



# Gravitational waves from primordial fluctuations

Guillem Domènech (INFN Padova)

Chennai Symposium on Gravitation and Cosmology, February 3rd, 2022

[Check my review paper: 2109.01398!](#)

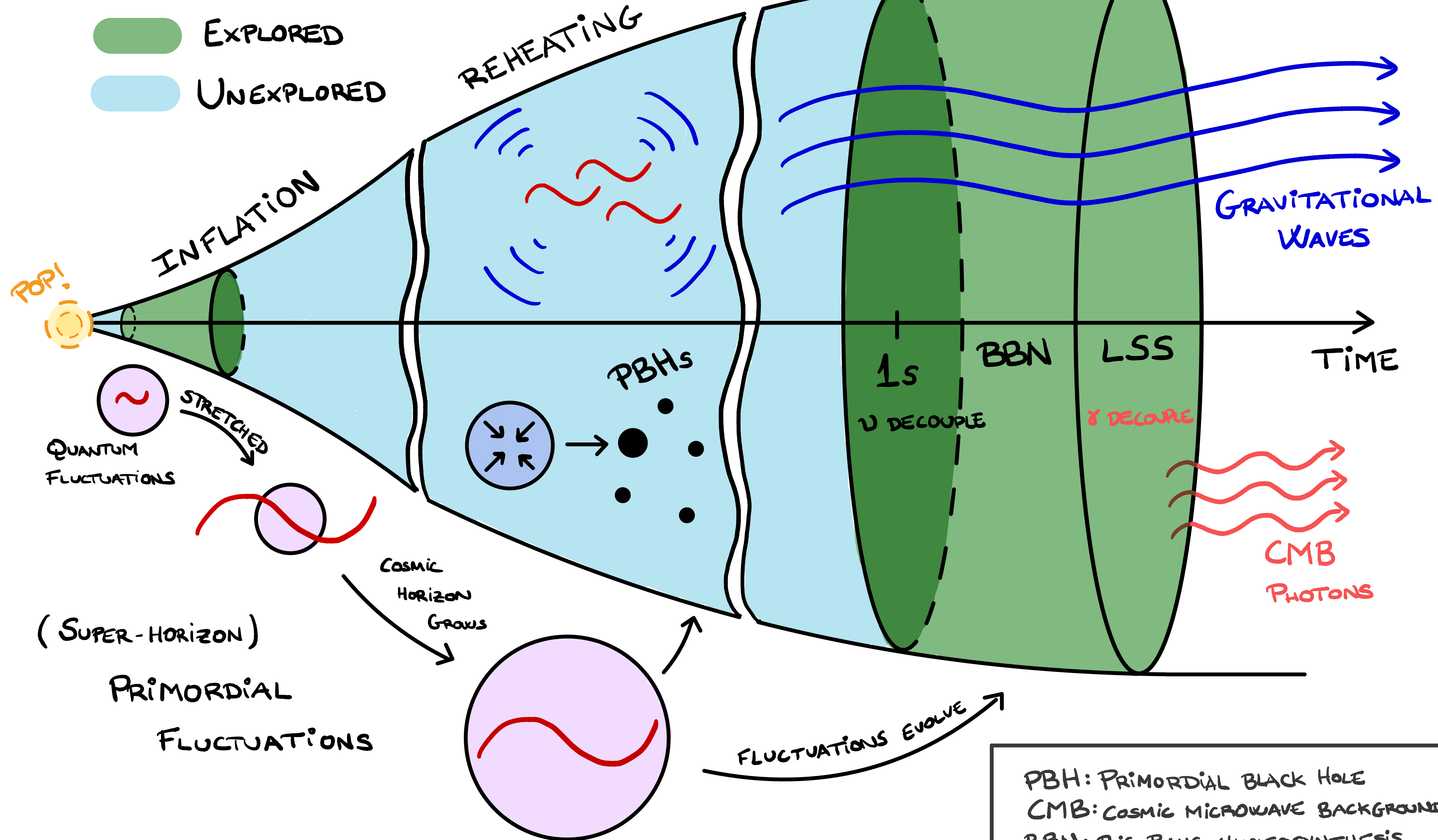
[domenech@pd.infn.it](mailto:domenech@pd.infn.it)

<https://domenechcosmo.netlify.app>



# THE EARLY UNIVERSE

by GUILLEM DOMÈNECH



PBH: PRIMORDIAL BLACK HOLE  
CMB: COSMIC MICROWAVE BACKGROUND  
BBN: BIG BANG NUCLEOSYNTHESIS  
LSS: LARGE SCATTERING SURFACE

# Cosmological sources of GWs

## New physics during inflation:

### Quantum fluctuations:

- Primordial tensor fluctuations
- GWs induced by primordial scalar fluctuations (“guaranteed” signal from the CMB)

### Additional sources:

- SU(2) gauge fields
- Axion fields

## Also... New physics after inflation (but before BBN):

### Phase transitions:

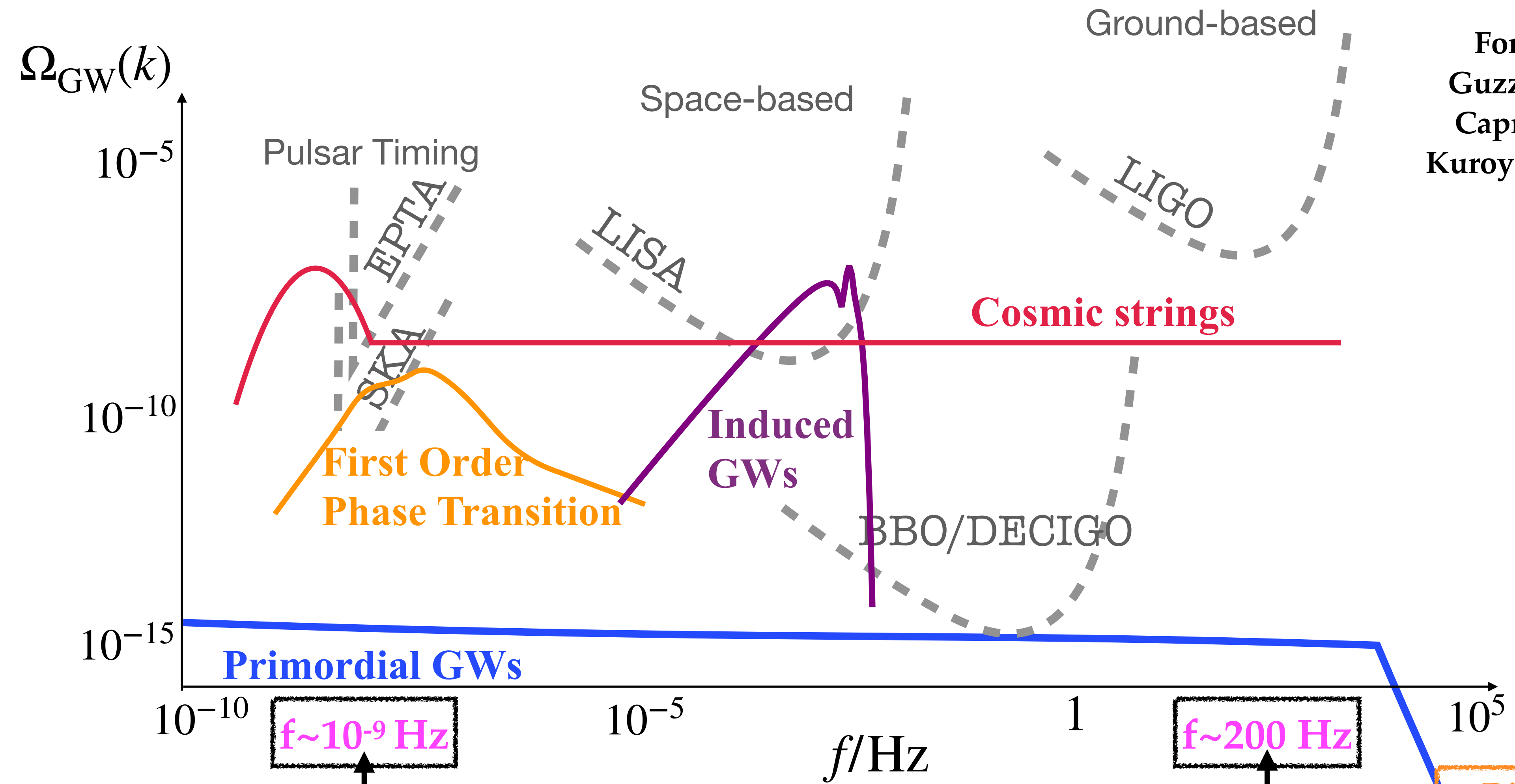
- Strong first order phase transitions
- Topological defects like cosmic strings

### Preheating and reheating:

- Parametric resonances

For more see e.g.  
Guzzetti+1605.01615  
Caprini+1801.04268  
Kuroyanagi+1807.00786

# Cosmological Stochastic GW background



For more see e.g.  
 Guzzetti+1605.01615  
 Caprini+1801.04268  
 Kuroyanagi+1807.00786

$f \sim 10^{-9}$  Hz  
 $\leftrightarrow$   
 $T \sim 100$  MeV

$f \sim 200$  Hz  
 $\leftrightarrow$   
 $T \sim 10^{10}$  GeV

**Disclaimer:** all these spectra are sensitive to the expansion history of the early universe

[Figure by Shi Pi]

# Overview

- Gravitational Waves induced by primordial fluctuations  
(Review of induced GWs in “standard” (radiation dominated) cosmology)
- Induced GWs and early universe’s expansion histories  
(Go beyond “standard” cosmology with general expansion histories)

**Talk based on:**

[GD, 1912.05583]

[GD, S.Pi, M.Sasaki, 2005.12314]

[GD and S.Pi, 2010.03976]

[GD and C.Lin, M.Sasaki, 2012.08151]

[V.Atal, GD: 2103.01056]

[GD and V.Takhistov, M.Sasaki, 2105.06816]

[GD, 2109.01398]

[L.Witkowski, GD, J.Fumagalli, S.Renaux-Petel, 2110.09480]

[GD, S.Passaglia, S.Renaux-Petel, 2112.10163]

- Induced GWs and early isocurvature fluctuations  
(Go beyond “standard” cosmology with initial conditions)

**GWs induced by  
primordial fluctuations**

# Non-Linear Theory of Gravitational Instability in the Expanding Universe

Kenji TOMITA

*Research Institute for Theoretical Physics  
Hiroshima University, Takehara, Hiroshima-ken*

(Received January 5, 1967)

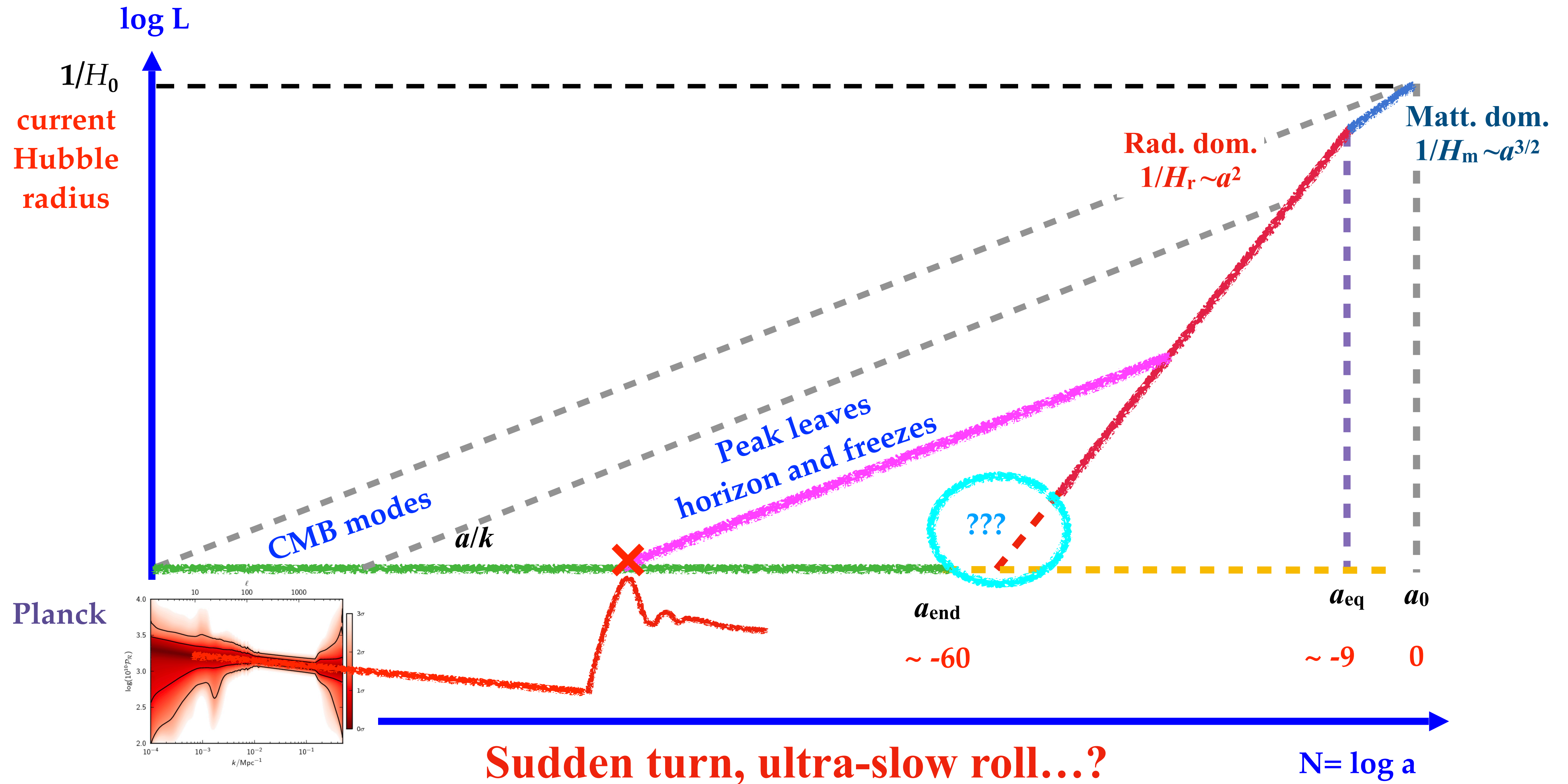
The gravitational instability in the expanding universe is studied in the second-order approximation. This work is an extension of Lifshitz's linearized theory on the basis of general relativity. Basic equations are formulated generally, but their analysis is confined to a special case where pressure effects are negligible and the spatial curvature of the unperturbed model universe is zero. The results show that the second-order density contrast tends to accentuate the increase of the first-order density contrast with time, unless the linear dimension of the perturbation is too great. Moreover it is shown that gravitational wave is induced by deformed density perturbations even if the first-order metric perturbation includes no part of gravitational wave. If time is reversed, our results will be applicable to the problem of the gravitational instability in the contracting universe or in the collapsing star.

