PBH and induced GW from single field inflation and the small scale imprints of reheating

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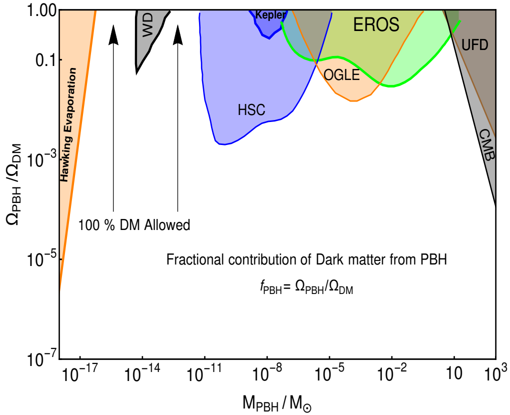
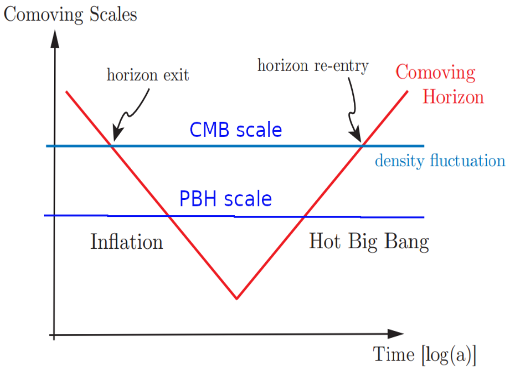
Overview

1. PBH and ISGWB from USR models
2. Effects of reheating
3. Summary

Primordial black holes (PBH) as CDM and its formation from inflation

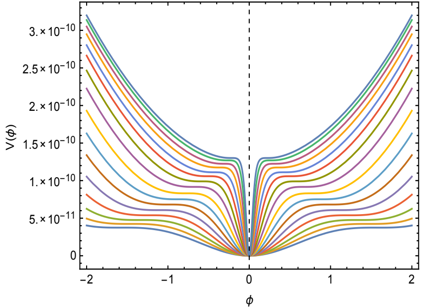


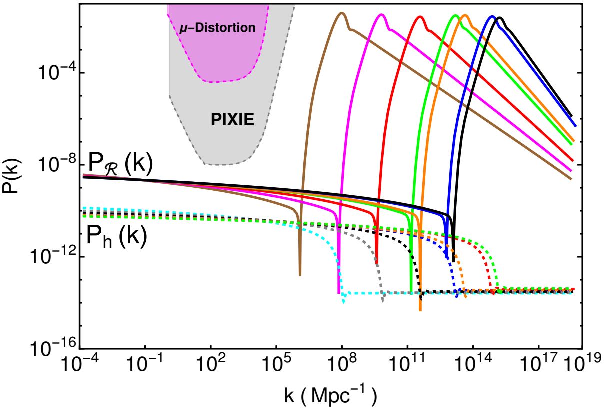


Conditions on curvature perturbation

- $\mathcal{R} \sim 0.1$; $P_{\mathcal{R}} \sim 10^{-2}$ at PBH scale ($\delta_c \sim w$)
- $\mathcal{R} \sim 0.0001$; $P_{\mathcal{R}} \sim 2.25 \times 10^{-9}$ at CMB scale

Inflationary model and scalar power spectra





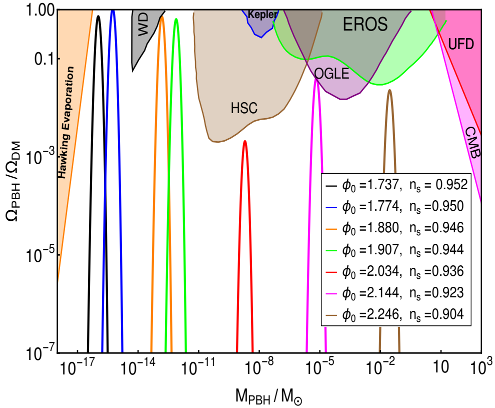
$$V(\phi) = V_0 \frac{a\phi^2 + b\phi^4 + c\phi^6}{(1+d\phi^2)^2}$$

