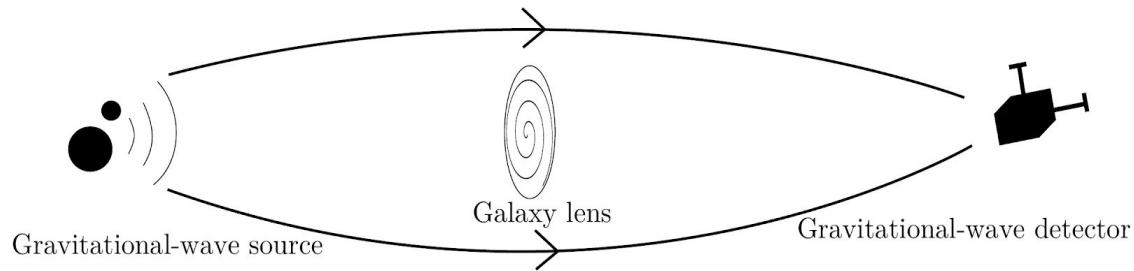
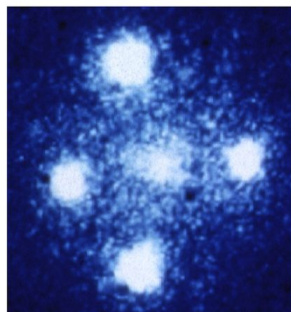


# Search for lensing signatures in gravitational-wave observations



ESA/Hubble & NASA



NASA, ESA, and STScI



NASA, ESA, Hubble SM4 ERO Team, ST-ECF

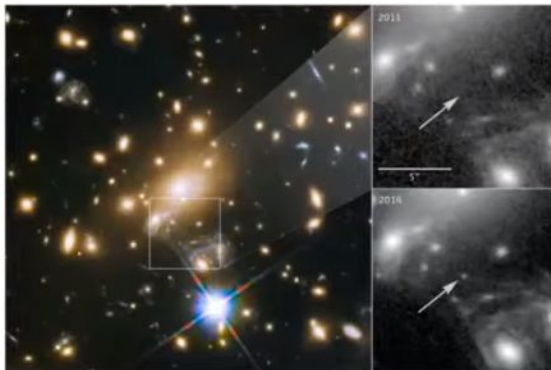
4th February 2022  
CSGC, IIT Madras

*On behalf of LVK lensing group*

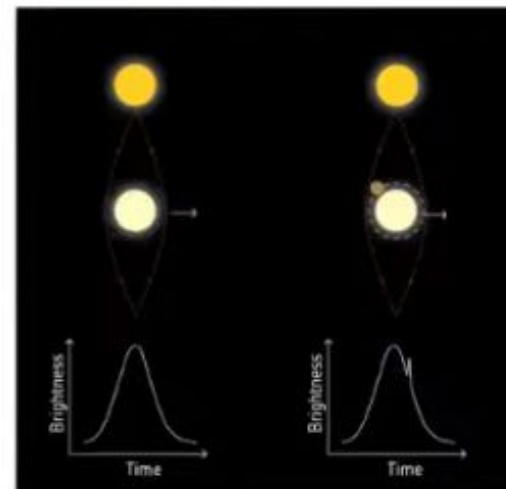
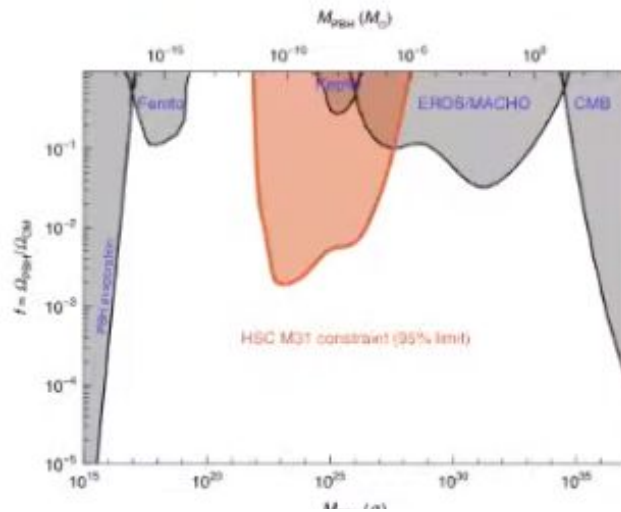
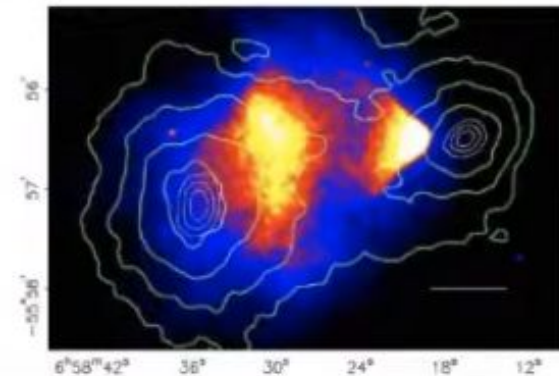
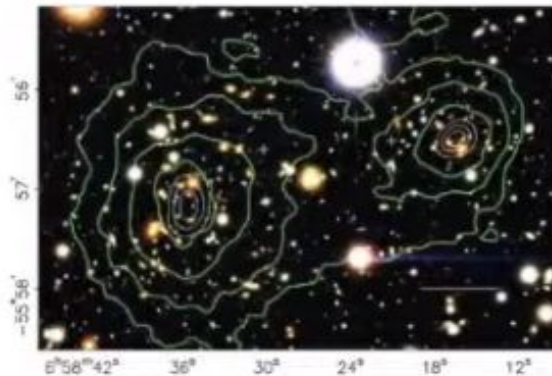
Presented by: Srashti Goyal (PhD Student, International Center for Theoretical Sciences, Bangalore)