# Induced GWs history

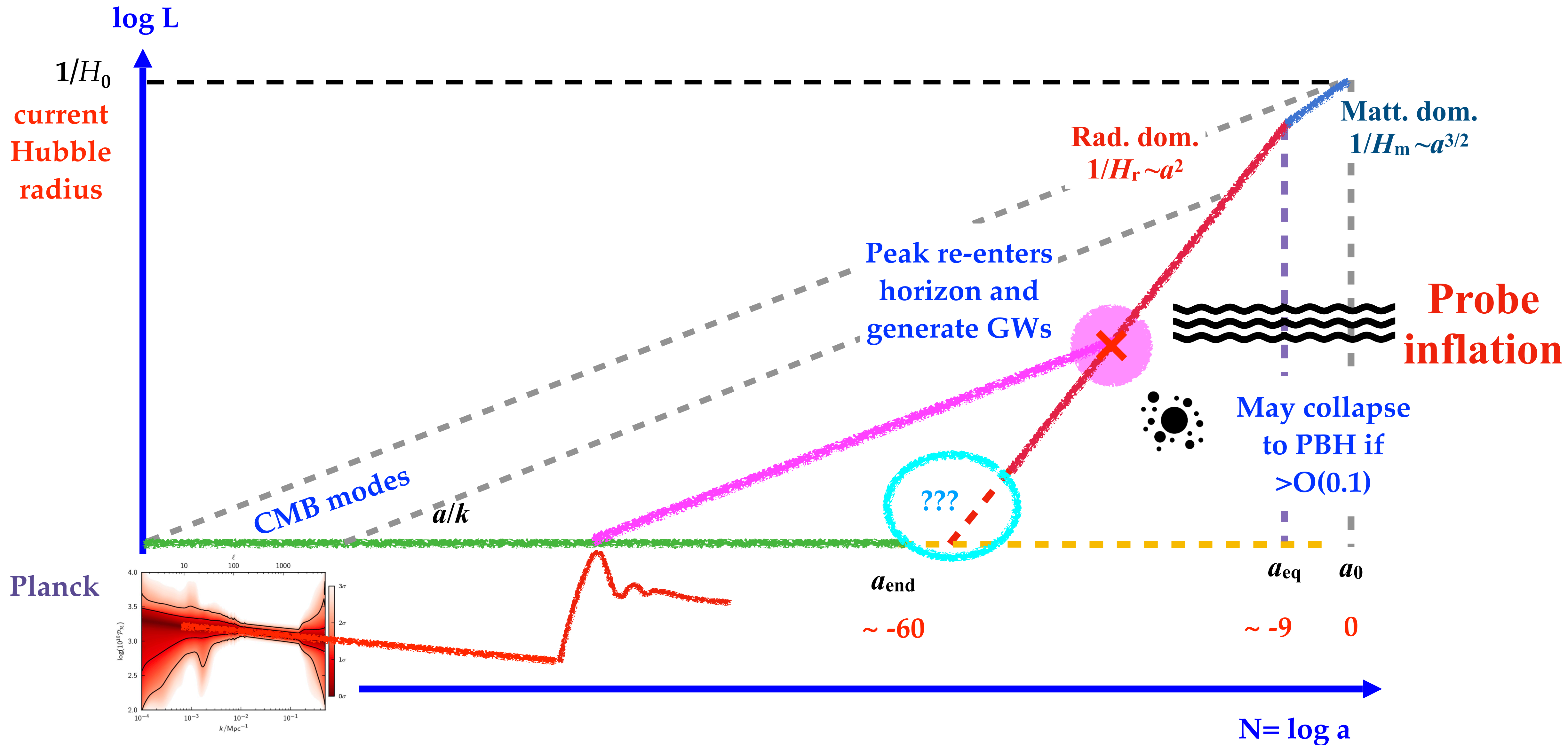
- First pointed out by K. Tomita in 1971 [*Prog. Theor. Phys.* 45, 1747 (1971)]
- Followed by Matarrese, Pantano, Saez in 1993 [*Phys.Rev.Lett.* 72 (1994) 320-323]
- Also Matarrese, Mollerach, Bruni in 1997 [*Phys.Rev.D* 58 (1998) 043504]
- Then Ananda, Clarkson and Wands in 2006 [gr-qc/0612013]
- And Baumann, Ichiki, Steinhardt and Takahashi in 2007 [hep-th/0703290]
- Saito and Yokoyama in 2008: **induced GWs  $\Leftrightarrow$  PBHs!** [0812.4339]
- ...After the first LIGO detection the publication number keeps growing!



# Probing last stages of inflation

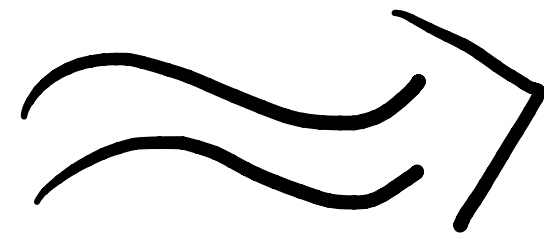
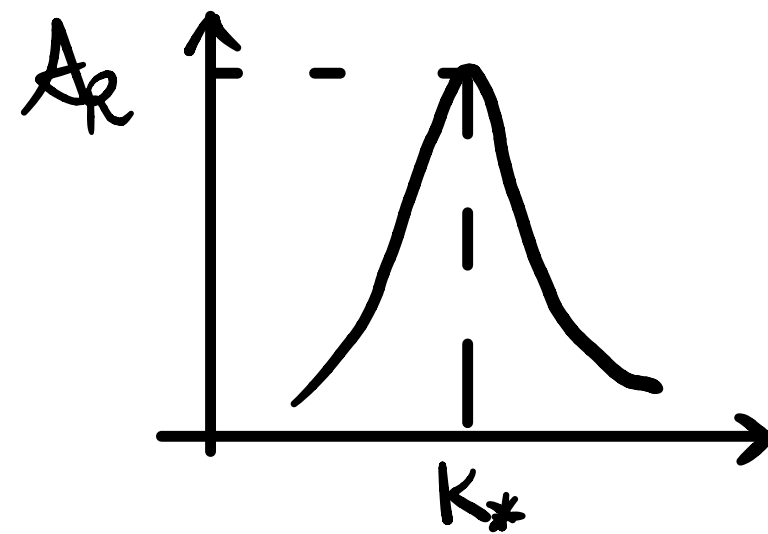


# Probing last stages of inflation

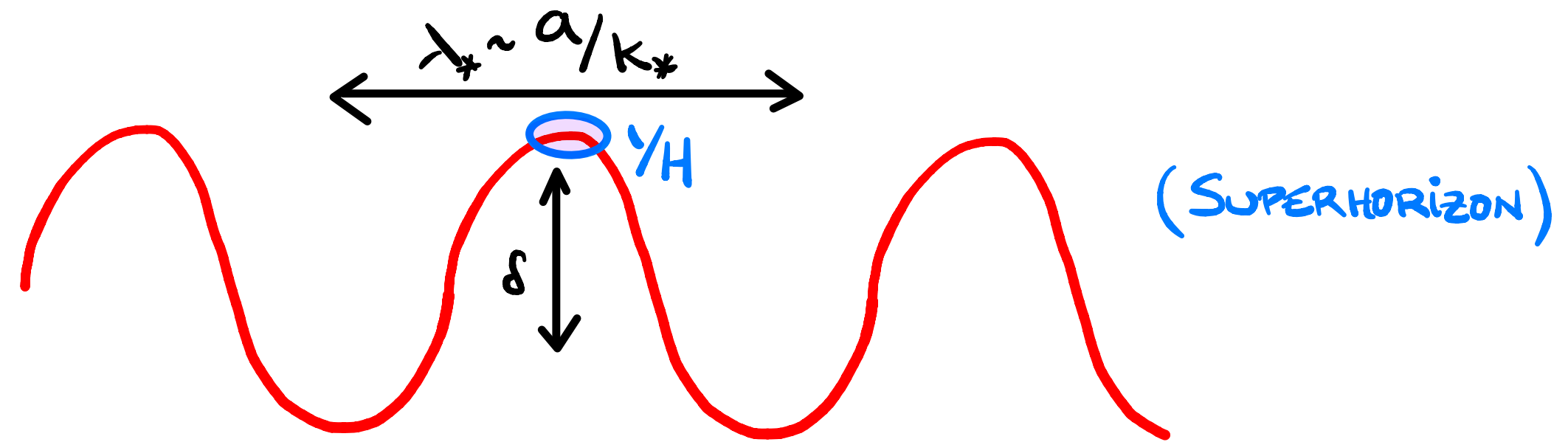


# GENERATION OF GWs AND PBHs

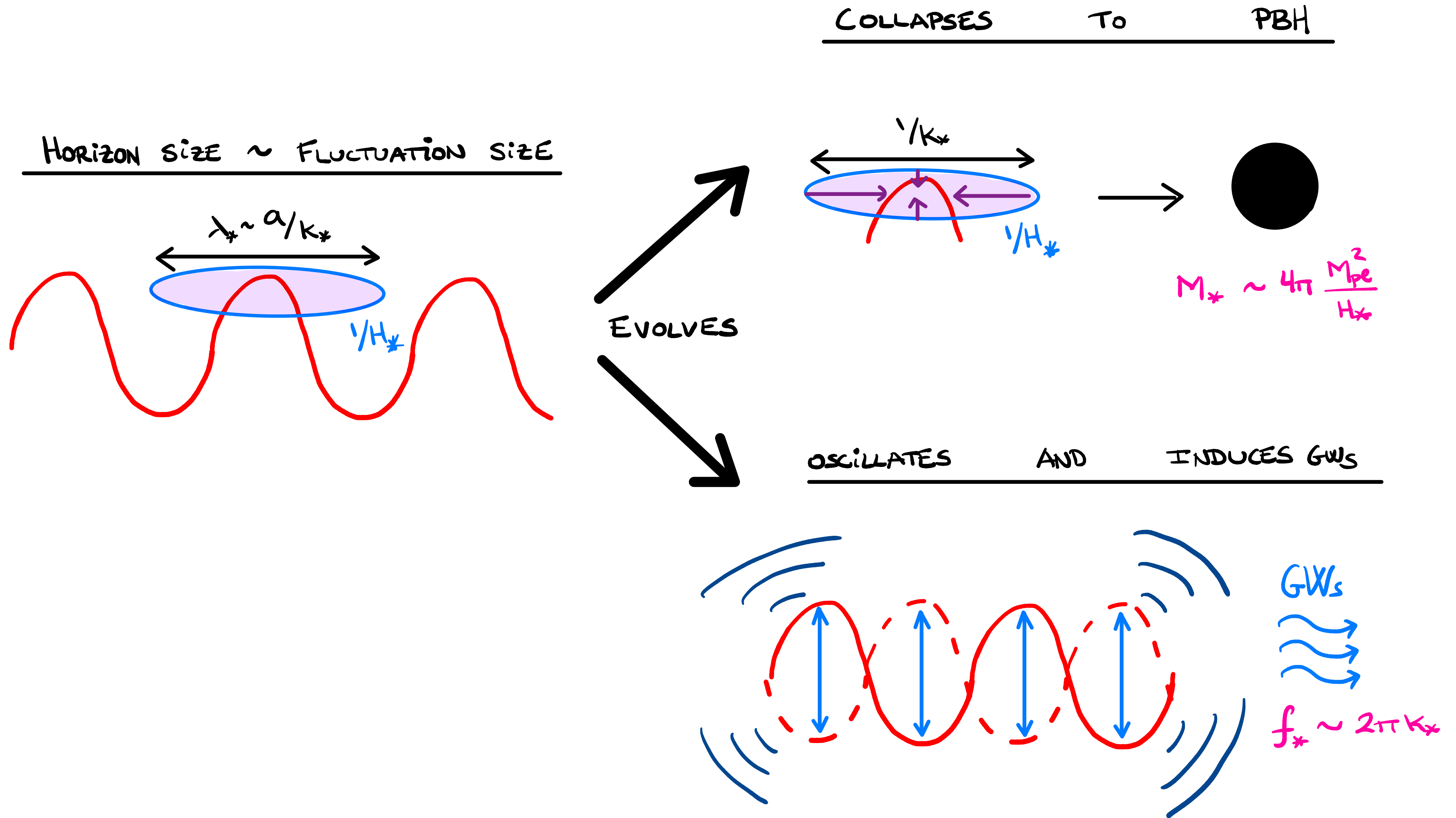
(SHARP) PEAK  
IN POWER SPECTRUM



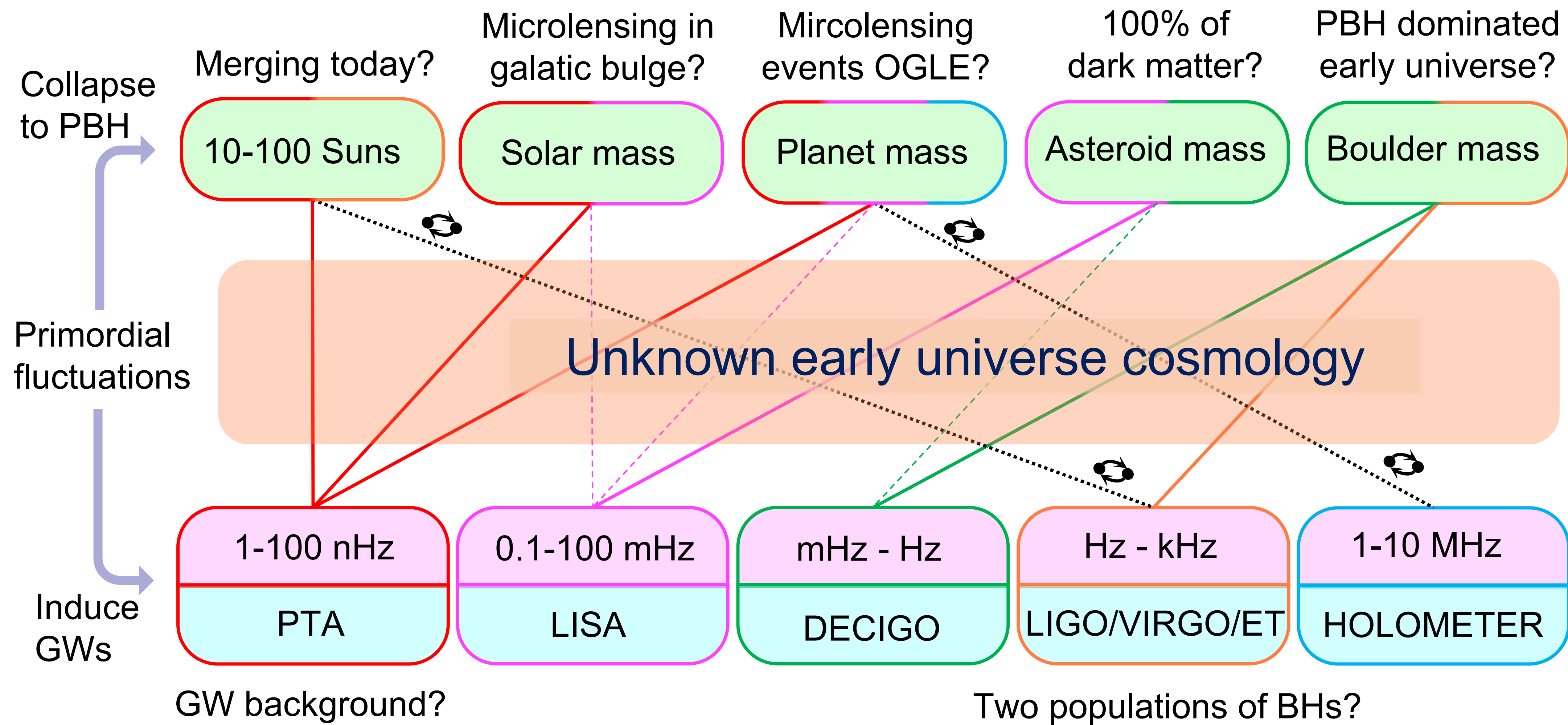
(INITIAL) FLUCTUATIONS ON SCALES  $\lambda_*$   
WITH  $P(\delta) \sim e^{-\frac{\delta^2}{2A_R}}$



# GENERATION OF GWs AND PBHs



# The PBH — induced GW connection



$$f_{\text{GW}} \sim \text{Hz} \left( \frac{M_{\text{PBH}}}{10^{16} \text{g}} \right)^{-1/2}$$