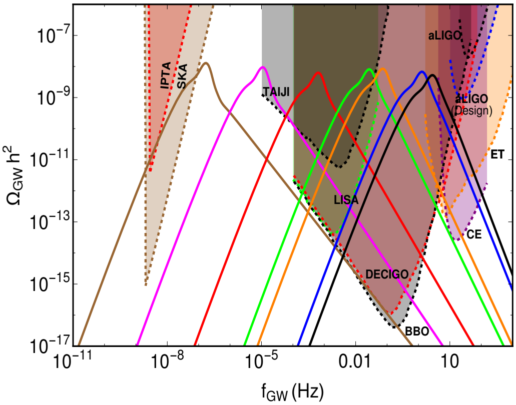
$$V'(\phi_0) = 0 \qquad V''(\phi_0) = 0$$

Mukhanov-Sasaki equations

$$\mathcal{R}_k{}'' + 2\left(\frac{z'}{z}\right)\mathcal{R}_k{}' + k^2\mathcal{R}_k = 0; \quad z = a\dot{\phi}/H = a\phi_N \qquad P_{\mathcal{R}} = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2$$

Primordial black holes and stochastic GW background



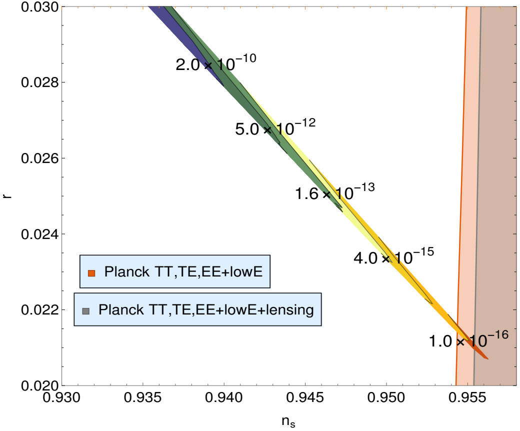


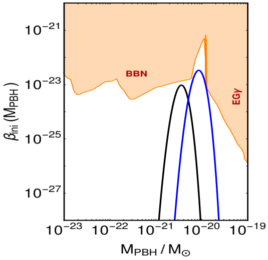
$$h''_{\mathbf{k}}(\tau) + 2\mathcal{H}h'_{\mathbf{k}}(\tau) + k^2 h_{\mathbf{k}}(\tau) = 4S_{\mathbf{k}}(\tau)$$

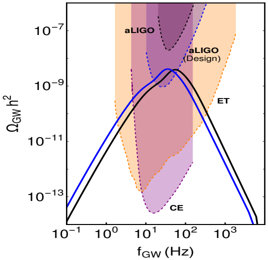
$$\Omega_{\rm GW}(\eta, k) \equiv \frac{1}{\rho_c}\frac{{\rm d}\,\rho_{\rm GW}}{{\rm d}\ln k} = \frac{1}{6}\left(\frac{k}{\mathcal{H}}\right)^2 \int_0^\infty {\rm d}v \int_{|1-v|}^{1+v} {\rm d}u \left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2 \times \overline{I^2_{\rm RD}(v,u,x)}\mathcal{P}_{\mathcal{R}}(kv)\mathcal{P}_{\mathcal{R}}(ku)$$

[Mollerach et al. 2003; Ananda et al. 2006; Baumann et al. 2007

Scalar index and low mass PBHs..