# Astronomy with gravitational lensing(bending) of light

- Look far
- Map Dark Matter
- Discover exoplanets

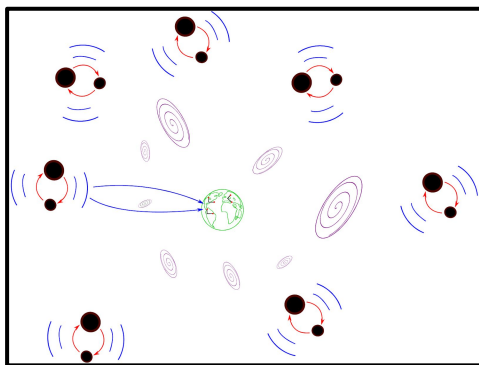


A distant star (right) at a distance of 9 billion light years is magnified more than 2,000 times by a galaxy cluster (right), making it visible from Earth.



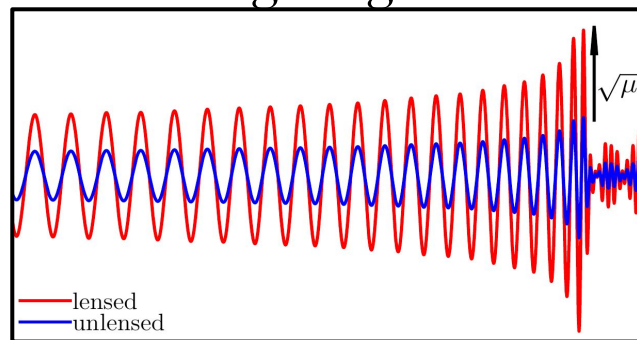
# O3a: 1st LVC search of gravitational-wave lensing

Arxiv: [2105.06384](https://arxiv.org/abs/2105.06384)