# Induced GWs amplitude

**After inflation:** 1st order: Free wave propagating

$$(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} = 0$$

Curvature  
perturbation



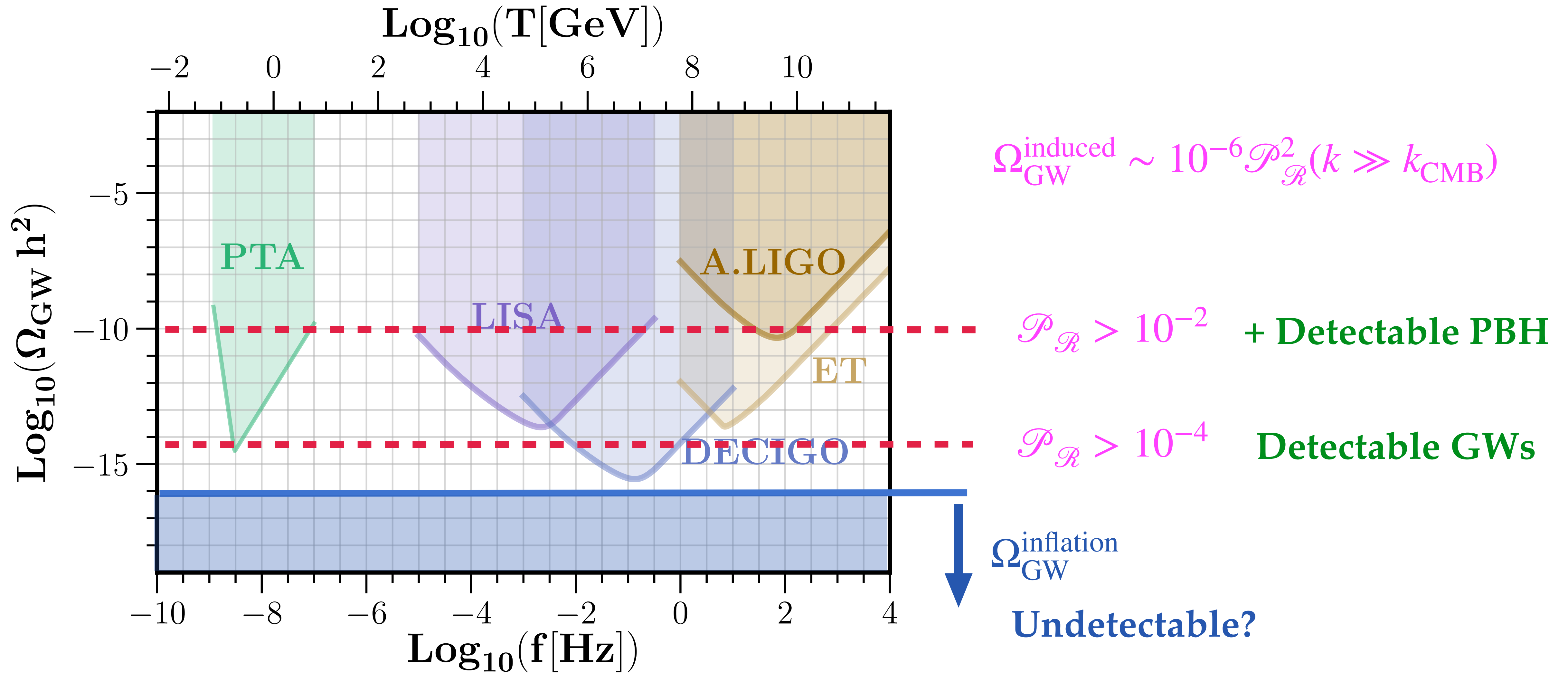
2nd order: Massless field with source  $(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} \sim \widehat{TT}_{ij}^{ab} (\partial_a \Phi \partial_b \Phi)$

$$\Omega_{\text{GW}}(k) = \frac{d\rho_{\text{GW}}}{d \ln k} = \frac{k^2}{12\mathcal{H}^2} \mathcal{P}_h(k, \tau)$$

$$\Omega_{\text{GW}}^{\text{induced}} \sim \frac{1}{12} \Omega_{r,0} \mathcal{P}_{\mathcal{R}}^2 \sim 10^{-6} \mathcal{P}_{\mathcal{R}}^2 (k \gg k_{\text{CMB}})$$

Density ratio of radiation today  $\Omega_{r,0} \sim 4 \times 10^{-5}$

# Primordial fluctuations — Induced GWs amplitudes



Power-law integrated sensitivity curves: Thrane & Romano 1310.5300

# Induced GWs and the early universe's expansion histories



# Induced GWs spectrum

**After inflation:** **1st order:** Free wave propagating  $(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} = 0$

**2nd order:** Massless field with source  $(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} \sim \widehat{TT}_{ij}^{ab} (\partial_a \Phi \partial_b \Phi)$

**Effects from “unknown” cosmological phases:**

1. Equation of state parameter  $w$ :
  - A. Expansion History (H)
  - B. Evolution of  $\Phi$
2. Propagation speed of fluctuations  $c_s$ : C. Resonances ( $f_h \sim 2f_\Phi$ )

$k \sim 2c_s k_p$

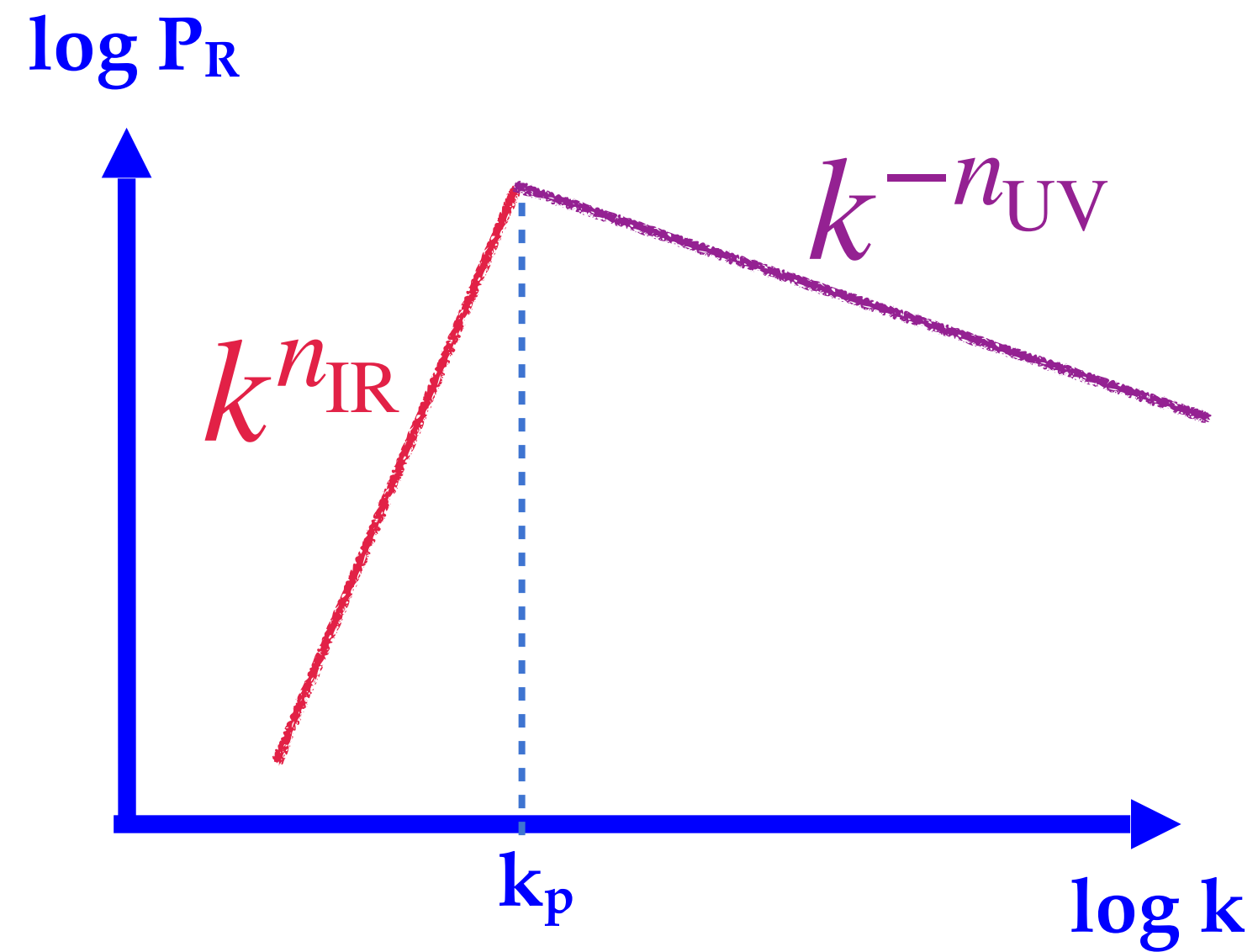
$(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} \sim \widehat{TT}_{ij}^{ab} (\partial_a \Phi \partial_b \Phi)$

**1st example:**

**Broken power-law**

# Induced GWs spectrum

[V.Atal & GD: 2103.01056]



$n_{UV} \propto f_{NL}$  (= local non-Gaussianity, or  $\eta$  parameter)

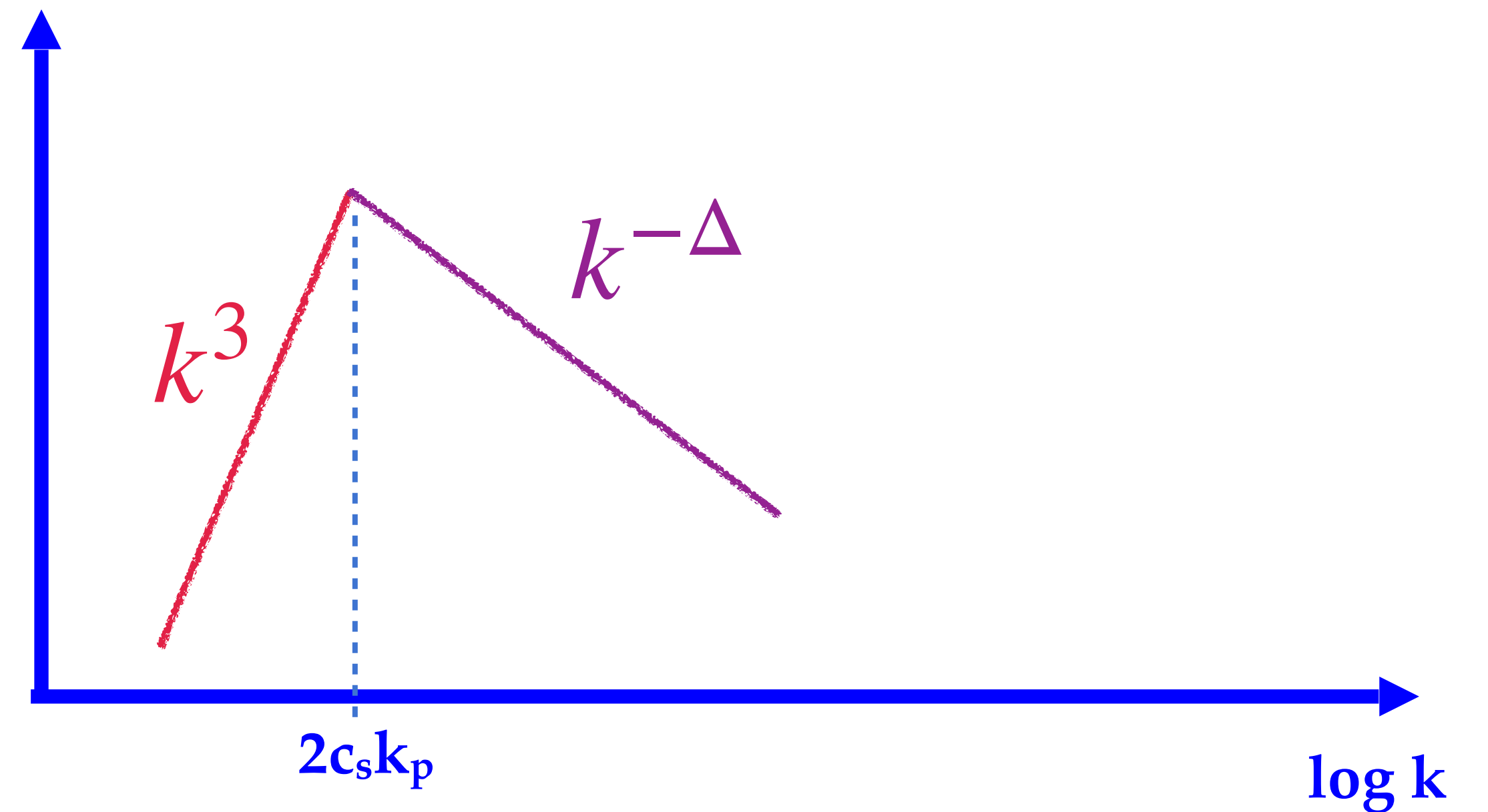
$$\Delta = \begin{cases} 2n_{UV} & n_{UV} < 4 \\ 4 + n_{UV} & n_{UV} > 4 \end{cases}$$

## Broken-power law

E.g. single field inflation models

$w = c_s^2 = 1/3$   
Radiation

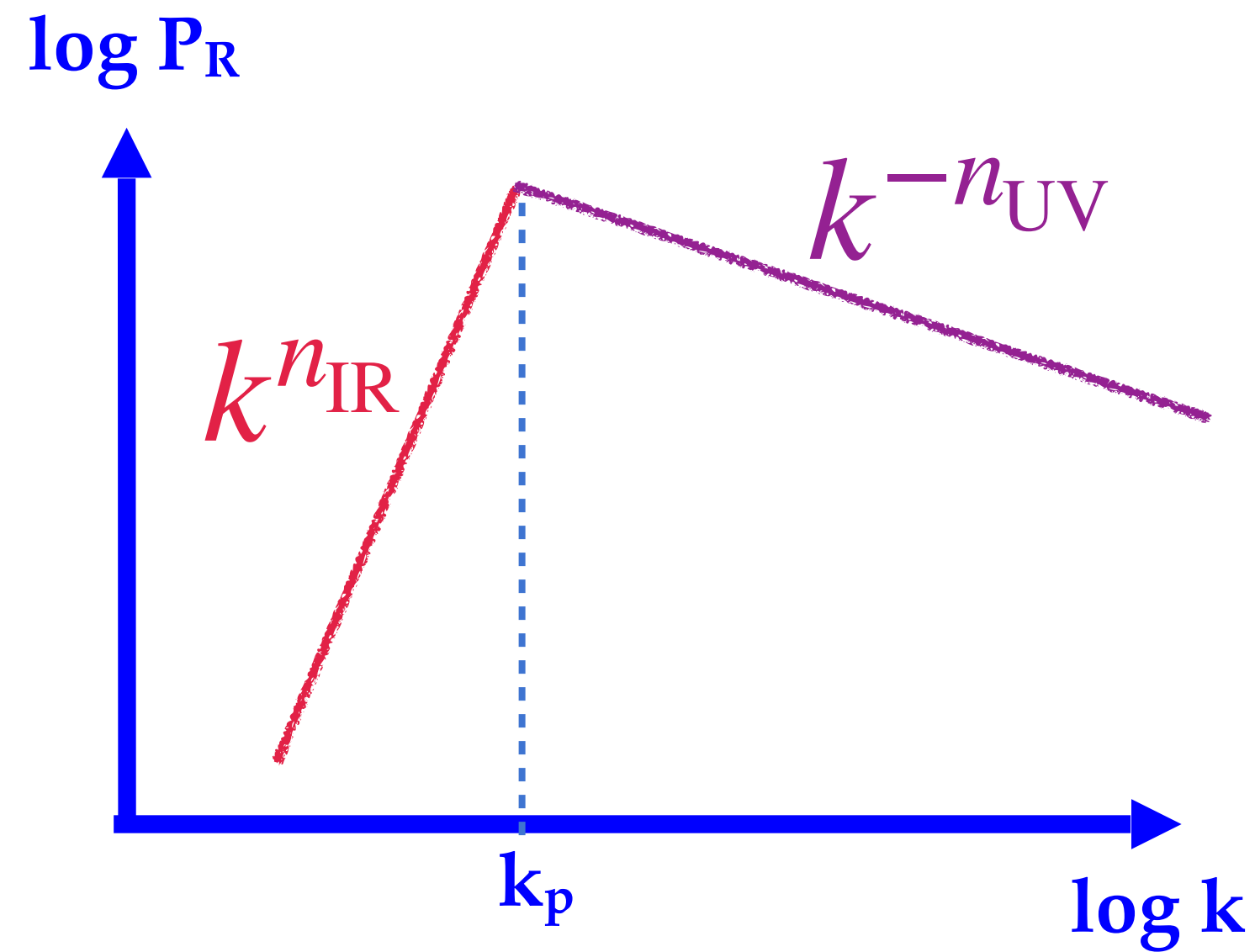
$\log \Omega_{GW}$



## Broken-power law

# Induced GWs spectrum

[V.Atal & GD: 2103.01056]



