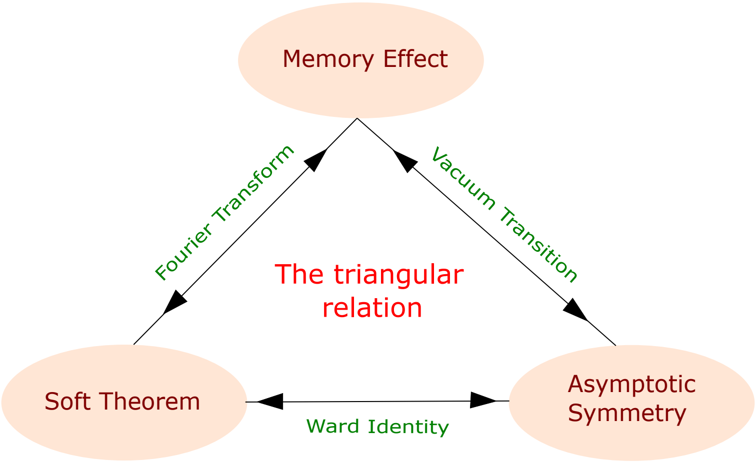
Black Holes with Supertranslation Memories

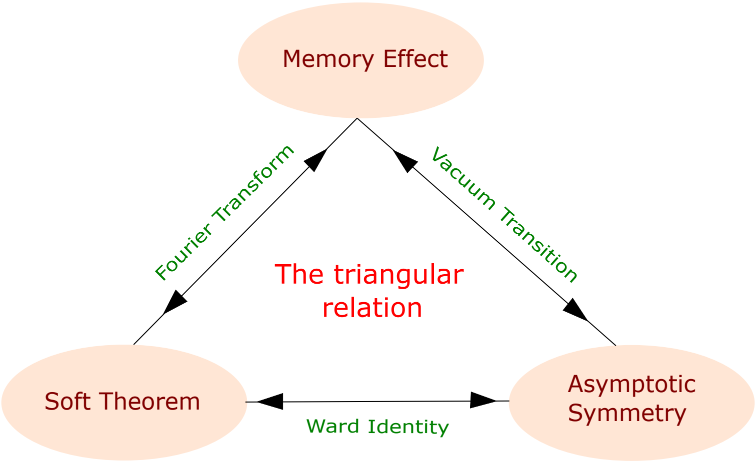
Srijit Bhattacharjee
IIIT Allahabad

2nd Chennai Symposium on Gravitation & Cosmology

February 2, 2022



Infrared Triangle: Strominger et al. (2017)



Infrared Triangle: Strominger et al. (2017)

More on soft theorems in Alok Laddha's talk on Feb 3.

In this talk

- Asymptotic symmetries and Memory effect
- Near horizon memory effect
- Detection of Supertranslated black holes via standard GR tests
- Conclusions

Symmetries of flat spacetime

- In a general spacetime the symmetries are obtained by solving the Killing's equation:

$$\mathcal{L}_X g_{\mu\nu} = 0$$

If the metric is invariant under local diffeos. generated by the vector field X, then the solutions of this equation indicate the symmetries.

- Killing's equation for $g = \eta$:

$$\partial_\mu X_\nu + \partial_\nu X_\mu = 0$$

- Generators: $X^\alpha = \delta^\alpha_\mu a^\mu$, and $X^\mu = \eta^{\mu\nu} b_{\nu\sigma} x^\sigma$.

- **Poincare group:** Translations $\ltimes L$

Asymptotic Symmetries

- A general spacetime (M, g) may or may not possess any symmetries that are available in flat space.