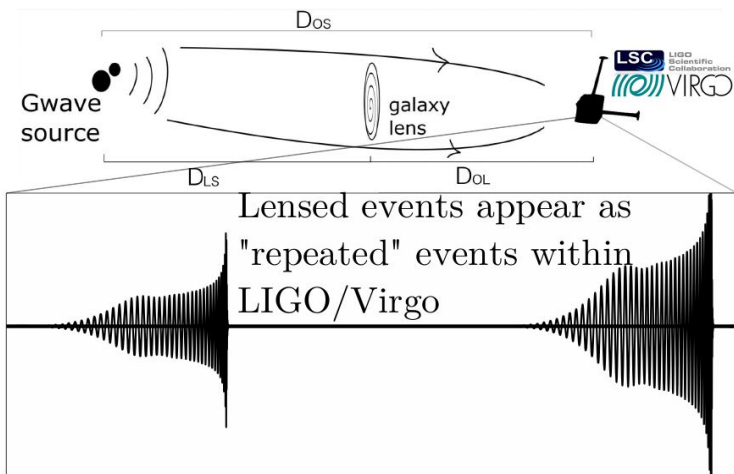


### I. Lensing statistics

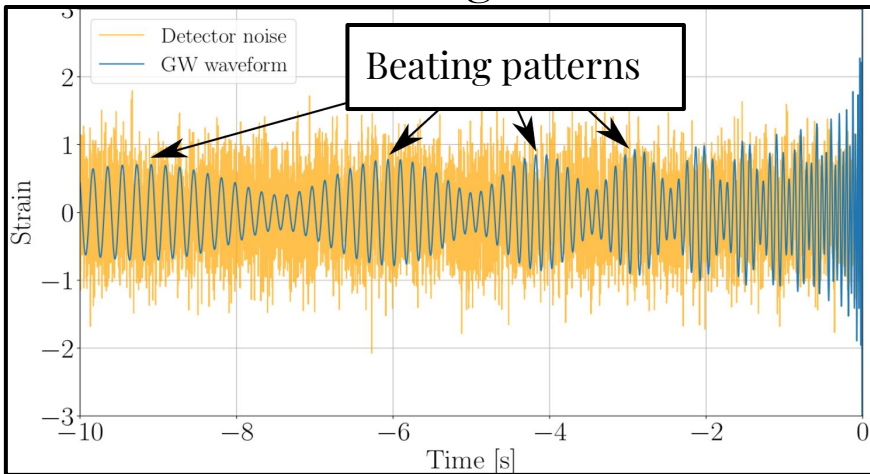
## II. Lensing magnification



## III. Multiple images



## IV. Microlensing distortions

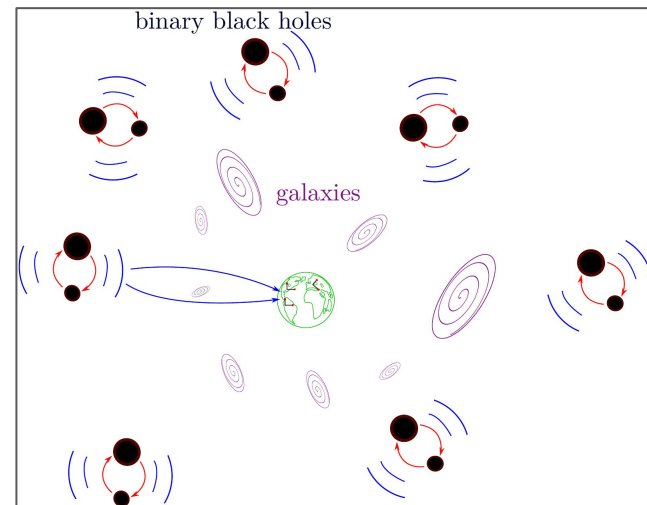
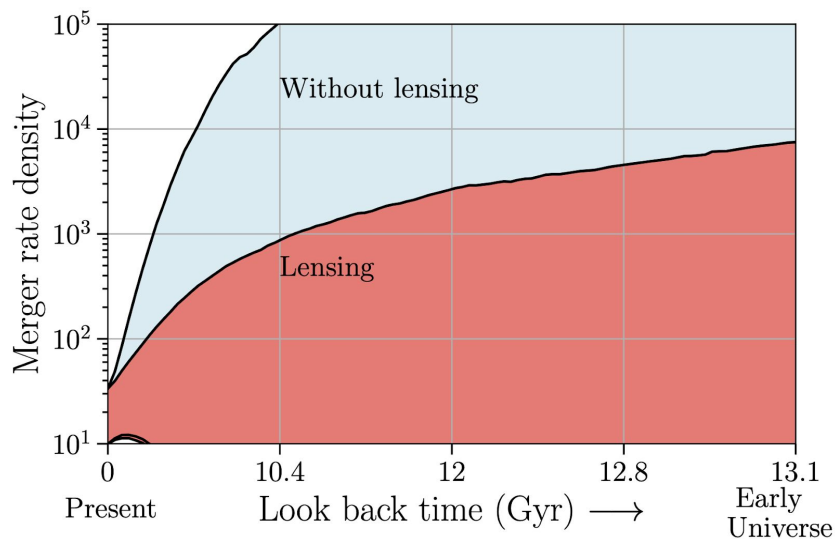


# I. Lensing statistics (strong lensing)

- Given our understanding of
  - binary black hole population
  - lenses populations

*We predict the rate of lensing*

- $1:10^{3-4}$  events are expected to be lensed  
(vital for putting priors in follow up analyses)



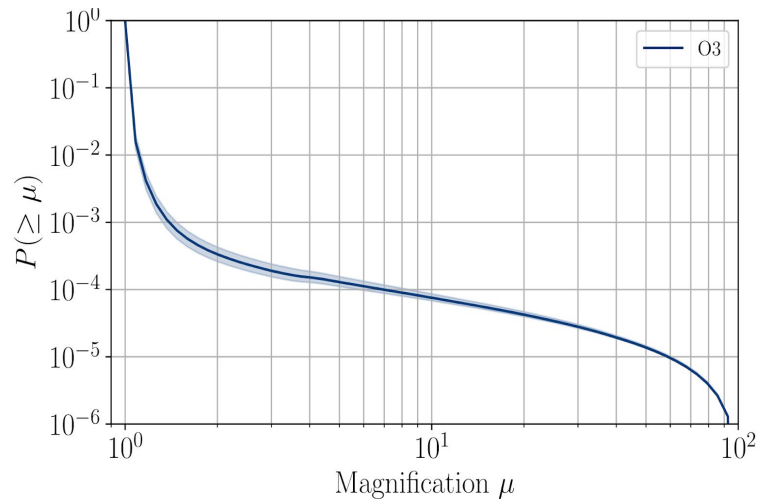
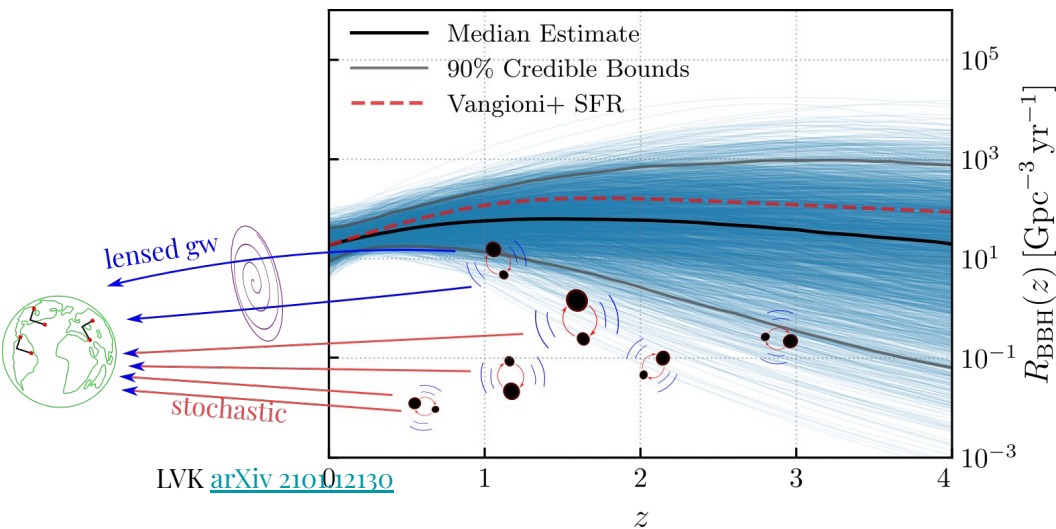
- Rate of mergers happening at early times are not well constrained currently

