On the equation of state during reheating

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Based on a work with S Anand and L Sriramkumar[arXiv:20XX.XXXXX]

Introduction: Inflation and Reheating.

- CMB constrains on reheating.
- On the equation of state.
- Summary and outlook.

Introduction

What is Inflation?

- Inflation is a period of exponential expansion of the universe in a state of negative pressure. i.e., $\ddot{a} > 0$,
- The 'right amount of inflation' produced a homogeneous and isotropic universe to act as an initial condition for hot big bang.
- The quantum fluctuations in the inflaton generates the seed for large scale structures.
- The inflaton decay during the reheating phase to produce all the matter and radiation content of the universe

Inflation and scalar field

- A period of negative pressure is easily achieved with a scalar field.

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi); \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- Equations of motion:
  - Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3\mathrm{M}_\mathrm{p}^2}\rho = \frac{1}{3\mathrm{M}_\mathrm{p}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\dot{H} + H^2 = \left(\frac{\ddot{a}}{a}\right) = -\frac{1}{6\mathrm{M}_\mathrm{p}^2}\left[\rho + 3p\right]$$

  - Klein-Gordon Equation
    $\ddot{\phi} + 3H\dot{\phi} + V' = 0.$
- Slow-roll parameters:

$$\epsilon_V = \frac{\mathrm{M}_\mathrm{p}^2}{2}\left(\frac{V'}{V}\right)^2; \quad \eta_V = \mathrm{M}_\mathrm{p}^2\frac{V''}{V}$$

- Observables in terms of slow-roll parameters
  - spectral tilt
    $n_s = 1 - 6\epsilon_V + 2\eta_V = 0.9649 \pm 0.0042$
  - scalar-to-tensor ratio $r = 16\epsilon_V < 0.1$

- The power spectrum

$$\Delta^2_{\mathcal{R}} = \frac{1}{12\pi^2}\frac{V^3}{\mathrm{M}_\mathrm{p}^6(V')^2} = A_s\left(\frac{k}{k_*}\right)^{n_s-1}$$

- The amount of expansion:

$$\Delta N = \ln\left(\frac{a_\mathrm{end}}{a_k}\right) = \int_{\phi_k}^{\phi_\mathrm{end}}\frac{1}{\sqrt{2\epsilon_V}}\frac{|d\phi|}{\mathrm{M}_\mathrm{p}}$$

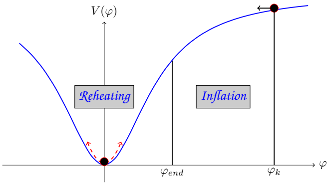


Figure: Inflation and reheating

Inflationary Reheating and related issues

- The typical scale of inflation is around the $\sim \mathrm{GUT\ scale}(\sim 10^{16}$GeV) while BBN requires a radiation dominated phase at around 10 MeV.
- There is a huge energy gap between inflation and BBN unconstrained from data.
- Thermalisation 'erase' information of the initial states.

- How to constrain the reheating phase from CMB data[1]?

[1] Kamionkowski, M., *PRL* **2014**, *113*, 041302, Martin, J., *PRL* **2015**, *114*, 081303.

CMB constrains on Reheating

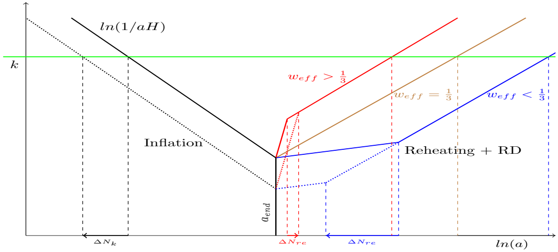


Figure: The comoving scales connects the inflationary phase with the CMB.

The relation connecting the reheating tempereture with present CMB temperature:

$$T_{\rm re} = \left(\frac{43}{11 g_{\rm re}}\right)^{\frac{1}{3}} \left(\frac{a_0 T_0}{k}\right) H_k e^{-N_k} e^{-N_{\rm re}}$$

$$N_{\rm re} = \frac{4}{3 w_{\rm eff} - 1}\left[N_k - 61.6 - \ln(H_k) + \frac{1}{4}\ln(\rho_k)\right]$$

