Detecting non-quadrupole GW modes by stacking multiple inspiralling BBH signals.

Anand Sengupta, IIT Gandhinagar

In collaboration with Soumen Roy (IITGn), KG Arun (CMI)

https://arxiv.org/abs/1910.04565

Motivation

- The measurement of the higher-order modes of a compact binary
  - can resolve the correlation between distance - inclination angle.
  - can constrain possible jets in the case of neutron star-blackhole binaries.

    K. G. Arun, H. Tagoshi, C. K. Mishra, and A. Pai, Phys. Rev. D90, 024060 (2014)
  - can resolve the two states of gravitational wave polarization.

    O. Jennrich, et al., 1st International LISA Symposium on Gravitational Waves Oxfordshire, England, July 9-12, 1996, Class. Quant. Grav. 14, 1525 (1997).
- Tests of GR.
  - Consistency tests (Masses and spins across harmonics)

    S. Dhanpal et al., Phys. Rev. D99, 104056 (2019)

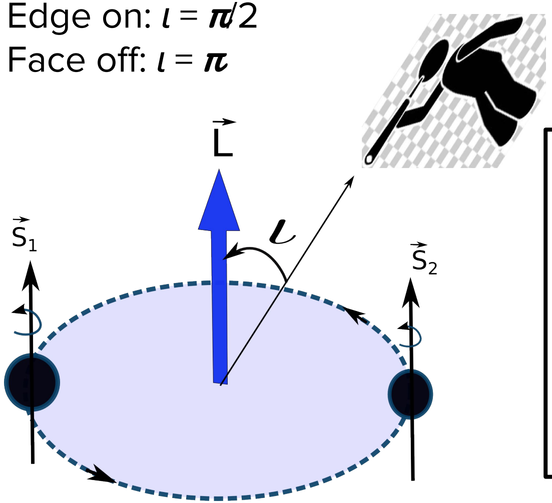
Signal model

Gravitational wave modes

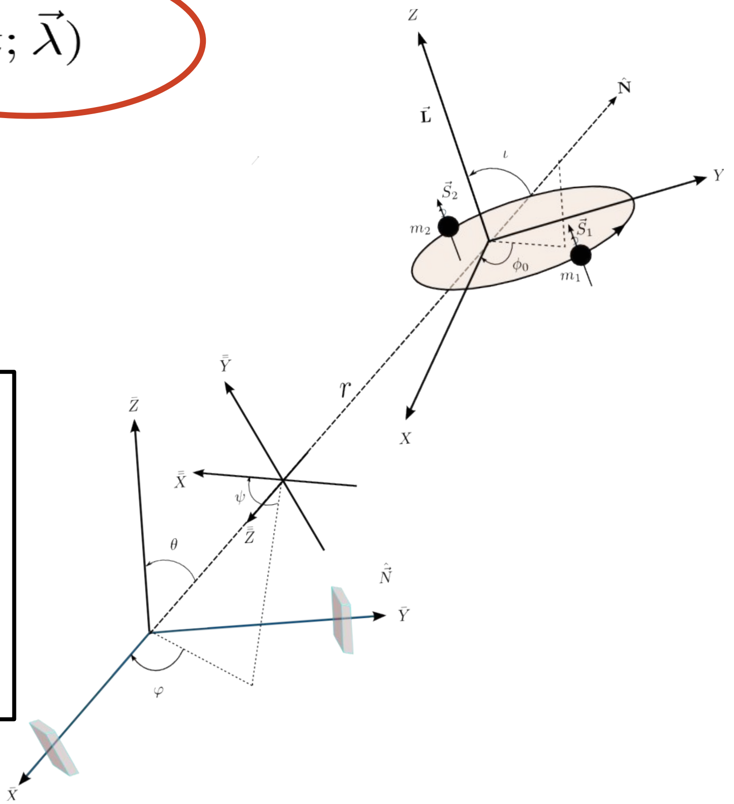
$$h(t;\iota,\phi_0,\vec{\lambda}) = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} {}_{-2}Y^{\ell m}(\iota,\phi_0)\, h_{\ell m}(t;\vec{\lambda})$$