*Non-detection of lensed events may help us to constrain models of merger rate densities at high redshifts*

# I. Lensing statistics (stochastic)

*Implications on unresolvable CBCs from stochastic background searches..*

- Knowledge on detected CBCs from **population studies**
- Magnification model from [Dai et al 2017](#):  
parametric fit to weak ([Takahashi et al 2011](#))  
and strong regime ([Hilbert et al 2008](#))
- Based on method in ([Buscicchio et al 2020](#))



## Conclusions

- Expected rate
  - $P(\mu > 2) = 4 \cdot 10^4$
  - $P(\mu > 10) = 1 \cdot 10^4$
- Merger rate rate uncertainties in blue shading

# II. Lensing magnification

*Lensing magnification can create apparently high-mass events..*

- Analyze under the hypothesis that they originate from intrinsically lower-mass population:
  - Binary black holes with a mass gap at 50 or 65 solar masses
  - Binary neutron stars from the galactic double neutron star population

Events analysed:-

**BNS:** GW190425

**NSBH:** GW190426\_152155

**BBHs:** GW190521, GW190602\_175927, GW190706\_222641

- Reported intrinsic source properties and expected magnification under the strong lensing hypothesis
  - BBHs: Moderate magnifications of order 10 originating from  $z \sim 1-2$ , while BNS and NSBH would require high magnifications of order 100 or more.
- **No compelling evidence of lensing magnification. Important to categorise such events for follow up analysis.**

# III. Search for multiple images

$$\tilde{h}_j^L(f; \theta, \mu_j, \Delta t_j, \Delta \phi_j) = \sqrt{|\mu_j|} h(f; \theta, \Delta t_j) e^{i\Delta \phi_j \text{sign}(f)}$$

Magnification

Time delay

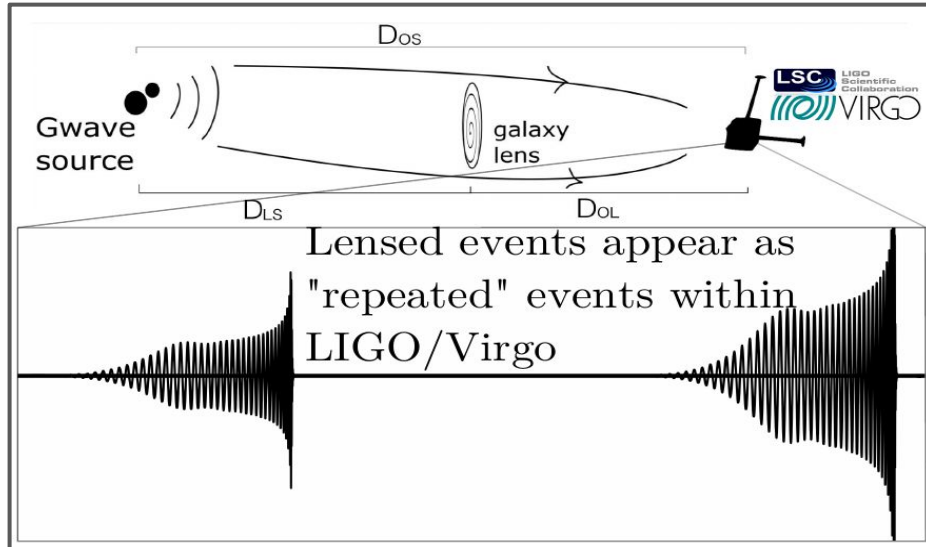
Morse phase

Image types:

**Type I:** minima  $\Delta \phi_j = 0$

**Type II:** saddle point  $\Delta \phi_j = \pi/2$

**Type III:** maxima  $\Delta \phi_j = \pi$



- The inferred luminosity distance and coalescence time would be different for lensed images of an event.
- While parameters such as masses, sky location and spins are expected to be the same for the two events
- Challenging to identify handful of lensed signals amongst thousands of unlensed ones.

# IIIA. Posterior-overlap based multi-image analysis [Haris et al., 2018]

*Check the consistency of posteriors between the pair of events.. fast!*

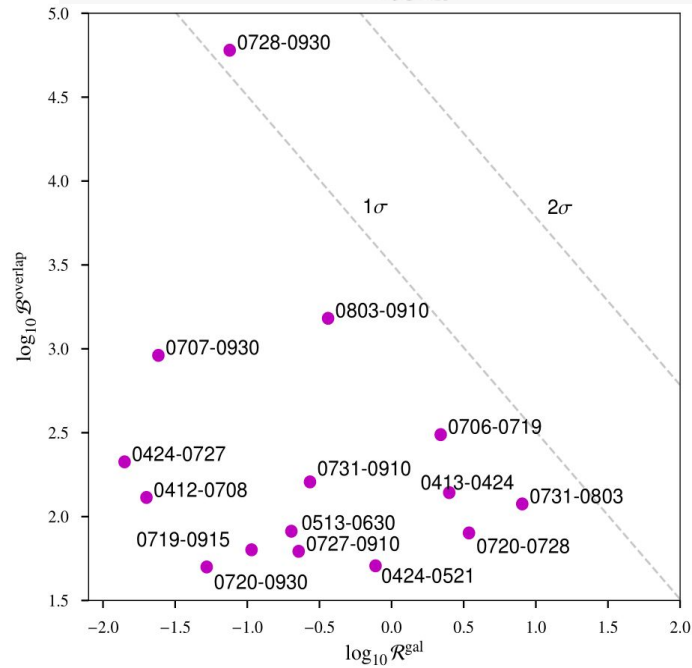
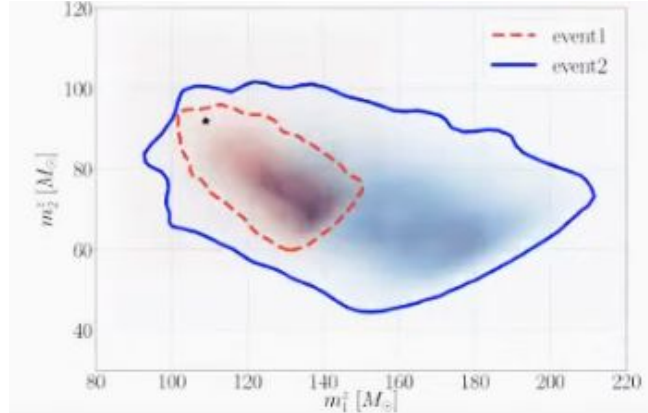
Lensed hypothesis v/s unlensed hypothesis  
for a selected pair of signals:

