Cosmological imprints of dynamical gauge fields during inflation

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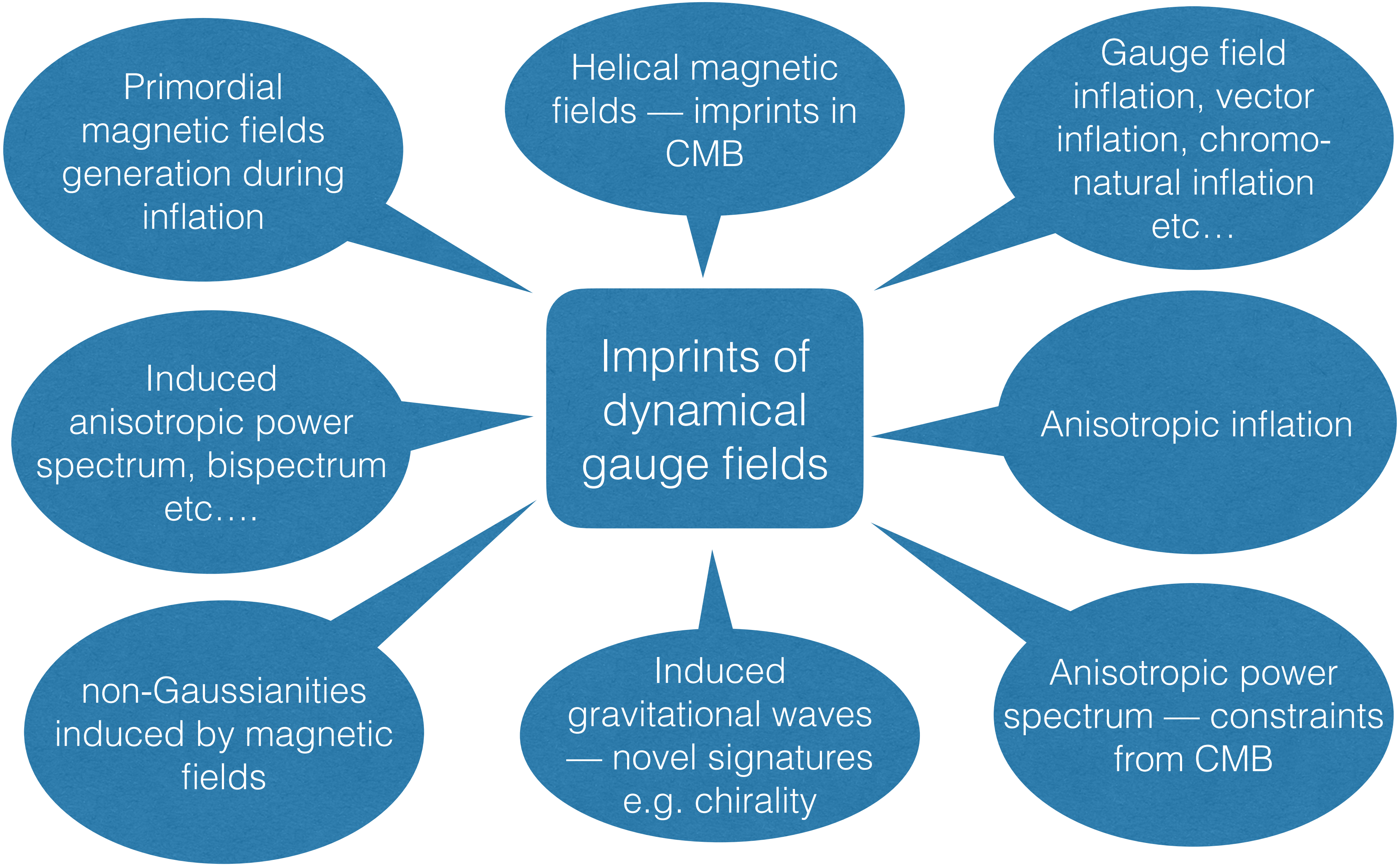
Outline of the talk

- Dynamical gauge fields during inflation — interesting signatures
- Cosmological magnetic fields
    - Inflationary generation mechanisms
    - Stringent constraints — from background and perturbations
- Non-Gaussian imprints — novel consistency relations
    - Cross-correlations with curvature perturbations
    - Cross-correlations with gravitons
- Conclusions