$$n_{UV} \propto f_{NL}$$

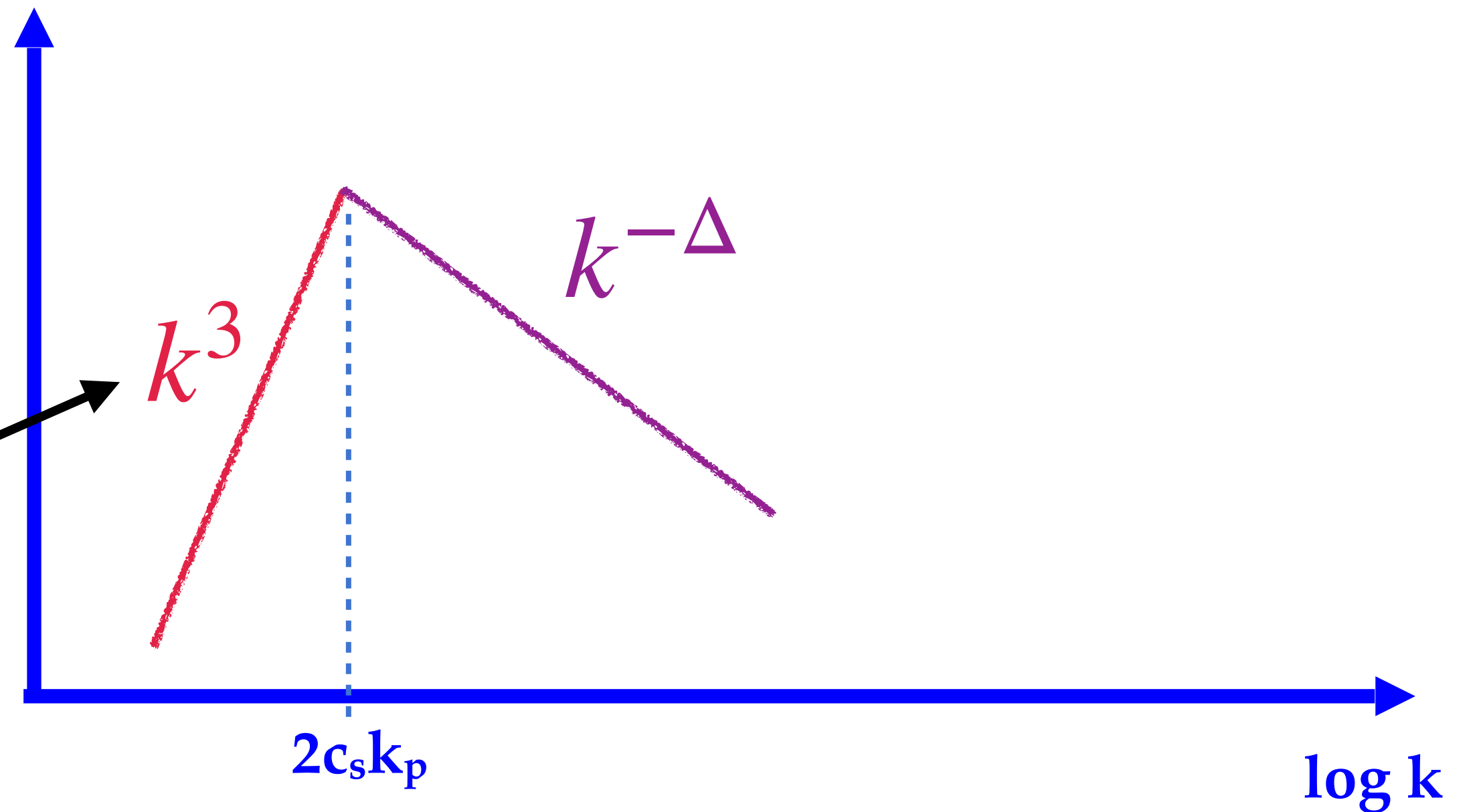
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## Broken-power law

E.g. single field inflation models

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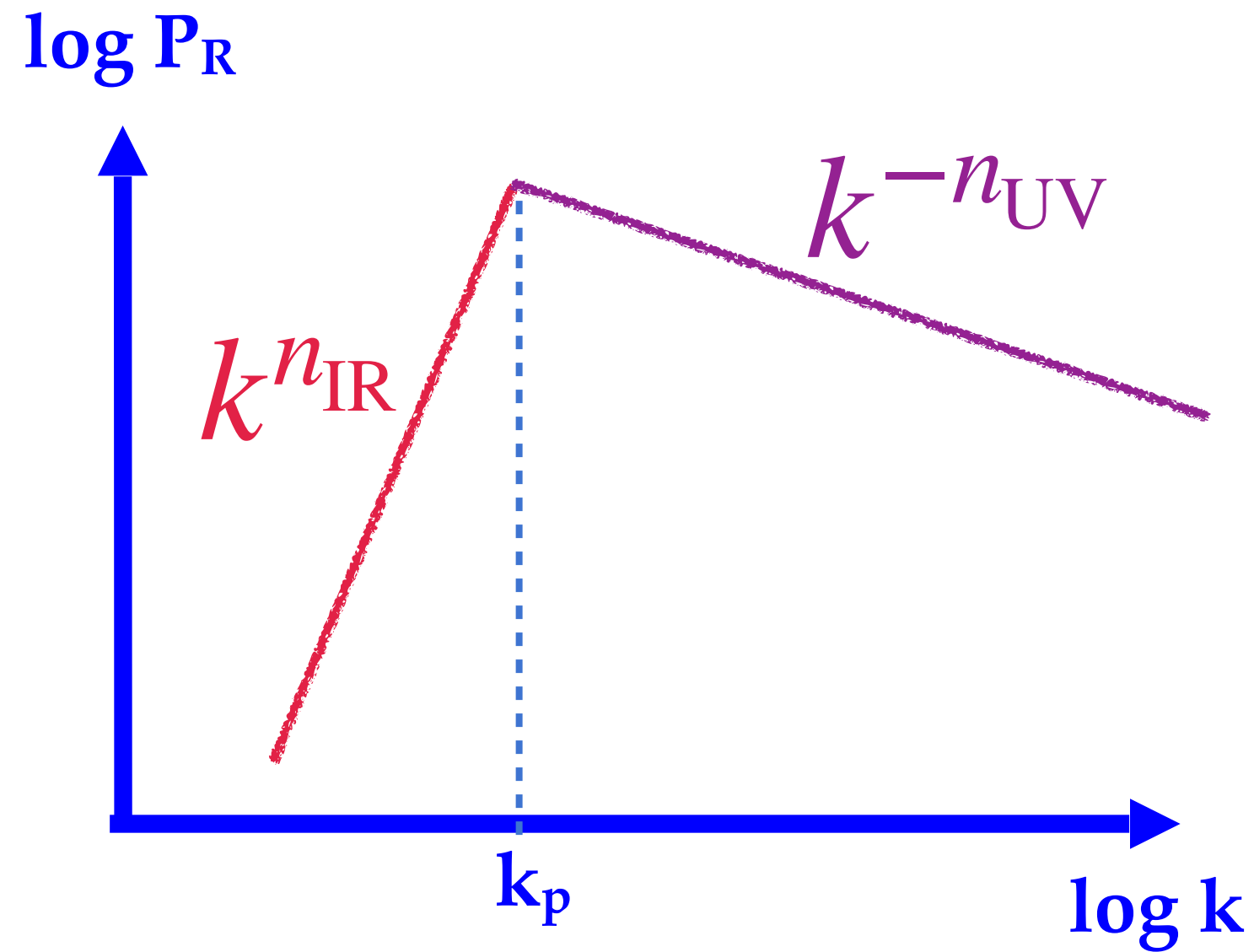


**“Universal” IR slope**  
[Cai, Pi & Sasaki, 1909.13728]

## Broken-power law

# Induced GWs spectrum

[V.Atal & GD: 2103.01056]



$$n_{UV} \propto f_{NL}$$

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## Broken-power law

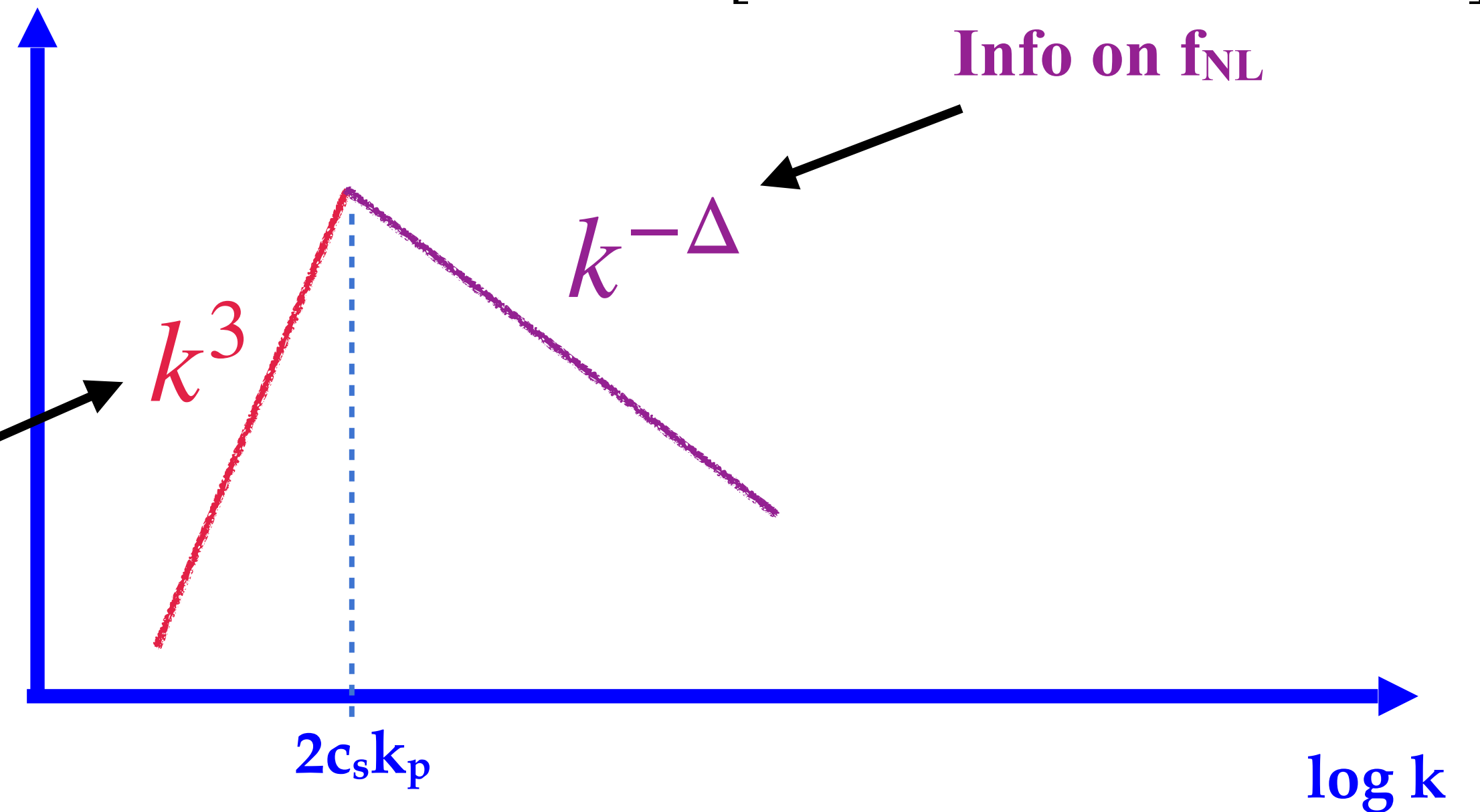
E.g. single field inflation models

$w = c_s^2 = 1/3$   
Radiation

$\log \Omega_{GW}$

[V.Atal & GD: 2103.01056]

