FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

Princeton University

Princeton gravity Initiative

DAAD
Deutscher Akademischer Austausch Dienst
German Academic Exchange Service

Black holes beyond General Relativity: shadows, stability, and nonlinear evolution

work with
Astrid Eichhorn
Roman Gold
Philipp Johannsen
Heloise Delaporte

work with
Sebastian Garcia-Saenz
Jun Zhang

work with
Hyun Lim

Aaron Held

DAAD PRIME Fellow, currently at **The Princeton Gravity Initiative**

03rd February 2022, Second Chennai Symposium on Gravitation and Cosmology

Black-hole phenomenology in **EFTs** **beyond GR**

(ng)EHT collaboration

Part I: background solutions

LIGO and Virgo collaborations

Part II: linear dynamics

Part III: nonlinear dynamics

leading-order EFT corrections

$$\mathcal{L}_{\text{EFT}} = \frac{1}{16\pi G} R + \alpha R_{ab} R^{ab} - \beta R^2$$

ghost-free theories

Lovelock's theorem → field equations of GR

+ **no other DOF**
+ four dimensions
+ quasi-local action

w/ Astrid Eichhorn
JCAP 05 (2021) 073
EPJC 81 (2021)
...

w/ Jun Zhang
(to appear)

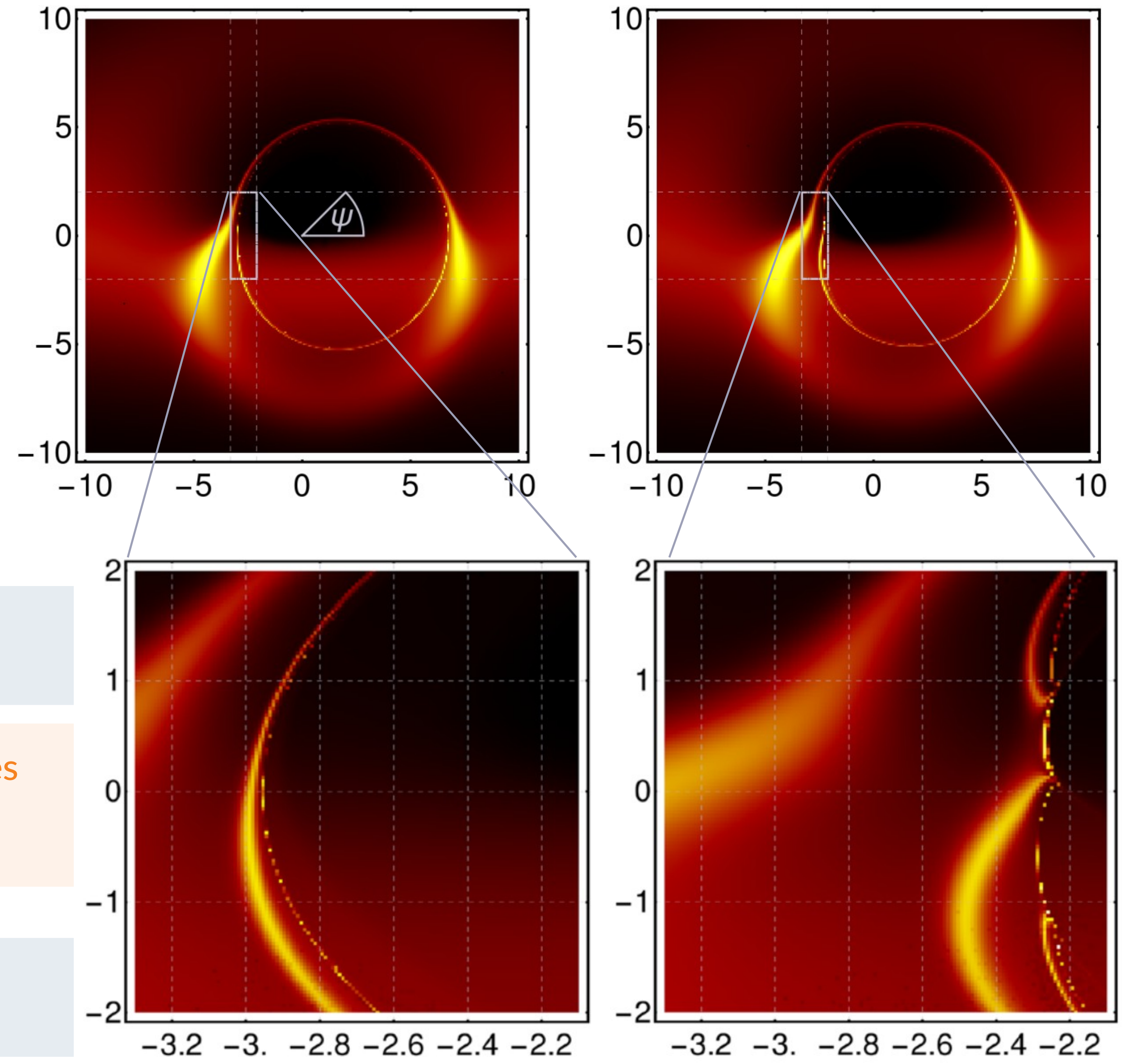
w/ Hyun Lim
PRD 104 (2021) 8
...

w/ Sebastian Garcia-Saenz & Jun Zhang
PRL 127 (2021) 13
...

w/ Hyun Lim & Claudia de Rham

Part I
Image features from fundamental principles

Eichhorn, Held, JCAP 05 (2021) 073
Eichhorn, Held, Eur.Phys.J.C 81 (2021)
Eichhorn, Held, Johannsen (to appear)
Eichhorn, Held, Delaporte (to appear)



principled approach:

