# Tests of General Relativity using Compact Binaries

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Chennai Mathematical Institute

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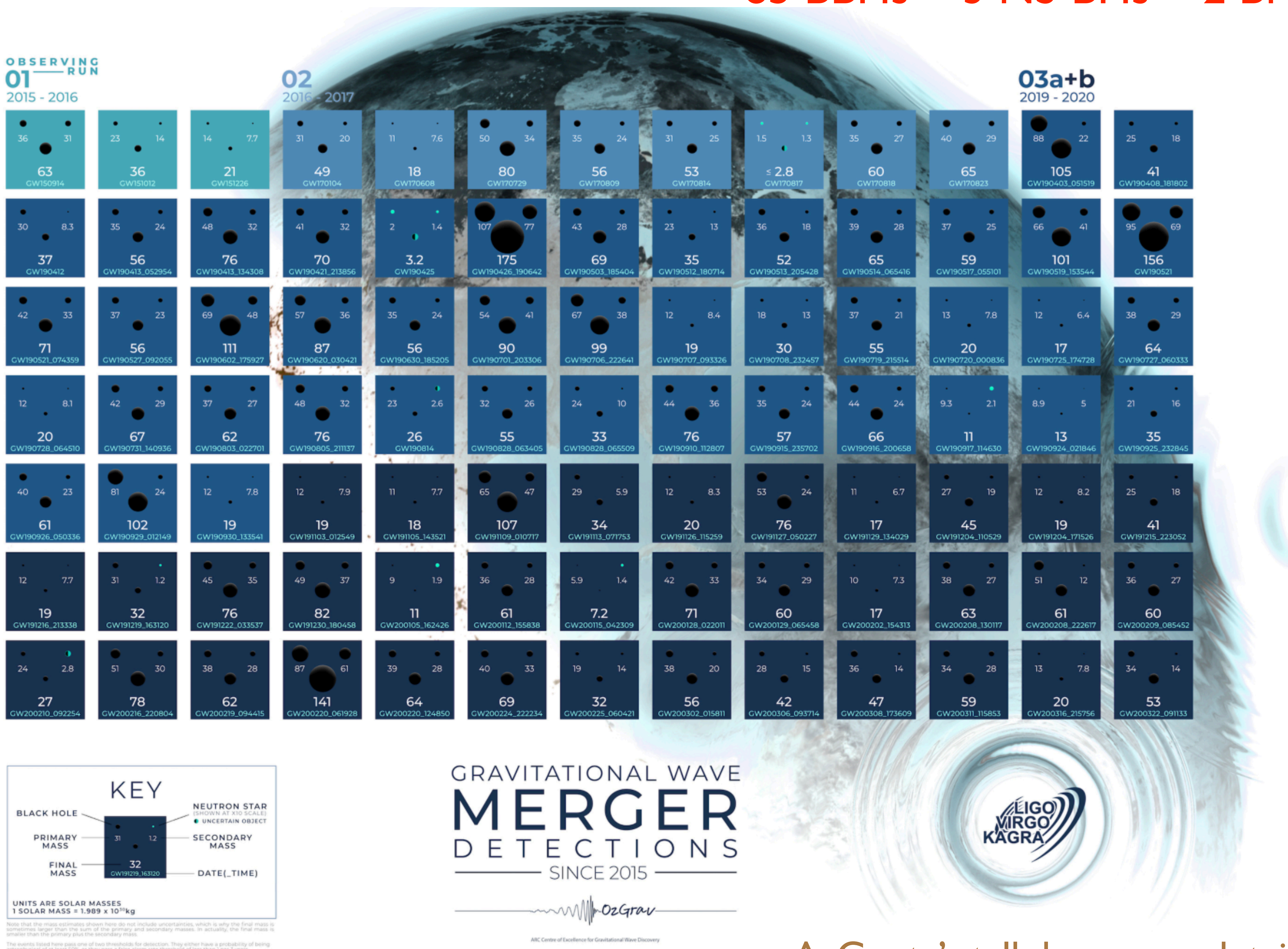
CHENNAI MATHEMATICAL INSTITUTE

LIGO INDIA

LIGO–India Scientific Collaboration

# 85 BBHs + 3 NS-BHs + 2 BNS




A. Gupta's talk has more details

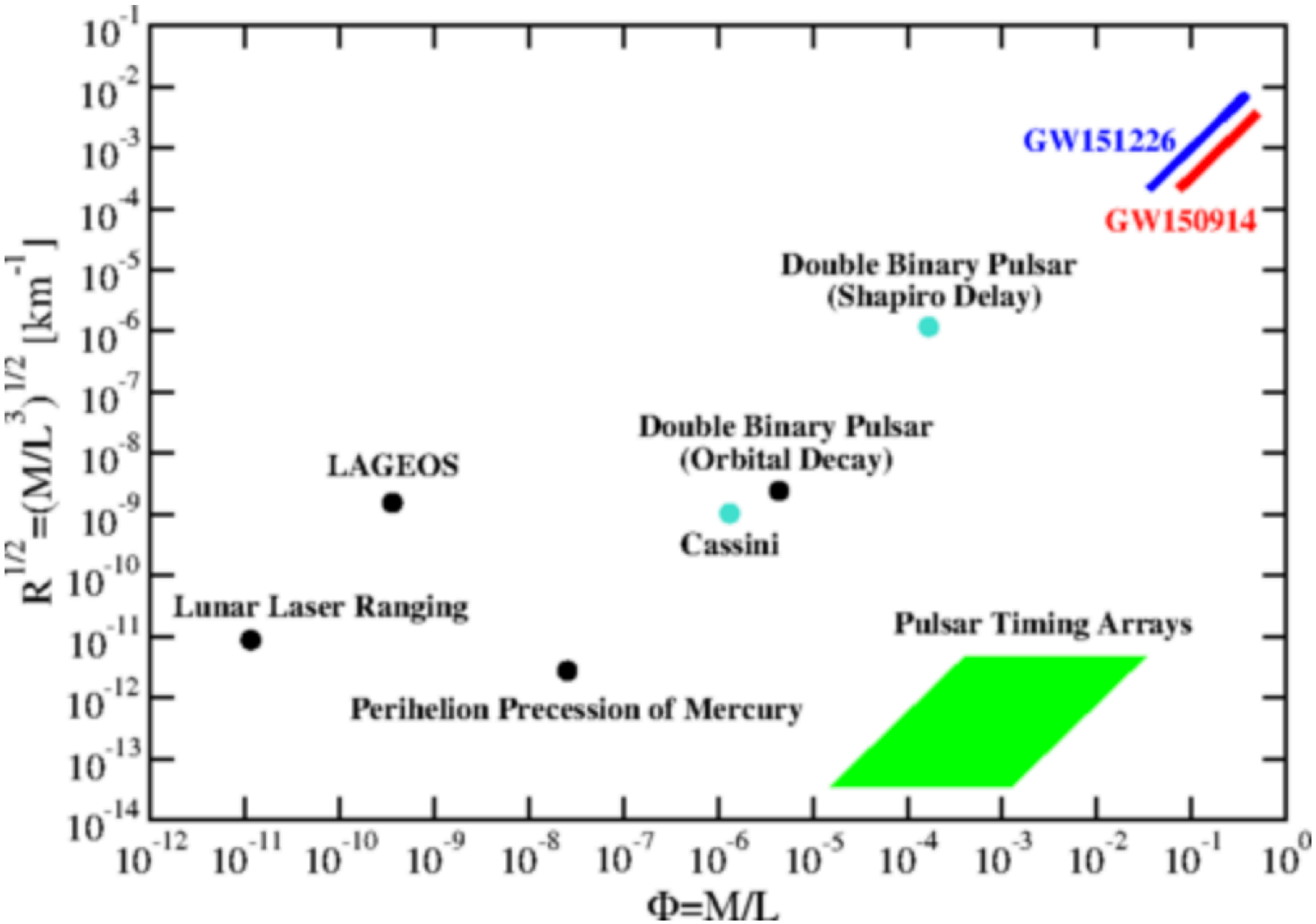
# Why should we test GR?