-2 spin weighted spherical harmonics

Face on: $\iota = 0$
Edge on: $\iota = \pi/2$
Face off: $\iota = \pi$



- $(\ell, |m|) = (2, 2)$ Quadrupole, dominant mode
- $(\ell, |m|) \neq (2, 2)$ Non-quadrupole, subdominant higher order modes (HOM)

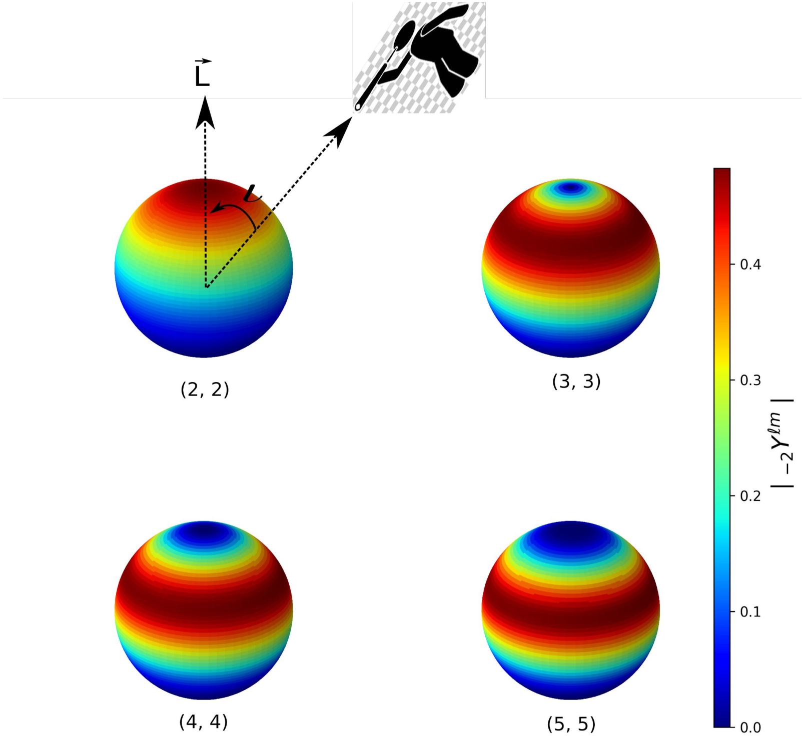


-2 spin-weighted spherical harmonics

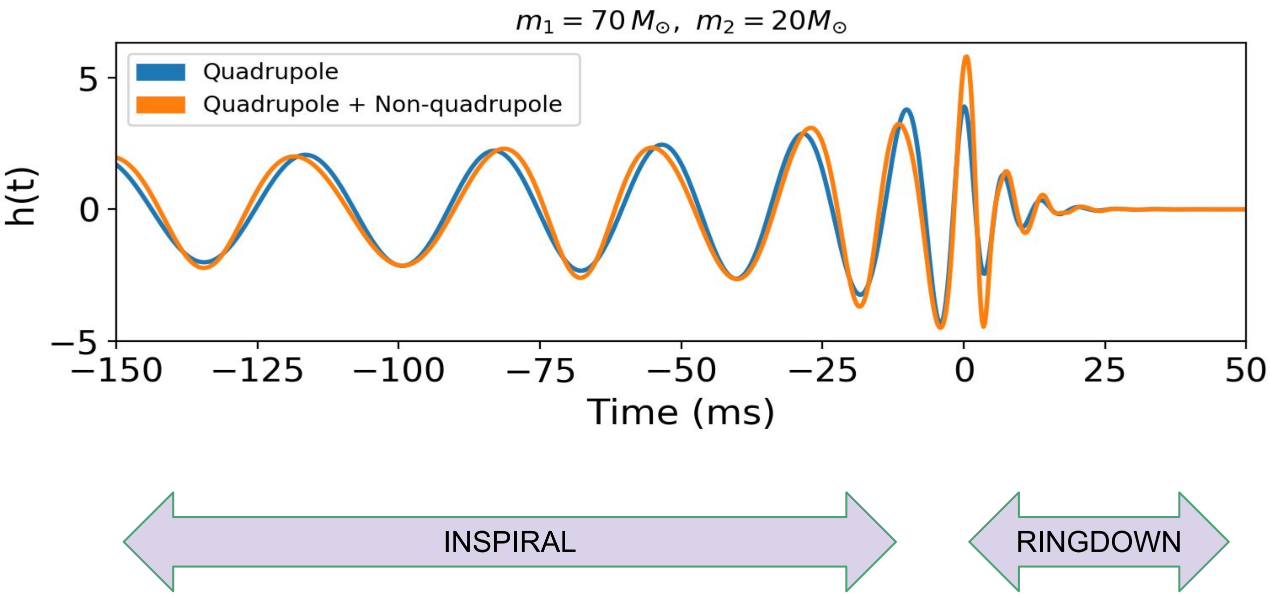
- Spin-weighted spherical-harmonics ${}_{s}Y^{lm}$ are commonly used to separate the angular dependence of the gravitational radiation