Dynamical gauge fields — cosmological imprints



Dynamical gauge fields — cosmological imprints

Primordial
fields

Gauge field
inflation, vector
chromo-
nflation
…

Anisotr
spectrum
e

c inflation

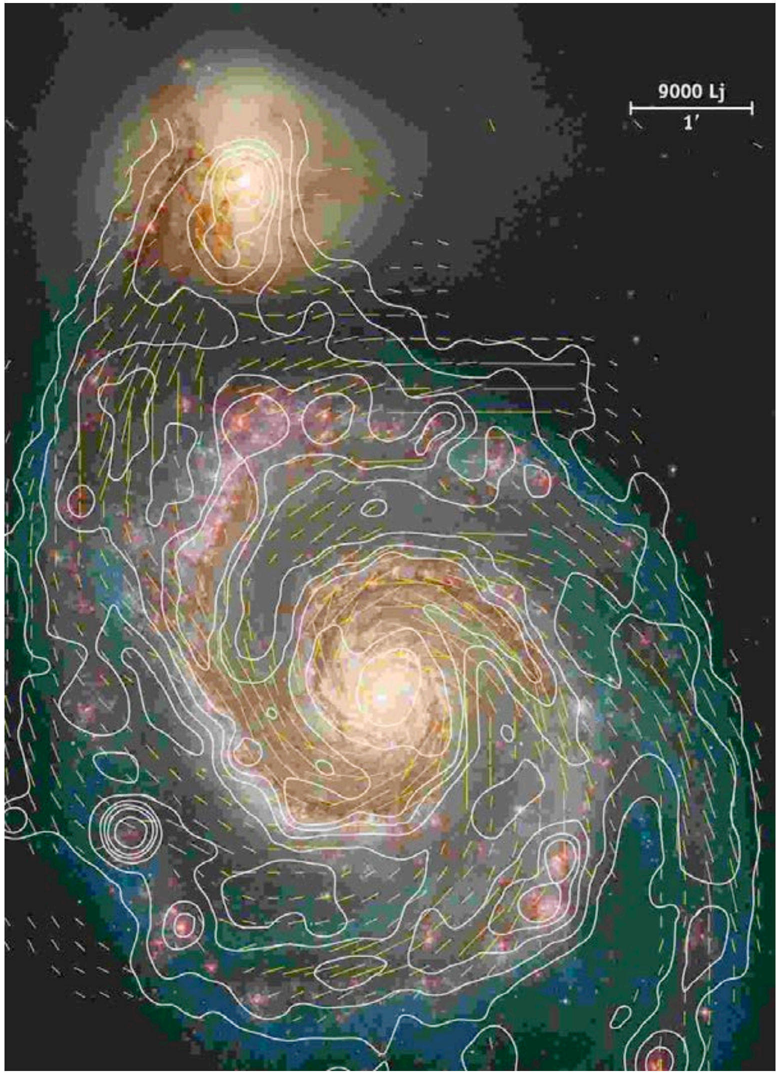
non-Ga
induced by magnetic
fields

pic power
spectrum — constraints
from CMB

Whether or not the gauge field takes a vacuum expectation value (vev) during inflation !!

