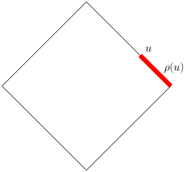


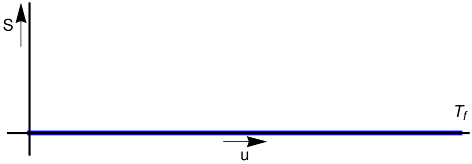


A more realistic Page curve

- But, the Page curve relies on an incorrect assumption of the factorization of the bulk Hilbert space.
- We should not insist that somehow our naive notions of factorization will be correct, or the lack of factorization can be ignored.

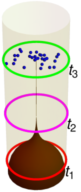
Our arguments suggest a more realistic Page curve is this

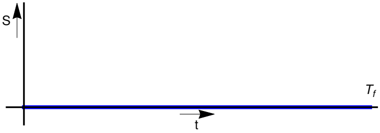




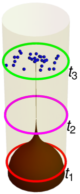
Page curve in AdS/CFT

This is also the prediction for small black holes forming and evaporating in AdSCFT.





Hardness of recovering information

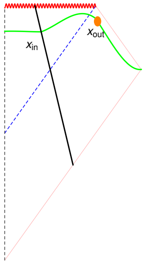


- Before the black hole forms, at t_1, information is easier to recover.
- After black hole formation, at t_2, information is hard to recover; requires S-pt correlators.
- After the black hole evaporates, at t_3, information remains hard to recover; still requires S-pt correlators.

Cloning Paradox

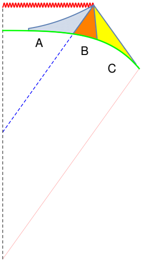
Hawking radiation appears to contain the same information as the infalling matter although both are intersected by the same nice slice.

[Susskind, Thorlacius, Uglum, 1993]



$$|\Psi\rangle \to |\Psi\rangle \otimes |\Psi\rangle ?$$

Strong Subadditivity Paradox



$$\mathrm{A} : r_h - \delta < r < r_h;$$
$$\mathrm{B} : r_h - \delta < r < r_h + \delta;$$
$$\mathrm{A} : r_h + \delta < r < \infty.$$

After the Page time, new radiation in B is entangled with A, and also with C. Seems to violate strong subadditivity of entropy:

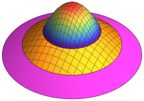
$$S_A + S_C \leq S_{AB} + S_{BC}.$$

[Mathur, AMPS, 2009–12]

Conclusion

An important principle in quantum gravity is that specifying the wave-function outside a bounded region leaves no freedom inside the region.

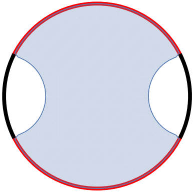
For quantum information purposes, this lack of independence cannot be ignored even when gravity is weak.



Conclusion

- For black holes, this principle implies that we shouldn’t ask “how the information comes out”?
- Rather, the information never goes in, in a theory of gravity.
- In particular, this suggests a trivial Page curve for the fine-grained entropy of BH radiation.

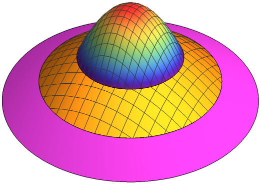
An important open question in AdS



- Even in AdS quantum information can be delocalized in stranger ways: "Entanglement wedge reconstruction".
- Can we understand this using canonical arguments?

Generalizations

- Can we generalize our construction to include massive particles and to work in higher dimensions in flat space — probably yes!
- What about de Sitter space?



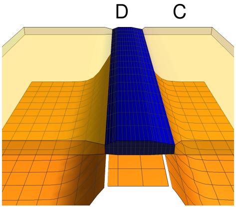
Thank you!

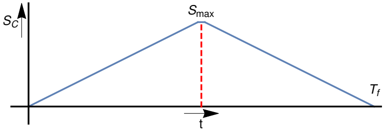
Appendix

AdS Page curve?

- To obtain a Page curve, we need a setup in which the Hilbert space factorizes.
- Conceptually the simplest setup is to consider a plasma-ball solution

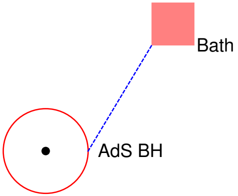
[Aharony, Minwalla, Wiseman, 2005]





Page curve in AdS/CFT

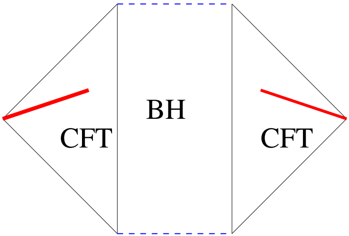
The recent “derivations” of holographic Page curves consider an initial state that decays into a bath.

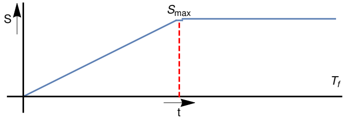


[Pennington, Almheiri, Mahajan, Maldacena..., 2019]

- We then compute the entropy of the bath.
- But Bath $\neq$ radiation since the bulk-boundary mapping is not local. Bath captures regions inside the black hole and misses some regions outside.

Real black holes?

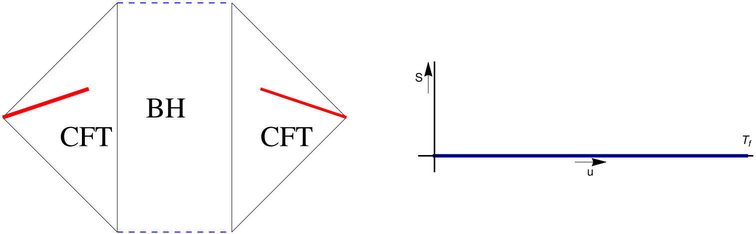




- If we consider the red strips then the entropy is believed to obey the semi-Page curve on the right.

- But this crucially relies on gravity being non-dynamical in part of the space.

Evaporation in our world



- In our world, we expect the trivial Page curve for the fine-grained entropy.
- Incorrect to suggest that gravity is weak on the red-slices and so should obey the non-dynamical curve.
- Gravity localizes quantum information unusually; neglecting this repeatedly leads to paradoxes.