- $${}_{-2}Y^{lm}\,(\iota,\varphi)= (2l+1/4\pi)^{1/2}\, d_{2}^{\;lm}(\iota)\;\; e^{im\varphi}$$

Wigner d function



Subtle changes to the signal



BBH searches in LIGO data

Templated searched for dominant quadrupolar ($\ell=2, m=2$) GW signals in the data.

Non-quadrupolar modes are much weaker. Hard to detect it from a single observation, unless we are lucky to have a relatively strong source with asymmetric masses.

A suitable **combination** of several sources could unravel these weak signals.

Stacking!

Stacking algorithms already explored in the context of post-merger ringdowns.

Stacking works for ringdowns very well

H. Yang, Kent Yagi, et al. Phys. Rev. Lett. 118, 161101 (2017)

$$h_{\ell m,j}(t) = A_{\ell m,j} e^{-\gamma_{\ell m,j} t} \sin(\omega_{\ell m,j}\, t - \phi_{\ell m,j}) \, .$$

To perform the alignment, out of the set of N events, one arbitrarily picks one (e.g. the *i*-th one) as the base case, and shift/rescale all others to give the same expected secondary mode phase offset. And now add!

As a result, there is a coherent mode stacking which ultimately increases the SNR of the subdominant ringdown modes.

Why it won't work for inspirals

The crucial impediment in combining inspiral signals is the time-varying instantaneous frequency, which changes quite rapidly in the late-inspiral stages.

Template model: properties of HOM

Gravitational wave modes

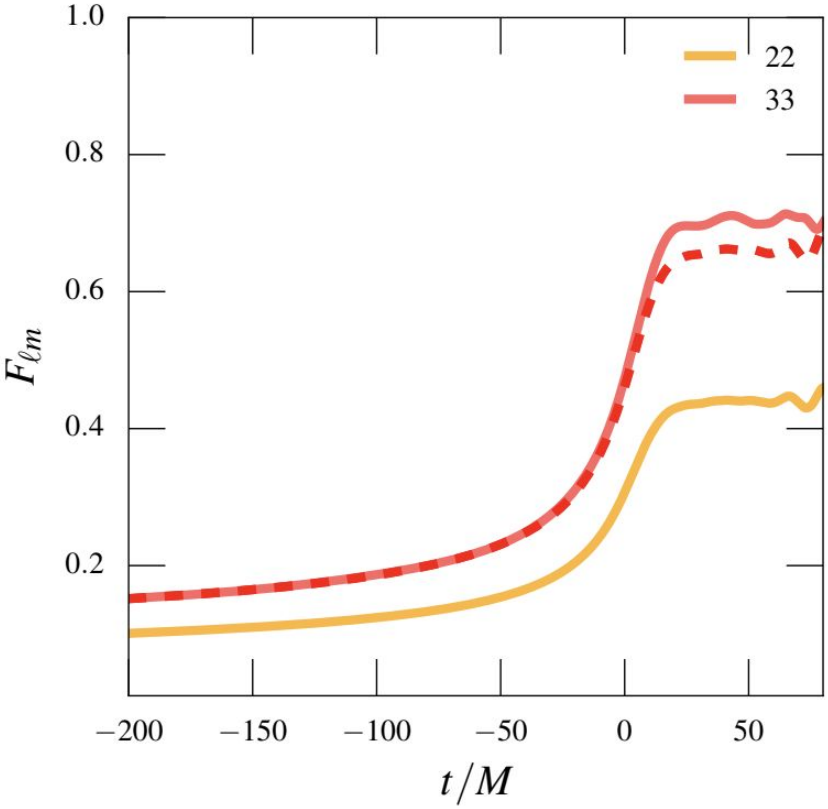
$$h(t;\iota,\phi_0,\vec{\lambda}) = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} {}_{-2}Y^{\ell m}(\iota,\phi_0)\, h_{\ell m}(t;\vec{\lambda})$$