- Schwarzschild spacetime has only two Killing vectors:

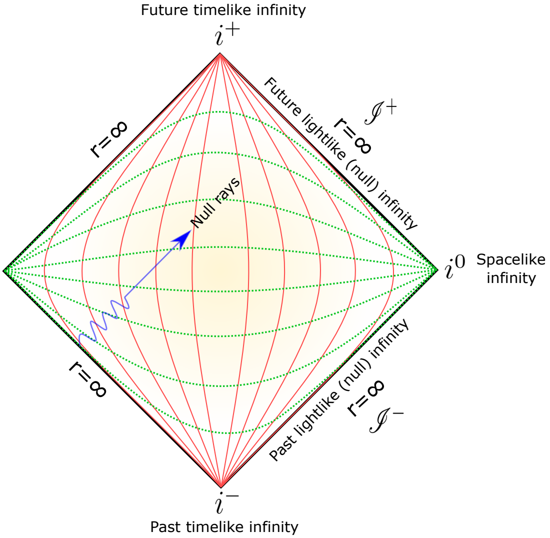
$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

$K_1 = \partial_t$ and $K_2 = \partial_\phi$

- The gravitational field sufficiently far from an isolated source $r \to \infty$ should be weak and the spacetime should look like flat.
  Eg. Schwarzschild, Kerr spacetime look like Minkowski spacetime.

- Asymptotic flatness can be achieved by imposing suitable fall-off conditions on the metric. The structure of metric satisfying AF boundary conditions near the null infinities carried out by Bondi-van der Burg-Matzner-Sachs (1962).

Asymptotic Infinities in 4D



Asymptotic Symmetries

- The symmetries that asymptotically preserve the asymptotic structure of metric in AF spacetimes do not resemble that of a flat spacetime at the *null infinities*.

- Surprisingly, Bondi-van der Burg-Metzner-Sachs (BMS) showed that the symmetries get enhanced. In fact it becomes infinite dimensional.

  [Bondi et al., Sachs (1962)]

- The enhanced symmetry group is called asymptotic symmetry group (ASG) or BMS group. A new set of infinite dimensional symmetries emerged, called as *supertranslations*.

- **BMS$_4$ group:** Supertranslations $\ltimes L$

AF metric in Bondi coordinates

- AF spacetimes near I^+ (Bondi, 1962):

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} +$$
$$\frac{2m_B(u,z,\bar{z})}{r}du^2 + D^z C_{zz}dudz + D^{\bar{z}}C_{\bar{z}\bar{z}}dud\bar{z} + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 +$$
$$\frac{1}{r}\left(4/3(N_z + u\partial_z m_B) - 1/4\partial_z(C_{z\bar{z}}C^{z\bar{z}})\right) + c.c. + \ldots$$

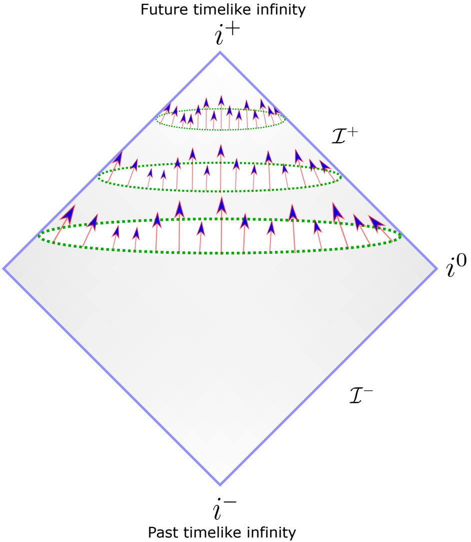
- The fall-off conditions

$$g_{uu} = -1 + \mathcal{O}(1/r) \quad ; g_{ur} = -1 + \mathcal{O}(1/r^2) \;\; ; \;\; g_{uz} = \mathcal{O}(1)$$
$$g_{zz} = \mathcal{O}(r) \;\; ; \;\; g_{z\bar{z}} = r^2\gamma_{z\bar{z}} + \mathcal{O}(1) \;\; ; \;\; g_{rr} = g_{ra} = 0$$

$$u \to u - f(z,\bar{z}) \quad ; r \to r - D^z D_z f$$
$$z \to z + \frac{1}{r}D^z f \;\; ; \;\; \bar{z} \to \bar{z} + \frac{1}{r}D^{\bar{z}} f; \;\; N_{zz} = \partial_u C_{zz} \simeq F_{uz} = \partial_u A_z.$$

