Going Beyond Einstein: Theoretical & Observational Constraints

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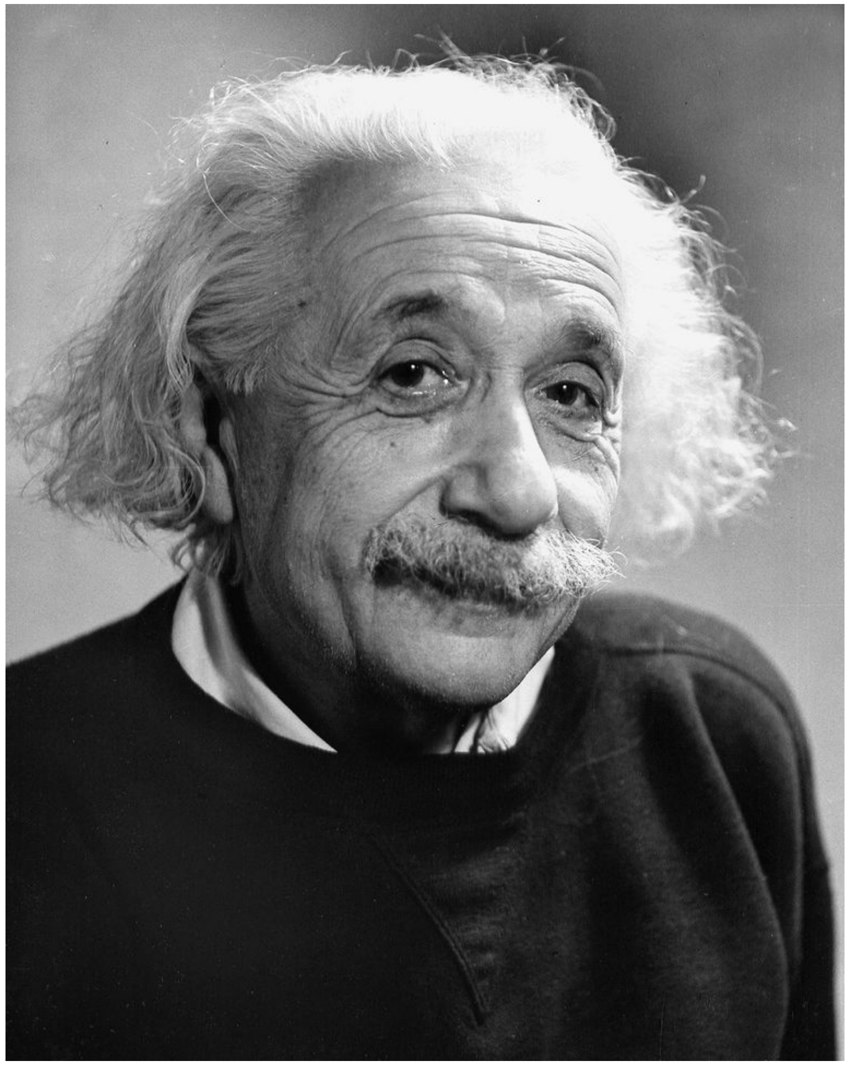
100 + n years of General Relativity:



The space time is a Lorentzian manifold $\left(M, g_{ab} \right)$

The curvature is determined from Einstein equation:

$$R_{ab} - \frac{1}{2} R \, g_{ab} + \Lambda \, g_{ab} = 8\pi T_{ab}$$

The motion of the test matter in the curved geometry is described by the geodesic equation:

$$\frac{d^2 x^a}{d\tau^2} + \Gamma^a_{bc} \frac{dx^b}{d\tau} \frac{dx^c}{d\tau} = 0$$

Remarkable success of General Relativity:

The `classic' tests:

1. Precession of perihelion of mercury.
2. Bending of Light.
3. Shapiro time delay and the speed of gravity.
4. Tests of Equivalence Principle

All these tests show that the Post Newtonian parameters agree with the GR prediction with `good' accuracy

The recent tests:

1. Gravitational Waves aka LIGO
2. Imaging the Event Horizon
3. Gravity Probe B experiments

All these tests show that the system is consistent with the prediction of GR!

Why do we still expect physics beyond GR:

General Relativity is `Perturbatively' Non Renormalizable: It may make sense only an effective field theory, with new counter-terms and couplings at each new loop order.

Higher curvature terms in action:

$$L = \frac{1}{16\pi}\left(R + \alpha\ \mathrm{O}(R^2) + \beta\ \mathrm{O}(R^3) + \ldots\right)$$

Point of View : Stop at finite number of terms, treat this theory as a classical theory with new phenomenological constants with appropriate dimensions of length.