-2 spin weighted spherical harmonics

- Expression of the modes: $h_{lm}(t) = A_{lm}(t)e^{i\varphi_{lm}(t)}$
- Instantaneous inspiral-merger phase: $\varphi_{lm}(t) = \frac{m}{2}\varphi_{22}(t)$
- Time varying frequency: $f_{lm}(t) = \frac{d\varphi_{lm}(t)}{dt} = \frac{m}{2} f_{22}(t)$

$f_{lm}(t) = \alpha f_{22}(t)$
Access (l, m) mode by setting α = m/2

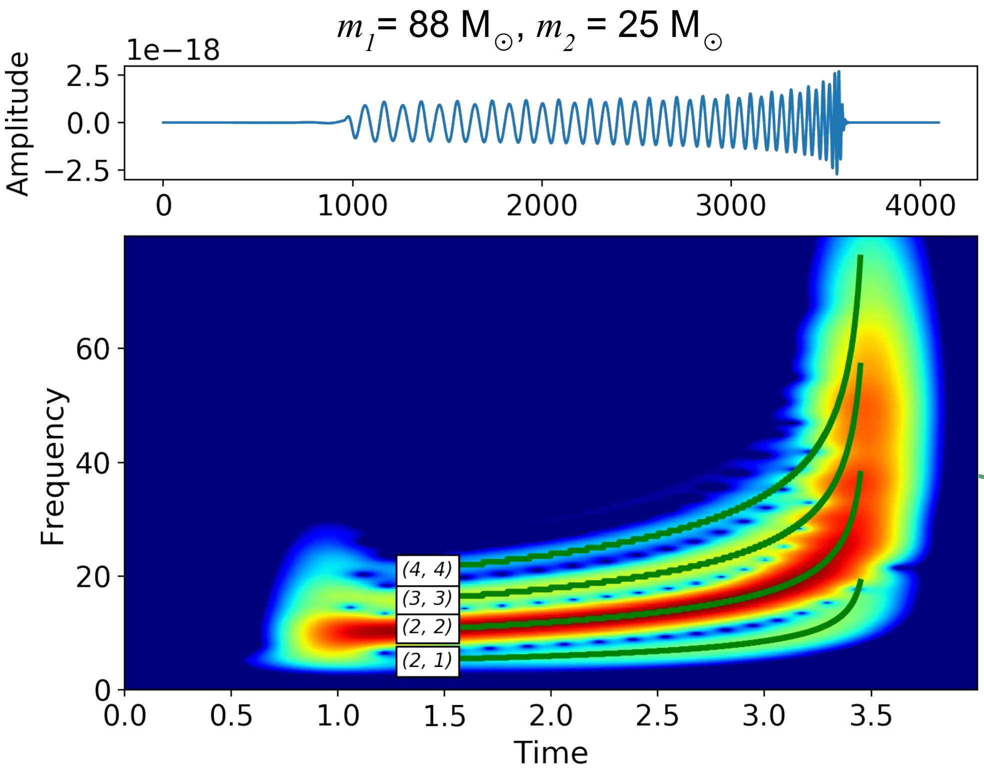
m/2 is the key factor to access all the modes



$$f_{lm}(t) = \frac{d\varphi_{lm}(t)}{dt} = \frac{m}{2} f_{22}(t)$$

Image Credit: Ajit Mehta (2019)

Template model: Use of spectrogram



$$f_{lm}(t) = \alpha f_{22}(t)$$

- $f_{21}(t) = 1/2\, f_{22}(t)$
- $f_{22}(t) = 2/2\, f_{22}(t)$
- $f_{33}(t) = 3/2\, f_{22}(t)$
- $f_{44}(t) = 4/2\, f_{22}(t)$