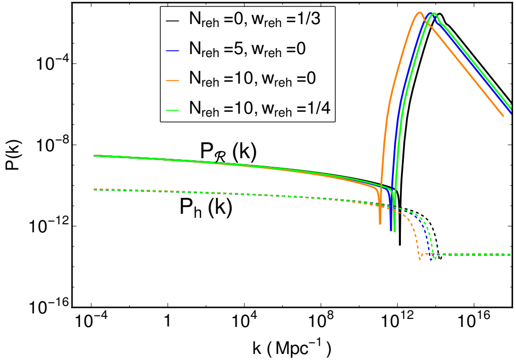


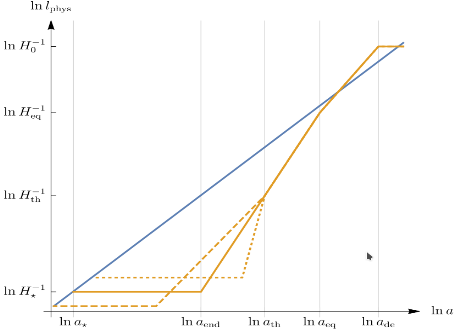


Effects of reheating

We assume reheating phase with a constant equation of state w_{reh}, and duration N_{reh}.

Remapping of scales : $k_e = k_{no-reheating} \times e^{-\frac{1}{4}N_{reh}(1-3w_{reh})}$





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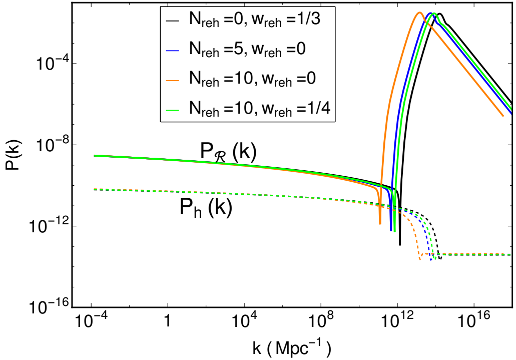
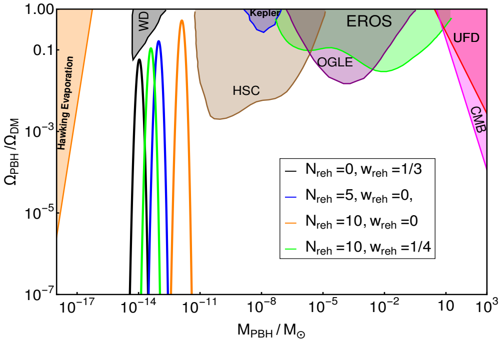




[**NB**, R. K. Jain; JCAP 01(2020), 037]

Effects of an early Matter dominated (eMD) reheating on ISGWB

Dependence on kernel I(u,v,x)

$$\Omega_{\rm GW}(\eta, k) = \frac{1}{6}\left(\frac{k}{\mathcal{H}}\right)^2 \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2 \overline{I^2_{\rm eMD+RD}(v,u,x)}\mathcal{P}_\mathcal{R}(kv)\mathcal{P}_\mathcal{R}(ku)$$

$$I(u,v,x,x_r) \simeq I_{RD}(u,v,x,x_r) = \int_{x_r}^{x} d\bar{x}\frac{a(\bar{x})}{a(x)} f(u,v,\bar{x},x_r) kG(\bar{x},x)$$

[Inomata, Kohri, Nakama, Terada; Phys. Rev. D 100, 043532 (2019)]
[Domènech, Sasaki ; Phys. Rev. D 103, 063531 (2021)]

$$f(u,v,\bar{x},x_r) = \frac{4}{9}\Big[(\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(u\bar{x},ux_r)((\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(v\bar{x},vx_r)$$
$$+\mathcal{T}(v\bar{x},vx_r))+\mathcal{T}(u\bar{x},ux_r)((\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(v\bar{x},vx_r)+3\mathcal{T}(v\bar{x},vx_r))\Big]$$