There is always a possibility of new degrees of freedom beyond the metric:

Scalar-Tensor Theories, Einstein-Aether theory, Non Minimal coupling between matter and geometry.

Exotic possibilities:

Violation of Lorentz invariance/equivalence principle. Massive graviton, Infinite derivative theories, non local action!

But there are strong constraints on any extension of physics beyond GR. (BGR-Physics)

Constraints from Consistency!

Consider the challenges to Higher curvature theories:

General form of the Lagrangian:

$$L = \frac{1}{16\pi}\left(R + \alpha\ \mathrm{O}(R^2) + \beta\ \mathrm{O}(R^3) + \ldots\right)$$

In 4 dimensions, the most general quadratic Lagrangian is

$$L = \frac{1}{16\pi}\left(R + \alpha\ R^2 + \beta\ \mathrm{R}_{ab}R^{ab}\right)$$

Unlike GR, this theory does not contain only the usual massless (long-range) spin-2 graviton field but also, in general, two massive (short-range) fields with spins 0 and 2.

Expand the metric over the flat space: $g_{ab} = \eta_{ab} + \chi\ \eta_{ab} + h^{(E)}_{ab} + \psi_{ab}$

The field equations can be expanded as, *Audretsch et. al. PRD 1977,*

$$\Box\, h^{(E)}_{\mu\nu} = -2\kappa(T_{\mu\nu} - \tfrac{1}{2}T\eta_{\mu\nu}),$$
$$(\Box - m_0^2)\chi = -\tfrac{1}{3}\kappa T, \quad m_0^{-2} := 6\alpha + 2\beta,$$
$$(\Box - m_1^2)\psi_{\mu\nu} = 2\kappa(T_{\mu\nu} - \tfrac{1}{3}T\eta_{\mu\nu}), \quad m_1^{-2} := -\beta,$$

The theory has ghosts unless: $\beta < 0;\ 3\alpha + \beta > 0$

An interesting theory: $L \sim R + \alpha\left(R^2 - 3R_{ab}R^{ab} \right);\ \alpha > 0$

This is a theory which has no scalar mode, but has a massive graviton mode which is not a ghost

Important constraint: The Positive mass theorem, stability of the flat space!

The Positive mass theorem in GR asserts that assuming the dominant energy condition, the mass of an asymptotically flat spacetime is non-negative; furthermore, the mass is zero only for Minkowski spacetime. *Schoen, Richard; Yau, Shing-Tung 1979, Witten 1981*

What is known about the Positive mass theorem of such higher curvature theories?

There is one old result for the theory: $L \sim R + \frac{1}{2\beta^2} R^2$ *Strominger PRD 1984*

The theory have non-negative energy (both ADM and Bondi mass), provided there exists a spacelike hypersurface on which $R > -\beta^2$

In fact, this is identical to the condition that the theory does not have any ghost!

For any f(R), it is possible to extend this result provided the condition f'(R) > 0 holds

It will be interesting to understand if the positive mass theorem holds for

$$L \sim R + \alpha\left(R^2 - 3\mathrm{R}_{ab}R^{ab}\right);\ \alpha > 0$$

Akash Mishra, Rajes Ghosh, SS, Upcoming

But the equation of motion is still higher order!

In higher dimension there is a better candidate:

$$L \sim R + \alpha\left(R^2 - 4\mathrm{R}_{ab}R^{ab} + R_{abcd}R^{abcd} \right)$$

The Einstein Gauss Bonnet Gravity is free from ghosts, the field equation is of second order in time!

The are well studied black hole solutions.

The entropy for these black holes are given by:

$$S = \frac{1}{4}\int\left(1 + 2\alpha\, {}^{D-2}R\right)dA$$

It is possible to decrease this entropy in black hole mergers ! *SS, Wall PRD 2010*

The theory only make sense as an effective theory! *Saugata Chatterjee, Maulik Parikh CQG 2013*

Even perturbatively, there are bounds on the coupling constant

Consider GB black holes in asymptotically AdS spacetimes:

$$f(r) = k + \frac{r^2}{4\alpha}\left[1 - \sqrt{1 - \frac{8\alpha}{l^2}\left(1 - \left(\frac{r_0}{r}\right)^4\right)}\right] \qquad k = 0, \pm 1 \qquad \lambda_{GB} = 2\alpha / l^2$$

Validity of the second law requires: $\boxed{\lambda_{GB} < \frac{9}{100}}$ *Fairoos C, Avirup Ghosh, SS PRD 2018*

This coincide with the avoidance of causality violation in the boundary theory.