“direct calculation in specific theories beyond GR”

principled-parameterized approach

“identify characteristic image features in families of spacetimes based on fundamental principles”

parameterized approach:

“parameterize all deviations from GR”

Constructing regular spinning black holes: the principled-parameterized approach

Eichhorn, **Held**, JCAP 05 (2021) 073
Eichhorn, **Held**, Eur.Phys.J.C 81 (2021)

$$ds_{\text{Kerr}} = -\frac{\left(\Delta - a^2 \sin^2\theta\right)}{\Sigma} du^2 + 2\, du dr - \frac{2a \sin^2\theta \left(a^2 - \Delta + r^2\right)}{\Sigma} du d\varphi - 2a \sin^2\theta dr d\varphi + \Sigma\, d\theta^2 + \frac{\sin^2\theta \left(\left(a^2 + r^2\right)^2 - a^2 \Delta \sin^2\theta\right)}{\Sigma} d\varphi^2$$

$$\Sigma = r^2 + a^2 \cos^2\theta$$

$$\Delta = r^2 - 2\, G\, M\, r + a^2$$

Kerr spacetime (metric)

Constructing regular spinning black holes: the principled-parameterized approach

Eichhorn, **Held**, JCAP 05 (2021) 073
Eichhorn, **Held**, Eur.Phys.J.C 81 (2021)

$$\left[\frac{\mathbb{I}}{2^4\,3}\right]^3 = \left[\frac{\mathbb{J}}{2^5\,3}\right]^2 = \left[\frac{G\,M}{(r - i\,a\cos\theta)^3}\right]^6$$

& all other Riemann invariants vanish

Kerr spacetime **(invariants)**

Cartan '28
Karlhede '80
MacCallum, Skea, McLenaghan, McCrea
Zakhary, McIntosh '97
Carminati, McLenaghan '91
Held '21 (2105.11458)

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i) **Regularity**
(all curvature invariants are regular everywhere)

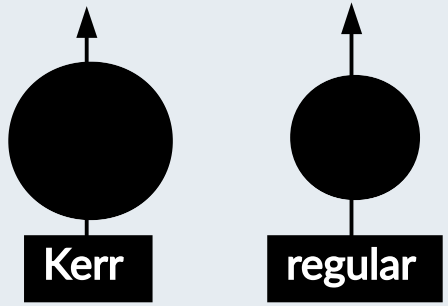
ii) **Newtonian limit**

iii) **Simplicity**
(a single new-physics scale)

iv) **Locality**
(new physics is tied to local curvature scales)

➢ **the horizon shrinks**

$$M \to M(r,\theta) \to \begin{cases} M \text{ for } r \to \infty \\ 0 \text{ for } r \to 0 \end{cases}$$



Constructing regular spinning black holes: the principled-parameterized approach

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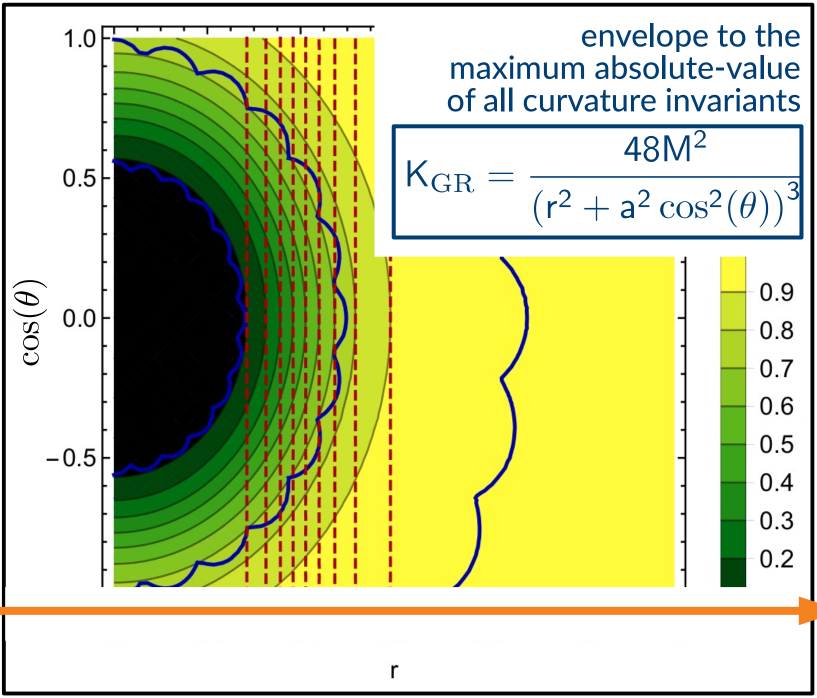
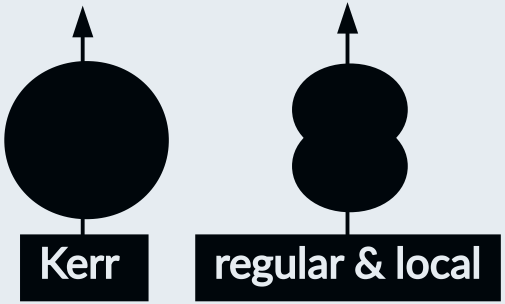


- **the horizon shrinks**

$$M \to M(r,\theta) = \frac{M}{1 + K_{\mathrm{GR}}\,\ell_{\mathrm{NP}}^4}$$



- **dependence on polar angle**

Broderick et. Al ‘21
Wong ‘21

N=1 (with temporal variability)