Collect the energies along tracks by varying α: $[0.2 \leq \alpha \leq 3.2]$

Clearly seen that (2, 2) power has overlap with subleading modes

Template model

Consistency among different sources

Time-frequency map using S-transform

Window function

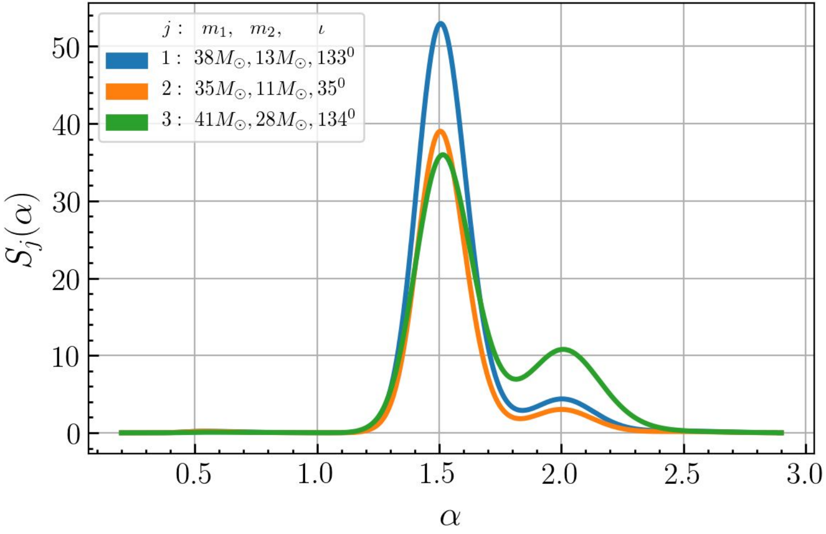
$$\tilde{x}(\tau, f) = \left| \int_{-\infty}^{\infty} x(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(t-\tau)^2 f^2}{2}} e^{-2\pi i f t} dt \right|^2$$

Signal

Collect the energies along scaled (2, 2) track

$$S_j(\alpha) = \sum_{\tau = t_c - \Delta\tau}^{t_c} \tilde{s}_j \left(\tau,\ f = \alpha f_{22}(\tau; \vec{\lambda}_j) \right)$$

One can stack the energy along the "α" bin to boost the effective statistic



Stacking sources is possible!

Now to pose the detection problem

- $S_j(\alpha)$, $N_j(\alpha)$, $Y_j(\alpha)$

- Imagine that we have already detected the quadrupole mode of several events. Their masses and spins have been detected from PE studies using (2,2) mode.

- Given a on-source segment

  Does $Y_j(\alpha)$ contain $N_j(\alpha) + S_j(\alpha)$?

Noise model
Distribution of summed noise pixels

- Time-frequency map of whitened Gaussian time series: $n(\tau) \sim \mathcal{N}(0, \sigma_0)$

$$X_n(t, f) = \int_{-\infty}^{\infty} n(\tau) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau - t)^2 f^2}{2}} e^{-2\pi i f \tau} d\tau$$

  - Covariance: $\sigma^2(t, f) = \langle X_n(t, f), X_n^*(t, f) \rangle = \frac{|f|}{2\sqrt{\pi}}$

- Summed noise energy along "α-track": $N_j(\alpha) = \sum_{t=t_c - \Delta t}^{t_c} \tilde{n}_j \left(t, f = \alpha f_{22}(t; \vec{\lambda}_j) \right)$

  - $N_j(\alpha)$ is the summation of "n" Gamma (Γ) random variables
  - For large "n", one can appeal to the central limit theorem (CLT)

- $N_j(\alpha)$ ~ Gaussian (μ, σ^2)

