Holography from WDW equation

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Summary

In a theory of quantum gravity (in AdS): If two wave functionals coincide near the boundary on an infinitesimal time interval, they coincide everywhere else.

If $\langle \Psi_1 | H^m O_1 \cdots O_p H^n | \Psi_1 \rangle = \langle \Psi_2 | H^m O_1 \cdots O_p H^n | \Psi_2 \rangle \ \forall \ n, m, p \implies |\Psi_1\rangle = |\Psi_2\rangle$.

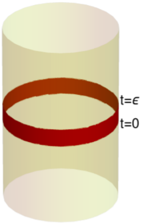
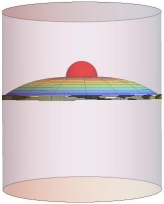


Figure: $H, O_i \in$ Red region

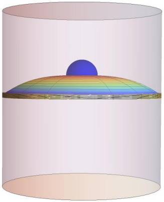
Toy Example

Consider two different excitations

$$|\psi_1\rangle = e^{i\lambda \int \phi_1(r) f(r)} |0\rangle\,, \qquad |\psi_2\rangle = e^{i\lambda \int \phi_2(r) f(r)} |0\rangle\,. \tag{1}$$



(a) Field ϕ_1 & Smearing function $f(r)$



(b) Field ϕ_2 & Smearing function $f(r)$

Measure $\langle\psi_i|HO_i(t)|\psi_i\rangle$ for $i = 1, 2$ where O_i is $\lim_{r\to\infty} r^{\Delta}\phi_i$ with $t \in [0, \epsilon]$

$$\langle\psi_i|HO_i(t',\Omega')|\psi_i\rangle \sim \lambda \int f(r) \frac{r^{d-1}}{\sqrt{1+r^2}} \sin t' \left[\frac{1}{\sqrt{1+r^2}\cos(t') - r\Omega\cdot\Omega'}\right]^{\Delta+1}$$

What one would find is

- $\langle\psi_1|HO_1|\psi_1\rangle$ at $O(\lambda)$ completely recovers $f(r)$.
- Similarly, $\langle\psi_2|HO_2|\psi_2\rangle$ at $O(\lambda)$ completely recovers $f(r)$.
- But $\langle\psi_i|HO_j|\psi_i\rangle = 0$ for $i \neq j$.

Therefore, by measuring such correlators in a thin time band on the boundary, one can extract information about the state.

This is not possible in classical GR.

This toy example shows how information is (de)localized in gravity in a non-trivial manner. We will study this is more detail via the WDW equation.

Why only in Gravity?

1. Consider an LQFT, like a scalar field theory. There is no way to distinguish a state $|\psi\rangle$ from $U_{bulk}|\psi\rangle$ via boundary measurements. Because $[U_{bulk}, O_{bndy}] = 0$

$$\langle\Psi|O_{bndy}|\Psi\rangle = \langle\Psi|U^{\dagger}_{bulk}O_{bndy}U_{bulk}|\Psi\rangle \tag{2}$$

   Such commutators are non-zero in gravity.

2. Similar problem also exists in Gauge theories where U_{bulk} can be constructed out of any gauge invariant operator, eg. $F_{\mu\nu}F^{\mu\nu}$.
   In gravity, there are no local gauge invariant operators.

3. It is possible to construct *split states* in theories other than gravity.

4. Note that we allow ourselves to measure Energy and also the correlation functions involving Energy.
   This is because the bulk state need not just be an energy eigenstate.

WDW

In a theory of gravity, Energy can be measured from the boundary.
Performing an ADM decomposition

$$H = \int_{\Sigma} \left(N\mathcal{H} + N^i \mathcal{H}_i \right) + H_\partial \tag{3}$$

where

$$\mathcal{H} = 2\kappa^2 g^{-1} \left(g_{ik} g_{jl} \pi^{kl} \pi^{ij} - \frac{1}{d-1} (g_{ij} \pi^{ij})^2 \right) - \frac{1}{2\kappa^2} (R - 2\Lambda) + \mathcal{H}^{matter}$$

$$\mathcal{H}_i = -2 g_{ij} D_k \frac{\pi^{jk}}{\sqrt{g}} + \mathcal{H}_i^{matter}$$

The primary constraints further impose secondary constraints

$$\mathcal{H} = 0, \qquad \mathcal{H}_i = 0. \tag{4}$$

Therefore the Hamiltonian itself is a boundary term H_∂.

WDW

1. Quantum mechanically the allowed wave functionals satisfy

$$\begin{aligned} \mathcal{H}\Psi[g,\phi] &= 0, \qquad \text{(WDW eq)} \\ \mathcal{H}_i\Psi[g,\phi] &= 0. \end{aligned} \tag{5}$$

   These are analogous to a wave functional satisfying the Gauss law in EM

$$\left(\nabla.\Pi^{em} - e\rho\right)\Psi[A,\phi] = 0.$$

   and they describe the gauge invariance of the wave function. We shall refer to them as the *point wise constraints.*

2. Equations (5) are very hard to solve in general and therefore we will study them perturbatively in $\kappa = \sqrt{8\pi G}$.

Perturbative WDW

1. We expand the metric perturbatively as [Kuchar]

$$g_{ij} = \gamma_{ij}^{AdS} + \kappa h_{ij} \tag{6}$$

and similarly expand the constraints.