$$\mathcal{B}^{\text{overlap}} = \int d\Theta \frac{p(\Theta|d_1) p(\Theta|d_2)}{p(\Theta)},$$

$$\mathcal{R}^{\text{gal}} = \frac{p(\Delta t|\mathcal{H}_{\text{SL}})}{p(\Delta t|\mathcal{H}_{\text{U}})},$$

Prior ratio of time delay distributions  
(based only on galaxy lenses!)

Using posteriors of 39 events, **19 event pairs with high bayes factors given to Joint-PE analysis.**





# III.B joint parameter estimation analysis

*This analysis includes information from the Morse phase which depends on the type of image... by the joint parameter estimation for the event pairs under the lensed hypothesis.*

## LALInference pipeline

*[Methods paper Liu et al. 2020 ([ApJ](#))]*

- Aligned-spin quadrupole radiation included (IMRPhenomD, 11 param.)
- Morse phase equivalent to shift coalescence phase
- Treat each phase shift separately
- Applied to O1+O2 data

## HANABI pipeline

*[Methods paper Lo&Magaña 2021 ([arXiv](#))]*

- Higher modes + precession included (IMRPhenomXPHM, 15 param.)
- Morse phase added in frequency domain
- Sample over image types
- **Includes source and lens population priors**
- **Includes selection effects**

# III.B Joint parameter estimation results

Event 1	Event 2	$\log_{10} \mathcal{R}^{\text{gal}}$	$\log_{10}(C_{\text{U}}^{\text{L}})$ LALINFERENCE ( $\Delta\phi$ : 0, $\pi/2$ , $\pi$ , $3\pi/2$ )	$\log_{10}(C_{\text{U}}^{\text{L}} _{\text{pop}})$ HANABI	$\log_{10}(\mathcal{B}_{\text{U}}^{\text{L}})$ HANABI
GW190412	GW190708_232457	-1.7	(+1.0, -9.7, -22.8, -4.4)	-5.6	-8.0
<b>GW190421_213856</b>	<b>GW190910_112807</b>	-	(+4.5, +2.5, -1.5, -0.0)	0.67	<b>-1.8</b>
<b>GW190424_180648</b>	<b>GW190727_060333</b>	-1.9	(+4.9, +0.0, +1.1, +4.0)	0.96	<b>-1.5</b>
<b>GW190424_180648</b>	<b>GW190910_112807</b>	-	(+2.5, +4.7, +4.3, +1.6)	0.62	-1.8
<b>GW190513_205428</b>	<b>GW190630_185205</b>	-0.7	(+0.8, +4.3, -1.9, -6.5)	-0.39	-2.8
GW190706_222641	GW190719_215514	0.34	(+2.4, +2.4, -0.0, -0.5)	0.81	<b>-1.7</b>
GW190707_093326	GW190930_133541	-1.6	(-4.6, -4.3, -3.5, -4.1)	-8.2	-11.
<b>GW190719_215514</b>	<b>GW190915_235702</b>	-1.	(+3.5, -2.1, -0.1, +4.1)	1.4	<b>-1.1</b>
GW190720_000836	GW190728_064510	0.54	(-1.4, -0.9, -4.5, -5.4)	-6.0	-8.5
GW190720_000836	GW190930_133541	-1.3	(-3.5, -2.8, -3.9, -3.9)	-8.2	-11.
GW190728_064510	GW190930_133541	-1.1	(-3.6, -2.5, -3.1, -2.9)	-7.	-9.8
GW190413_052954	GW190424_180648	0.4	(+0.6, -0.9, +0.4, -0.0)	0.35	-2.1
<b>GW190421_213856</b>	<b>GW190731_140936</b>	-2.1	(+3.1, -1.9, +2.5, +5.2)	1.7	<b>-0.79</b>
<b>GW190424_180648</b>	<b>GW190521_074359</b>	-0.1	(+1.3, +3.8, +3.7, +4.4)	-0.64	<b>-3.1</b>
<b>GW190424_180648</b>	<b>GW190803_022701</b>	-2.1	(+4.2, +1.9, +2.6, +3.1)	0.81	<b>-1.7</b>
GW190727_060333	GW190910_112807	-0.6	(+1.8, +3.3, +3.7, +3.4)	0.12	-2.3
<b>GW190731_140936</b>	<b>GW190803_022701</b>	0.9	(+4.1, +3.2, +2.2, +3.4)	1.1	<b>-1.3</b>
<b>GW190731_140936</b>	<b>GW190910_112807</b>	-0.6	(+0.1, +4.5, +0.8, -7.2)	0.92	<b>-2.1</b>
<b>GW190803_022701</b>	<b>GW190910_112807</b>	-0.4	(+4.0, +5.5, +4.7, +2.6)	1.5	<b>-0.98</b>