No reason why GR should be the correct theory of gravity!

and

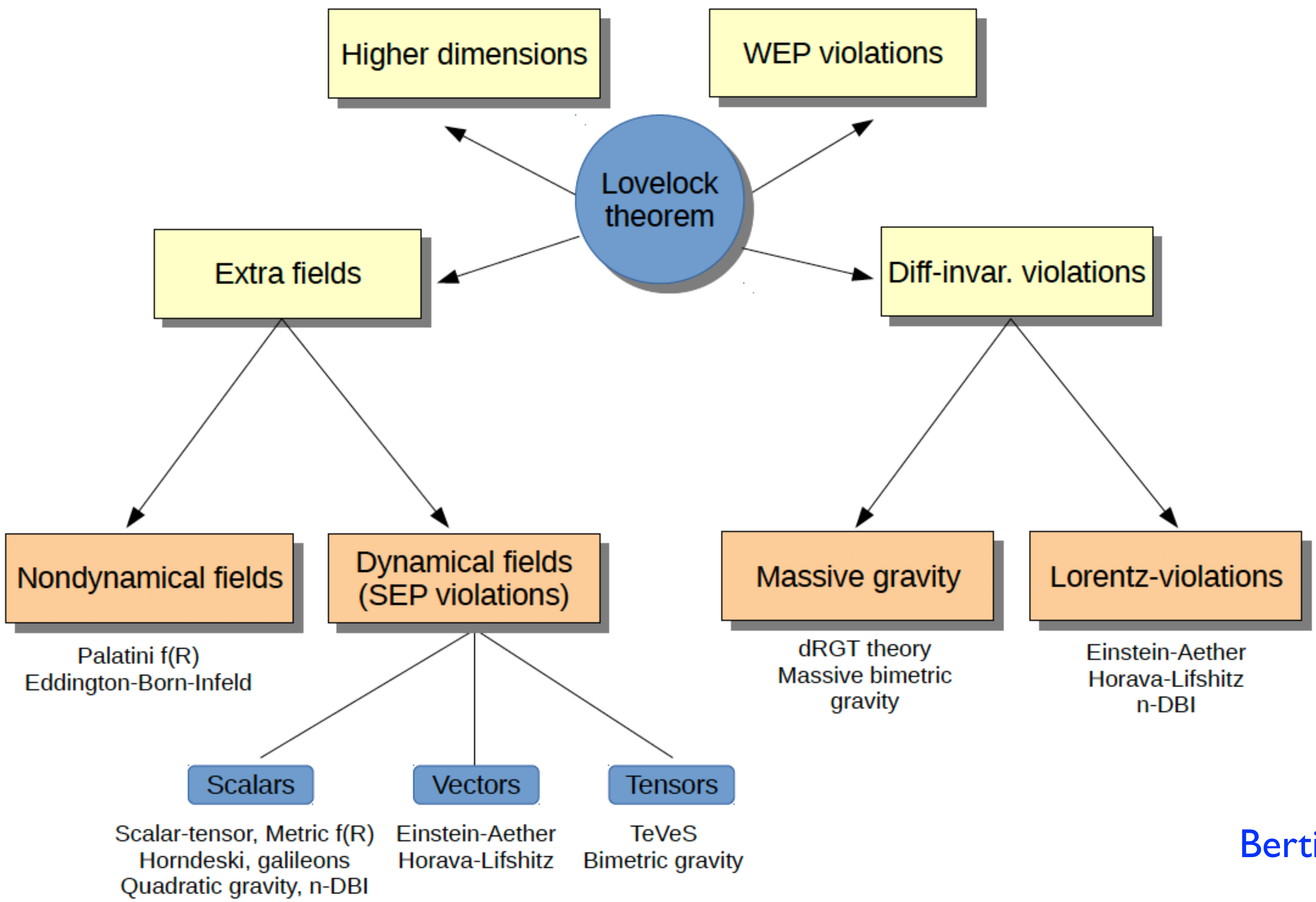
We can test it in several ways!

# An overview of various tests




Yunes, Pretorius, Yagi 2016

# Modifications to GR




Berti+ 2015

# GWs in modGR

Difference from GR in terms of

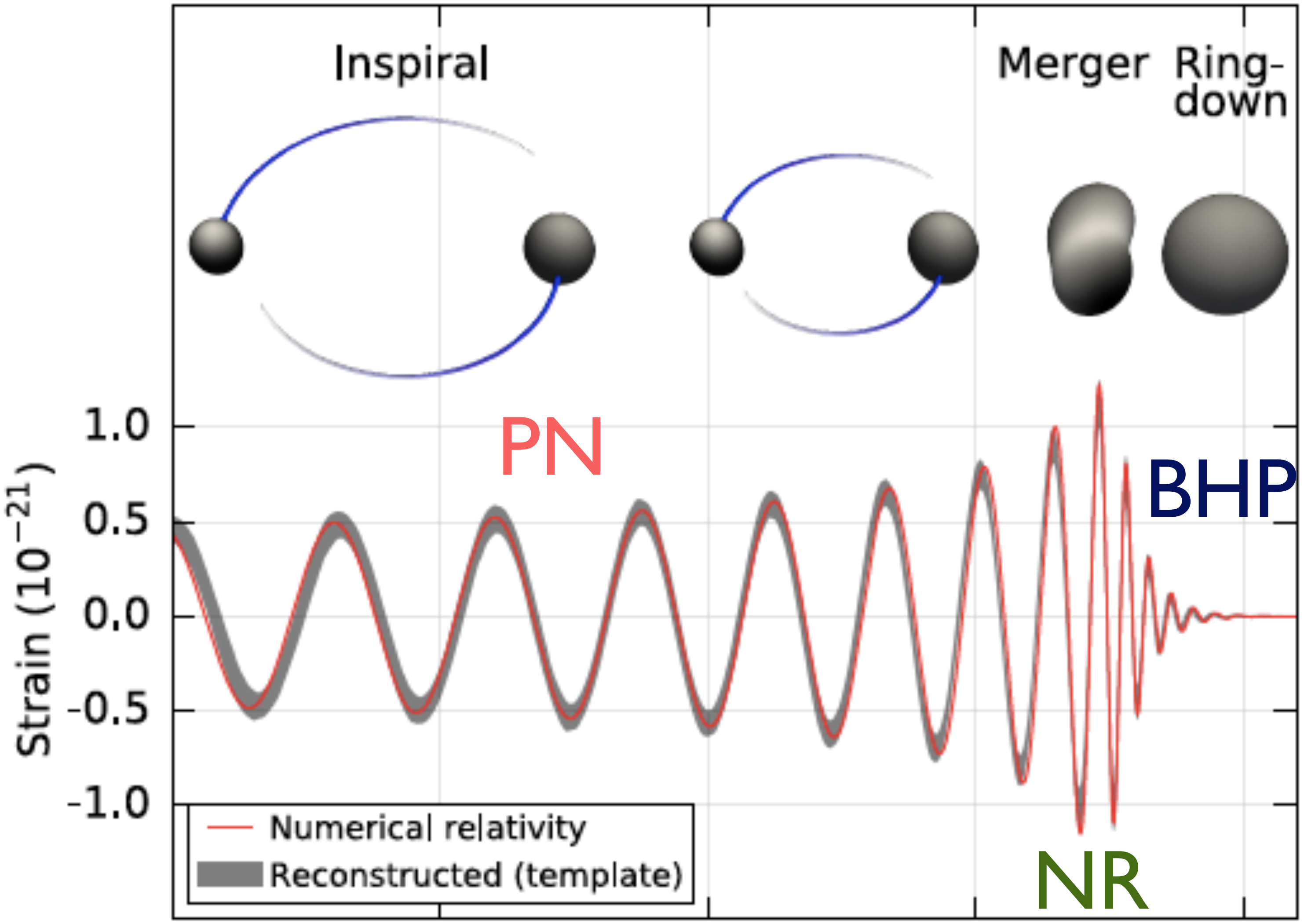
- How GWs are generated.
- How GWs propagate.
- GW polarization.