Noise model: correlation in $N_j(\alpha)$

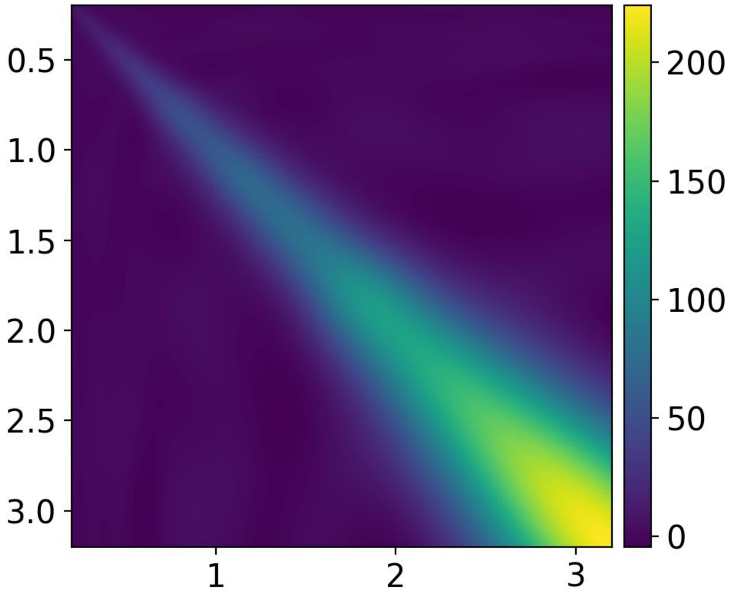
- $N_j(\alpha)$ is correlated with $N_j(\alpha+\Delta\alpha)$, since both are computed with same noise realization and also, two consecutive tracks are very near.

- Estimate the covariance matrix using 1000 noise realizations for each events

- Cholesky decomposition for decorrelating the noise model $\Sigma_j^{-1} = L_j^T L_j$

- Decorrelated noise $L_j N_j(\alpha)$ follows $\mathcal{N}(0, 1)$

$$\Sigma_j(\alpha, \alpha') := \mathbb{E}\left[(N_j(\alpha) - \mu_j(\alpha)) \; (N_j(\alpha') - \mu_j(\alpha'))^T \right]$$



Detecting HOM in additive correlated Gaussian noise (single loud event)

Binary hypothesis

$$\begin{cases} \mathcal{H}_0 : Y(\alpha) = N(\alpha), \\ \mathcal{H}_1 : Y(\alpha) = N(\alpha) + S(\alpha) \end{cases}$$

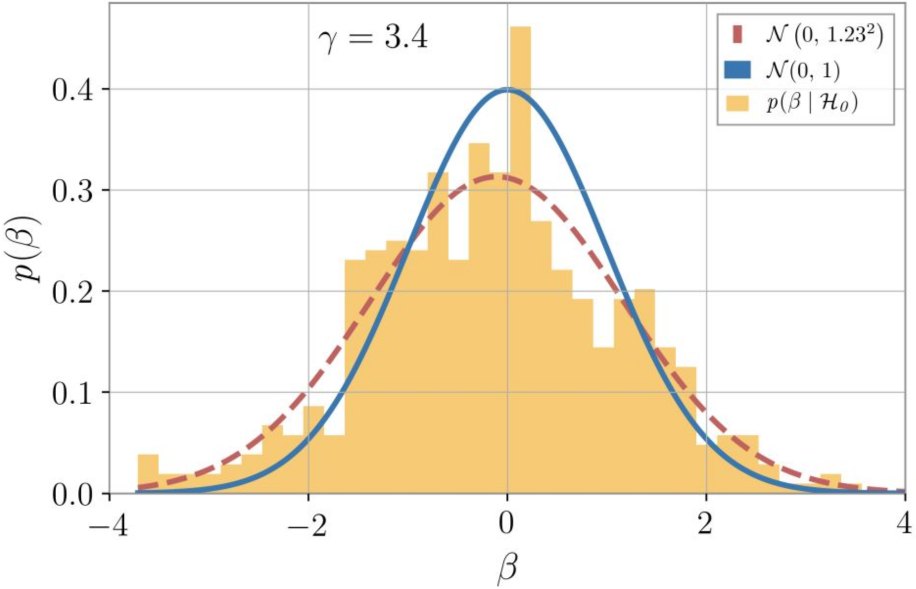
Logarithmic likelihood ratio

$$\Lambda[Y(\alpha)] = \langle Y(\alpha) \mid S(\alpha) \rangle - \frac{1}{2} \|S(\alpha)\|^2$$