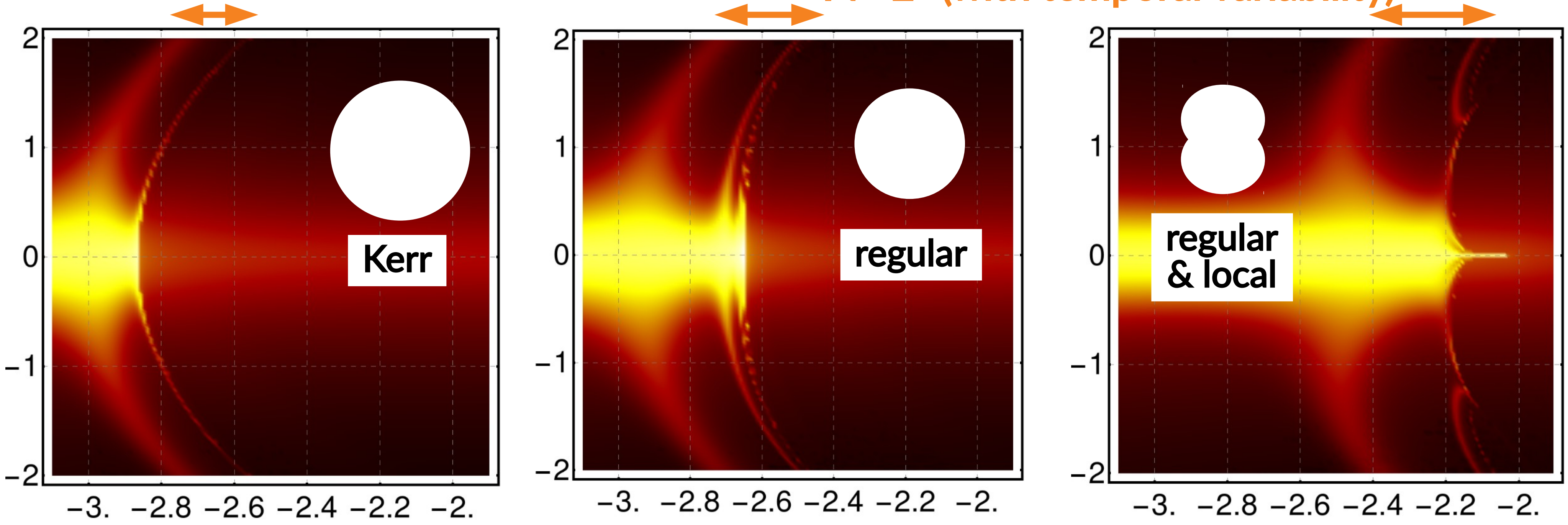
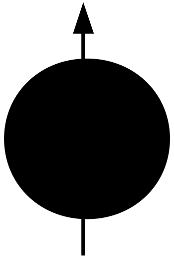


image features due to increased compactness	1) Shadow shrinks 2) Photon rings are stretched out	
prograde image features distinguishing local vs non-local models	3) No asymmetry 4) No cusps or dent	3) Asymmetry 4) **Cusps and dent**

Circularity

$$\xi_1^{[\mu}\xi_2^{\nu}\mathrm{D}^{\kappa}\xi_1^{\lambda]} = 0 \quad \text{at at least one point}$$
$$\xi_2^{[\mu}\xi_1^{\nu}\mathrm{D}^{\kappa}\xi_2^{\lambda]} = 0 \quad \text{at at least one point}$$
$$\xi_1^{\mu}\mathrm{R}_{\mu}{}^{[\nu}\xi_2^{\kappa}\xi_1^{\lambda]} = 0 \quad \text{everywhere}$$
$$\xi_2^{\mu}\mathrm{R}_{\mu}{}^{[\nu}\xi_1^{\kappa}\xi_2^{\lambda]} = 0 \quad \text{everywhere}$$



vacuum GR solutions are **circular**

Kerr

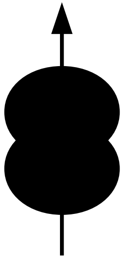
Papapetrou ‘66
Kundt et.Al ‘66
Wald ‘84

general parametrizations of stationary axisymmetric black holes are **circular**

parameterized approach

Johannsen Phys.Rev.D 88 (2013)
Benenti-Francaviglia ‘79
Konoplya et.Al., Phys.Rev.D 93 (2016)
Papapetrou ‘66

see also
Ben Achour et.Al. ‘20
(non-circular solutions to scalar-tensor theory)
Ioka, Sasaki, ‘03 ‘04
(non-circular neutron stars)



new physics tied to local curvature scales leads to **non-circularity**

principled-parameterized approach

Eichhorn, Held
JCAP 05 (2021) 073
EPJC 81 (2021)

Part II.
Linear stability of Black Holes

Held, Zhang (to appear)
Gracia-Saenz, Held, Zhang, PRL 127 (2021) 13

cf. also Lu, Perkins, Pope, Stelle ‘17
Held, Zhang (to appear)