- $x = \eta k$ $\quad x_r = \eta_r k$ $\quad \frac{a(\eta)}{a(\eta_r)} = 2\frac{\eta}{\eta_r} - 1$ $\quad \mathcal{H} = aH = \frac{1}{\eta-\eta_r/2}$

Scalar transfer function $\mathcal{T}_k(\eta)$ and Kernel I(u,v,x,x_r)

Oscillating terms

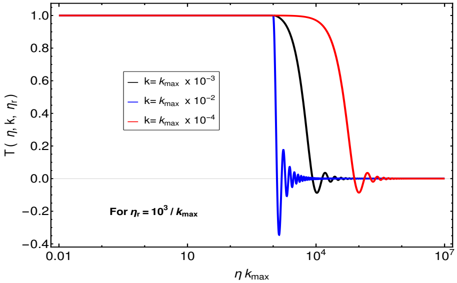
$$\mathcal{I} = I(u, v, x, x_r) \times (x - x_r/2)$$
$$\mathcal{I} = \mathcal{I}_s \sin(x) + \mathcal{I}_c \cos(x) + 4 \text{ other terms}$$

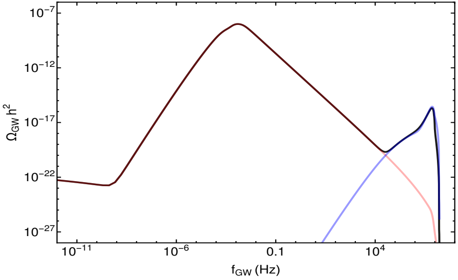
Oscillation avarage

$$\overline{\mathcal{I}^2} = \frac{1}{2}\left(\mathcal{I}_s^2 + \mathcal{I}_c^2\right)$$

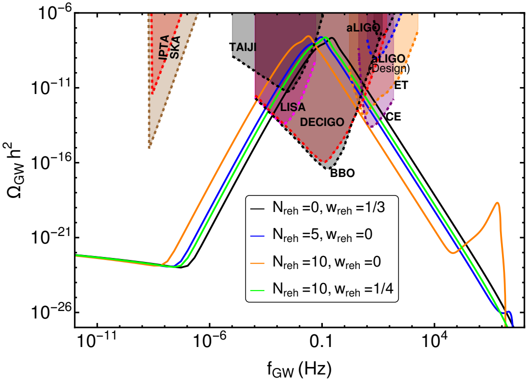
Different k regiemes

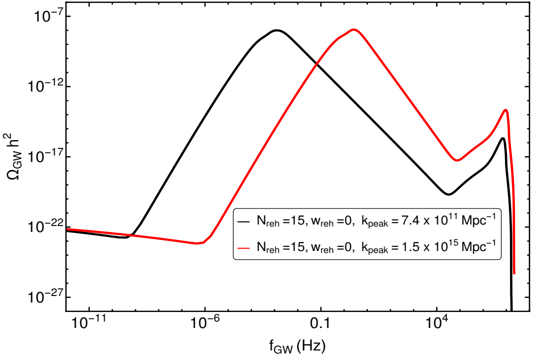
$$\mathcal{I}_s \simeq \mathcal{I}_{ss} + \boxed{\mathcal{I}_{sl} x_r^4} + \text{other terms}$$
$$\mathcal{I}_c \simeq \mathcal{I}_{cs} + \boxed{\mathcal{I}_{cl} x_r^4} + \text{other terms}$$





ISGWB for different reheating histories





[**NB**, R. K. Jain; Phys. Rev. D 104, 023531 (2021)]

Summary of Results

- It is possible to produce PBHs in different mass ranges, and ISGWB in different frequncy bands in these class of models.
- Low mass PBHs are favoured due to n_s tension.
- Even for monochromatic case, a matter dominated reheating phase results in higher PBH mass range with more abundant PBH formation, lower frequency range for primary ISGWB peak.
- A matter dominated reheating phase with a sudden eMD-RD transition can lead to a second ISGWB peak at very high frequency, whose amplitude is determined from reheating history as well as the PBH mass range.