A glance at the derivation

- The perturbation modes we observe are the ones that are comparable to the horizon:

$$k = a_k H_k, \tag{1}$$

$$\ln\left(\frac{k}{a_k H_k}\right) = 0, \tag{2}$$

$$\ln\left(\frac{a_e}{a_k}\frac{a_{\rm re}}{a_e}\frac{a_0}{a_{\rm re}}\frac{k}{a_0 H_k}\right) = 0, \tag{3}$$

$$N_k + N_{\rm re} + \ln\left(\frac{a_0}{a_{\rm re}}\right) + \ln\left(\frac{k}{a_0 H_k}\right) = 0 \tag{4}$$

- The reheating entropy is conserved in the CMB

$$a_{\rm re}^3 g_* T_{\rm re}^3 = a_0^3 g_0 T_0^3 \tag{5}$$

$$= a_0^3\left(2T_0^3 + 6\times\frac{7}{8}T_{\nu,0}\right) \tag{6}$$

  Neutrino temperature in terms of photon temperature: $T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_0$

- Reheating temperature and present CMB temperature

$$\frac{T_{\rm re}}{T_0} = \left(\frac{43}{11 g_*}\right)^{1/3}\frac{a_0}{a_{\rm re}} \tag{7}$$

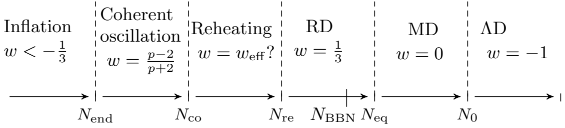
On the equation of the state:

What we know (and don't know?)

- CMB measurements can constrain reheating phase provoded we know the effective equation of state during reheating.

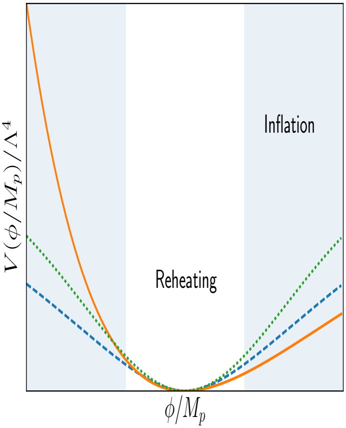
$$w_{\mathrm{eff}} = \frac{1}{N_{\mathrm{re}}} \int^{\mathrm{N_{re}}} w(N')\mathrm{d}N'$$

- Each epoch in cosmic evolution is charecterised by its equation of state.



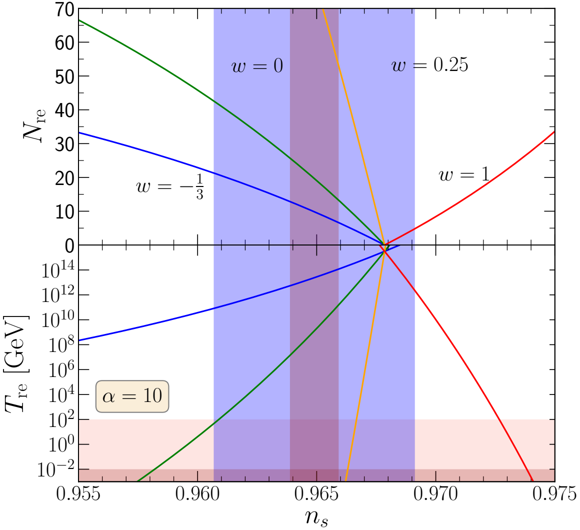
On the equation of the state:

'The zeroth order approximation'



- Around the minima of the potential $V(\phi) \propto \phi^p$
- During the oscillation phase of inflaton: $w_{\rm co} = \frac{p-2}{p+2}$
- For the most conservative bound, we may take $-\frac{1}{3} \leq w_{\rm re} \leq 1$
- Full numerical simulation shows equation of state gradually changes from $w_{\rm co}$ at the end of coherent oscillation to radiation-like eos $w_{\rm rad} = 1/3$ at the end of reheating.