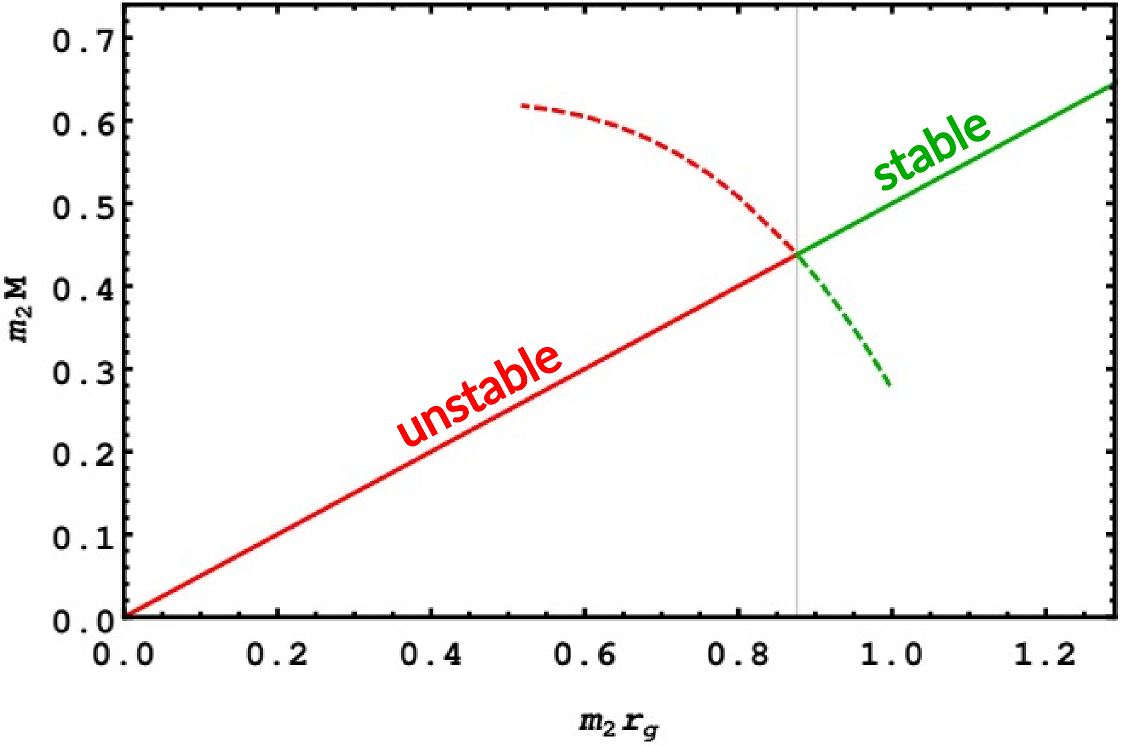
Effective field theory of gravity

$$\mathcal{L}_{\rm EFT} = \frac{1}{16\pi\,\mathrm{G}}\mathrm{R} + \mathcal{O}(\text{curvature}^2)$$

General Relativity (GR)

$$= \frac{1}{16\pi \mathrm{G}}\mathrm{R} + \alpha \mathrm{R}_{ab}\mathrm{R}^{ab} - \beta \mathrm{R}^2 + \mathcal{O}(\text{curvature}^3)$$

$$= \ \ldots$$



Dynamics: linear DoF

$$\mathcal{L}_{QG} = \frac{1}{16\pi G} R + \alpha R_{ab} R^{ab} - \beta R^2$$

$$\mathcal{L}_{QG} = \frac{1}{M_{Pl}^2} \left[\frac{1}{2} R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

- ➢ massless spin-2 (graviton) h_{ab}
- ➢ massive spin-0 ϕ
- ➢ massive spin-2 ψ_{ab}

Decomposition (background)

- spherical harmonics $Y_{\ell m}(\theta, \phi)$
- axisymmetric perturbations $m = 0$
- focus on the monopole $\ell = 0$

$$h_{ab}^{(\text{polar})} = e^{-i\omega t} \begin{pmatrix} AH_0 & H_1 & 0 & 0 \\ H_1 & H_2/B & 0 & 0 \\ 0 & 0 & r^2\mathcal{K} & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta\, \mathcal{K} \end{pmatrix} Y^{\ell=0}$$

$$\psi_{ab}^{(\text{polar})} = e^{-i\omega t} \begin{pmatrix} AF_0 & F_1 & 0 & 0 \\ F_1 & F_2/B & 0 & 0 \\ 0 & 0 & \mathcal{M} & 0 \\ 0 & 0 & 0 & \sin^2\theta\, \mathcal{M} \end{pmatrix} Y^{\ell=0}$$

$$\frac{d^2}{dr_*^2}\psi(r) + \psi(r)\left[\omega^2 - V(r)\right] = 0$$

GR-background: Brito, Cardoso, Pani ‘13
non-GR: Held, Zhang (to appear)