- Supertranslations: $\zeta = f(z,\bar{z})\partial_u + \frac{1}{r}(D^z f\partial_z + D^{\bar{z}} f\partial_{\bar{z}}) + D_z D^z f\partial_r + \cdots$
- Supertranslations are *angle dependent* translations, form an infinite dimensional abelian Normal subgroup of BMS group.

Supertranslations



A supertranslation shifts retarded time u individually at each angle on I^+.

Superrotations

Recently, the BMS group has been further enhanced to accommodate another class of infinite symmetries. These act as local diffeomorphisms (globally non-invertible) on the celestial sphere at null infinities. These are known as *superrotations*.

Barnich, Troessaert; PRL, (2010)

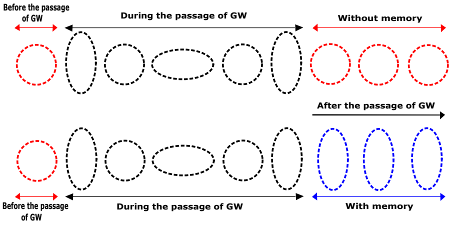
Extended BMS group: ST $\ltimes$ *local* conformal transformations of celestial sphere.

Consequences of Supertranslations

- Supertranslations act non-trivially on physical data. It relates two physically inequivalent solutions.
- If the spacetime initially is in a vacuum $C_{zz} = 0$, then ST will generate a non zero C_{zz} with same energy.
- Memory effect and clock desynchronization upon passage of gravity wave.
- ST charges are conserved. Conservation for every f: $Q_f^+|_{I_-^+} = Q_f^-|_{I_+^-}$
- In the presence of black holes ST charge should contain contribution from the future event horizon also $Q_f^+|_{I_-^+} + Q_f^{\mathcal{H}^+} = Q_f^-|_{I_+^-}$.
- Hawking, Perry Strominger proposal (2016): The horizon supertranslation charges create soft gravitons and black holes carry infinite number of soft hairs. Charge conservation will imply exact correlation between early and late time Hawking quanta. A possible resolution of Information puzzle.

Memory Effect

- **GW-memory** is a non-oscillatory part to GW amplitude generating a permanent displacement for test detectors induced by gravitational waves.
  [Zel'dovich-Polnarev (1974), Braginsky-Thorne (1987), Christodoulou (1991)]



Effect of GW propagating through a ring of test mass

- Memory effect could be detected in near future through aLIGO or LISA detectors. [P D Lasky et al. PRL (2016); Favata CQG (2010); PRD (2011)]

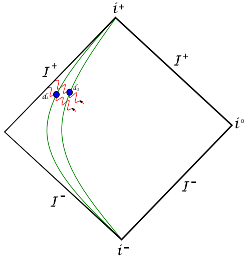
Memory effect and Supertranslation

- Gravitational memory effect estimates the shift between test detectors situated at asymptotic null infinity.

- The detectors positioned near the future null infinity interact with GWs passing through them.

- The interaction between GWs and detector induces a permanent displacement in the setup after the passage of GWs.

- It has been established that there is a direct relation between displacement memory and BMS supertranslations.