# What can change in modified gravity?

- How GWs are generated.
  - *Presence of additional fields, curvature corrections, higher D etc*
- How GWs propagate.
  - *Dispersion, Birefringence,…*
- GW polarization.
  - *Non-GR modes of polarizations*

*All these can be put to test.*

# Compact binary dynamics



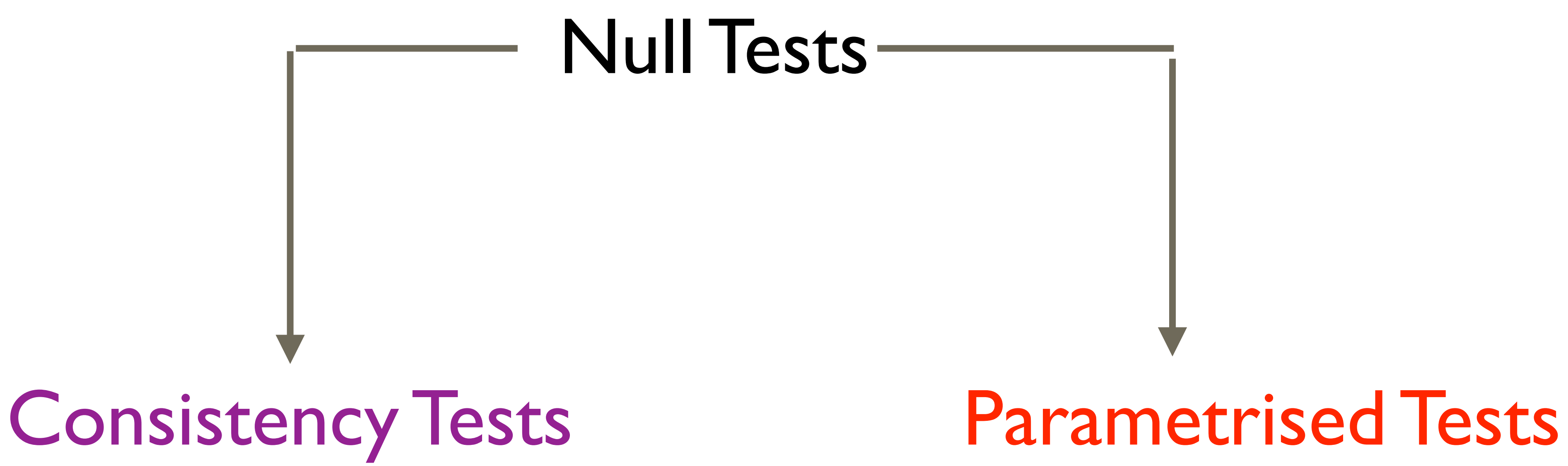

LVC, Phys. Rev. Lett. 116, 061102 (2016)

# How can we test?

- *Use specific predictions/waveforms of modified theories and*
  - *Constrain the additional parameters using the GW data*
  - *Carry out model selection between competing models.*
- *There are hurdles to do this*
  - Lack of accurate waveforms in modGR theories
  - No well-posed initial value problem for certain modGR
  - Nevertheless, recent developments in the case of Scalar-Tensor theories are encouraging.

# Alternative: Null Tests of GR

- Our theoretical limitations should not stop us from *testing GR.*
- Use the best use of our understanding of the GR dynamics and waveform.
- Ask whether there are "deviations" from the GR waveform that the data supports.



# This talk

- Three different tests of GR:
  - Their basic philosophy
  - What do they search for
  - Latest bounds from them
  - What are the Pro/Con of these tests.

# ***Residuals Test***

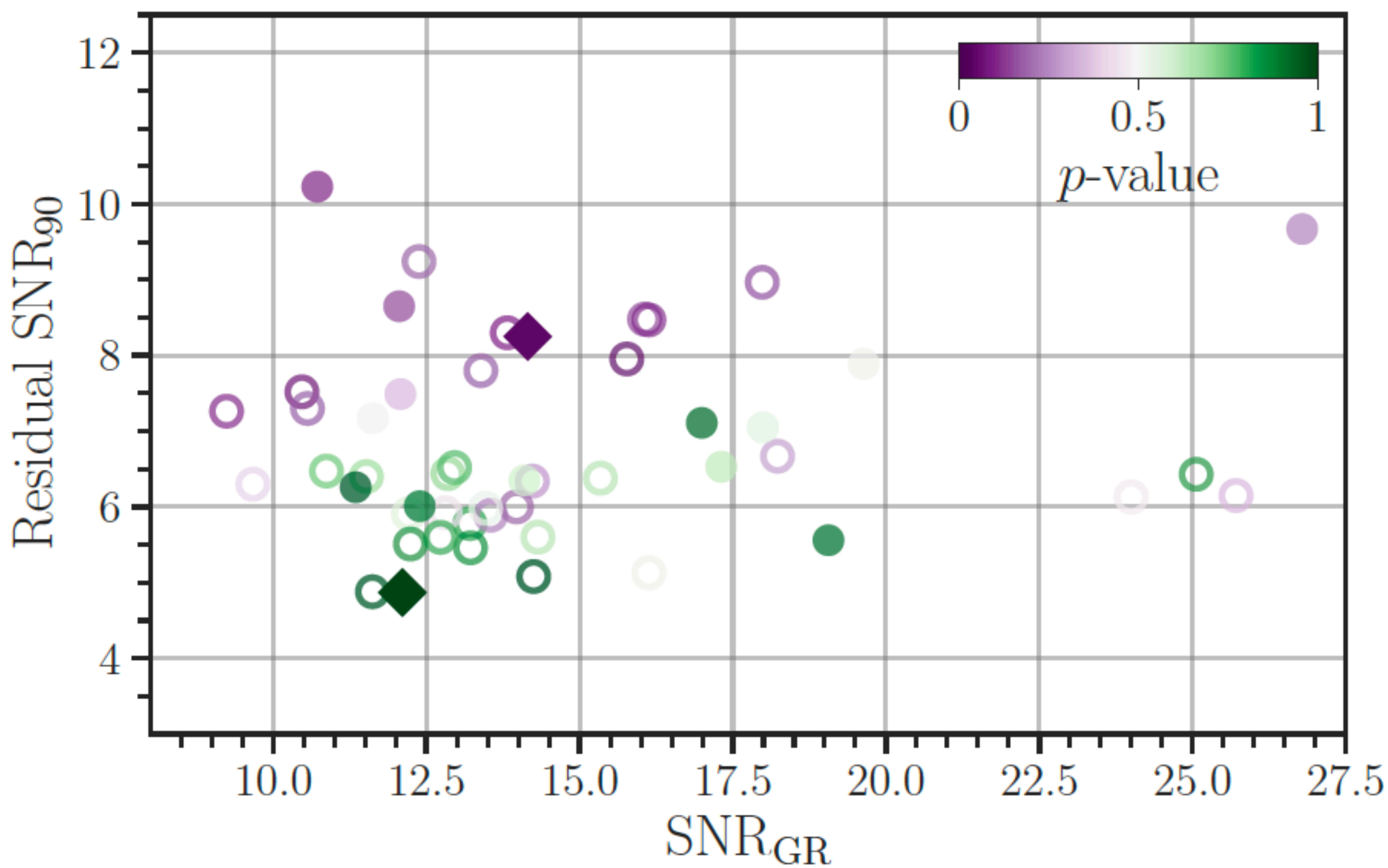
# Residuals Test: Philosophy

- Subtract off the "best-fit" (max likelihood) waveform from the data.
- Ask if there is any coherent remnant power in the "residuals".
- If the residual is consistent with detector noise, statistical consistency with GR.
- Use p-values to quantify the consistency.