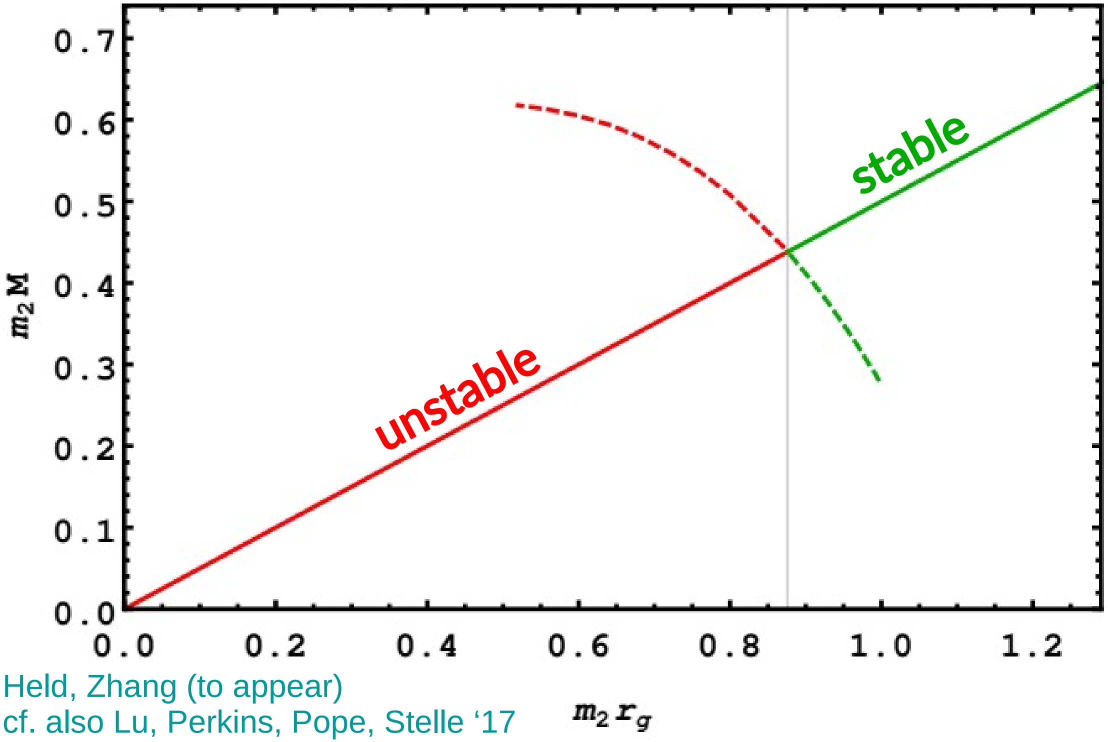
Boundary conditions:

- purely ingoing waves at the horizon
- outgoing waves at asymptotic infinity define QNMs
- ingoing waves at asymptotic infinity define bound states

- positive imaginary part signals instability
- negative imaginary part signals stability

Part II. Linear stability of spherically-symmetric black holes in quadratic gravity

cf. also Brito et.Al Phys. Rev. D 88, (2013)
Collingbourne Math. Phys. 62, (2021)
Held, Zhang (to appear)



Held, Zhang (to appear)
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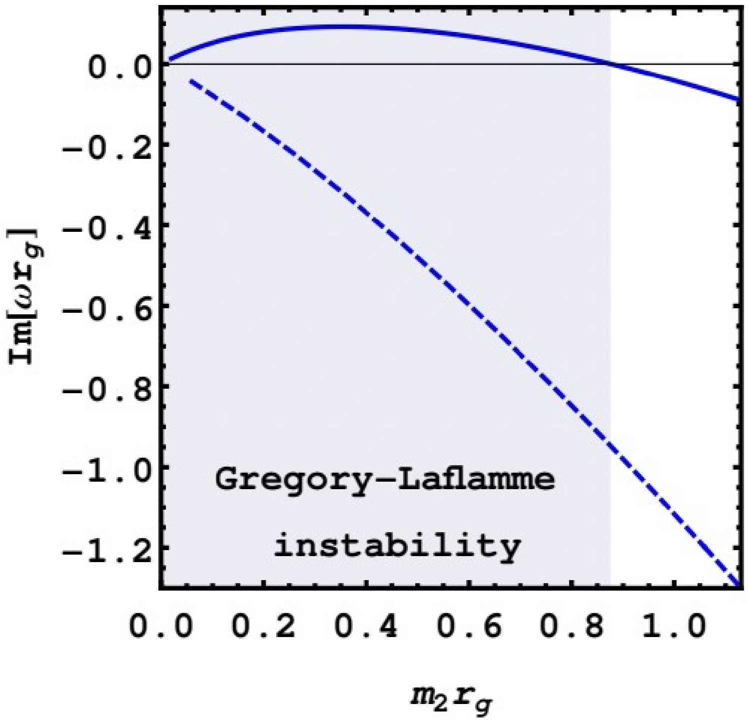

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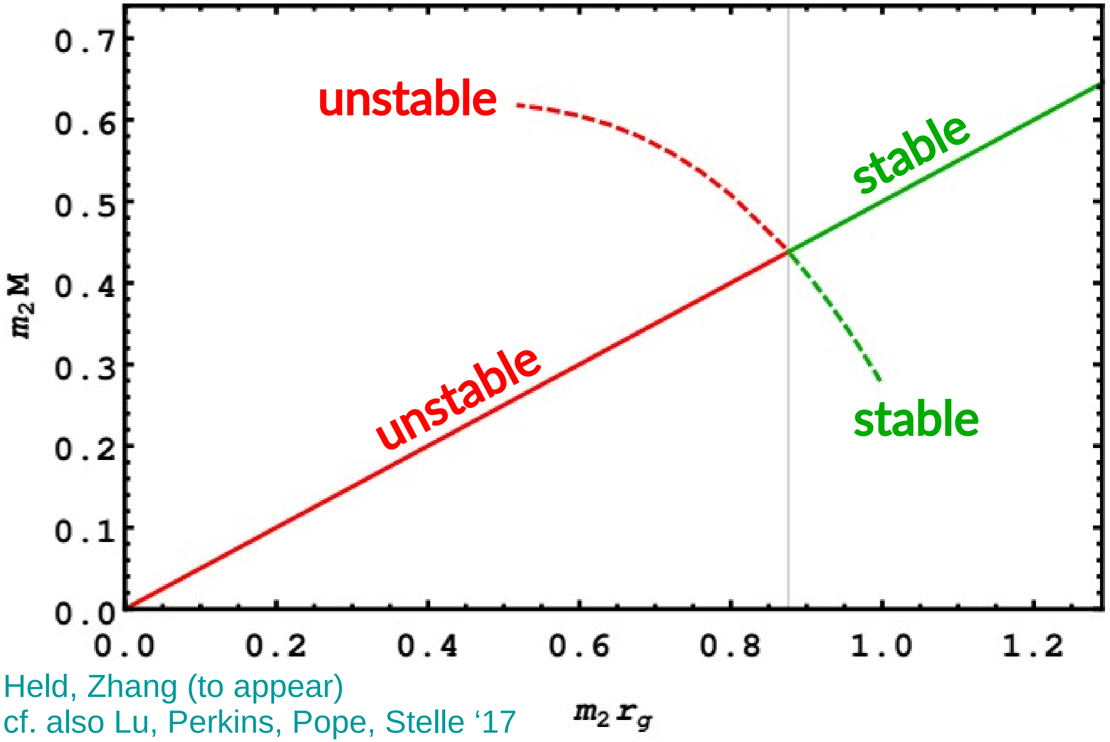