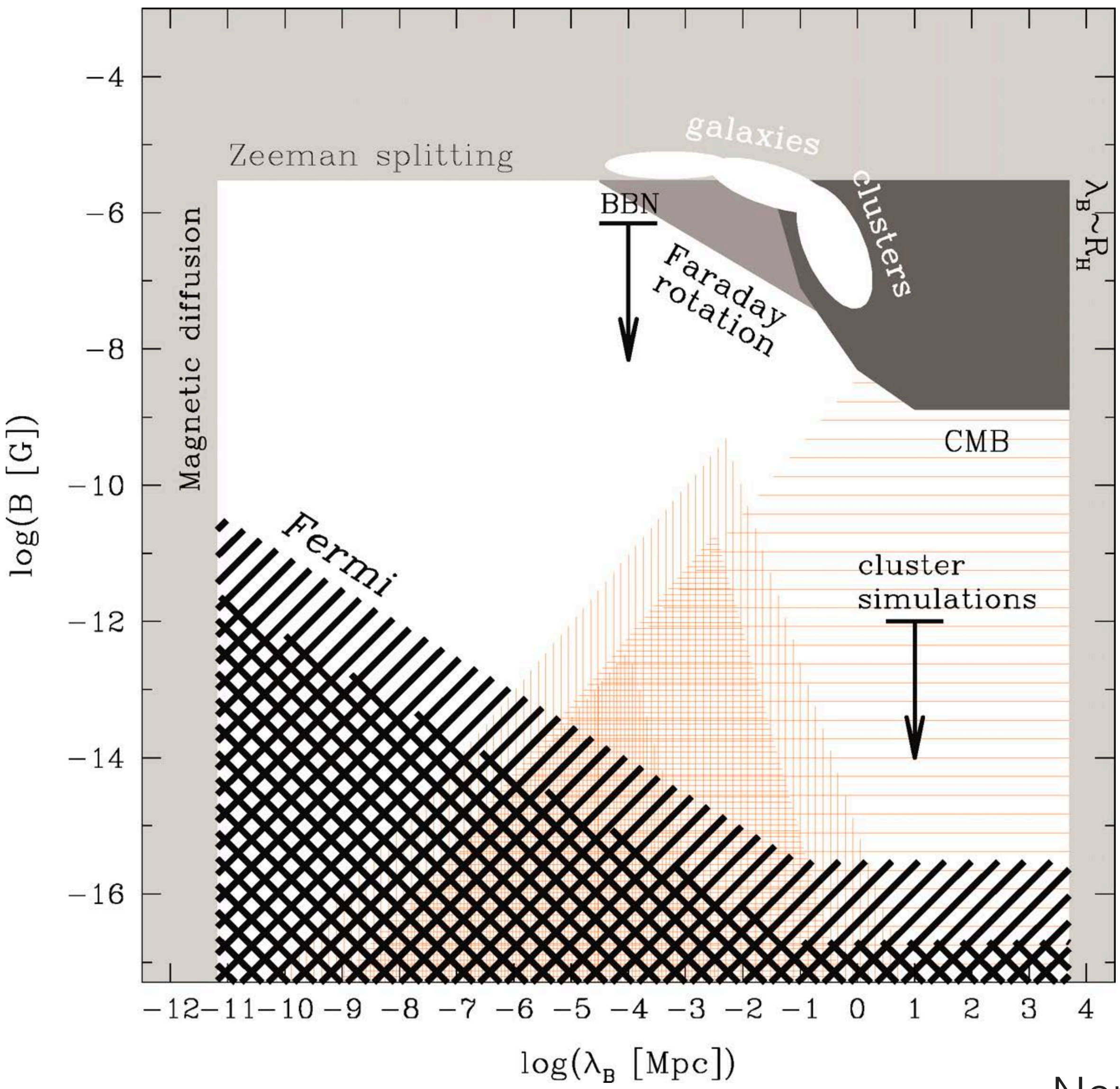
Cosmological magnetic fields

- Our observed universe is magnetized on all scales.
- All the bound structures — stars, galaxies and clusters carry magnetic fields, also present in the intergalactic medium.
- Stars: B ~ 0.1 – few G.
- Galaxies: B ~ 1 – 10 μG with coherence length as large as 10 kpc.
- Clusters: B ~ 0.1 – 1 μG, coherent on scales up to 100 kpc.
- Intergalactic medium: $B \gtrsim 3 \times 10^{-16}\,\mathrm{G}$ on scales of ~ 1 Mpc.



Neronov & Vovk, 2010

Constraints on cosmic magnetic fields



Neronov & Vovk, 2010

Primordial magnetic fields from inflation

- Inflationary mechanisms — most interesting due to the very nature of inflation.
- Standard Maxwell action is conformally invariant — the electromagnetic fluctuations do not grow in any conformally flat background like FRW.
- A necessary condition — break conformal invariance of the Maxwell theory. (Turner & Widrow, 1988)
- Various possible couplings:
  - Kinetic coupling: $\lambda(\phi, \mathcal{R}) F_{\mu\nu} F^{\mu\nu}$
  - Axial coupling: $f(\phi, \mathcal{R}) F_{\mu\nu} \tilde{F}^{\mu\nu}$
  - Mass term: $M^2(\phi, \mathcal{R}) A_\mu A^\mu$