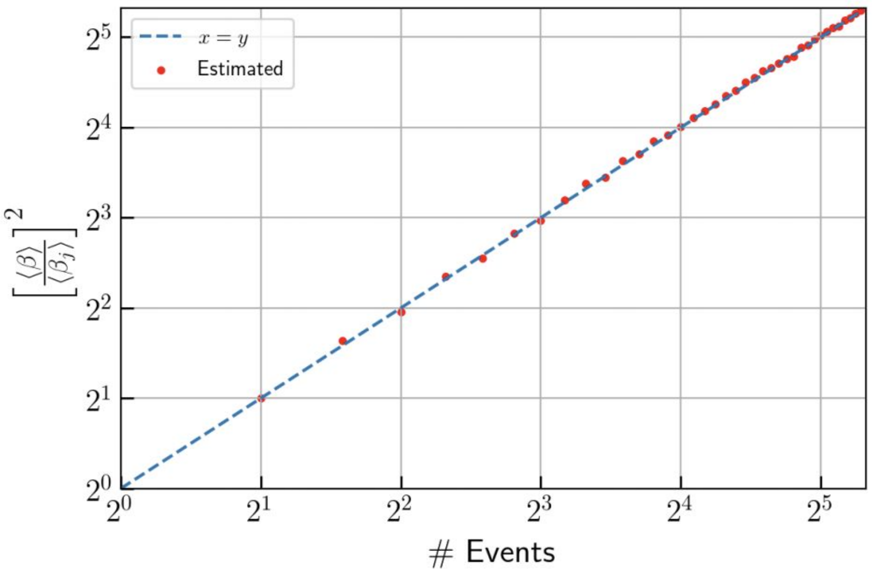
Final statistic

$$\beta[Y(\alpha)] = \left\{ \Lambda\left[Y(\alpha)\right] + \gamma^2/2 \right\} / \gamma$$

Signal power: $\gamma = \|S(\alpha)\|$



Scaling of the detection statistic with number of events



For n_0 identical events,

$$\langle\beta\rangle / \langle\beta_j\rangle = \sqrt{n_0}.$$

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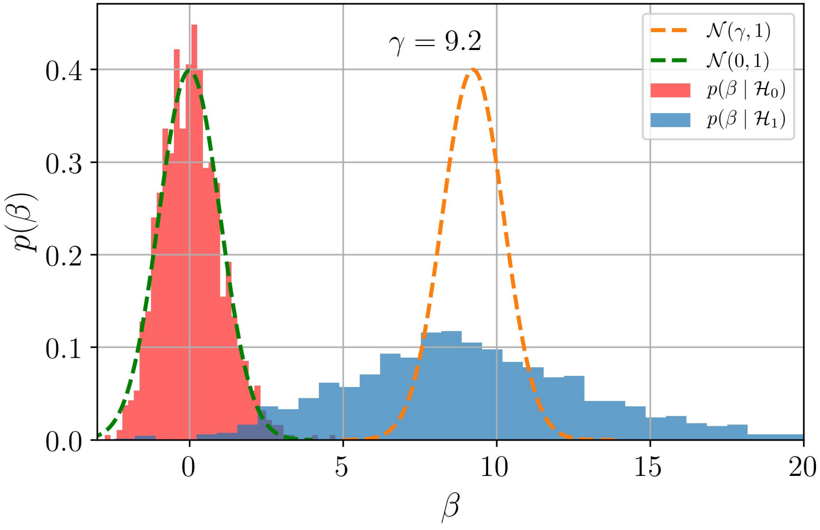


