# Tidal vs absolute acceleration effects on Rindler probes

Chennai Symposium on Gravitation and Cosmology

Hari K

based on K. Hari and D. Kothawala, Phys. Rev. D 104, 064032 (2021);
arXiv:2106.14496 [gr-qc]

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# Outline

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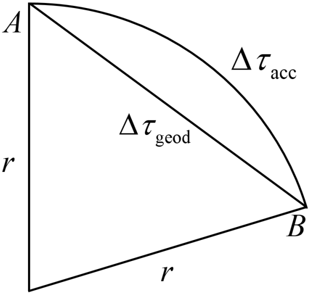
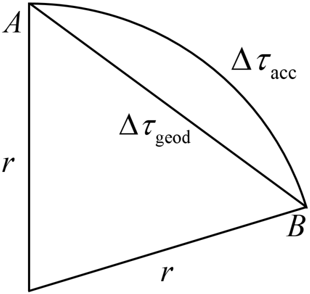
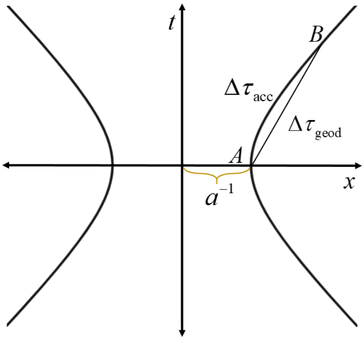
- With the available series expansion, it not clear about the curvature contribution to this relation.




Analogue from 2D - circle,

$$\Delta\tau_{\mathrm{geod}} = 2r\sin\left(\frac{\Delta\tau_{\mathrm{acc}}}{2r}\right)$$

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For Minkowski spacetime,

$$\Delta\tau_{\text{geod}} = \frac{2}{a}\sinh\left(\frac{a\Delta\tau_{\text{acc}}}{2}\right)$$

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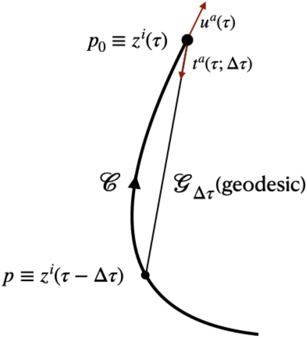
# Setting up the problem

- The accelerated trajectory $\mathcal{C}(\tau)$, 4-velocity $u^i(\tau)$ and $a^i = \nabla_u u^i$.

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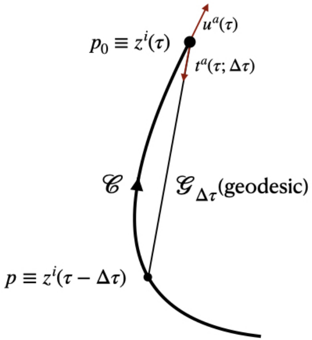


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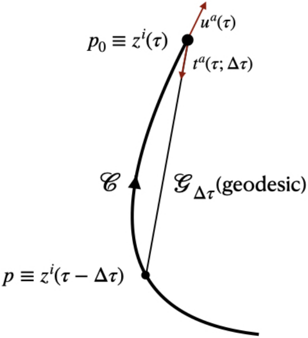
$$\eta_{ab}\widehat{t^a}(0;\Delta\tau)\widehat{t^b}(0;\Delta\tau) = -1$$

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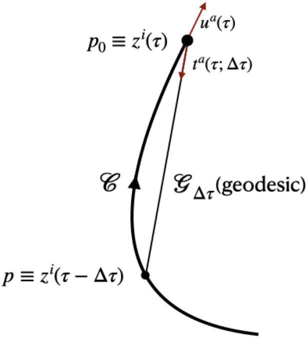
- Equating $\hat{z}^a(-\tau_{\rm acc}) = \hat{x}^a(p)$

$$(\tau_{\rm geod})^2 = \eta_{ab}\hat{z}^a(-\tau_{\rm acc})\hat{z}^b(-\tau_{\rm acc})$$

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$$\hat{z}^k(\tau) \quad = \quad \sum_{n=0}^{\infty} \frac{\tau^n}{n!} \left[\frac{\mathrm{d}^n \hat{z}^k}{\mathrm{d}\tau^n}\right]_{\tau=0}$$

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- Apply i) and ii), and relate $\mathrm{d}\hat{z}^i/\mathrm{d}\tau$ with $\hat{u}^i$ and $\mathrm{d}^2\hat{z}^i/\mathrm{d}\tau^2$ with $\hat{a}^i$.

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- Higher derivatives are computed from successive differentiation of definition of acceleration.