Hofman et. al. PRD 2008, Brigante et. al. JHEP 2008, Buchel et. al. PRD 2010,

There may be similar bounds on higher Lovelock theories!

There are more stringent bounds

In GR, light going near a massive body would suffer a time delay relative to the flat space.

In Gauss Bonnet gravity, we can have either time delay or time advancement irrespective of the sigh of higher curvature coupling.

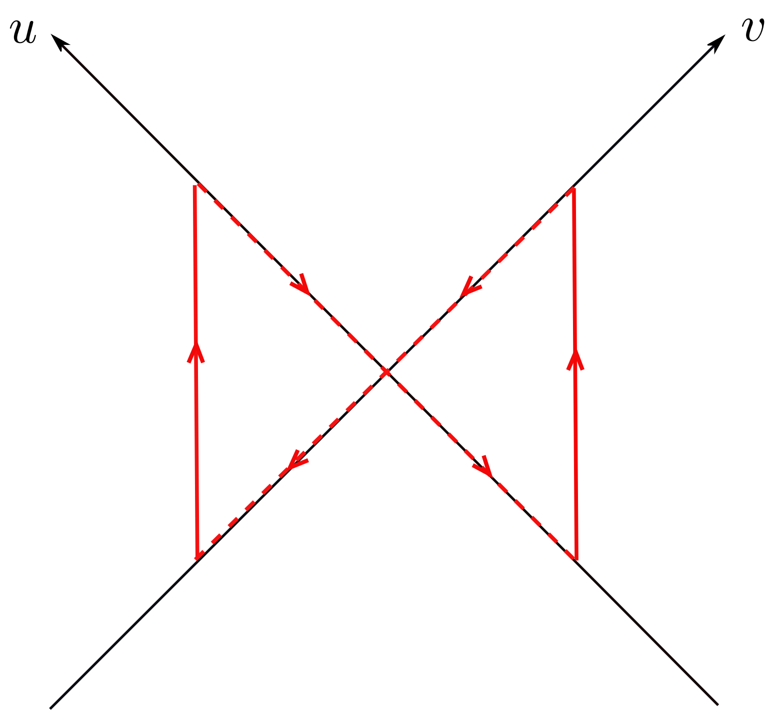
The delay for gravitational perturbations in a shock wave background is given by:

$$= \frac{4\Gamma(\frac{D-4}{2})}{\pi^{\frac{D-4}{2}}} \frac{G|P_u|}{b^{D-4}} \left[1 + \frac{4\lambda_{GB}(D-4)(D-2)}{b^2} \left(\frac{(\epsilon.n)^2}{\epsilon.\epsilon} - \frac{1}{D-2}\right)\right]$$

Camanho at. al. JHEP 2014

We can create time advancement when impact parameter b ~$\sqrt{\lambda_{GB}}$

Using two such shock waves, we can then prepare a time machine violating causality.



Closed time like curve without any violation of the energy conditions!

Claim: Finite truncation does not lead to a viable classical theory.

All possible terms to all orders are necessary, including higher spin fields.
This may be equivalent to String Theory!

Objection: It may not be possible to create such shock wave spacetimes from generic initial data on an initial spacelike Cauchy surface.

Papallo & Reall, PRD 2015

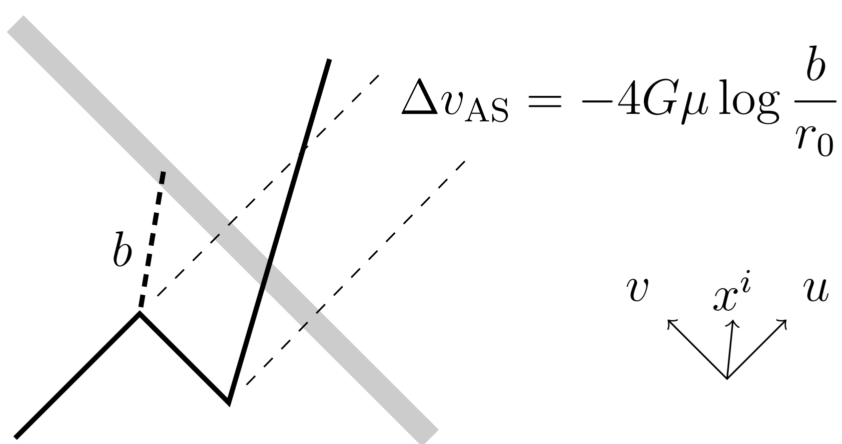
There are many interesting features of this time machine construction:

Even in GR, it is possible to have time advancement but only in four dimension.