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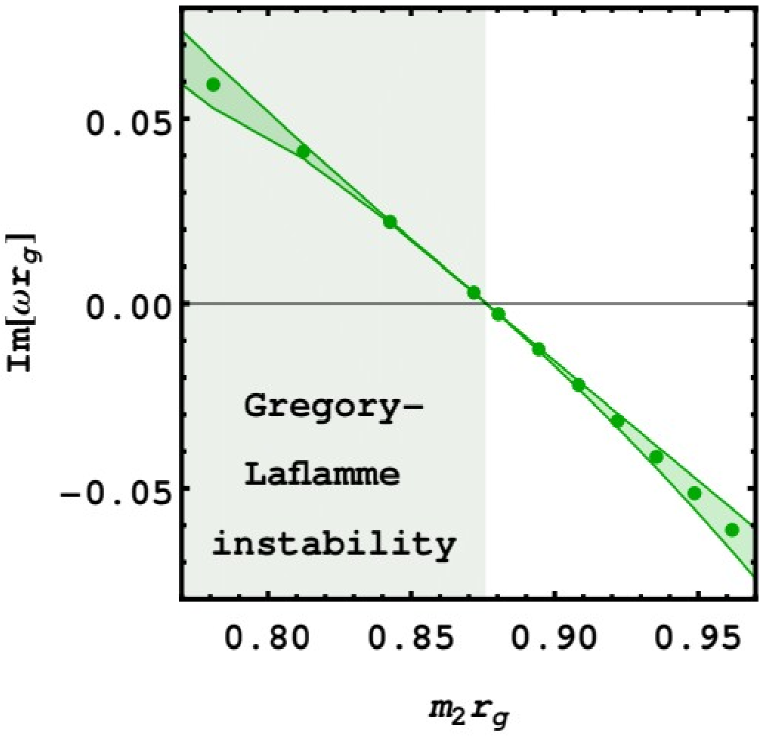
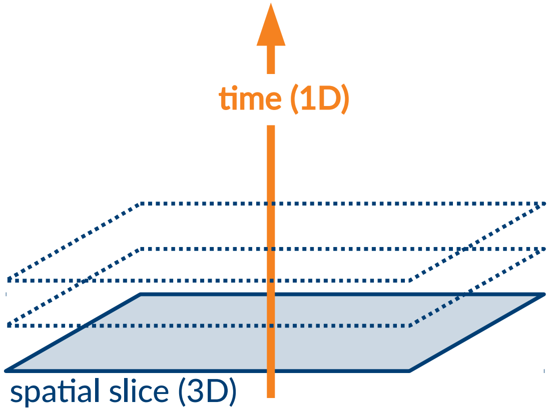


A well-posed initial value problem (IVP) …



" An initial value problem is well-posed if a solution "

- **exists for all future time**
- is **unique**
- and depends **continuous**ly on the initial data

… for General Relativity

Formal proof of existence and uniqueness
Yvonne Choquet-Bruhat '52

(3+1) numerical evolution
Frans Pretorius '05
Baumgarte, Shapiro, Shibata, Nakamura '87-'99
Sarbach et.Al '02-'04

… for Quadratic Gravity

Formal proof of existence and uniqueness
Noakes '83

spherical symmetry: **Held**, Lim, PRD 104 (2021) 8
(3+1): **Held**, Lim, (to appear)

Well-posed IVP in Quadratic Gravity

Noakes, JMP 24, 1846 (1983)

4th order, quasilinear equations of motion

Step 1:
use linear DOFs
in harmonic coords
to cast to 2nd order

$$g^{\alpha\beta}\partial_\alpha\partial_\beta g_{\mu\nu} = \mathcal{F}_1(g, \mathcal{R}, \widetilde{\mathcal{R}}, \partial) \quad \text{(massless spin-2)}$$
$$\Box\mathcal{R} = \mathcal{F}_2(g, \mathcal{R}, \widetilde{\mathcal{R}}, \partial) \quad \text{(massive spin-0)}$$
$$\Box\widetilde{\mathcal{R}}_{\mu\nu} = \mathcal{F}_3(g, \mathcal{R}, \widetilde{\mathcal{R}}, \partial) \quad \text{(massive spin-2)}$$