**Coherence ratio:**  
Overlap information

**Population-weighted  
coh. ratio:** overlap +  
prior BBH and lens  
population

**Bayes factor:** overlap  
+ pop. prior +  
selection effects

**No evidence of  
strongly lensed  
super-threshold  
pairs in GWTC-2**

Table 3

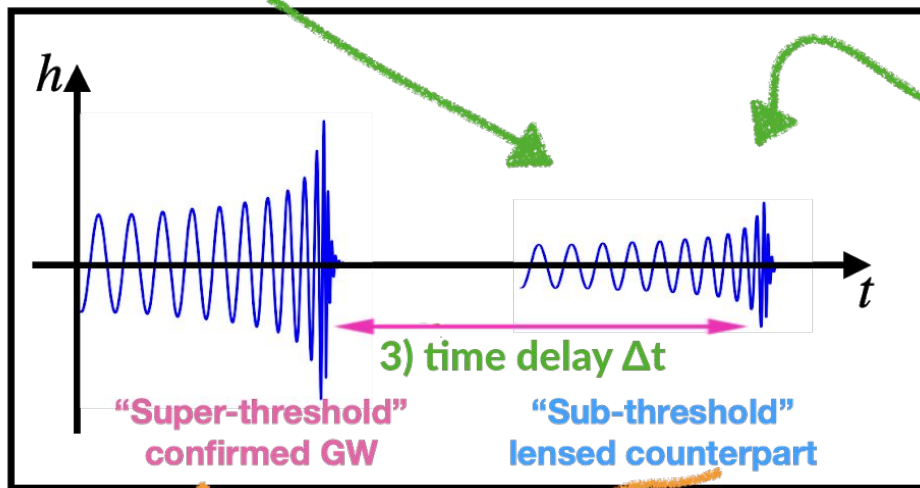
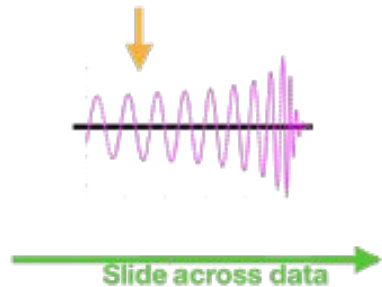
# III.C Search for sub-threshold lensed images

*Strongly lensed events could have **fainter counterparts** not yet identified in wide parameter space searches..*

- Targeted searches can reduce the noise background thanks to a smaller trials factor when only looking for lensed waveforms that are identical up to the 3 points discussed before:

1) An overall scaling factor

Gravitational wave waveform Template



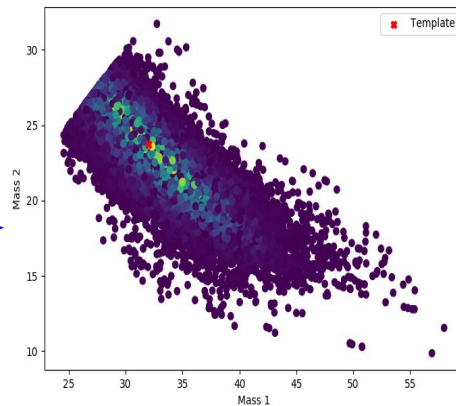
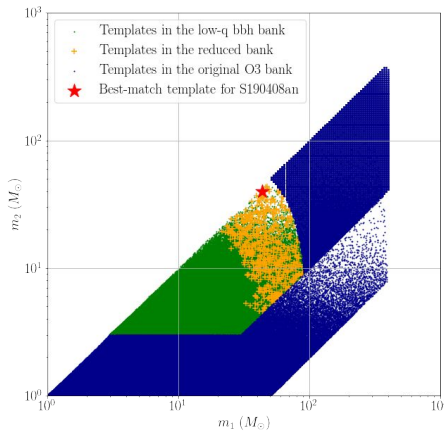
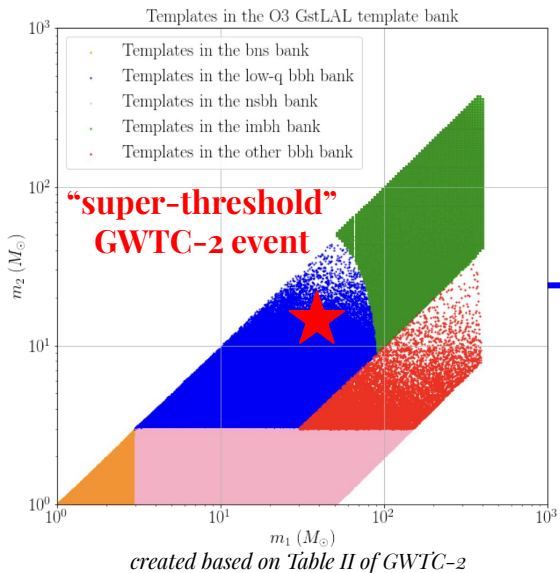
2) A Morse phase factor

(not explicitly covered in these searches)

Image credits: Alvin K.Y. Li

They share **similar intrinsic parameters!**  
(e.g. masses, spins...)