Vallisneri, 2012

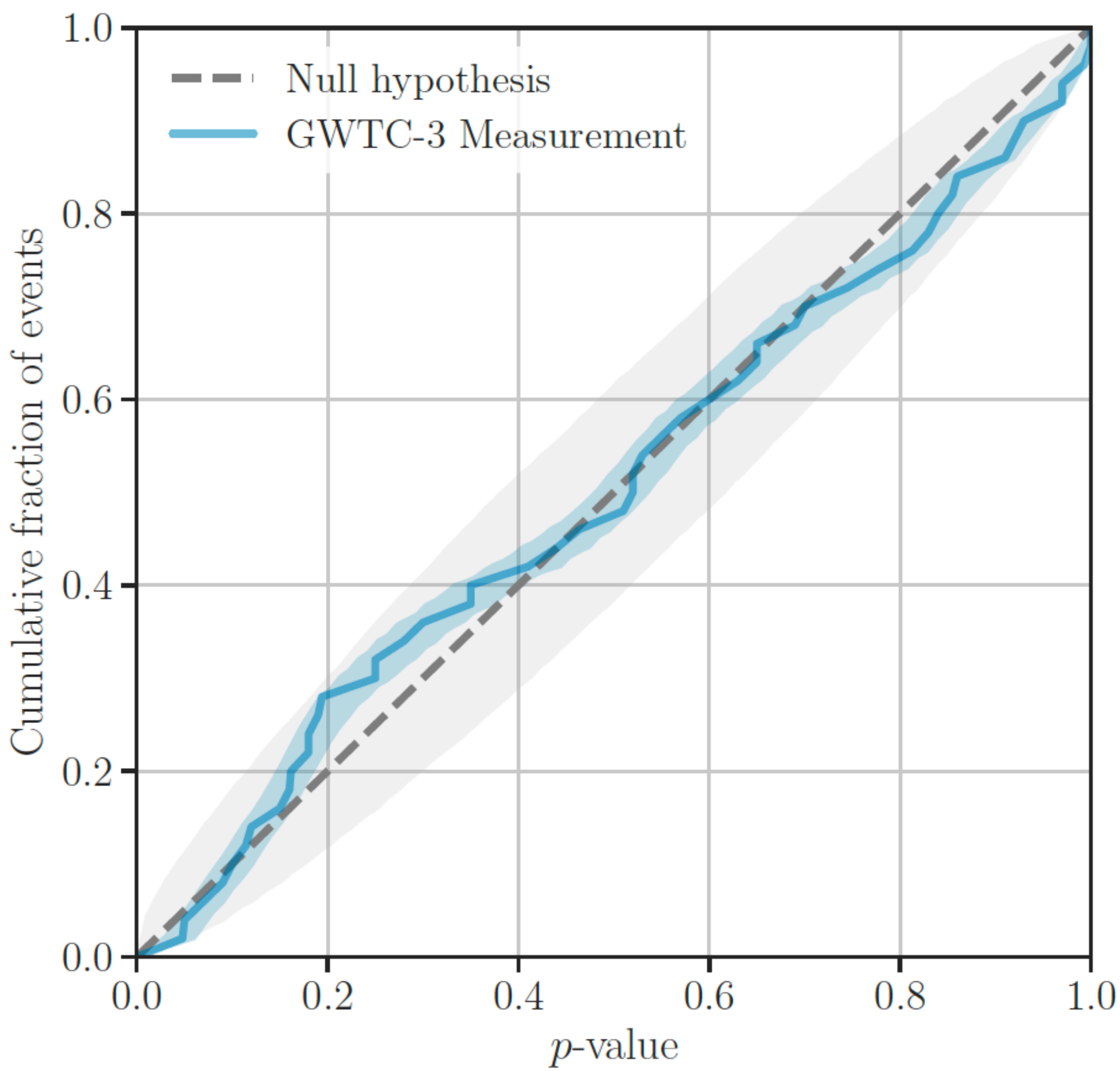
# Residuals Test

90% upper limit on Res. SNR




LIGO-Virgo-KAGRA collab, arXiv:2112.06861

# Residuals Test: Philosophy




LIGO-Virgo-KAGRA collab, arXiv:2112.06861

# Pros and Cons

- Pros
  - Seemingly the "most general", clean test we can perform!
  - Non-parametric method.
- Cons
  - Not (necessarily) the "most sensitive" test we can perform.
  - Very sensitive to waveform systematics (depends on what waveform you subtract off).

# *Parameterized Tests of Inspiral*

# Parametrised tests

- Introduce *physically motivated* free parameters to the waveform and bound them using the data.
- The free parameters capture *how different is the signal from the predictions of GR*.
- Can be mapped onto specific alternative theories, if they have a corresponding prediction.
- Widely used in solar system (PPN) and binary pulsar (PPK) tests.
- You can parameterize inspiral, ringdown etc. (Shilpa's talk on RD)

# PN phasing formula

$$\tilde{h}(f) = \mathcal{A}(f)\, e^{i\Phi(f)}$$

$$\Phi(f) = 2\pi f t_c - \phi_c + \frac{3}{128\,\eta\, v^5} \sum_{k=0}^{N} [\phi_k + \phi_{kl} \ln v]\; v^k$$

$$\phi_k(m_1, m_2, \chi_1, \chi_2, \cdots)$$
$$\phi_{kl}(m_1, m_2, \chi_1, \chi_2, \cdots)$$

$$v = (\pi\, m\, f)^{1/3}$$

$$\eta = \frac{m_1\, m_2}{(m_1 + m_2)^2}$$

PN coefficients are functions of source parameters and contain various physical effects in the compact binary dynamics

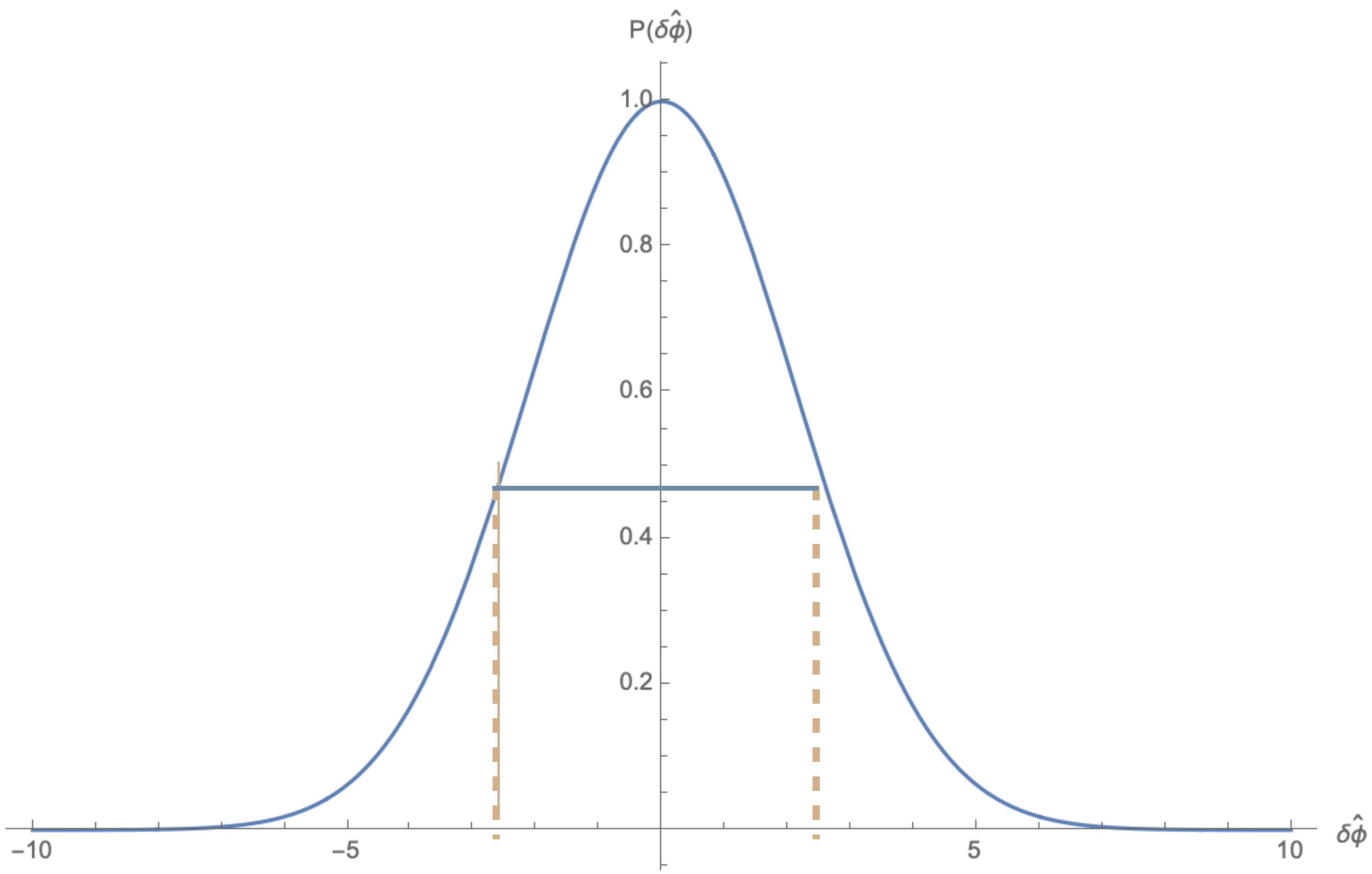
# Physical effects in the PN phasing

| PN Order | Effect |
|---|---|
| 0PN | Chirp Mass |
| 1PN | Possibility to measure component masses,<br>Periastron Advance, |
| 1.5PN | Tails of GWs, Spin-orbit interaction |
| 2PN | Spin-spin interaction, spin-induced quadrupole |
| 2.5PN | Black hole Horizon Flux (spinning) |
| 3PN | Tails of Tails, Tail^2 |
| 3.5PN | Spin-induced octupole |
| 4PN | Black hole Horizon Flux (nonspinning) |
| 5PN | Tidal interactions |