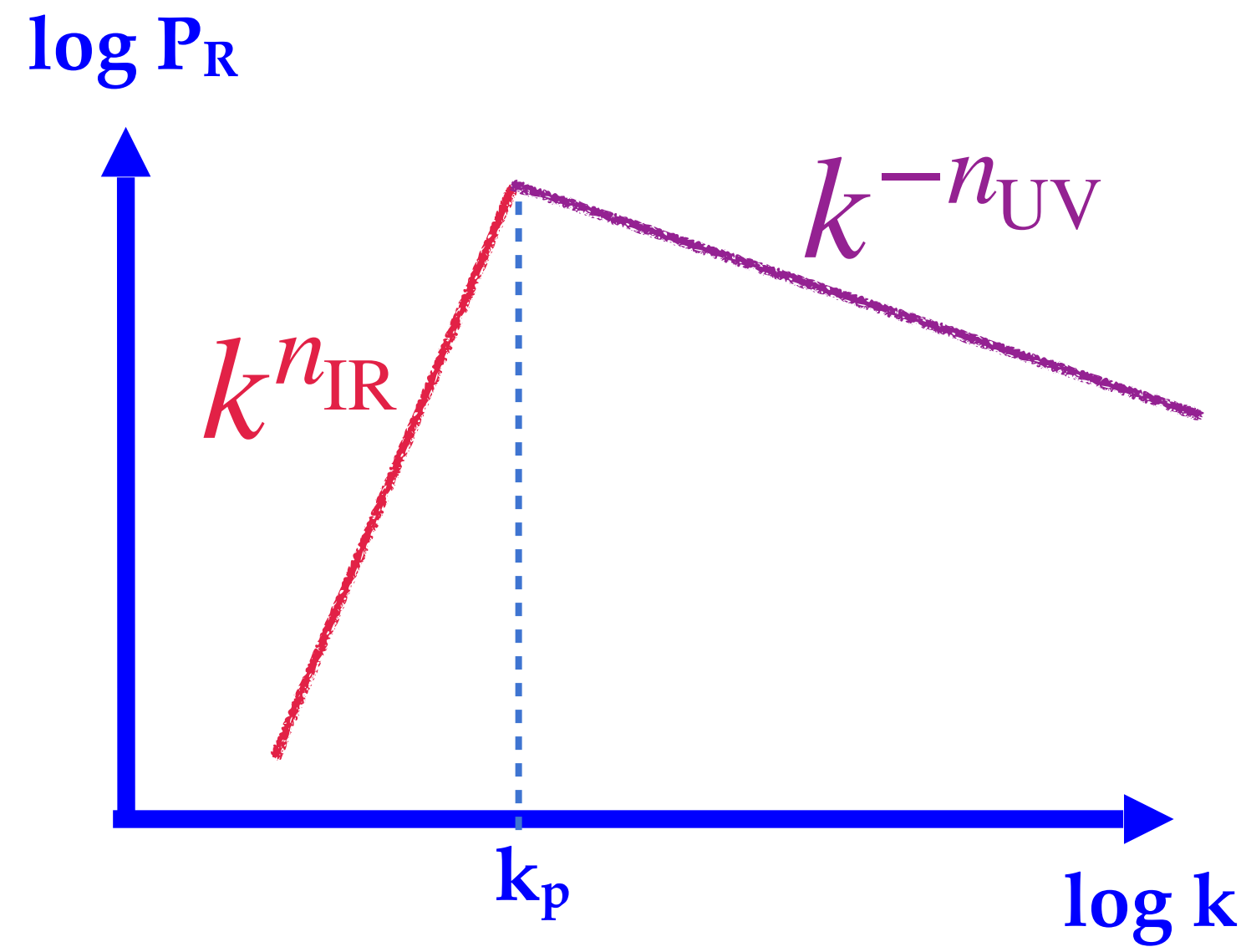
Info on  $f_{NL}$



**“Universal” IR slope**  
[Cai, Pi & Sasaki, 1909.13728]

## Broken-power law

# Induced GWs spectrum

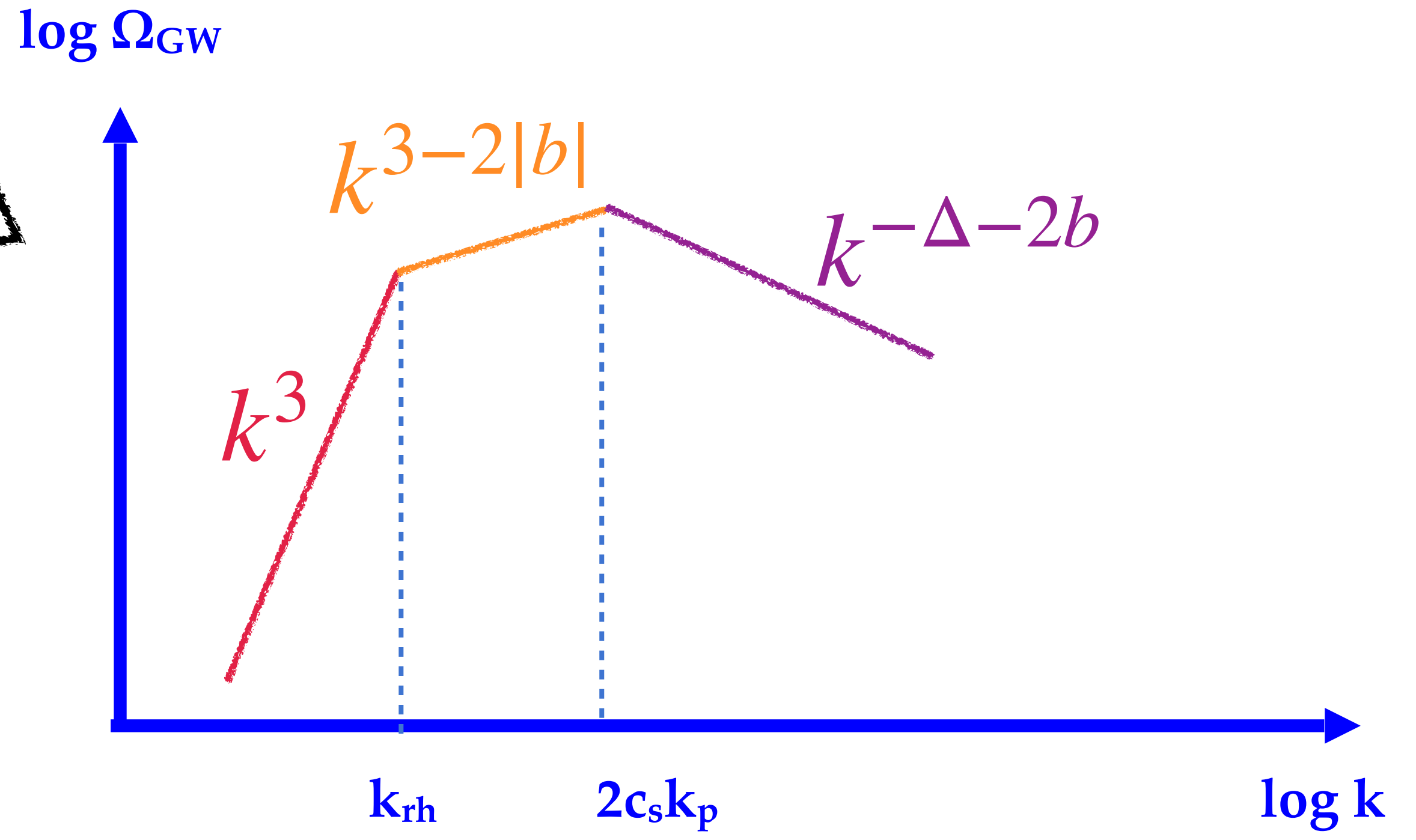
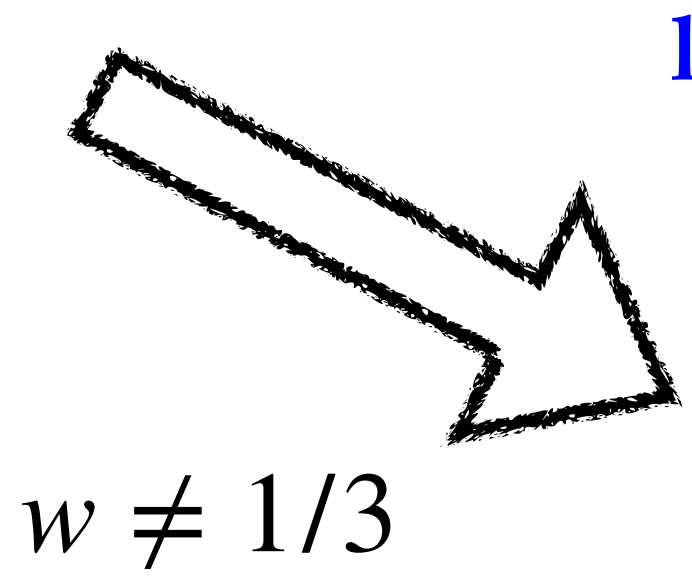


$$n_{UV} \propto f_{NL}$$

$$\Delta = \begin{cases} 2n_{UV} & n_{UV} + b < 4 \\ 4 + n_{UV} & n_{UV} + b > 4 \end{cases}$$

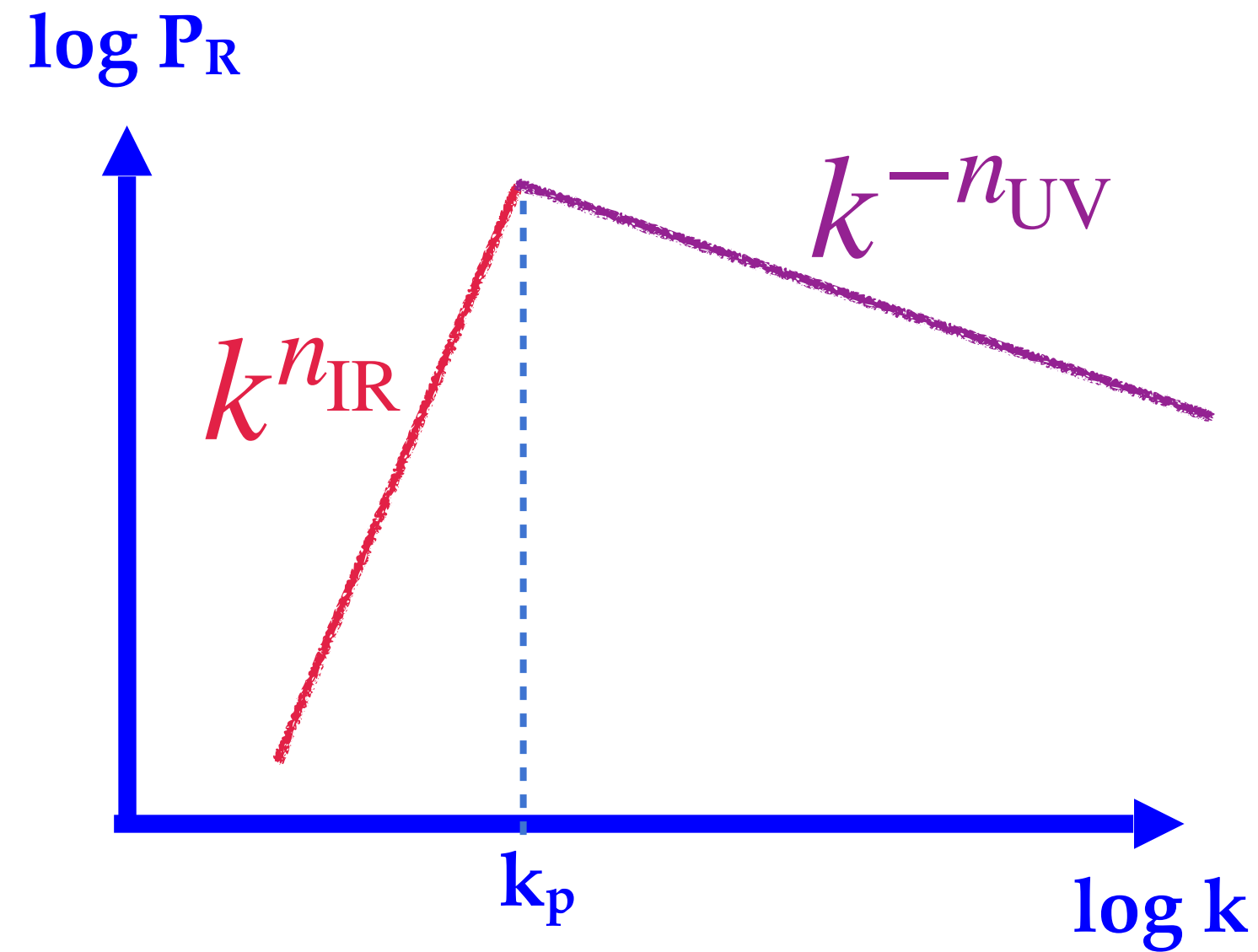
**Broken-power law**

$$b = \frac{1 - 3w}{1 + 3w}$$



**Double broken-power law**

# Induced GWs spectrum



Broken-power law

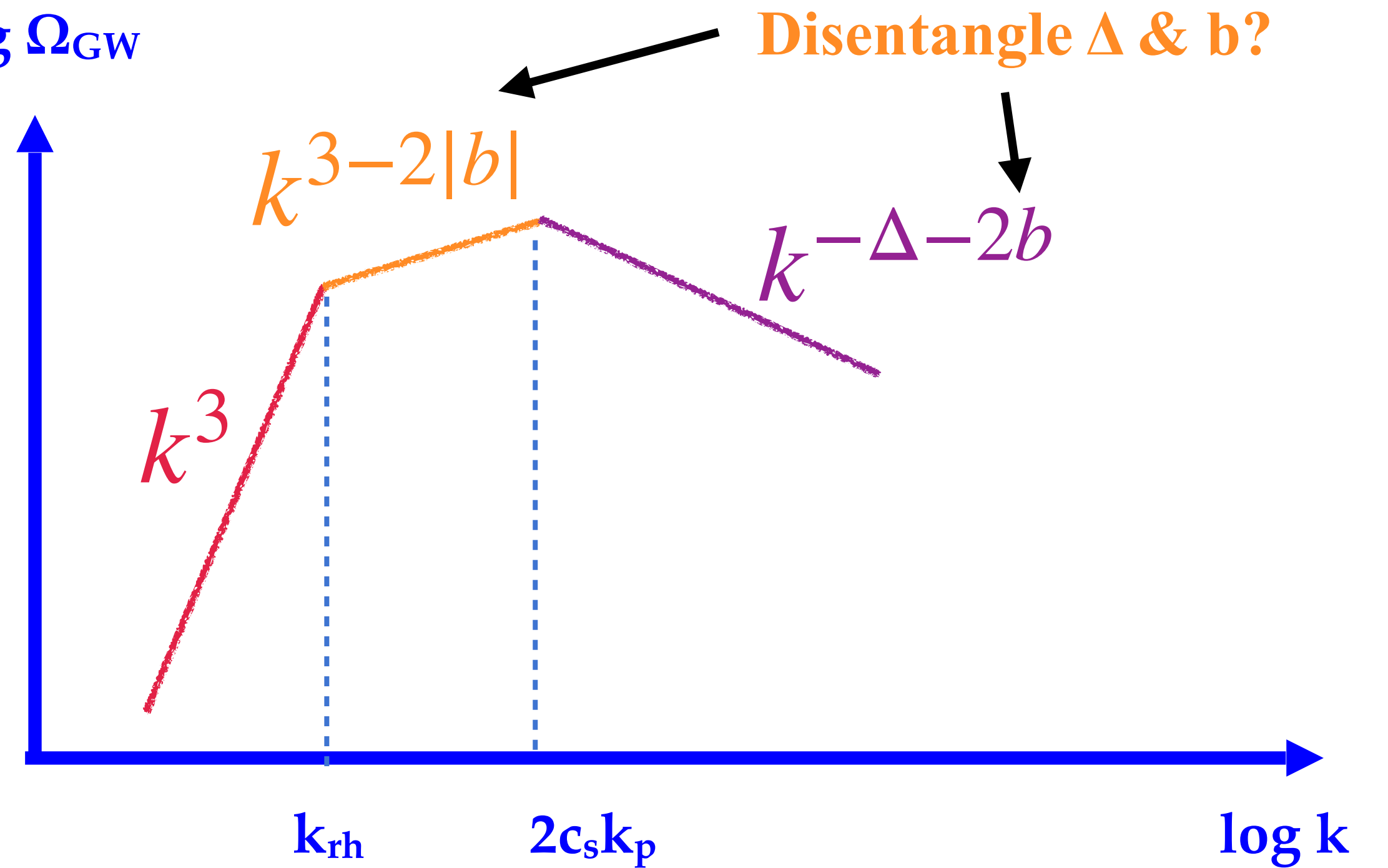
$$b = \frac{1 - 3w}{1 + 3w}$$

$$n_{UV} \propto f_{NL}$$

$$\Delta = \begin{cases} 2n_{UV} & n_{UV} + b < 4 \\ 4 + n_{UV} & n_{UV} + b > 4 \end{cases}$$