- Let us consider the asymptotic form of the Bondi metric:

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + \frac{2m_B}{r}du^2 + D^zC_{zz}dudz + D^{\bar{z}}C_{\bar{z}\bar{z}}dud\bar{z} + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + \ldots$$

- Solving geodesic deviation equation (GDE) gives separation $(s^z, s^{\bar{z}})$ between two nearby geodesics after the passage of GW,

$$r^2 \gamma_{z\bar{z}} \partial_u^2 s^{\bar{z}} = -R_{uzuz} s^z \quad ; \quad R_{uzuz} = -\frac{r}{2} \partial_u^2 C_{zz}$$

- As a result, the separation between two near by geodesics is,

$$\Delta s^{\bar{z}} = \frac{\gamma^{z\bar{z}}}{2r} \Delta C_{zz} s^z$$

- The change in C_{zz} due to burst in the radiation is

$$\Delta C_{zz}(z, \bar{z}) = 2\mu D_z^2 G(z, \bar{z}; z_{rad}, \bar{z}_{rad}) - \frac{\mu}{2\pi} \int d^2 z' \gamma_{z'\bar{z}'} D_z^2 G(z, \bar{z}; z', \bar{z}')$$

- One can determine the supertranslation which would produce the same change in the separation of test detectors. Under a supertranslation parametrized by a function f, the change in the metric is related to

$$\Delta C_{zz} = \mathcal{L}_f C_{zz} = -2D_z^2 f$$

- Comparing the $\Delta C_{zz}(z, \bar{z})$ expression above, one obtains

$$f(z, \bar{z}) = \mu G(z, \bar{z}; z_{rad}, \bar{z}_{rad}) - \frac{\mu}{4\pi} \int d^2 z' \gamma_{z'\bar{z}'} G(z, \bar{z}; z', \bar{z}')$$

[Strominger-Zhiboedov (2016)]

Near horizon Supertranslations

- Horizon serves as a null boundary. Does there a similar structure emerge?
- Similarities between the near horizon region and far region may tempt to formulate a *flat space holography*.
- Motivation also comes from HPS proposal to resolve information loss puzzle.

- Near horizon AS and their memories have been recovered in different setups.

- Carrying out similar asymptotic analysis as done by BMS.

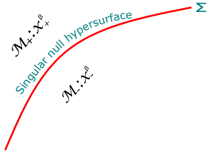
Koga (2001), Donnay et al. (2016)

- Null boundaries are endowed with a universal intrinsic structure $[l, \kappa]$. The symmetries that preserve this structure form the BMS group.

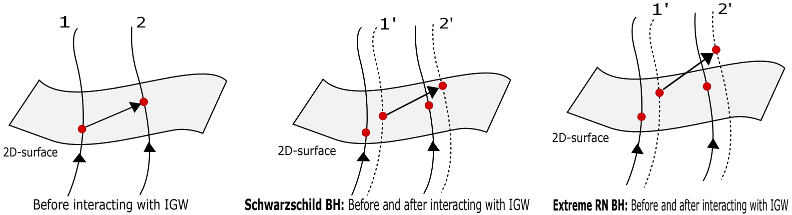
Chandrashekharan et al. JHEP (2018); Ashtekar et al. GRG (2018); Rahman-Wald, PRD (2020),..

- BMS like symmetries are recovered as the soldering freedom when two BH sptms are stitched across a null hypersurface.

Blau et al. JHEP (2016); **SB**, AB, PRD (2018)



Memory due to interaction of IGW & timelike geodesics



Timelike geodesics 1 & 2 get displaced upon interacting with IGW, depicted as 1' & 2' with a new relative deviation vector

- $$X_A = X_{A(0)} + \frac{U}{2}\gamma_{AB}X^B_{(0)} + UdX^-_{A(0)}/dU$$
- $\gamma_{\theta\theta} = 2\psi(V)'\left(F_{\theta\theta} + \frac{F_\theta^2\psi(F)''}{\psi(F)'} - \frac{M}{\psi(F)'} - \frac{M}{\psi(V)'}\right)$
- For null geodesics transversely passing through a horizon shell, the effect of ST is encoded in the jumps of the expansion and shear of the null congruence.