Shock wave solution in GR in D = 4:

Aichelburg-Sexl metric

$$ds^2 = -2du\,dv + f(r)\delta(u)du^2 + dx_1^2 + dx_2^2\ ,\ \ f(r) = -4G\mu\log(r/r_0)^2$$



Geodesic of a masses particle with time advancement.

But, in 4D, this can not be used to create a time machine, the massless particle will be absorbed by the either of the shocks.

Hollowood & Shore JHEP 2016

The time advancement from shock wave scattering may be understood as the higher derivative corrections to the graviton three-point coupling.

We like to understand what happens to the classical argument for theories for which there is no correction to the graviton three-point coupling.

$$L \sim R + \alpha\left(R^2 - 3R_{ab}R^{ab}\right);\ \alpha > 0$$

$$\sqrt{g}R|_{h^3} \quad = \quad h_{\mu\nu}\,(h,^{\mu\nu}h) + 2h_{\mu\nu},^{\sigma}\, h^{\nu\rho},^{\mu}\, h_{\rho\sigma} \qquad \sqrt{g}R_{\mu\rho\nu}{}^{\sigma}R^{\mu\rho\nu}{}_{\sigma}|_{h^3} \quad = \quad h_{\mu\nu},_{\rho}{}^{\lambda}\, h_{\rho\sigma}\, h^{\sigma\lambda},^{\mu\nu}$$

$$\sqrt{g}R_{\mu\nu}R^{\mu\nu}|_{h^3} = \sqrt{g}R^2\Big|_{h^3} = 0$$

Forger et. al. PLB 1996

The theory has an exact shock wave solution $ds^2 = -du\ dv + H(u,x_\perp)\ du^2 + dx_\perp^2$

$$H(u,x_\perp) = f(x_\perp)\ \delta(u)\ ,\quad u = t - z\ ,\quad v = t + z$$

$$f(\rho) = -\frac{16\pi G p}{\Omega_{D-3}}\left[\frac{(-2\beta)^{2-D/2}}{\Gamma(D/2-1)}\left(\frac{\rho}{\sqrt{-\beta}}\right)^{2-D/2} K_{2-D/2}\left(\frac{\rho}{\sqrt{-\beta}}\right) + \frac{1}{(4-D)}\left(\frac{\rho}{\rho_0}\right)^{4-D}\right]$$

The time machine construction in this theory will be interesting…..

Rajes Ghosh, Alok Laddha, SS, Ongoing

Observational Constraints

Test of the Newtons Law:

A theory of the form: $L = \frac{1}{16\pi}\left(R + \alpha\, R^2 + \beta\, \mathrm{R}_{ab} R^{ab}\right)$

Produces generic correction to the Newton's law. These lead to the strongest bound of the couplings:

$$V(r) = \frac{GM}{r}\left[1 + \alpha \exp\left(-\frac{r}{\lambda}\right)\right].$$

The torsion balance experiments give: $\lambda \leq 8 \times 10^{-5}\, m$

For our theory, the form will be:

$$U(\mathbf{x}) \sim \frac{U_0}{|\mathbf{x}|} + U_1 \frac{e^{-\lambda_1 |\mathbf{x}|}}{|\mathbf{x}|} + U_2 \frac{e^{-\lambda_2 |\mathbf{x}|}}{|\mathbf{x}|}$$

$$\lambda_1^2 = -\frac{1}{\beta} \qquad \lambda_2^2 = \frac{1}{2(3\alpha + \beta)}$$

C. D. Hoyle et all, PRD 2004,
Berry et. al. PRD 2014

Setting $\beta = 0$, **we get a bound:** $\alpha \leq 2 \times 10^{-9}\, m^2$

Remember the Planck Length: $l_p^2 \sim 10^{-70} m^2$

$$\alpha / l_p^2 \sim 10^{60} !$$

There is something apparently interesting at the current limit!

$$\frac{l_p}{\Lambda^{1/2}} \sim 10^{-9} m^2 !$$

The local experiments based on the Newton's law can not bound the individual couplings!

Can we find a testable effect which is sensitive to individual couplings?

$$L = \frac{1}{16\pi}\left(R + \alpha\ R^2 + \beta\ \mathrm{R}_{ab}R^{ab}\right)$$

Effect ~ $O(\beta) + O(\alpha^2) + O(\alpha\beta)$

Gravitational Time delay!

In general relativity, all massless fields follow the same light cone!

Gravity travels at the speed of light!

In a generic theory of modified gravity, this is no longer true, about a curved background, the light cone structure depends on the helicity of the propagating modes.

Gravity and light see different background metrics!