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# Results

- Implemented CADABRA[3] for the calculation and obtained the relation to $O(\Delta\tau^{10})$:

$$
\begin{aligned}
\tau_{\text{geod}}^2 = \tau_{\text{acc}}^2 &+ \frac{1}{12} a^2 \tau_{\text{acc}}^4 + \frac{1}{360}\left(a^4 + 3a^2 \mathscr{E}_n\right)\tau_{\text{acc}}^6 \\
&+ \frac{1}{20160}\left(a^6 + 17a^2\mathscr{E}_n^2 + 18a^4\mathscr{E}_n\right)\tau_{\text{acc}}^8 \\
&+ \frac{1}{1814400}\left(a^8 + 81a^6\mathscr{E}_n + 339a^4\mathscr{E}_n^2 + 155a^2\mathscr{E}_n^3\right)\tau_{\text{acc}}^{10} \\
&+ O(\tau_{\text{acc}}^{12}) + \mathscr{R}_A
\end{aligned}
$$

Here, $\mathscr{E}_n := R_{0n0n} = R_{abcd}u^a n^b u^c n^d$ and $\mathscr{R}_A$ collectively represents all terms of Riemann tensor with at least one index, neither $0$ nor $n$

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- Main observation → exact summation to remarkable expression:

$$\tau_{\rm geod} \quad = \quad \frac{2}{\sqrt{-\mathscr{E}_n}}\sinh^{-1}\left[\sqrt{\frac{-\mathscr{E}_n}{a^2-\mathscr{E}_n}}\sinh\left(\frac{\sqrt{a^2-\mathscr{E}_n}\,\tau_{\rm acc}}{2}\right)\right]+\mathscr{R}_A$$

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# Application to well known spacetimes

- *Maximally symmetric spacetimes*: $\mathscr{R}_A = 0$ due to the structure of Riemann tensor,

$$\tau_{\rm geod} \quad = \quad \frac{2}{\sqrt{\Lambda}} \sinh^{-1}\left[\sqrt{\frac{\Lambda}{a^2+\Lambda}} \sinh\left(\frac{\sqrt{a^2+\Lambda}\,\tau_{\rm acc}}{2}\right)\right]$$

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- *Static, Spherically symmetric spacetimes*: For motion is $t - r$ plane, number of $n$ and $0$ on Riemann tensor, $\Rightarrow \mathscr{R}_A = 0$. Generally, $\nabla R_{abcd} \neq 0$, result will only be approximate.

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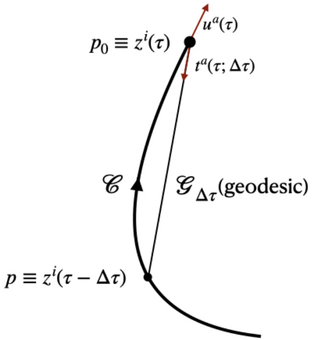
# Twin paradox

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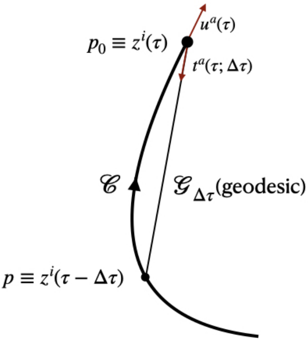


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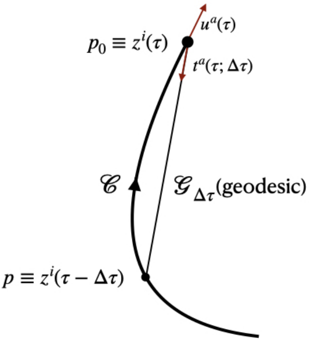


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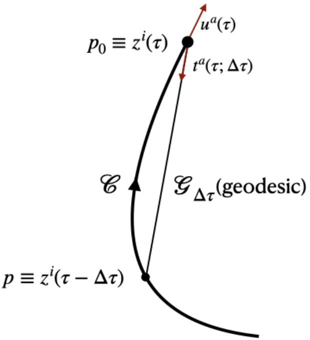


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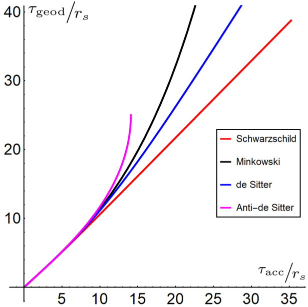


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# Unruh-de Witt detector

- Transition rate which is the time derivative of response function of the detector given by,

$$\dot{\mathcal{F}}(\omega) = 2\int_0^{\tau' - \tau_0} \mathrm{d}s\, \mathrm{Re}\left[e^{-i\omega s} G^{+}(\tau', \tau' - s)\right]$$