*Every order above 1PN contains more than one physical effect.*

# A GR null test of the PN phasing

Arun+2006, Yunes, Pretorius 2009, Li+ 2013




$$\phi_a \to \phi_a^{\rm GR} + \delta\phi_a$$

$$\phi_a \to \phi_a^{\rm GR}\left(1 + \frac{\delta\phi_a}{\phi_a^{\rm GR}}\right)$$

$$\downarrow$$

$$\delta\hat{\phi}_a$$

$$\delta\hat{\phi}_a^{\rm GR} \equiv 0$$

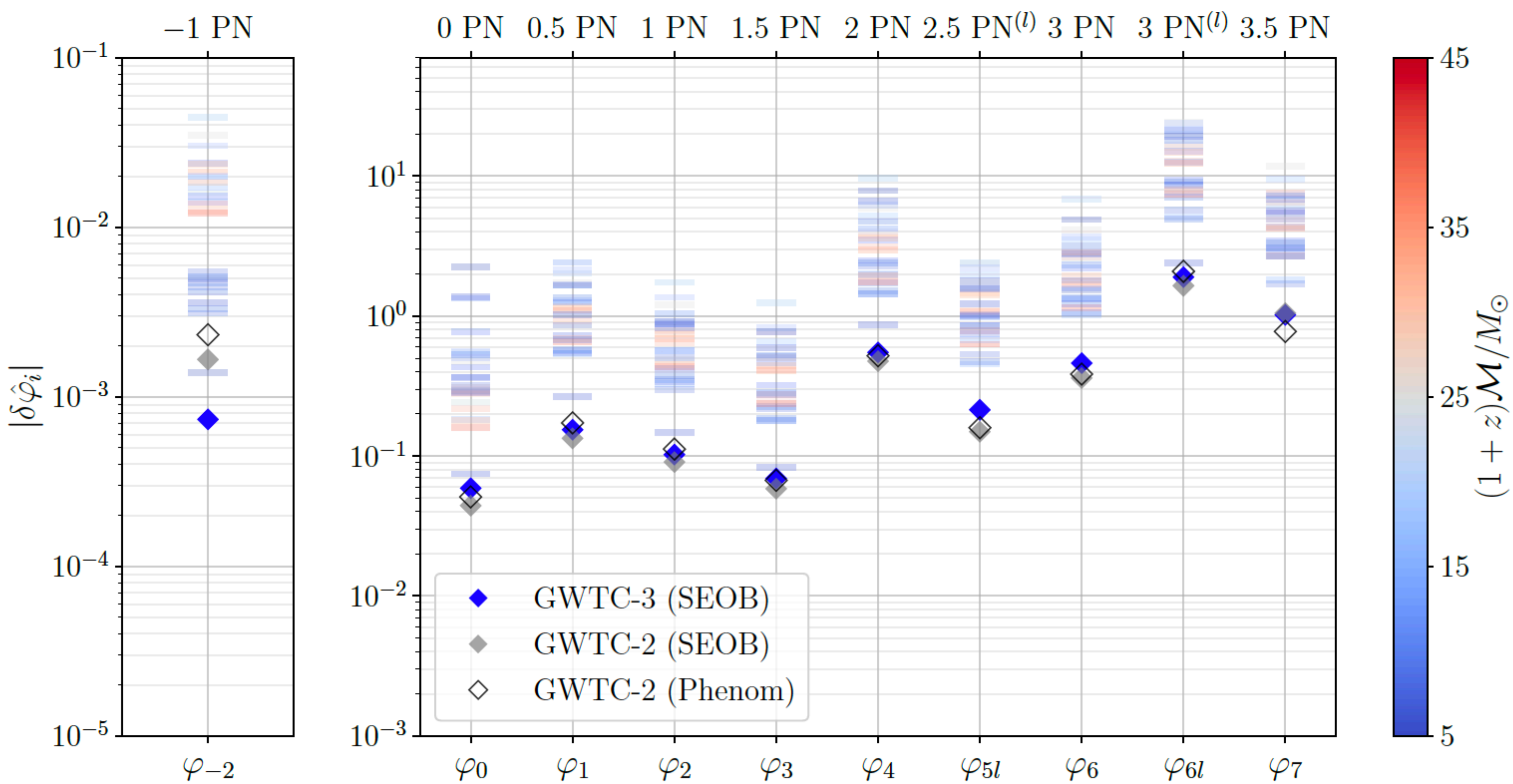
Measurement gives
the bound on the deformation parameter at a fixed confidence interval

# What does it test?

- How consistent are the physical effects (tails, spin interactions, tides,…) with GR.
- Are the phasing coefficients in agreement with the predictions of GR or is there any evidence for deviation?

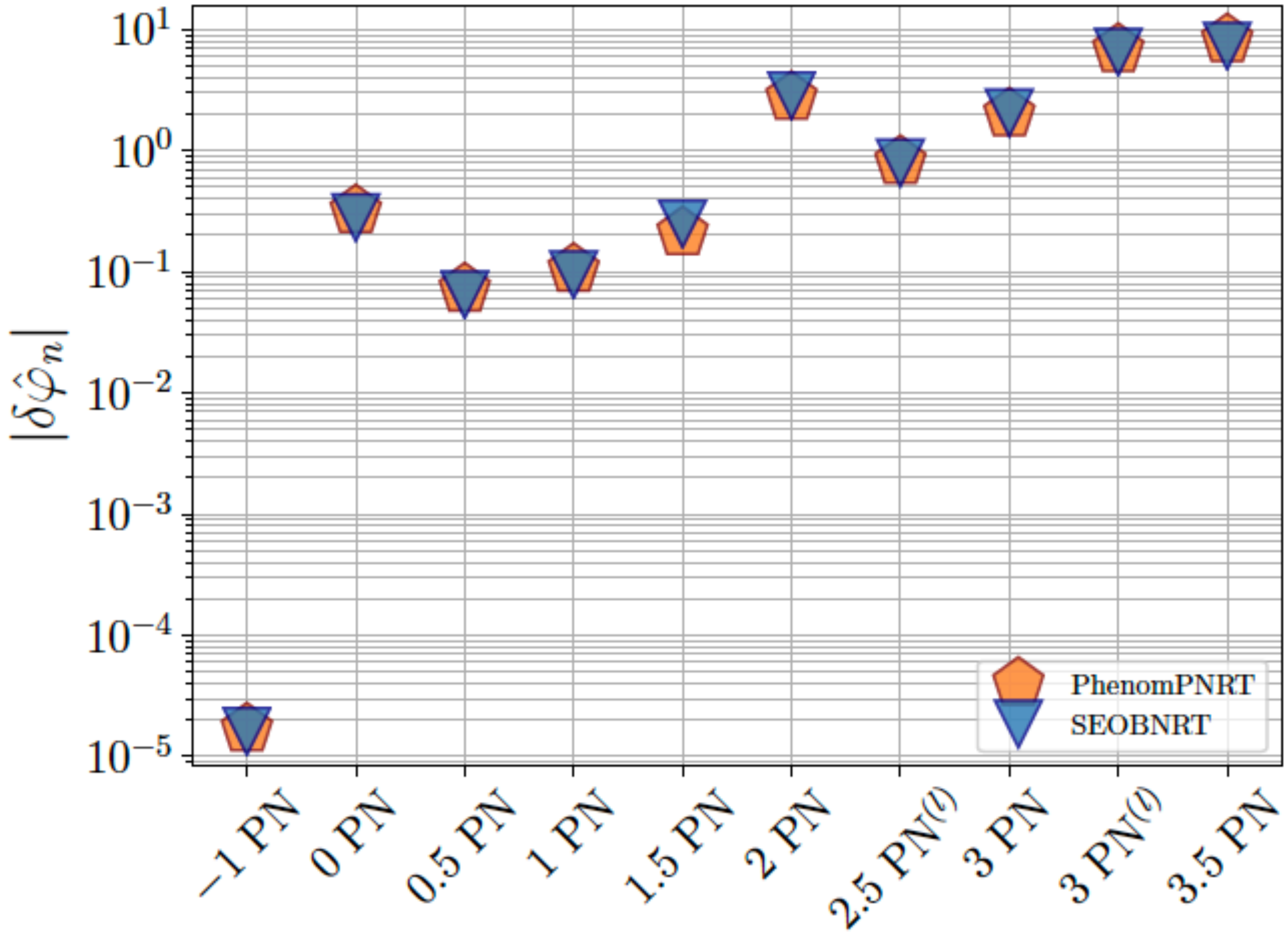
# Current bounds

## Combing BBH+NS-BH events from O1/O2/O3




LIGO-Virgo-KAGRA collab, arXiv:2112.06861

# Interesting special case: GW170817




*Bounds in presence of tidal interactions*

LIGO-Virgo, Phys. Rev. Lett. 123, 011102 (2019)