- Two matched-filter pipelines based on those already used in GWTC-2, two different targeted search strategies



## 1) GSTLAL lensing search

- based on GSTLAL pipeline [Sachdev+2019; Hanna+2020; Messick+2017]
- lensing adaptation following [Li+2019](#)
- targeted template banks based on recovery of injections with parameters drawn from GWTC-2 posterior samples

## 2) PyCBC lensing search

- based on PyCBC pipeline [Nitz+2017]
- lensing adaptation following [McIsaac+2019](#)
- a single template per target (max-posterior of GWTC-2 samples)

# III.C Search for sub-threshold lensed images

- *8 new triggers found with FAR < 1/16 yr* (6 of them unique)
- 6 candidate pairs followed up with LALInference joint PE assuming the triggers are astrophysical (we did not calculate a  $p_{\text{astro}}$  here for whether they are!)
- some pairs *consistent* with shared parameters, but compared with results shown before for GWTC-2 pairs, the obtained  $C_{\text{U}}^{\text{L}}$  give no evidence for lensed pairs

UTC time	GWTC-2 targeted event	$ \Delta t $ [d]	$(1+z)M$ [ $M_{\odot}$ ]	FAR [ $\text{yr}^{-1}$ ]		$O_{90\%CR}$ [%]	$\log_{10} C_{\text{U}}^{\text{L}}$ (LALINFERENCE) ( $\Delta\phi$ : 0, $\pi/2$ , $\pi$ , $3\pi/2$ )
				PYCBC	GstLAL		
2019 Sep 25 23:28:45	GW190828_065509	28.69	17.3	0.003	98.681	0.0%	–
2019 Apr 26 19:06:42	GW190424_180648	2.04	65.5	–	0.017	63.8%	(–5.8, –5.8, –5.9, –5.6)
2019 Jul 11 03:07:56	GW190421_213856	80.23	47.7	0.032	0.341	1.2%	(+2.3, +1.1, +1.1, +2.6)
2019 Jul 25 17:47:28	GW190728_064510	2.54	9.0	–	0.038	0.0%	–
2019 Jul 11 03:07:56	GW190731_140936	20.46	47.4	0.045	0.944	2.9%	(+2.6, –1.2, –1.6, +0.9)
2019 Aug 05 21:11:37	GW190424_180648	103.13	68.8	–	0.051	26.9%	(–1.1, +0.6, –0.3, –0.7)
2019 Jul 11 03:07:56	GW190909_114149	60.36	49.0	0.053	1.196	12.6%	(+3.5, +2.2, +3.4, +2.9)
2019 Sep 16 20:06:58	GW190620_030421	88.71	53.3	0.055	1.389	49.5%	(+1.7, +3.6, +2.1, –3.2)

- last pair (highest  $C_{\text{U}}^{\text{L}}$ ) has  $\log_{10}(\mathcal{B}_{\text{U}}^{\text{L}}) = -3.2$  from Hanabi

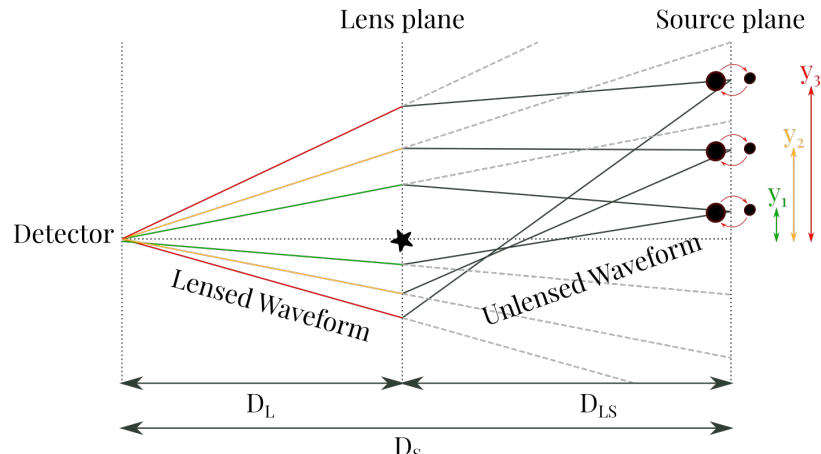
# IV. Microlensing search (wave-optics limit)

*Microlenses which are comparable to the GW wavelength can modulate the waveforms by frequency-dependent amplification factors...*

$$h^{ML}(f; \theta_{ML}) = h^U(f; \theta) F(f; M_L^z, y)$$

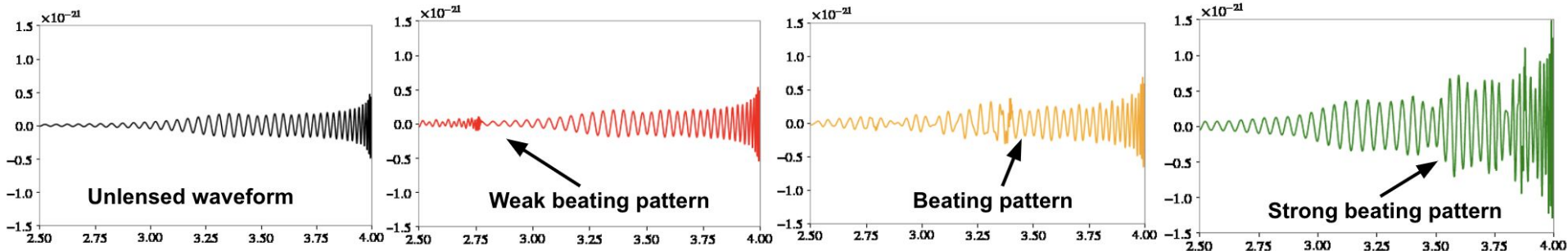
red-shifted lens mass

source position



- Lensed images with time delays shorter than the chirp time of the signal superpose to create **beating patterns** which are more significant when the GW passes closer to the lens (ie. smaller  $y$ ).