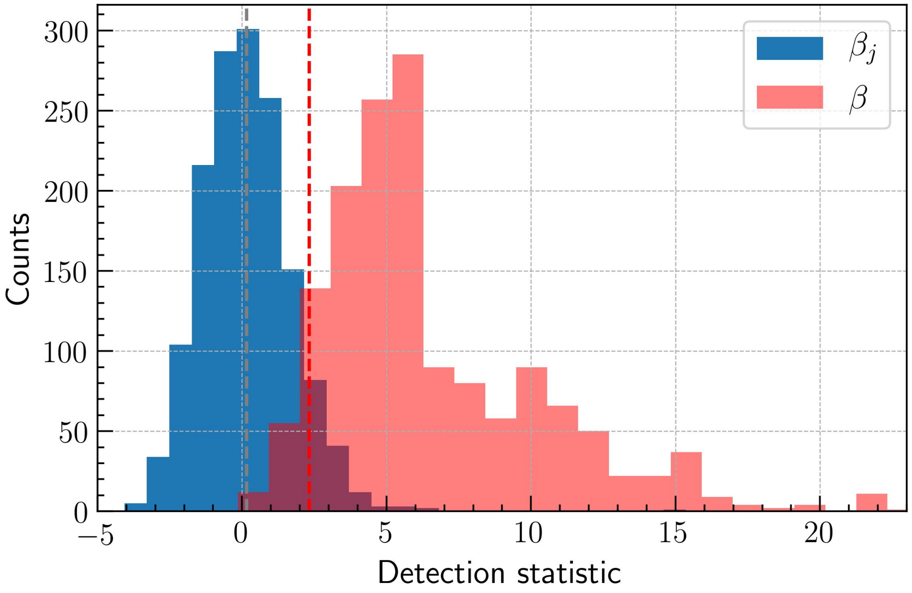
Detecting HOM with multiple events

- Generate 1000 events with a distribution of masses and distances as likely to be seen in advanced LIGO.

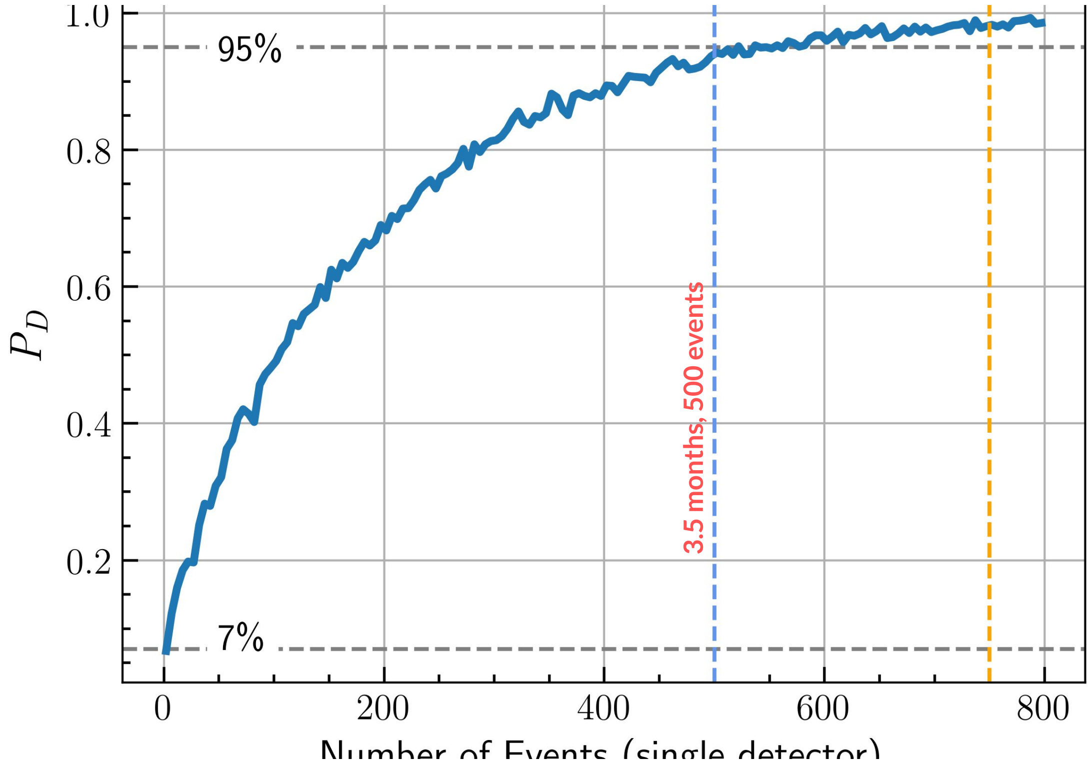
- Both the sky location of the source and orientation of the binaries are isotropically distributed.

- Probability of detection increases from 7% → 95%
  [at a fixed FAP of = 1%]

  500 events: 3.5 months at design sensitivity.



Number of events vs probability of detection



Conclusion

- Tests of GR with inspiral HOMs.
  - The structure of $Y_j(\alpha)$ itself could serve as a test of GR

- Ready to analyze O3 events for the presence of HOMs.
- **Reference:**

  Unveiling the spectrum of inspiralling binary black holes,
  Soumen Roy, Anand S. Sengupta, K. G. Arun, https://arxiv.org/abs/1910.04565