# Results from GW190814/GW190412

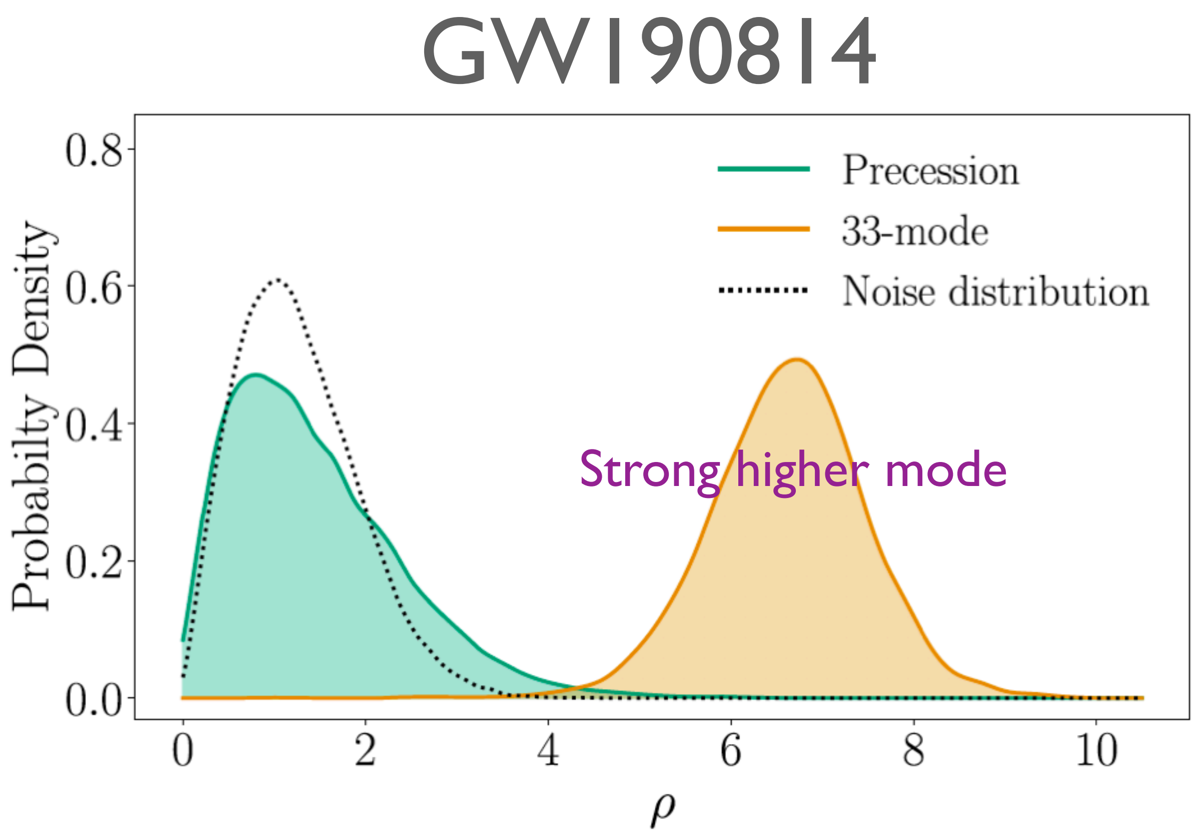
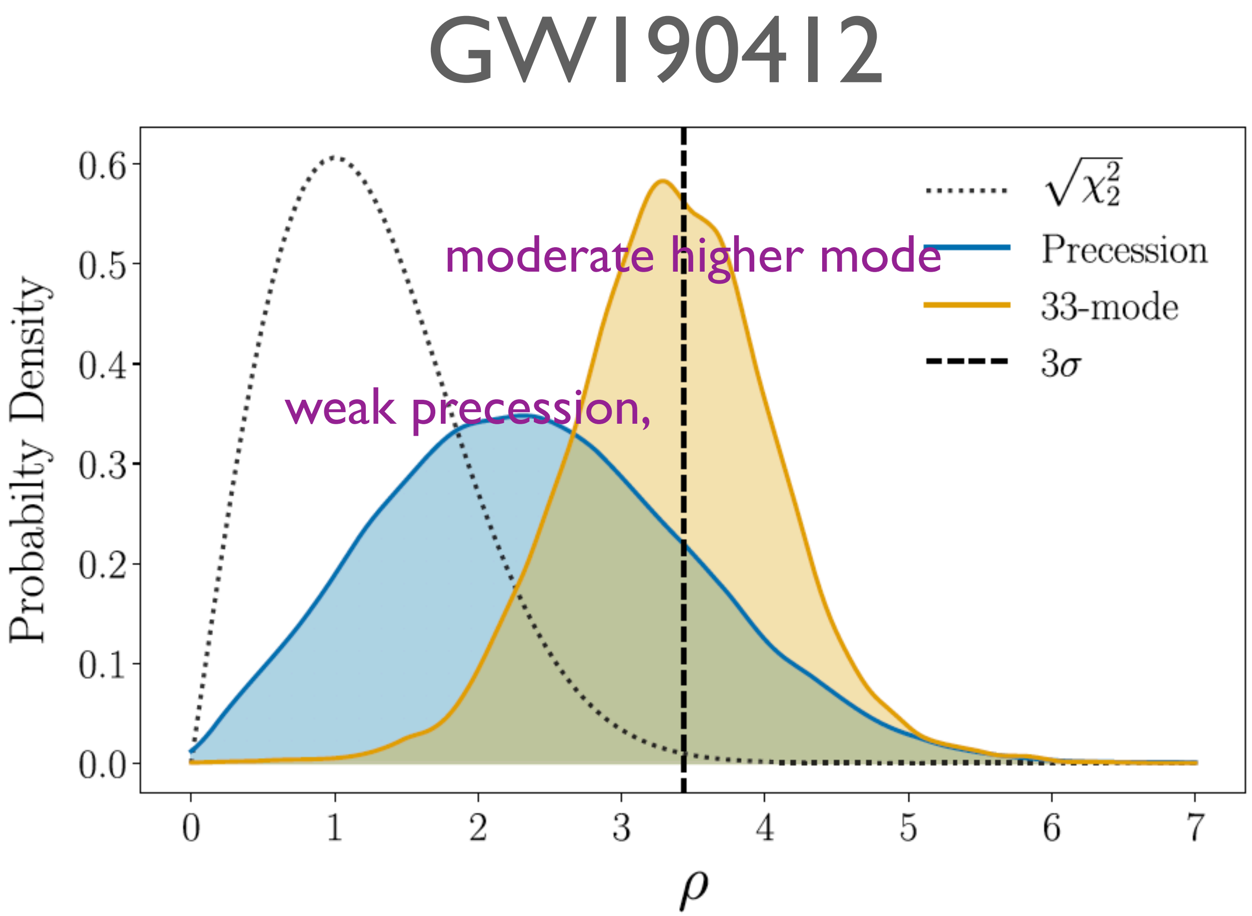



**Figure 5.** Posterior distributions for the precessing S/N, $\rho_p$ (green) and the optimal S/N in the (3, 3) subdominant multipole moment, $\rho$ (orange). The gray dotted line shows the expected distribution for Gaussian noise.