$w \neq 1/3$

$\log \Omega_{GW}$



Double broken-power law

**2nd example:**

**Very sharp peak**



# Sharp peaks

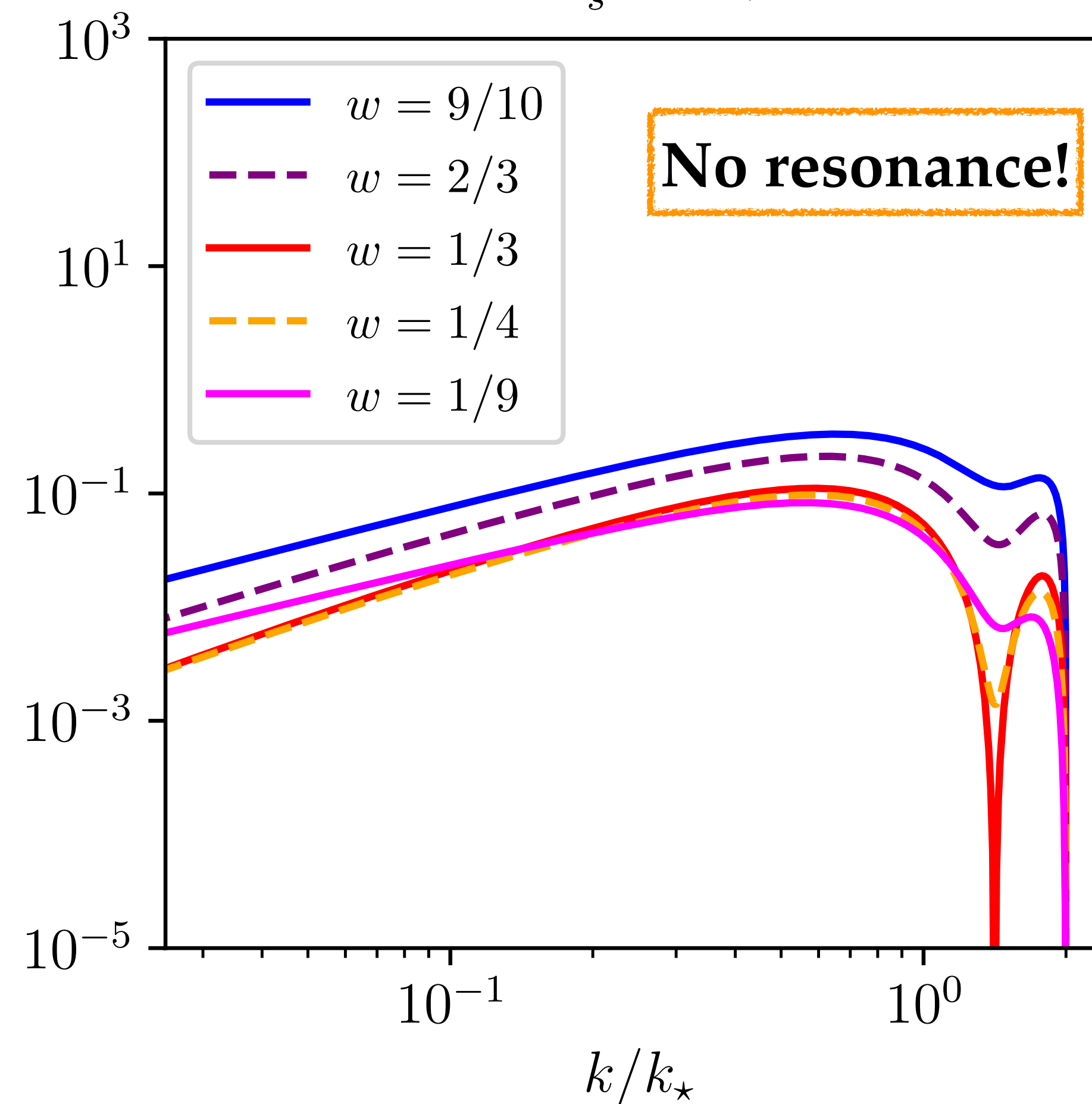
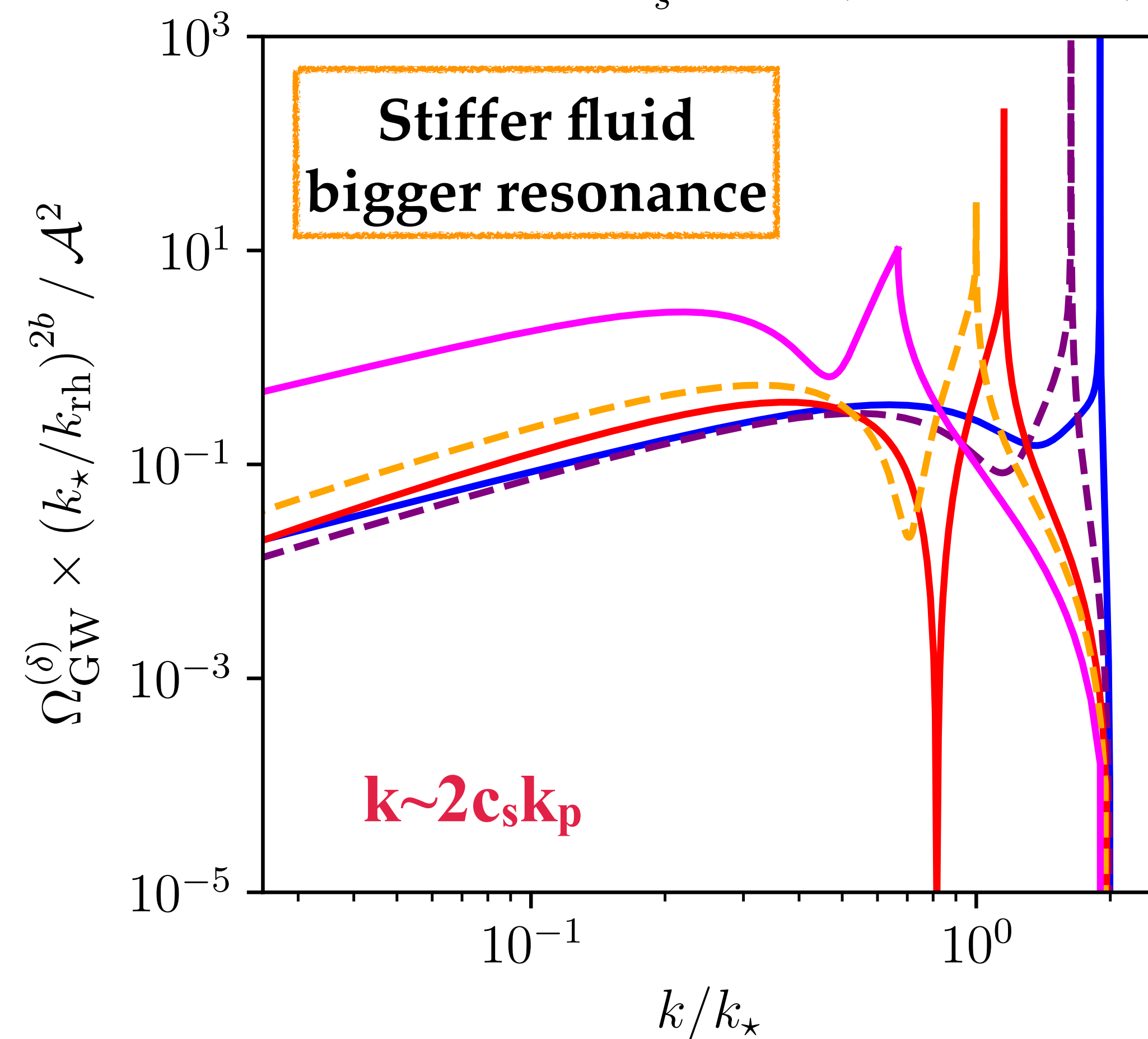
[GD, 1912.05583]

[GD, S.Pi, M.Sasaki, 2005.12314]

Infinitely sharp peak:  $\mathcal{P}_{\mathcal{R}}(k) = \mathcal{A}_{\mathcal{R}}\delta(\ln(k/k_*))$

$c_s^2 = w$  (Perfect fluid)

$c_s^2 = 1$  (Canonical scalar field)



**3rd example:**

**Oscillatory modulations**

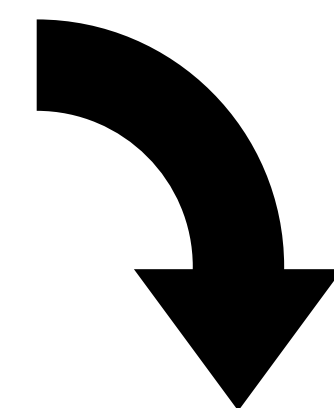
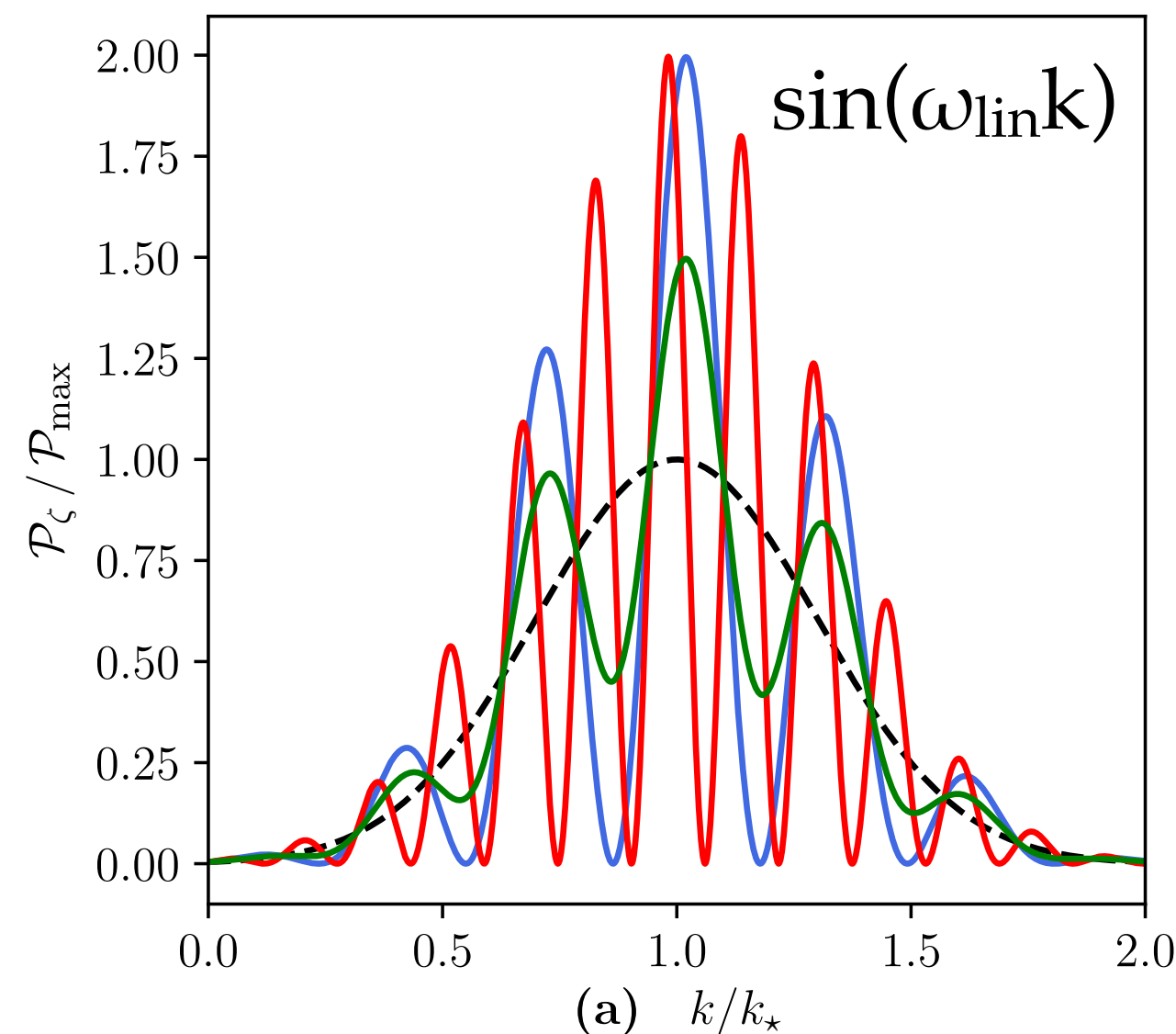
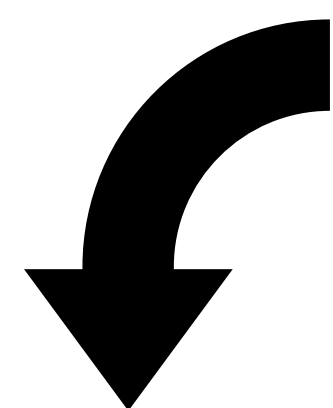
# Oscillatory features

[L.Witkowski, GD, J.Fumagalli, S.Renaux-Petel, 2110.09480]

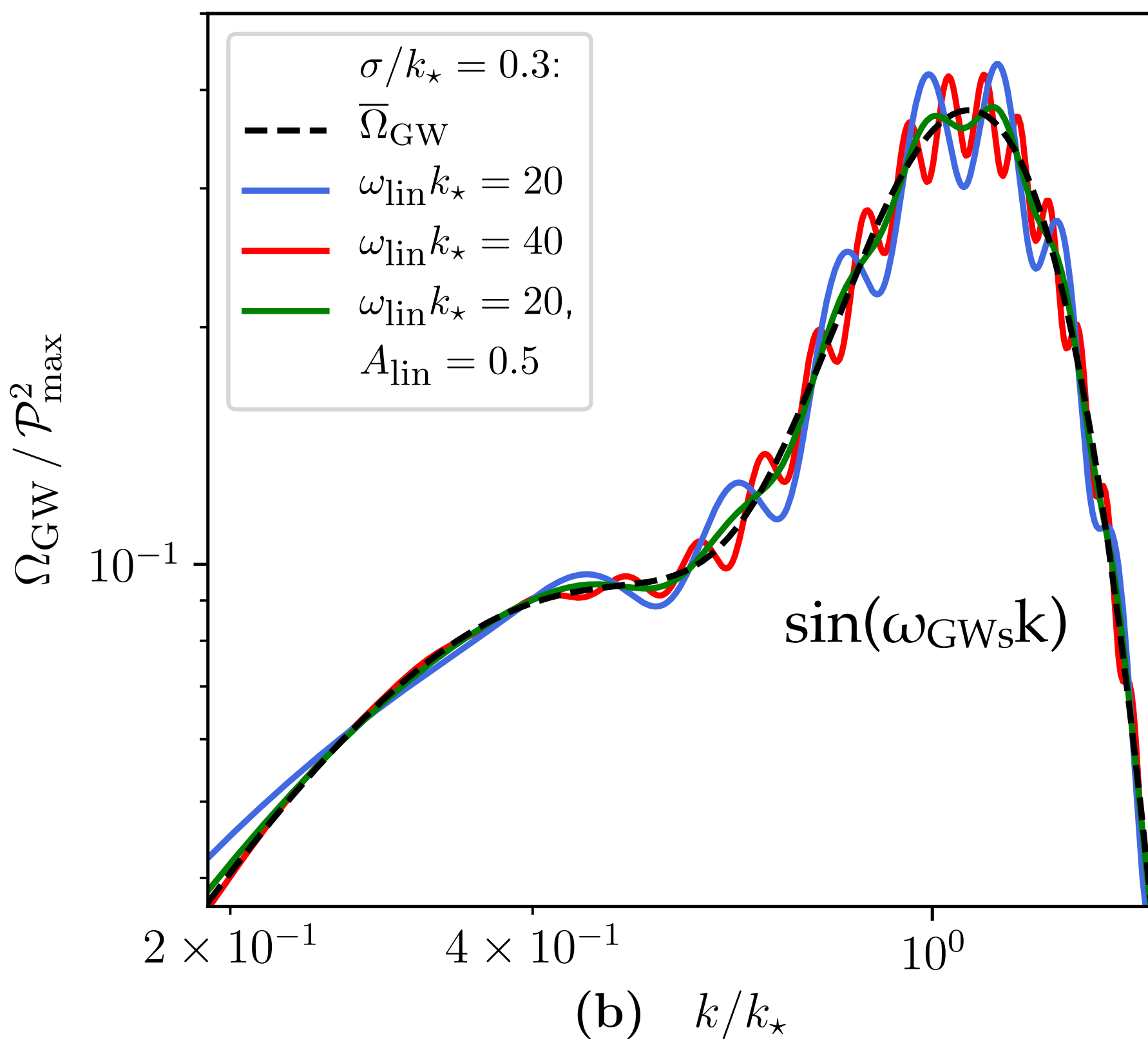
[J.Fumagalli, S.Renaux-Petel, L.Witkowski, 2004.08369,  
2012.02761,  
2105.06481]