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- The Wightmann two-point function in a Hadamard state is given by[5],

$$G^+(x, x') = \frac{1}{4\pi^2}\left(\frac{\Delta^{\frac{1}{2}}(x, x')}{\sigma_\epsilon^2(x, x')} + v(x, x')\ln\left[\sigma_\epsilon^2(x, x')\right]\right)$$

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- Computation similar to Rindler motion in Minkowski spacetime yields

$$\dot{\mathcal{F}}(\omega;\tau) \quad = \quad \frac{\omega}{2\pi}\left[\exp\left(\frac{\hbar\omega}{[k_{\mathrm{B}}T]_{\mathscr{E}_n}}\right)-1\right]^{-1} + (\mathscr{R}_A, \nabla R_{abcd} \text{ terms})$$

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- For maximally symmetric spacetimes[6], with curvature length scale $\Lambda$, $\mathscr{E}_n = -\Lambda$

$$[k_{\rm B}T]_{\mathscr{E}_n} = \frac{\hbar}{2\pi}\sqrt{a^2+\Lambda}$$

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- In $N \to 0$ limit, the equivalence principle holds.

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- Since $a \neq a_{\text{eff}}$, the observer can distinguish the acceleration and gravitational effects in the local frame, this is at variance with the equivalence principle.

# Outline

# Conclusion and summary

- We obtained a semi-analytic expression for the relation between the geodesic and proper time intervals which is analytic.
- It is a combination of hyperbolic functions and depends on acceleration $a$ only through the combination $q = \sqrt{a^2 - \mathscr{E}_n}$
- The expression gives novel insights into the role of tidal vs. absolute acceleration, which usual Taylor expansion will not give.
- For maximal symmetry, expression is exact and spherical symmetry, result is approximate.
- Classical $\rightarrow$ differential ageing of twins.
- Quantum $\rightarrow$ gives a thermal contribution to the *detector response* with a modified *Unruh temperature* $[k_{\mathrm{B}}T]_{\mathscr{E}_n} = (\hbar/2\pi)\sqrt{a^2 - \mathscr{E}_n}$.
- We also encounter variation from equivalence principle due to the contribution from the tidal part of the curvature.

# Thank You !

- For Maximally symmetric spacetime with constant curvature $\Lambda$,

$$
\begin{aligned}
R_{abcd} &= \Lambda\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right) \\
R_{ab} &= D_1 \Lambda g_{ab} \\
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◂ back

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back

- The bound in anti-de Sitter spacetime is;

$$\tau_{\text{acc}} \leq \frac{2}{\sqrt{a^2 - |\Lambda|}} \sinh^{-1}\left[ \sqrt{\frac{a^2 - |\Lambda|}{|\Lambda|}} \right] \leq \frac{2}{\sqrt{|\Lambda|}}$$

back

## *Hyperbolic motion and Rindler condition*

- Imposing Serret-Frenet formula to spacetime curves gives;

$$\begin{aligned} \nabla_{\boldsymbol{u}} u^k &= a n^k \quad ; \quad \nabla_{\boldsymbol{u}} a = 0 \\ \nabla_{\boldsymbol{u}} n^k &= a u^k \end{aligned}$$

- The Rindler conditions immediately help us take care of all covariant derivatives of $a^i$ along $u^i$ in Eq. 1, since they imply

$$\begin{aligned} (\nabla_{\boldsymbol{u}})^p a^k &= a^{2p} u^k \qquad (p = 1, 3, 5, ...) \\ (\nabla_{\boldsymbol{u}})^p a^k &= a^{2p-1} n^k \qquad (p = 2, 4, 6, ...) \end{aligned}$$

back

- The Wightmann two-point function for the Hadamard state is given by, $G^+(x,x') := \left\langle \mathsf{H} \middle| \hat{\phi}(x)\hat{\phi}(x') \middle| \mathsf{H} \right\rangle$ where $x$ is the coordinate position.

- The projection of this Wightmann function along the accelerated trajectory $G^+(\tau,\tau') = G^+(x,x')$, is used for transition rate.

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$$G^+(x,x') = \frac{1}{4\pi^2}\left(\frac{\Delta^{\frac{1}{2}}(x,x')}{\sigma_\epsilon^2(x,x')} + v(x,x')\ln\left[\sigma_\epsilon^2(x,x')\right]\right)$$

where $\Delta(x,x')$ is the VVD, $\epsilon$ is a small positive parameter, $\sigma_\epsilon^2(x,x')$ is the world function with $i\epsilon$ prescription given as, $\sigma_\epsilon^2(x,x') := \sigma^2(x,x') + 2i\epsilon\left[T(x) - T(x')\right]$, where $T(x)$ is increasing global time function and $v(x,x')$ is polynomial function of $\sigma^2(x,x')$

◂ back