3 microlens systems with same  $M_L^z$  and different source positions

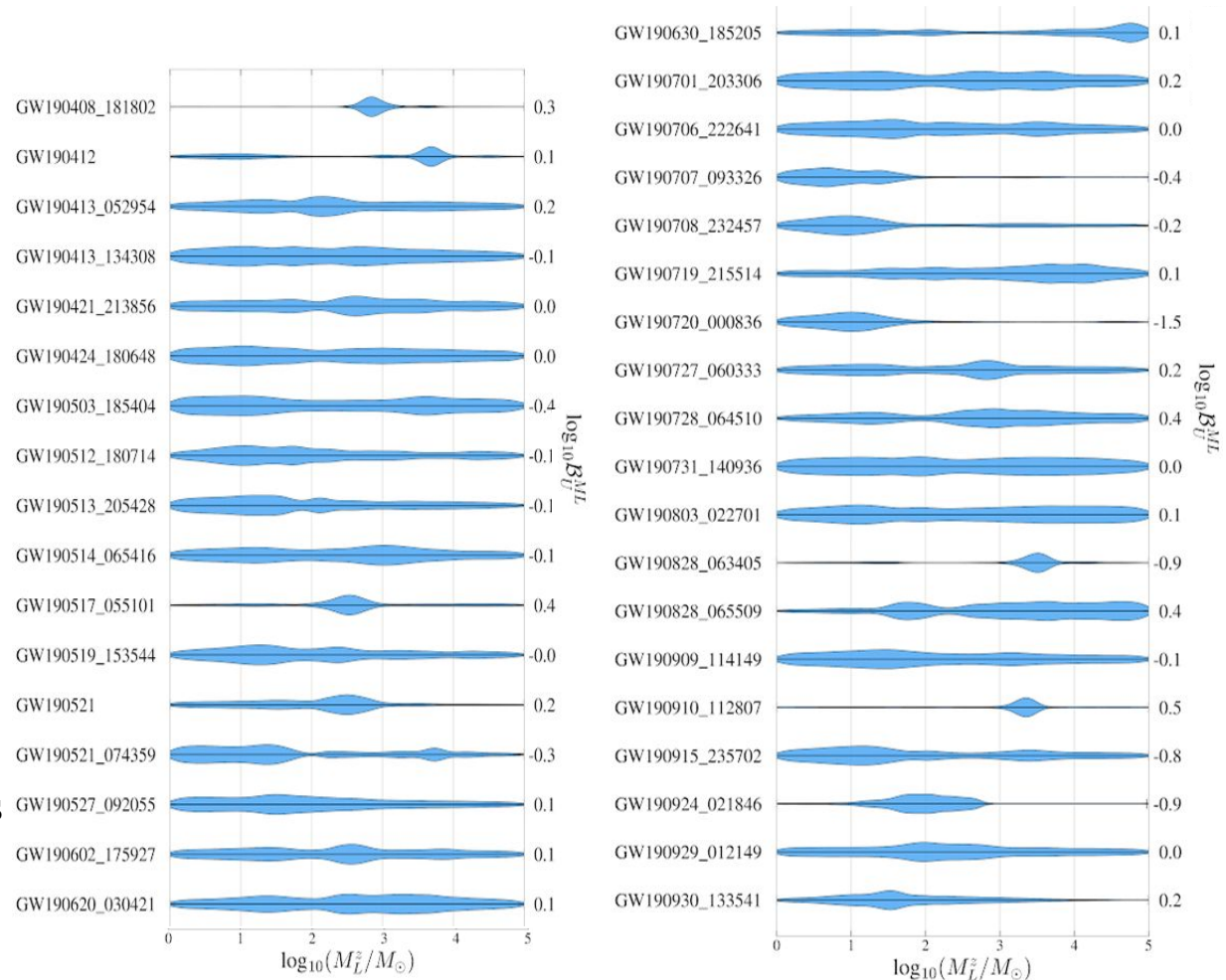


## Search Method:

- Investigate lensing signatures due to **isolated point mass lens** on O3a events by calculating Bayes' factors between the two (lensed & unlensed) hypotheses.

## Results:

- For none of the events are the posteriors well recovered, no high Bayes factors.
- Bayes factors for all events within the statistical fluctuations expected for unlensed events.
- **No microlensing effect observed.**



# Conclusions

Four gravitational-wave analyses on O3a data:

- Statistical forecasts, constraining the rate of lensing
- Analysis of high-mass events under the hypothesis that they might be lensed
- Three searches for multiple images from strong lensing
- Search for microlensing-induced beating patterns
  
- First LVC analysis on a topic that is expected to be pursued further with new data (see the [LVK white paper](#)), **stay tuned for O3b lensing analysis.**
  
- As the current detector network expands and its sensitivity increases, our chances to detect lensing will improve.

**Thank You for listening!**