2nd order, quasilinear + constraints

Step 2:
diagonalize by
adding appropriate
derivatives

$$g^{\alpha\beta}\partial_\alpha\partial_\beta g_{\mu\nu} = \mathcal{F}_1(g, \mathcal{R}, \widetilde{\mathcal{R}}, h, V, \partial) \quad \text{(massless spin-2)}$$
$$\Box\mathcal{R} = \mathcal{F}_2(g, \mathcal{R}, \widetilde{\mathcal{R}}, h, V, \partial) \quad \text{(massive spin-0)}$$
$$\Box\widetilde{\mathcal{R}}_{\mu\nu} = \mathcal{F}_3(g, \mathcal{R}, \widetilde{\mathcal{R}}, h, V, \partial) \quad \text{(massive spin-2)}$$
$$\Box h_{\mu\nu\gamma} = \mathcal{F}_4(g, \mathcal{R}, \widetilde{\mathcal{R}}, h, V, \partial) \quad (h_{\mu\nu\alpha} \equiv \partial_\alpha g_{\mu\nu})$$
$$\Box V_\mu = \mathcal{F}_5(g, \mathcal{R}, \widetilde{\mathcal{R}}, h, V, \partial) \quad (V_\alpha \equiv \partial_\alpha \mathcal{R})$$

2nd order, quasilinear, diagonal + constraints

Leray ‘53, Choquet-Bruhat et.Al ‘77

Leray’s theorem guarantees well-posed IVP for $\mathcal{C}^\infty$ initial data

Numerical Evolution of Quadratic Gravity (sph-symm)

Held, Lim, PRD 104 (2021) 8

Cartoon method to reduce to spherical symmetry

$$\mathbf{u} = (R,\, g_{tt},\, g_{tx},\, g_{xx},\, g_{yy}) \qquad \partial_t^2 \mathbf{u} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \partial_t \mathbf{u}\right)$$

$$\mathbf{v} = (\widetilde{R}_{tt},\, \widetilde{R}_{tx},\, \widetilde{R}_{xx}) \qquad \partial_t^2 \mathbf{v} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \partial_t \mathbf{u},\, \partial_t \mathbf{v},\, \partial_t^2 \mathbf{u}\right)$$

Diagonalization to quasi-linear 2nd-order form

$$\partial_t^2 \dot{\mathbf{u}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \partial_t \dot{\mathbf{u}},\, \partial_t \mathbf{v}\right) \qquad \partial_t \dot{\mathbf{u}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}}\right)$$

$$\partial_t^2 \mathbf{v} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \partial_t \dot{\mathbf{u}},\, \partial_t \mathbf{v}\right) \qquad \partial_t \mathbf{u} \equiv \dot{\mathbf{u}}$$

Reduction to 1^{st} order in time

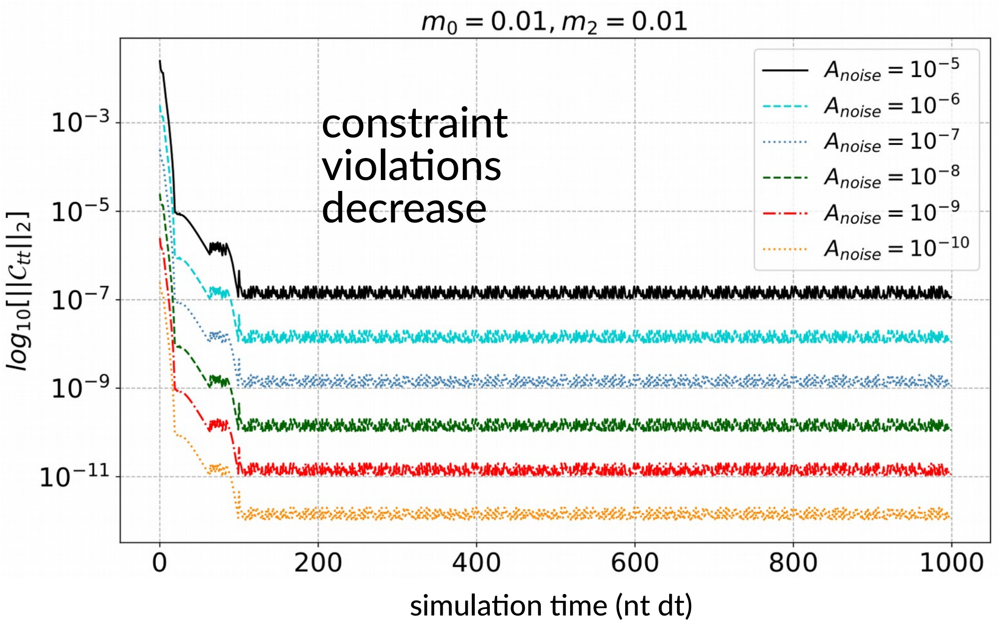
$$\partial_t \ddot{\mathbf{u}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \ddot{\mathbf{u}},\, \dot{\mathbf{v}}\right) \qquad \partial_t \dot{\mathbf{u}} \equiv \ddot{\mathbf{u}} \qquad \ddot{\mathbf{u}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}}\right)$$