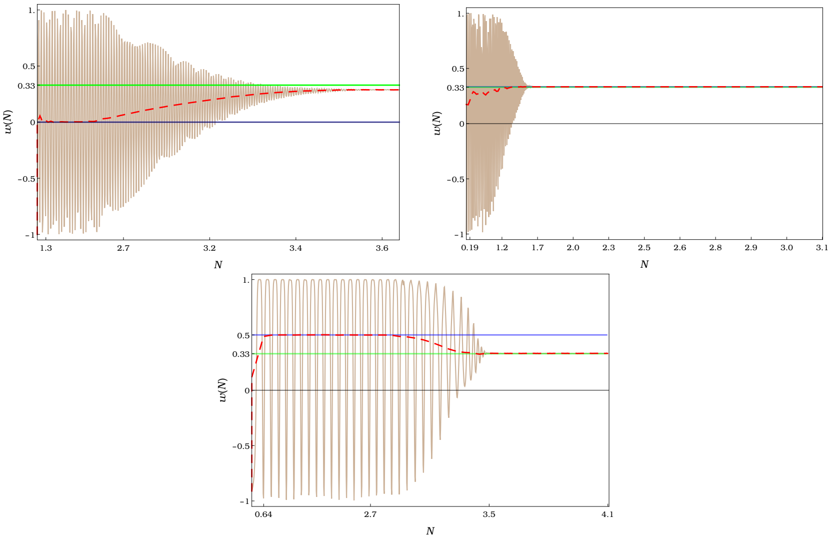
A typical result



On the equation of the state:

What we know from preheating studies?

(Equation of state during reheating must depend on the inflationary potential)



Parameterizing the equation of state during reheating

"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk"–Johnny von Neumann

- **Case A:** exponential form

$$w(N) = w_0 + w_1 \, \exp\left(-\frac{1}{\Delta}\,\frac{N}{N_{\rm re}}\right)$$

- **Case B:** tan-hyperbolic form

$$w(N) = w_0 + w_1 \, \tanh\left(\frac{1}{\Delta}\,\frac{N}{N_{\rm re}}\right)$$

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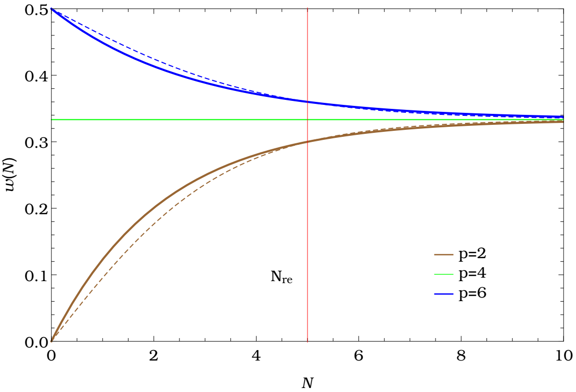
The parameterized equation can be written as inflationary parameters:

$$w(N,p) = \begin{cases} \frac{1}{3} + \frac{2}{3}\left(\frac{p-4}{p+2}\right)\exp\left(-\frac{1}{\Delta}\frac{N}{N_{\rm re}}\right), & \text{(case A)} \\ \\ \frac{p-2}{p+2} - \frac{2}{3}\left(\frac{p-4}{p+2}\right)\tanh\left(\frac{1}{\Delta}\,\frac{N}{N_{\rm re}}\right), & \text{(case B)} \end{cases}$$

with

$$\frac{1}{\Delta} = \begin{cases} \ln\left[\left(\frac{p-4}{p+2}\right)\left(\frac{2}{3\,w_{re}-1}\right)\right], & \text{(case A)} \\ \\ \tanh^{-1}\left[\left(\frac{3}{2}\right)\left(\frac{p-2-w_{\rm re}(p+2)}{p-4}\right)\right] & \text{(case B)} \end{cases}$$

The parametrized equation of state:



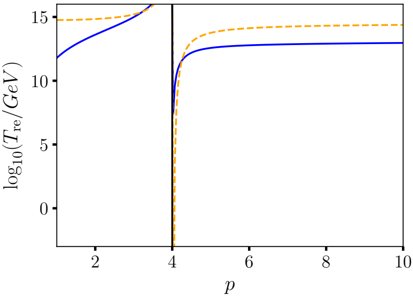
$$w_{\text{eff}} = \frac{1}{N_{\text{re}}} \int_{N}^{\text{N}_{\text{re}}} w(N') \text{d}N'$$

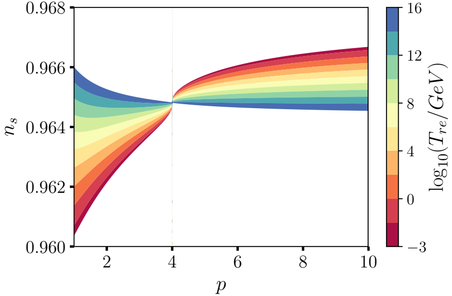
What is equation of state during reheating?

p	$w_p = \frac{(p-2)}{(p+2)}$	$w_{\rm eff}^{\rm exp}$	$w_{\rm eff}^{\rm tanh}$
1	-1/3	0.12	0.09
2	0	0.20	0.19
4	1/3	1/3	1/3
6	1/2	0.41	0.42
8	3/5	0.44	0.45
$p \to \infty$	1	0.53	0.56

- Equation of state is completely specified by the inflationary potential.
- Deviates significantly from 'zeroth order approximation'.