2. It is useful to decompose h_{ij} in terms of the Longitudinal and Transverse modes

$$h_{ij} = h_{ij}^{L} + h_{ij}^{T} + h_{ij}^{TT} \tag{7}$$

where $h_{ij}^{L} = \nabla_i \epsilon_j + \nabla_j \epsilon_i$ and the others satisfy [ADM]

$$\nabla^i h_{ij}^{T} = \nabla^i h_{ij}^{TT} = 0, \qquad h_{ij}^{TT} \gamma^{ij} = 0.$$

1. The point wise constraints, even in perturbation theory are hard to solve but the integrated constraints are much easier.
   An example of an integrated constraint: The Gauss law in EM

$$\int_{\partial\Sigma} d^{d-1}\Omega n^i \; \Pi_i^{em} \; \Psi[\vec{A},\phi] = e \int_{\Sigma} d^d x \; \rho\Psi[\vec{A},\phi].$$

2. In Gravity the integrated Hamiltonian constraint takes the form

$$H_{\partial}\psi_I = \int_{\Sigma} d^d x N\gamma \; \mathcal{H}_{bulk}\psi_I \tag{8}$$

where

$$H_{\partial} = \frac{1}{2\kappa}\int_{\partial\Sigma} d^{d-1}\Omega n_i \Big[N\nabla^j(h_{ij} - h\gamma_{ij}) - \nabla^j N(h_{ij} - h\gamma_{ij})\Big]$$

where h depends on h^T only and

$$\mathcal{H}_{bulk} = 2\Pi^2_{TT} - \frac{1}{8}h^{TT,ij}(\Delta_N + 2)h^{TT}_{ij} + \mathcal{H}^{matter}$$

The solutions of integrated constraints ψ_I depend on h^{TT}, ϕ (scalar matter) and H_∂.

Integrated constraints can be solved by diagonalizing the perturbation Ham ($\mathcal{H}_{bulk}$)

$$\psi_I^{E,\{a\}}[h^{TT}, \phi, H_\partial] = \psi_F^{E,\{a\}}[h^{TT}, \phi] \otimes |H_\partial = E\rangle \tag{9}$$

where $\psi_F^{E,\{a\}}[h^{TT}, \phi] = \psi_g[h^{TT}]\psi_m[\phi]$ are Fock space wave functionals with discrete E.

Their ground state wave functionals are Gaussian [Hartle, Kuchar]

$$\psi_0[\phi] = \exp\Big(- \frac{1}{2} \int d^d x \sqrt{\gamma}\ \phi \sqrt{-(\Delta_N - m^2)\phi}\Big),$$
$$\psi_0[h^{TT}] = \exp\Big(- \frac{1}{8} \int d^d x \sqrt{\gamma}\ h^{TTij} \sqrt{-(\Delta_N + 2)} h_{ij}^{TT}\Big).$$

1. The constraints show that the dynamical degree of freedom in the wave functional are correlated to the value of H_∂.

2. One can explicitly solve the point wise constraints till $O(\kappa)$

$$\Psi = e^{i\kappa S}\psi_F^{E,\{a\}} + O(\kappa^2) \tag{10}$$

3. We work with solutions within perturbation theory (till $O(\kappa)$).

Holography

For any density matrix

$$\rho(h',\phi',h,\phi) = \sum_{E,E',\{a\},\{a'\}} c(E,E',\{a\},\{a'\})\Psi^{E',\{a'\}}[h',\phi']^*\Psi^{E,\{a\}}[h,\phi] \qquad (11)$$

Imposing

$$\langle H_\partial^m O_1 \cdots O_p H_\partial^n \rangle_{\rho_1} = \langle H_\partial^m O_1 \cdots O_p H_\partial^n \rangle_{\rho_2} \qquad \text{for arbitrary } m, n, p$$

with $O_i = O(t_i, \Omega_i)$ for $t_i \in [0, \epsilon]$ implies

$$c_1(E,E',\{a\},\{a'\}) = c_2(E,E',\{a\},\{a'\}) \qquad (12)$$

The proof is sketched in the next slide.

The proof is as follows. Define $diff \equiv \langle \cdots \rangle_{\rho_1} - \langle \cdots \rangle_{\rho_2}$ and obtain

$$diff = \sum_{E,E',\{a\},\{a'\}} (c_1 - c_2) E^m E'^n \langle O(t_1,\Omega_1) \cdots O(t_q,\Omega_q) \rangle_\rho = 0 \qquad (13)$$

where,

$$\langle O(t_1,\Omega_1) \cdots O(t_p,\Omega_p) \rangle_\rho$$
$$= e^{iEt_1 - iE' t_q} \sum_{E_j,a_j} e^{iE_1 u_2 + iE_2 u_3 + \cdots + iE_{q-1} u_q} \langle E, a | O(0) | E_1, a_1 \rangle \cdots \langle E_{p-1}, a_{p-1} | O(0) | E', a' \rangle$$

where $u_i = t_i - t_{i-1}$.

Such correlators can be argued to be non-zero by using the edge of the wedge theorem and therefore we must have $c_1 = c_2$.

What we have shown is: If two wave functionals solving the WDW equation in AdS, coincide near the boundary on an infinitesimal time interval, they coincide everywhere else.

Natural extensions:

1. AdS_2 and AdS_3?
2. Black Holes?
3. Flat Space Holography?
4. De Sitter space?
5. Entanglement Wedge?

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