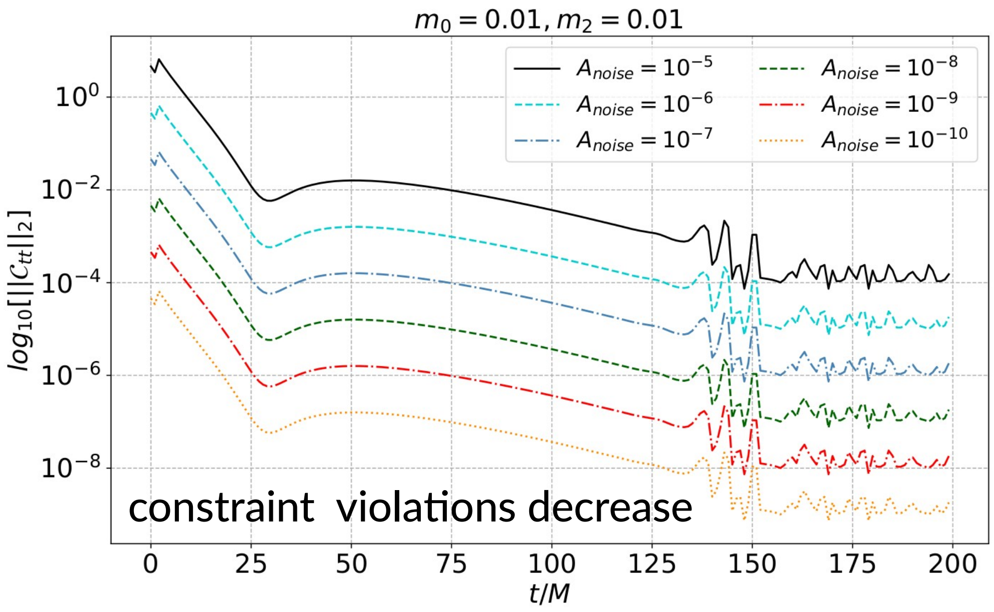
$$\partial_t \dot{\mathbf{v}} = \mathcal{O}\left(\mathbf{u},\, \mathbf{v},\, \dot{\mathbf{u}},\, \ddot{\mathbf{u}},\, \dot{\mathbf{v}}\right) \qquad \partial_t \mathbf{u} \equiv \dot{\mathbf{u}}$$

$$\partial_t \mathbf{v} \equiv \dot{\mathbf{v}}$$

Results: numerical stability ...

Held, Lim, PRD 104 (2021) 8



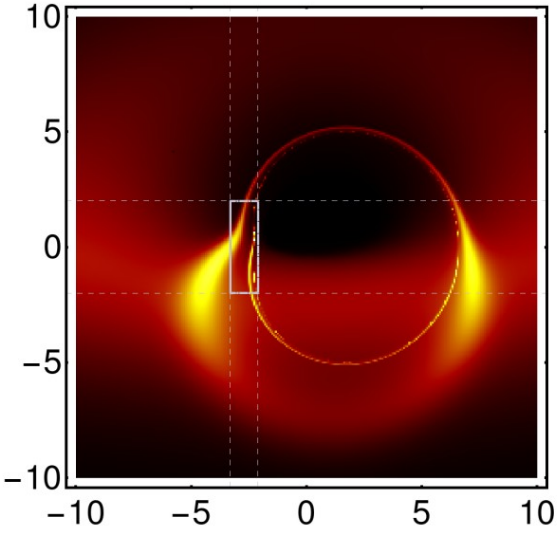


... in **flat spacetime** ... ... and about **Schwarzschild**

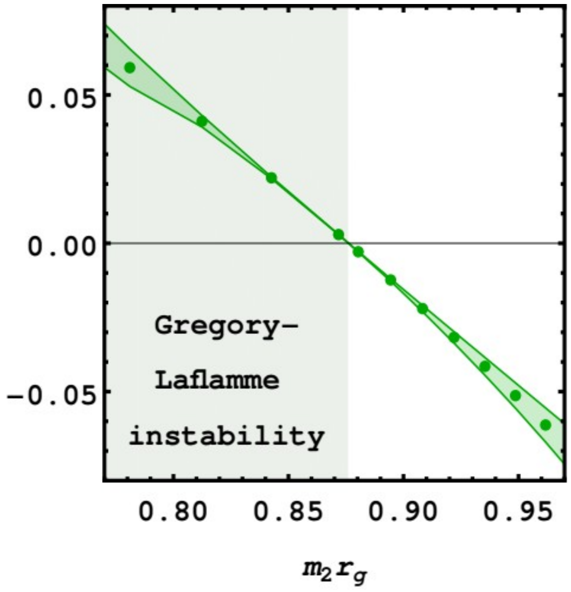
stay tuned for (3+1) evolution

Probing black holes …

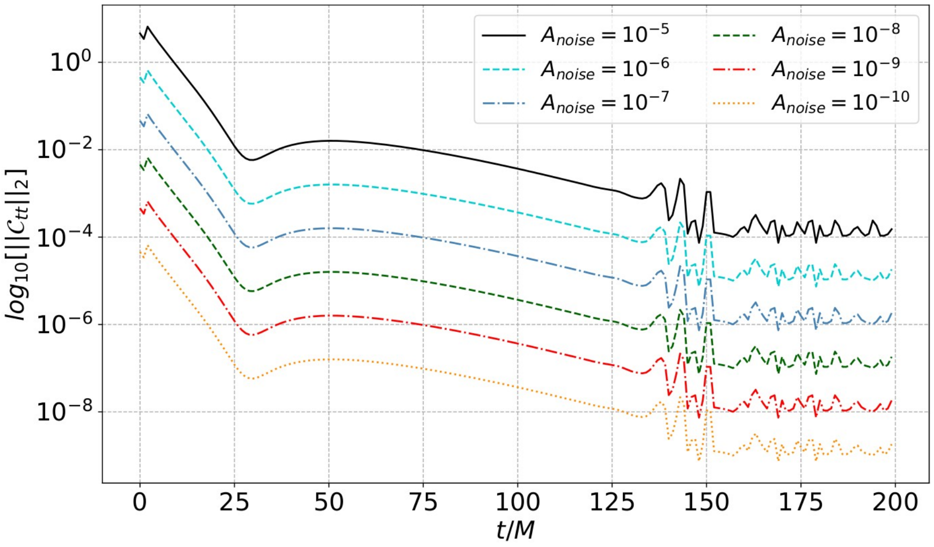
I. background spacetimes



II. linear dynamics



III. non-linear dynamics



… in the EFT of gravity

– thank you –