Consider the propagation of GW and EMW from a distance source in a FRW universe

EMW travels along the null geodesic of the background metric:

$$ds^2 = -dt^2 + a^2(t)\left(dr^2 + r^2 d\Omega^2\right).$$

But GW travels along the null geodesic of a different metric:

$$ds^2_{\text{eff}} = -U(t)dt^2 + a^2(t)V(t)\left(dr^2 + r^2 d\Omega^2\right)$$

Avirup Ghosh, Soumya Jana, Akash Mishra, SS, PRD 2019

where $U(t)$ and $V(t)$ come from the higher curvature terms. In the GR limit $U, V \to 1$.

Consider a GW event with an Electromagnetic counterpart:

Notations:

δt_E **Intrinsic time delay between GW and EMW**

δt_o **Observed time delay between GW and EMW**

$$\left(\frac{1}{a}\sqrt{\frac{U}{V}}\right)\Bigg|_{t_O}\delta t_O-\left(\frac{1}{a}\sqrt{\frac{U}{V}}\right)\Bigg|_{t_E}\delta t_E$$
$$=\int_{t_E}^{t_O}\frac{1}{a}\left(1-\sqrt{\frac{U}{V}}\right)dt.$$

Writing in terms of the redshift:

$$\delta t_O=\delta t_E\,(1+z_E)\sqrt{\frac{U_E V_O}{U_O V_E}}$$
$$+\sqrt{\frac{V_O}{U_O}}\int_0^{z_E}\frac{dz}{H(z)}\left(1-\sqrt{\frac{U(z)}{V(z)}}\right)$$

In the quadratic gravity, we can not solve the Friedmann equations

So, let us try for a perturbative approach, we expand in terms of the higher curvature couplings:

Interestingly, for pure Ricci^2 gravity: $U = V$

$$ds^2_{eff} = U(t)\, ds^2$$

For full quadratic theory: $L = \frac{1}{16\pi}\left(R + \alpha\, R^2 + \beta\, R_{ab}R^{ab}\right)$

$$\boxed{U = V + O(\beta) + O(\alpha\beta)}$$

This is exactly what we want to constrain an individual coupling

In GR: $\delta t_O = \delta t_E(1+z_E)$

For the quadratic gravity, it becomes:

$$\delta t_O = \delta t_E(1+z_E)\left[1+\beta\,(H'_G(z_E)H_G(z_E) - H'_G(0)H_G(0))\right] - \beta \int_0^{z_E} (1+z)H'_G dz \; + O(\alpha\beta) + O(\beta^2)$$

The time delay provides a tool to bound the coefficient β only!

Such a time delay is indeed observed:

Binary Neutron Star Merger: GW170817 and GRB 170817A *APJ 2017*

Observed Time Delay: +1.74 ± 0.05 Seconds

Part of this time delay (or entirely) could be due to astrophysical reasons, but part of this could also be due to violation of GR

Setting the intrinsic delay to zero, we can get a upper limit

$$\boxed{\frac{\beta H_0^2}{c^2} \leq 8.5 \times 10^{-16}}$$

This is a very weak bound on the higher curvature coupling compared to the local tests, but the situation is expected to be much better with more such sources!

But, let us remind ourselves the condition of the higher curvature coupling from theoretical considerations:

$$\beta < 0$$

The observed time delay can be written as a perturbative expression:

$$\delta t_o = f\,\delta t_E - \frac{3\beta H_0^2 z\ \Omega_m}{2} + O(\alpha\beta) + O(\beta^2)$$

In the absence of intrinsic delay, we must have: $\delta t_o > 0$

This implies that we should see EMW first and then GW. We see exactly the opposite!

So, there must be a lower limit to the intrinsic delay of 1.7 seconds!

This is an important information which can provide constrains on the GRB physics!

As an example, this provides an intriguing limit on the size of the prompt emission region of GRB 170817A.

Shoemaker et. al. PRD 2018

$$\Delta t_{\rm ast} = \frac{r_{\rm em}}{v_j}\left(1 - \frac{v_j}{c}\cos\theta_{\rm ob}\right)$$

$$r_{em} \geq c\Delta t_{\rm ast}(1 - \cos\theta_{\rm ob})^{-1} \simeq 9.9 \times 10^{11}\ {\rm cm}$$

This is a crucial input to test the standard GRB paradigm, the relativistic fireball model

Conclusions:

Physics beyond GR is constrained from various theoretical and observational tools

Aspects like black hole second law, causality & Positivity of ADM mass are important criterion to test any alternative gravity theory

Gravitational wave observations provides a new window to BGR physics

Thank U