LIGO/Virgo collab, ApjL,896, L44(2020)




LIGO/Virgo collab, PRD, 102, 043015 (2020)

*Bounds on deviation parameters from these are the first ever test of non-quadrupolar radiation.*

# Criticisms

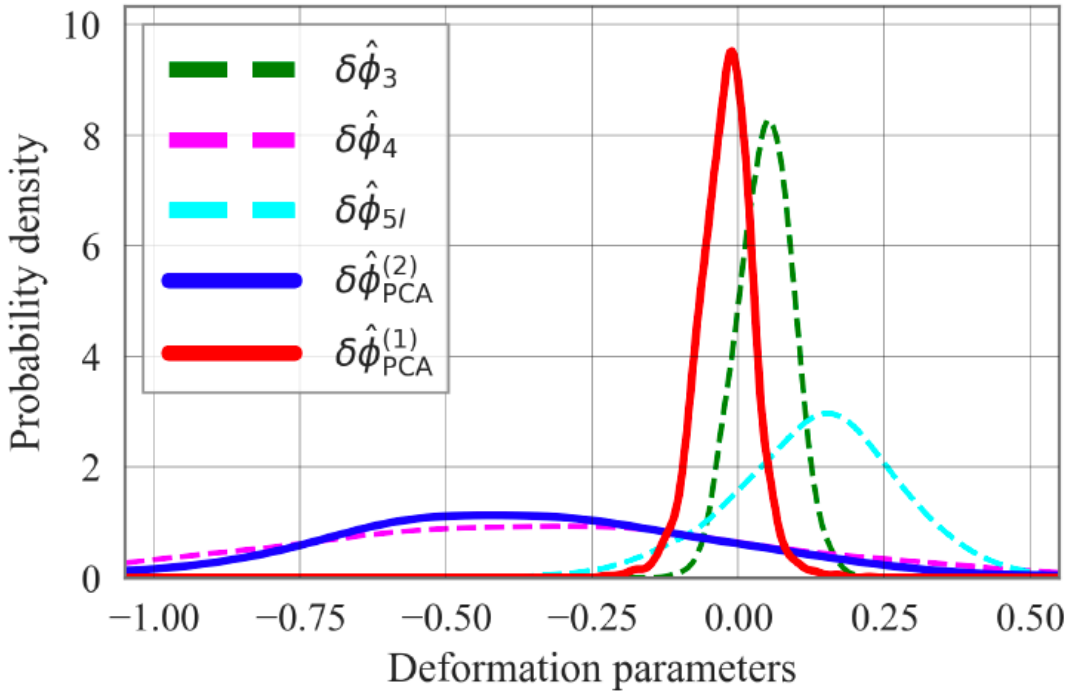
- What are we testing?
  - Physical effects in GR Vs specific modGR predictions?
  - Any theoretical backing for different modifications?
- Meaning of single parameter tests?
  - We test only one parameter at a time!
  - With the current sensitivity multi-parameter tests are uninformative (due to parameter degenarcies).
  - Multi-banding of GW signals may be required to carry out multiparameter tests (Gupta+2020, Datta+2020).

# Use of Principal Component Analysis

## Multiparameter Tests using PCA

## Joint bounds: O1/O2

- Identify the linear combinations of the eight PN deformation parameters that is best measured.




Pai, KGA, 2013, Saleem, Datta, KGA, Sathyaprakash, 2021

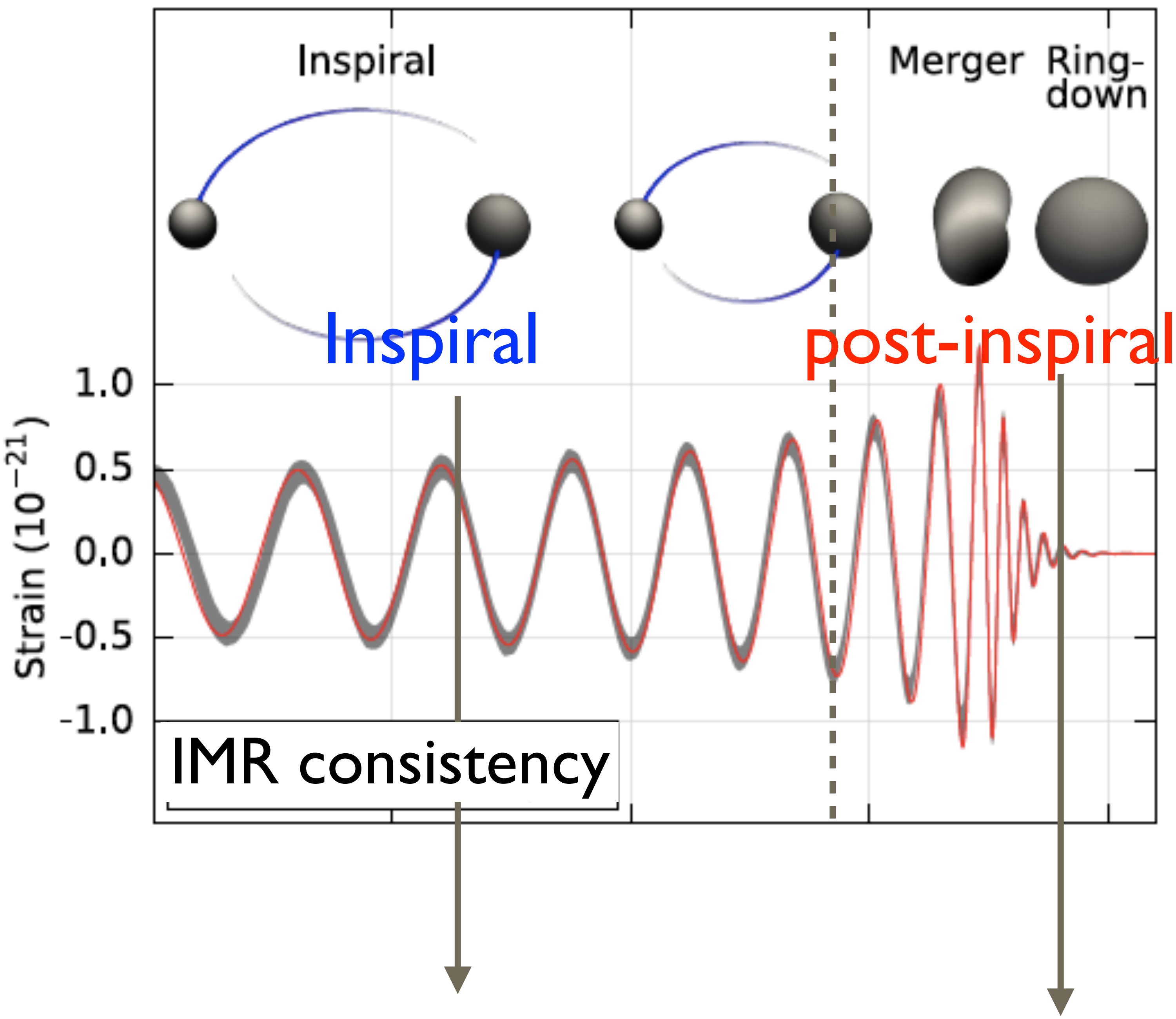
Saleem, Datta, KGA, Sathyaprakash, 2021.

*Can be a promising alternative, needs to explore more.*

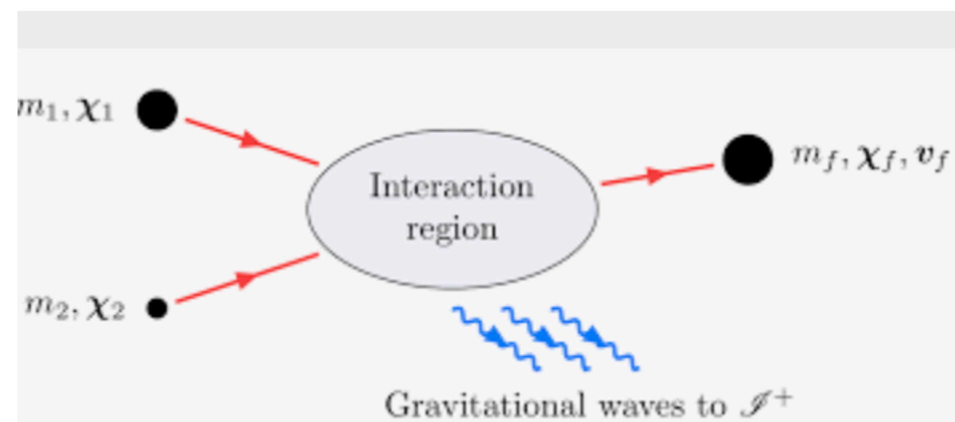
# *Inspiral-Merger-Ringdown Consistency*

# The test

*Consistency between the mass and spin of the remnant BH inferred independently from the inspiral and post-inspiral parts of the waveform.*