[**SB**, SK, AB, *PRD* (2019); **SB**, SK, *PRD* (2020)]

Near horizon Displacement Memory

- The near-horizon asymptotic form of the 4d metric is

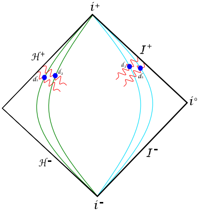
$$ds^2 = g_{vv} dv^2 + 2k dv d\rho + 2 g_{vA} dv dx^A + g_{AB} dx^A dx^B$$

with following fall-off conditions for the horizon $\rho = 0$:

$$g_{vv} = -2\kappa(v, x^A)\rho + \mathcal{O}(\rho^2) \quad ; \quad k = 1 + \mathcal{O}(\rho^2) \quad ; \quad \gamma_{AB} dx^A dx^B = \frac{4}{(1+\zeta\bar{\zeta})^2} d\zeta d\bar{\zeta}$$

$$g_{vA} = \rho\theta_A(v, x^A) + \mathcal{O}(\rho^2) \quad ; \quad g_{AB} = \Omega(v, x^A)\gamma_{AB} + \rho\lambda_{AB}(v, x^A) + \mathcal{O}(\rho^2)$$

[Donnay et al. PRL (2016) ; Donnay et al. JHEP (2016)]



$$\Delta S_E^{\bar{\zeta}} = \frac{\rho}{4}(1+\zeta\bar{\zeta})^2 H(\zeta, \bar{\zeta}) \Delta v S_E^{\bar{\zeta}} + \mathcal{O}(\rho^2); \; g_{va} = 0, \kappa \to 0$$

- It can be shown supertranslation $f = X(\zeta, \bar{\zeta}) = X_1(\zeta) + X_{2.}(\bar{\zeta})$ induces the displacement memory effect. **SB**, SK AB, JHEP (2021)

Standard GR tests to detect ST hair

- Can the supertranslation "hair" be detected through gravitational through light-bending or similar standard GR tests?
- We addressed this question for dynamical black holes.
- A black hole can be implanted linearized ST hair by sending shock waves:

$$\hat{T}_{vv} = \frac{\mu + \hat{T}(\Theta)}{4\pi r^2}\delta(v - v_0)$$

[HPS, (2016)]

- ST Vaidya black hole: $h_{\mu\nu} = \theta(v - v_0)\left(\mathcal{L}_{f=-C} g^V_{\mu\nu} + \frac{2G\mu}{r}\delta^v_\mu \delta^v_\nu\right)$.
- We have provided a first step (locating the photon sphere) to analyse the shadow of a ST black hole in a dynamical phase.

Mishra et al. PRD (2019)

Supertranslated Vaidya Black Hole

$$ds^2 = -g_{vv}dv^2 + 2dvdr + g_{v\theta}dvd\theta + r^2\tilde{g}_{\theta\theta}d\theta^2 + r^2\sin^2\theta\tilde{g}_{\phi\phi}d\phi^2$$

.

Supertranslated Vaidya Black Hole

$$ds^2 = -g_{vv}dv^2 + 2dvdr + g_{v\theta}dvd\theta + r^2\tilde{g}_{\theta\theta}d\theta^2 + r^2\sin^2\theta\tilde{g}_{\phi\phi}d\phi^2 + 2\epsilon\xi(r,v)d\theta d\phi - 2a\psi(r,v)drd\phi - 2a\chi(r,v)dvd\phi.$$

Supertranslated Vaidya Black Hole

$$ds^2 = -g_{vv}dv^2 + 2dvdr + g_{v\theta}dvd\theta + r^2\tilde{g}_{\theta\theta}d\theta^2 + r^2\sin^2\theta\tilde{g}_{\phi\phi}d\phi^2 + 2\epsilon\xi(r,v)d\theta d\phi - 2a\psi(r,v)drd\phi - 2a\chi(r,v)dvd\phi.$$

where

$$g_{vv} = 1 - \frac{2M(v)}{r} - \frac{M(v)}{r^2}f'' - \frac{M(v)\cot\theta}{r^2}f' - \frac{2\dot{M}(v)}{r}f,$$

$$g_{v\theta} = \csc^2\theta f' - 2\left(1 - \frac{2M(v)}{r}\right)f' - \cot\theta f'' - f''',$$

$$g_{\theta\theta} = 1 + \frac{f''}{r} - \frac{\cot\theta}{r}f',\ g_{\phi\phi} = 1 - \frac{f''}{r} + \frac{\cot\theta}{r}f'.$$