Application to inflationary models





- ✓ We have specified the effective reheating equation of state which only depends on the nature of inflaton oscillation during reheating around the minima of the potential.
- ✓ Taking the time varying nature in calculating the effective equation of state during reheating results in lower value of reheating temperature.
- ⚠ The presence of an extended period of non-perturbative processes such as parametric resonance will change the results significantly and we may loose the predictive power of CMB constrains on reheating.

Thank You!

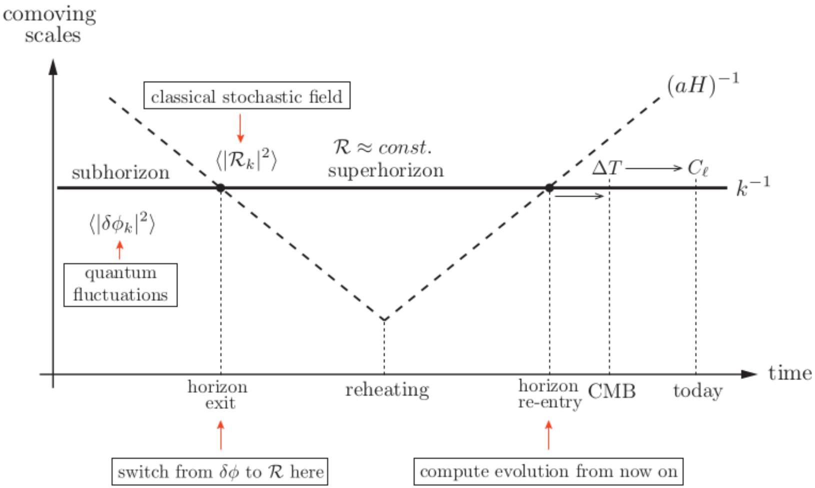


Figure 6.2: *Curvature perturbations during and after inflation: The comoving horizon $(aH)^{-1}$ shrinks during inflation and grows in the subsequent FRW evolution. This implies that comoving scales k^{-1} exit the horizon at early times and re-enter the horizon at late times. While the curvature perturbations $\mathcal{R}$ are outside of the horizon they don't evolve, so our computation for the correlation function $\langle|\mathcal{R}_k|^2\rangle$ at horizon exit during inflation can be related directly to observables at late times.*

Figure: From Baumann, Physics of Inflation

temperature and scale factor

$$\frac{k}{a_0 H_0} = \frac{a_k}{a_{end}} \frac{a_{end}}{a_{re}} \frac{a_{re}}{a_{eq}} \frac{a_{eq} H_{eq}}{a_0 H_0} \frac{H_k}{H_{eq}}$$

Assuming effective equation of state parameter

$$\rho_t(N) = \rho_{end} \exp\left(-3\int_0^N [1 + w(N')]dN'\right)$$

the efolding number

$$N_{re} = [3(1 + w_{re})]^{-1} \ln(\rho_{end}/\rho_{re})$$

the density of radiation energy density

$$\rho_{re} = (\pi^2/30) g_{re} T_{re}^4$$

Assuming, the reheating entropy is preserved in the CMB and neutrino backgrounf today

$$g_{s,re} T_{re}^3 = \left(\frac{a_0}{a_{re}}\right)^3 \left(2T_0^3 + 6 \times \frac{7}{8} T_{\nu 0}^3\right)$$

$$\frac{T_{re}}{T_0} = \left(\frac{43}{11 g_{s,re}}\right)^{1/3} \frac{a_0}{a_{eq}} \frac{a_{eq}}{a_{re}}$$

Which implies,

$$\frac{3(1 + w_{re})}{4} N_{re} = \frac{1}{4} \ln \frac{30}{g_{re}\pi^2} + \frac{1}{4} \ln \frac{\rho_{end}}{T_0^4} + \frac{1}{3} \ln \frac{11 g_{gs,re}}{43} + \ln \frac{a_{eq}}{a_0} - N_{RD}$$

We get the expression for N_{re}

$$N_{re} = \frac{4}{1 - 3w_{re}} \left[-N_k - \ln \frac{k}{a_0 T_0} - \frac{1}{4} \ln \frac{30}{g_{re}\pi^2} - \frac{1}{3} \ln \frac{11 g_{s,re}}{43} + \frac{1}{4} \ln \frac{\pi^2 r A_s}{6}\right]$$