Relies on Numerical Relativity
Fitting formulae that connect
the initial and final BH parameters




$$(M_f, \chi_f) \stackrel{?}{=} (M_f, \chi_f)$$

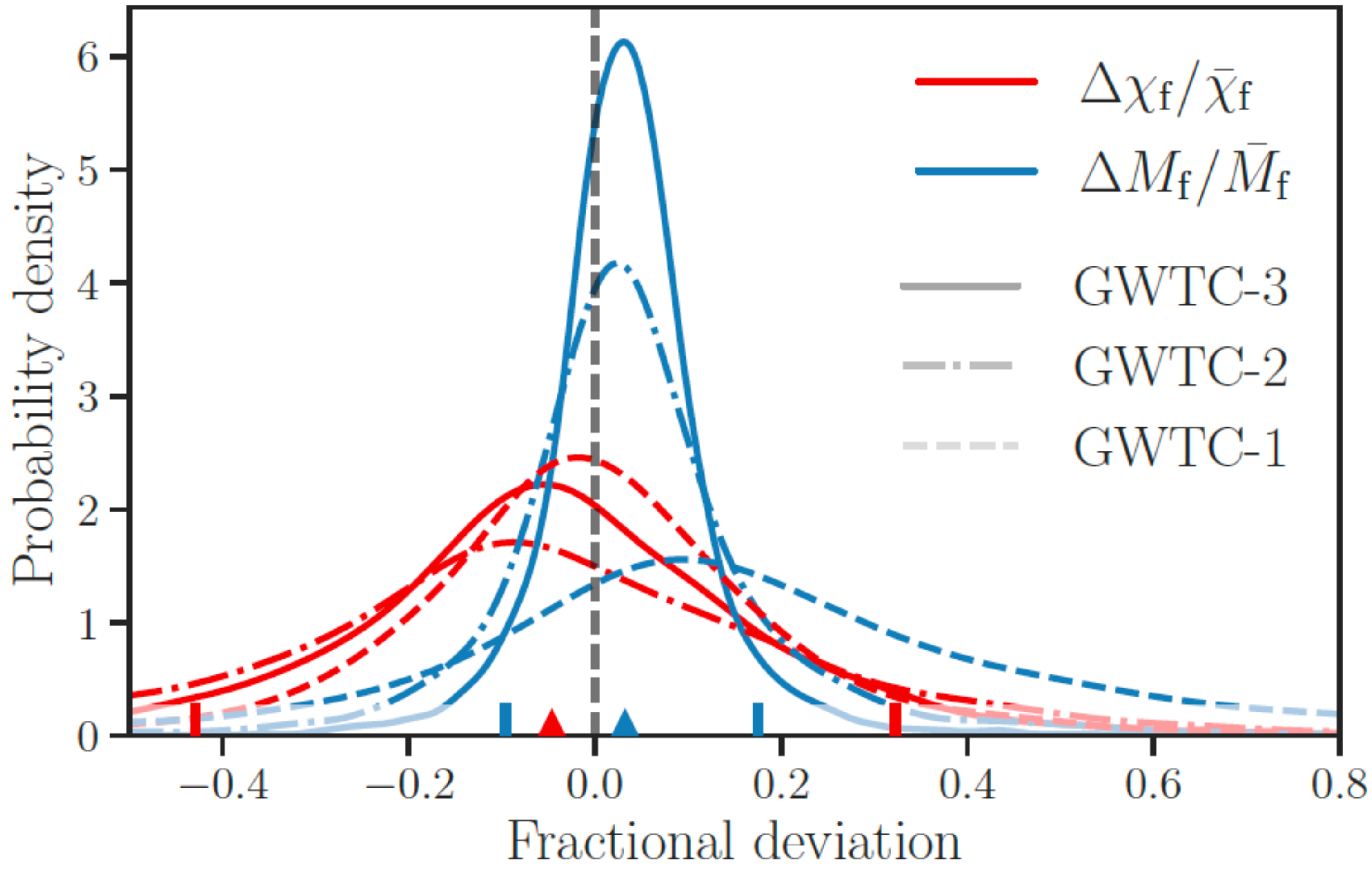
*Consistency between low- and high-frequency parts of the waveform*

Ghosh+ 2016

# What does it test

- Any anomalous loss of energy/angular momentum as the binary transits from inspiral to ringdown through merger.
- Powerful test of the Merger dynamics.

# Current bounds




LIGO-Virgo-KAGRA collab, arXiv:2112.06861

$$\frac{\Delta M_f}{\bar{M}_f} = 2\frac{M_f^{\rm insp} - M_f^{\rm postinsp}}{M_f^{\rm insp} + M_f^{\rm postinsp}}, \quad \frac{\Delta\chi_f}{\bar{\chi}_f} = 2\frac{\chi_f^{\rm insp} - \chi_f^{\rm postinsp}}{\chi_f^{\rm insp} + \chi_f^{\rm postinsp}},$$

# Pros and Cons

- Pros:
  - A powerful probe of the merger dynamics.
  - A smart combination of parametric and consistency tests.
- Cons:
  - Limited applicability as the events require good SNR in both parts of the waveform.
  - Uses NR fits to obtain final mass and spin currently (in future it should be replaced by RD-only estimation.)

# Status

arXiv.org > gr-qc > arXiv:2112.06861

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**General Relativity and Quantum Cosmology**

*[Submitted on 13 Dec 2021]*

## Tests of General Relativity with GWTC-3

The LIGO Scientific Collaboration, the Virgo Collaboration, the KAGRA Collaboration: R. Abbott, H. Abe, F. Acernese, K. Ackley, N. Adhikari, R. X. Adhikari, V. K. Adkins, V. B. Adya, C. Affeldt, D. Agarwal, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, T. Akutsu, P. F. de Alarcón, S. Albanesi, R. A. Alfaidi, A. Allocca, P. A. Altin, A. Amato, C. Anand, S. Anand, A. Ananyeva, S. B. Anderson, W. G. Anderson, M. Ando, T. Andrade, N. Andres, M. Andrés-Carcasona, T. Andrić, S. V. Angelova, S. Ansoldi, J. M. Antelis, S. Antier, T. Apostolatos, E. Z. Appavuravther, S. Appert, S. K. Apple, K. Arai, A. Araya, M. C. Araya, J. S. Areeda, M. Arène, N. Aritomi, N. Arnaud, M. Arogeti, S. M. Aronson, K. G. Arun, H. Asada, Y. Asali, G. Ashton, Y. Aso, M. Assiduo, S. Assis de Souza Melo, S. M. Aston, P. Astone, F. Aubin, K. AultONeal, C. Austin, S. Babak, F. Badaracco, M. K. M. Bader, C. Badger, S. Bae, Y. Bae, A. M. Baer, S. Bagnasco, Y. Bai, J. Baird, R. Bajpai, T. Baka, M. Ball, G. Ballardin, S. W. Ballmer, A. Balsamo, G. Baltus, S. Banagiri, B. Banerjee, D. Bankar, J. C. Barayoga, C. Barbieri, B. C. Barish, D. Barker, P. Barneo, F. Barone, B. Barr, L. Barsotti, M. Barsuglia, D. Barta, J. Bartlett, M. A. Barton, I. Bartos, S. Basak et al. (1582 additional authors not shown)

*No evidence for beyond-GR physics.*

# What does it mean?

- Our current sensitivities are not good enough to detect a GR violation, if present.
- Perhaps, GR violation happens in some extreme region of the parameter space (e.g., strong precession + high eccentricity)
- Maybe both!
- The search should go on.

# Looking forward

# Towards claiming a GR violation

- Needs to have excellent control over the waveform systematics.
  - Missing physical effects, such as eccentricity (Talk by Pankaj Saini).
  - Parameter degeneracies.
- Understanding the detector noise.
  - Nonstationarities and Non-Gaussianities in the data.
- Need to work towards controlling these.

# Improving the null tests

- Accurate GR waveforms with all the physical effects (tides, spin-effects, eccentricity, …) and improved detector characterization.
- Recasting some of the tests based on the developments in modGR theoretical modelling
  - Improve the efficiency of the tests by making use of the results from modGR theories.
- Assessment of the ability of various tests to detect a GR violation. (Johnson-McDaniel+2021)

# Conclusions

- The GR tests using GWs have found no evidence for any beyond-GR physics.
- A suite of tests, not necessarily independent, complement each other in this search.
- Improved tests in the future should capture the essence of analytical/numerical computations from modGR theories.
- Future improvements in waveform modelling and detector noise characterisation would be extremely crucial.