$\psi, \xi, \chi \rightarrow$ perturbations that vary smoothly w.r.t v from 0 to 1, and a, ϵ are very small. ST 'f' depends on θ only.

Photon Sphere of a Supertranslated Vaidya Black Hole

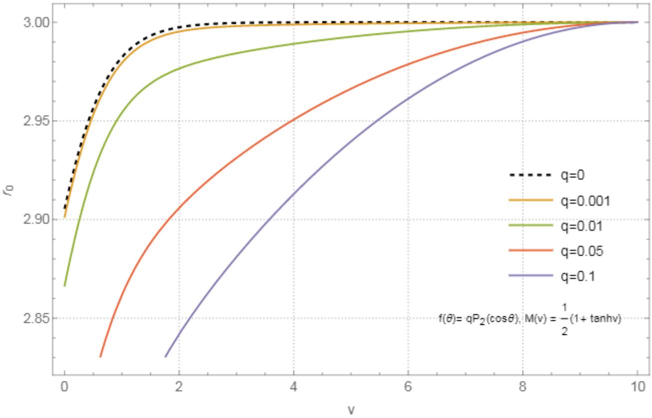
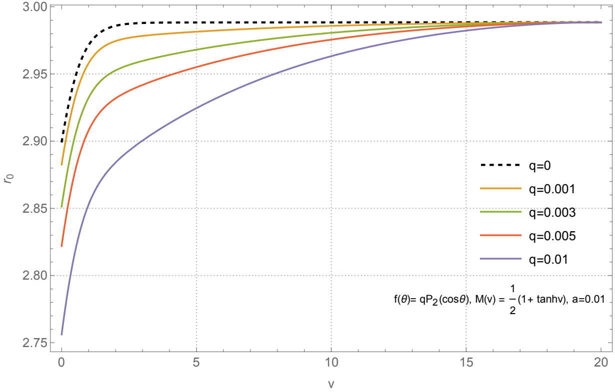


Fig. 6.1 $M(v) = 1/2(1 + \tanh(v)), a = 0, M_0 = 1, v_0 = 10.$

SK, SS, **SB**; arXiv:2110.03547 (2021)

Photon Sphere of a Supertranslated Vaidya Black Hole



$$M(v) = 1/2(1 + \tanh(v)), a = 0.01, M_0 = 1, v_0 = 20$$

SK, SS, **SB**; arXiv:2110.03547 (2021)

Conclusions

- Supertranslations generate memory effect both in the far as well as in the near horizon region of black holes.
- A supertranslated dynamical black hole may be detected through the study of its evolving photon sphere. It can have useful observational implication like determining the shadow of the same black hole.
- Lot of activities are going on to include gravitational memory to waveform catalogs (Boyle, Khera et al.,..). BMS like symmetries in Cosmology (Bonga-Prabhu).
- Spin memory effect is related to superrotations. It has not been addressed in detail [Pasterski et al. (2015), Nichols (2017)].
- There are recent efforts to recover BMS group at spatial infinity by Henneaux et al. (2020), and at time like infinity by Chakraborty et al. (2021) (Debodrina's talk).
- Supertranslations can be detected through gravitational memory effect in upcoming detectors.

Acknowledgements

- **Funding:** Science & Engineering Research Board (GoI) for the project titled *Near horizon structure of black holes.*
- **Image credit:** Shailesh Kumar
- **Collaborators:** Shailesh Kumar, Subhodeep Sarkar, Arpan Bhattacharyya

Thank you !!!