[Cai+1901.10152]

$$c_s^2 = w = 1/3$$



$$c_s^2 = w = 9/10$$

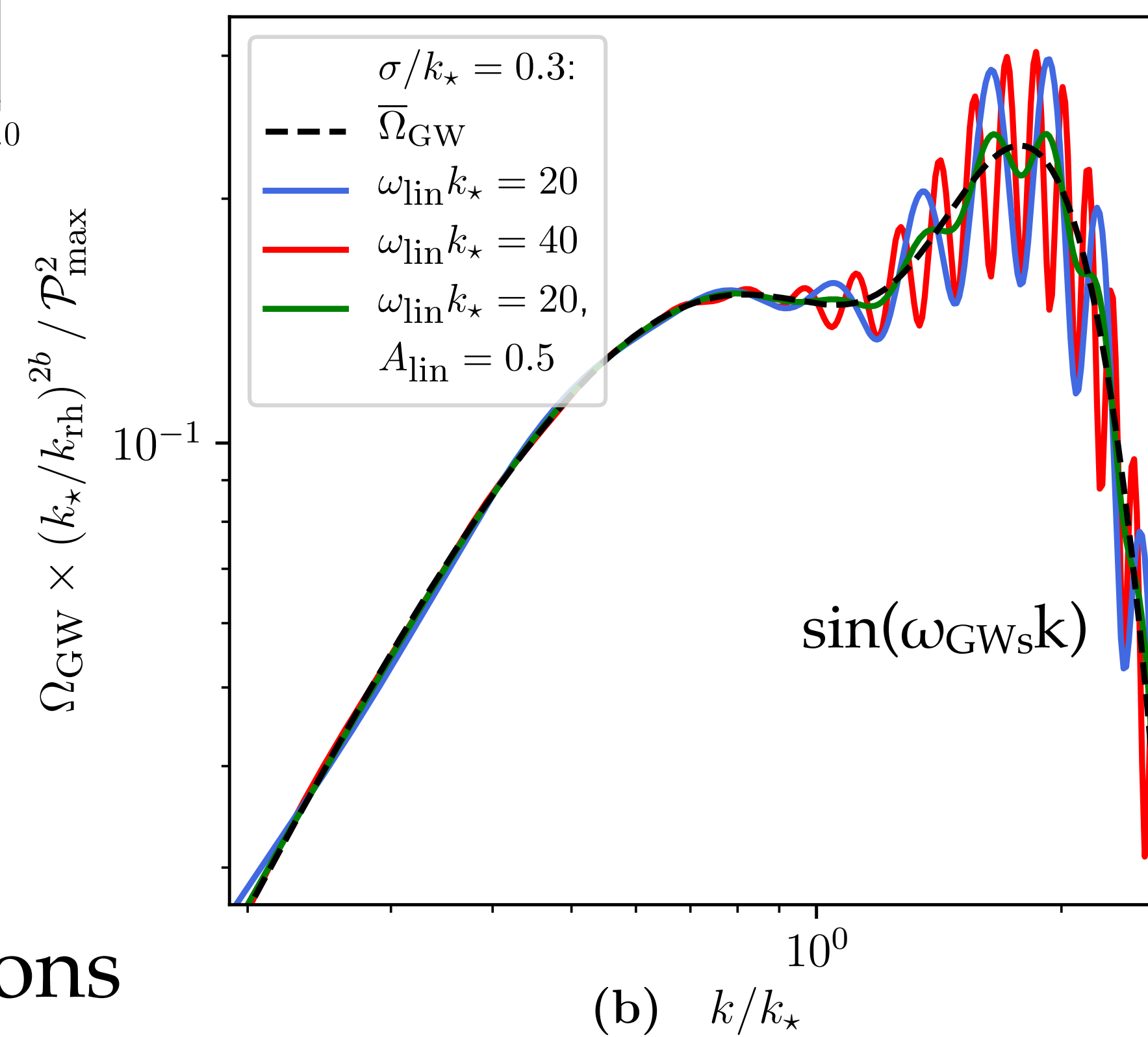


Disentangle  $\omega_{\text{lin}}$  &  $w$ ?

$$\omega_{\text{GWs}} = \omega_{\text{lin}} / c_s$$

Amplitude >20%  
=>  $w > 1/3!$

$c_s^2 = 1 \Rightarrow$  no oscillations

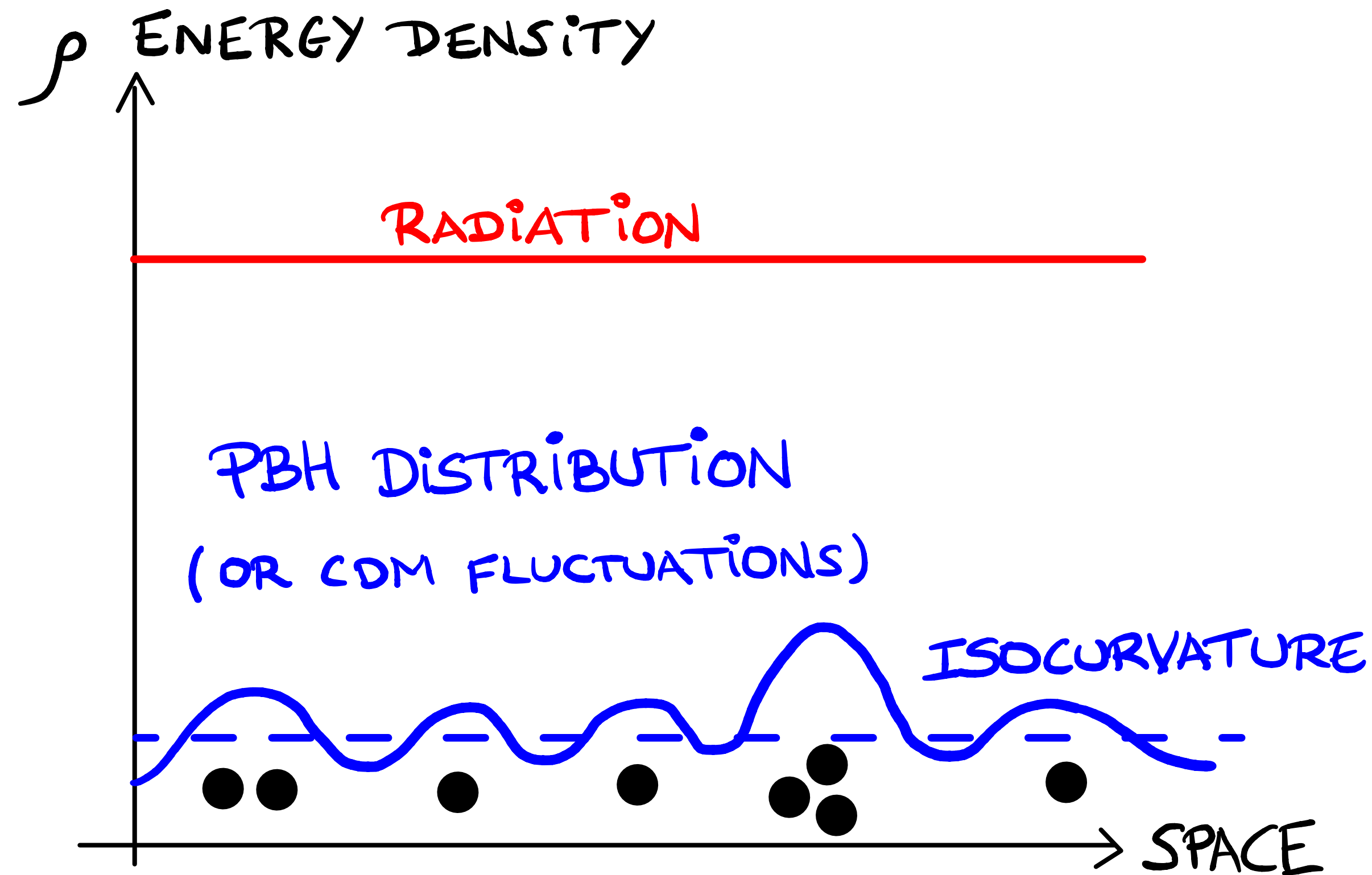


# **Induced GWs and early isocurvature fluctuations**

# ISOCURVATURE EVOLUTION

[T.Papanikolaou, V.Vennin, D.Langlois, 2010.11573]

[GD, C.Lin, M.Sasaki, 2012.08151]



## Short explanation:

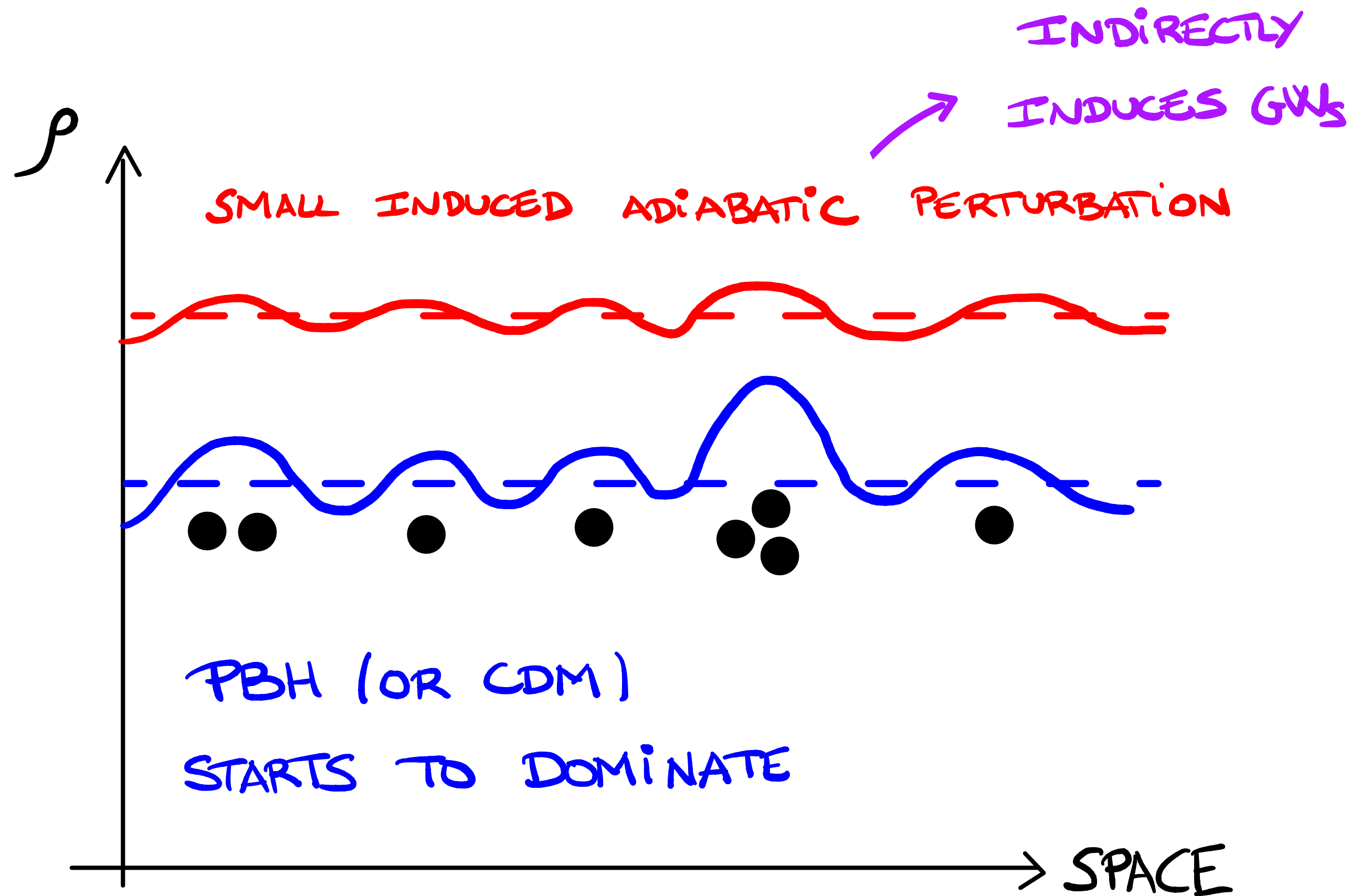
Initial conditions  
for a universe after PBH formation

The (inhomogeneous) PBH distribution  
give rise to density fluctuations

These fluctuations must be isocurvature  
as they form by the collapse  
of homogeneous radiation

**The evolution of isocurvature onwards  
also applies to CDM isocurvature!**

# ISOCURVATURE EVOLUTION



## Short explanation:

PBH (or CDM) energy density gains relative weight

Isocurvature induces curvature fluctuations

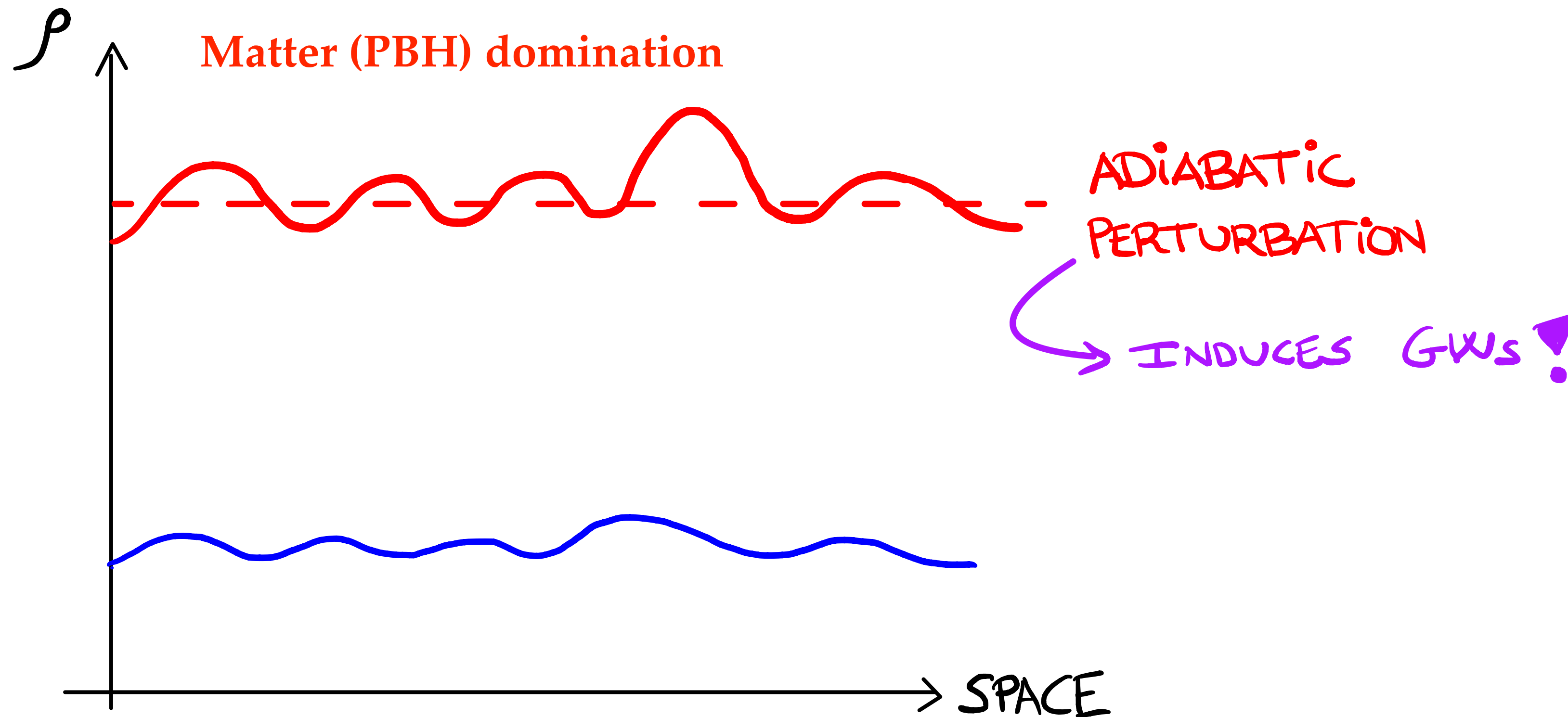
Curvature fluctuations induce GWs

The later a mode enters the horizon,  
the larger the curvature fluctuation induced

Relevant component for large CDM isocurvature!