Thank You

Press Schechter Formalism

(Radiation dominated Epoch $\quad w = 1/3 \quad R = 1/k = (aH)^{-1}$)

$$\sigma_\delta^2(R) = \frac{16}{81}\int \frac{dk}{k}(kR)^4 P_{\mathcal{R}}(k) W^2(k,R) \tag{1}$$

$$\beta_f(M) = \frac{1}{2}\mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\,\sigma_\delta(M(R))}\right) \tag{2}$$

$$M(R_f) = 4\pi\gamma M_{\mathrm{Pl}}^2\left(\frac{a_{eq}}{R_{eq}}\right)R_f^2$$

$$\beta_{eq}(M) = \beta_f(M)\left(\frac{a_{eq}}{a_f}\right) = \beta_f(M)\left(\frac{R_{eq}}{R_f}\right)$$

$$f_{PBH} = \frac{\beta_{eq}(M)}{\Omega_{DM}(M)}$$

Induced Stochastic GW Background (ISGWB)

$$h''_{\mathbf{k}}(\tau) + 2\mathcal{H}h'_{\mathbf{k}}(\tau) + k^2 h_{\mathbf{k}}(\tau) = 4S_{\mathbf{k}}(\tau)$$

$$S_k^s = \int \frac{\mathrm{d}^3 q}{(2\pi)^{3/2}} e_{ij}^s(k) q_i q_j \left[2\Phi_q \Phi_{k-q} + \frac{4}{3(1+w)}(\mathcal{H}^{-1}\Phi'_q + \Phi_q)(\mathcal{H}^{-1}\Phi'_{k-q} + \Phi_{k-q})\right]$$

$$\Omega_{\mathrm{GW}}(\tau, k) = \frac{1}{6}\left(\frac{k}{\mathcal{H}}\right)^2 \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{4v^2 - (1+v^2-u^2)^2}{4uv}\right)^2$$

$$\times \overline{I_{\mathrm{RD}}^2(v,u,x)} \mathcal{P}_{\mathcal{R}}(kv) \mathcal{P}_{\mathcal{R}}(ku)$$

$$I_{RD}(u,v,x) = \int_0^x d\bar{x} \frac{a(\bar{x})}{a(x)} f(u,v,\bar{x}) kG(\bar{x},x)$$

Transfer function in RD in presence of eMD

$$\mathcal{T}_k''(\eta) + 4\mathcal{H}\mathcal{T}_k'(\eta) + \frac{k^2}{3}\mathcal{T}_k(\eta) = 0$$

$$\mathcal{T}(x, x_r) = \frac{3\sqrt{3}\left[A(x_r)\, j_1\left(\frac{x - x_r/2}{\sqrt{3}}\right) + B(x_r)\, y_1\left(\frac{x - x_r/2}{\sqrt{3}}\right)\right]}{x - x_r/2}$$

$$A(x_r) = \frac{x_r}{2\sqrt{3}} \sin\left(\frac{x_r}{2\sqrt{3}}\right) - \frac{1}{36}(x_r^2 - 36)\cos\left(\frac{x_r}{2\sqrt{3}}\right)$$

$$B(x_r) = -\frac{1}{36}(x_r^2 - 36)\sin\left(\frac{x_r}{2\sqrt{3}}\right) - \frac{x_r}{2\sqrt{3}}\cos\left(\frac{x_r}{2\sqrt{3}}\right)$$

$x = \eta k$ $\quad x_r = \eta_r k$ $\quad \frac{a(\eta)}{a(\eta_r)} = 2\frac{\eta}{\eta_r} - 1$ $\quad \mathcal{H} = aH = \frac{1}{\eta - \eta_r/2}$ $\quad \mathcal{T}_k(\eta_r) = 1$ $\quad \mathcal{T}_k'(\eta_r) = 0$