1st possibility: GWs by CDM isocurvature

# ISOCURVATURE EVOLUTION



**Short explanation:**

PBH (or CDM) dominates the energy density

Curvature fluctuations induced GWs

**Relevant component for PBH dominated universe!**

**GWs induced in an early matter era get greatly enhanced for sudden reheating!**

[Inomata+1904.12878,1904.12879]

**Like PBH evaporation!**

**2nd possibility: GWs by PBH isocurvature**

**1st example:**

**GWs by CDM Isocurvature**

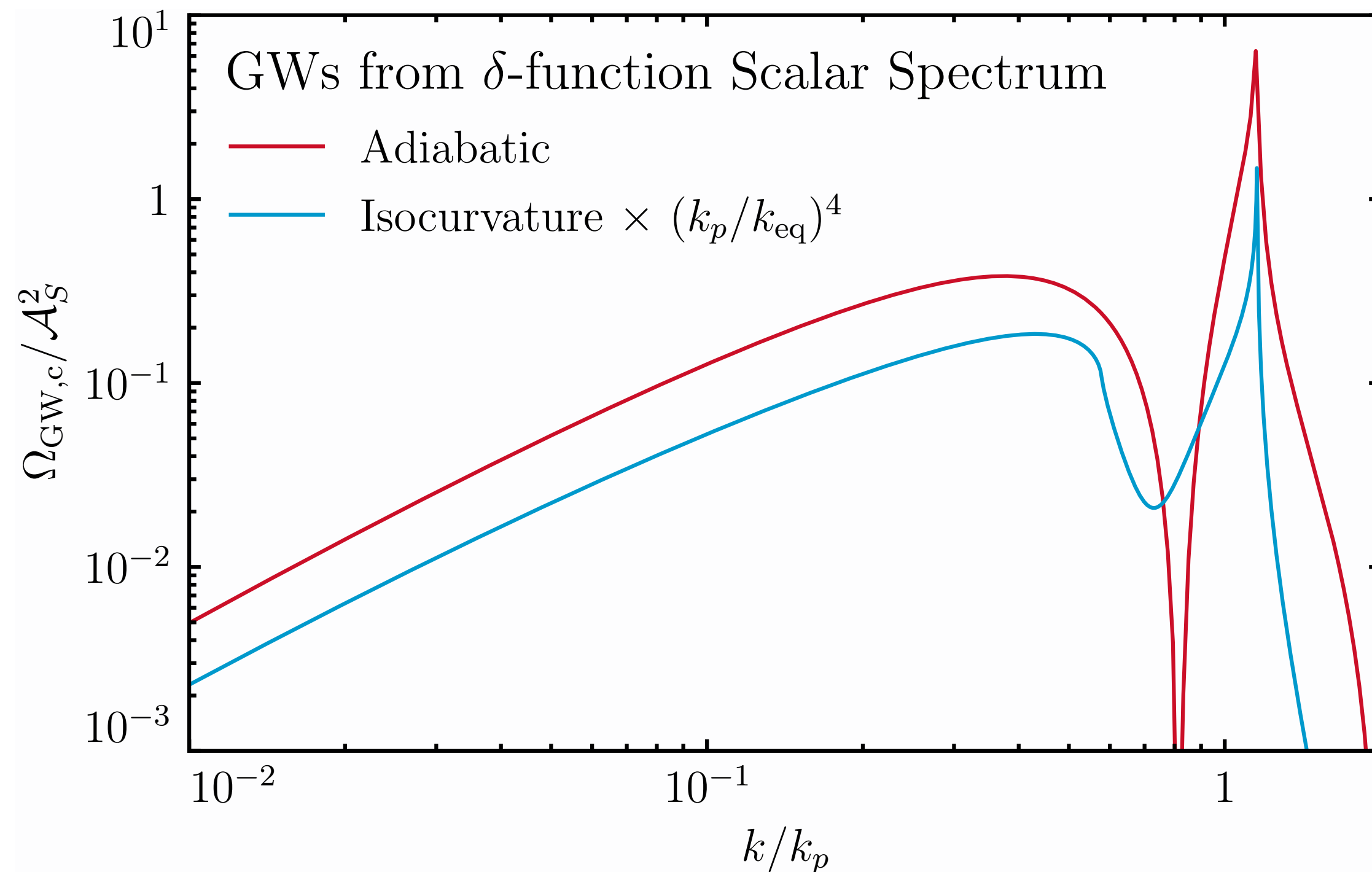


# GWs by CDM isocurvature

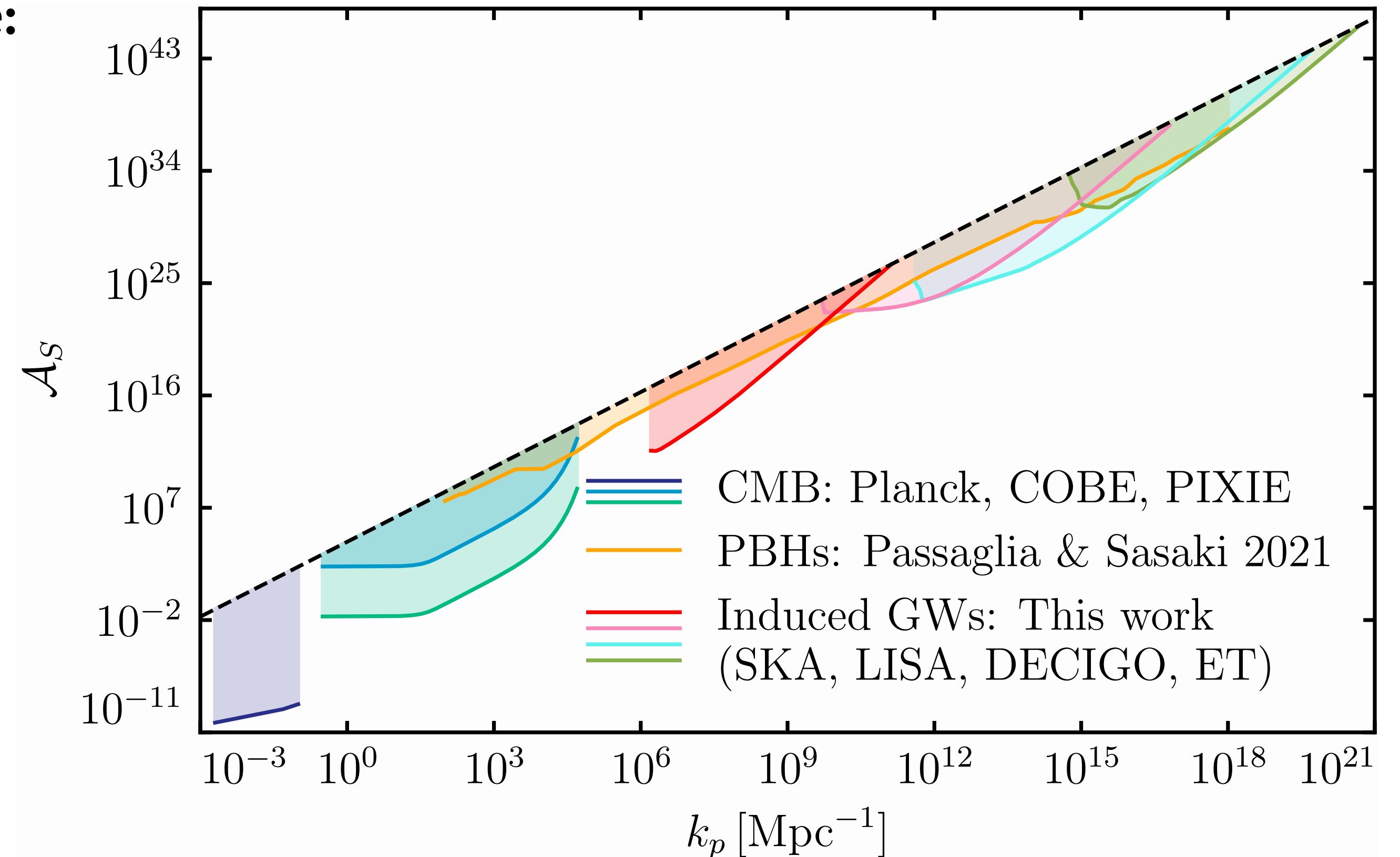
PBH counterpart in:  
[S.Passaglia, M.Sasaki, 2109.12824]

Large enough CDM isocurvature leads to a **detectable GW signal!**

Spectral shape different from the adiabatic one:



[GD, S.Passaglia, S.Renaux-Petel, 2112.10163]



**2nd example:**

**GWs by PBH density fluctuations**

# GWs from PBH density fluctuations

[K.Inomata+2003.10455]

[T.Papanikolaou+ 2010.11573]

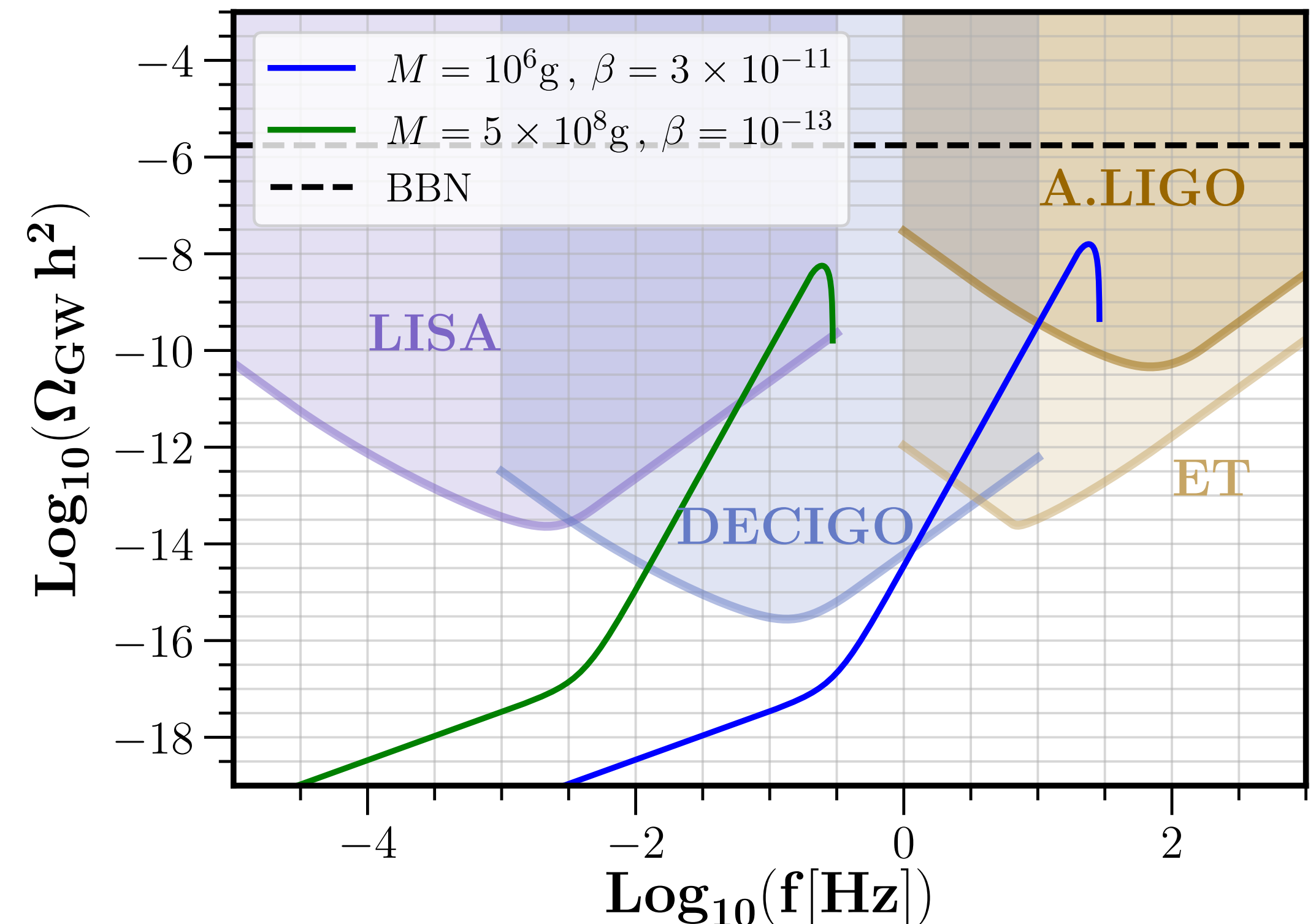
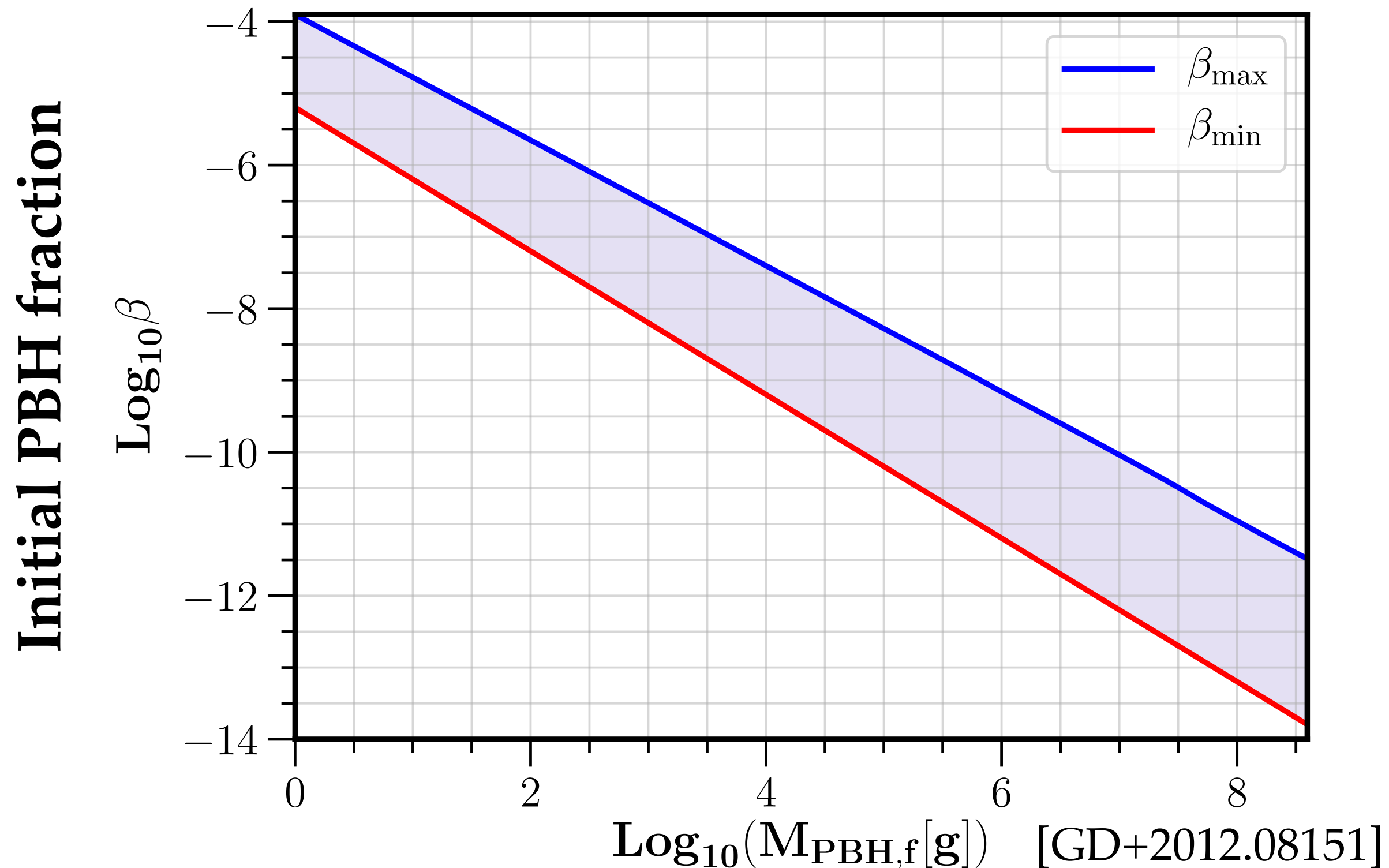
[GD+2012.08151]

Sudden evaporation of all PBHs, creates huge velocity fluctuations in the radiation fluid

and a **loud GW signal!**

We can use it to test the PBH dominated universe:

(If PBH spin, we can combine info on  $N_{\text{eff}}$  [GD+2105.06816])



# Summary

Induced GWs (together with PBHs) are a unique probe of:

- **Physics of inflation**

We need  
to know  
both!

Primordial spectrum      **Unique for small scales**  
Early CDM isocurvature      **New probe of isocurv.**  
Gaussian vs non-Gaussian

- **Physics after inflation  
(before BBN)**

Equation of state      **IR broken power-law**  
Sound speed of fluctuations      **Resonant peak**  
PBH dominated universe      **Constraints on  $\beta$**

≡≡≡ **The End** ≡≡≡