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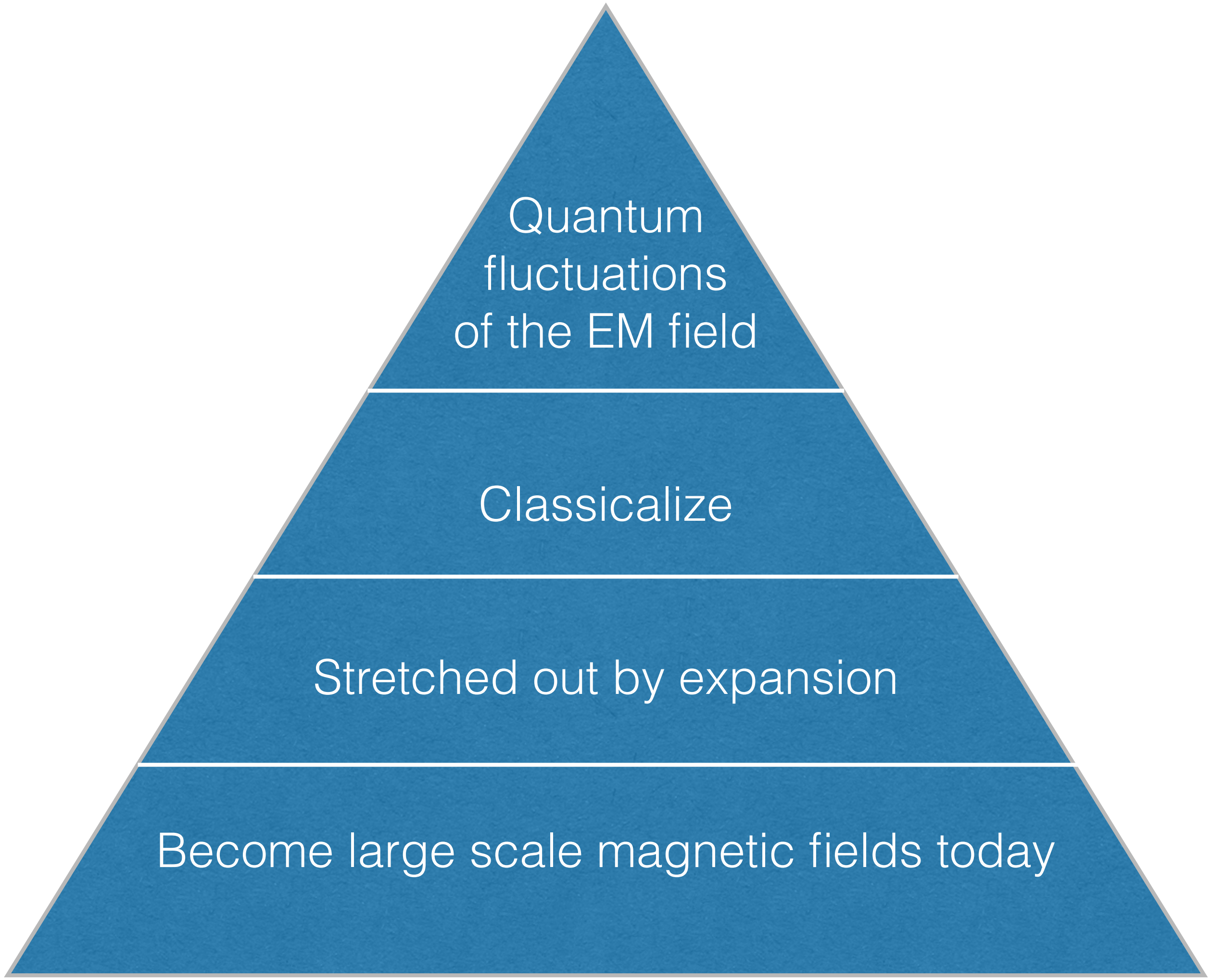
negative mass-squared needed for generating relevant, magnetic fields, breaks gauge invariance

Inflationary magnetogenesis



Inflationary magnetogenesis with kinetic coupling

Gauge-invariant kinetic coupling $\lambda(\phi, \mathcal{R}) F_{\mu\nu} F^{\mu\nu}$

$$\lambda(\eta) \propto a^{2\alpha} \propto \eta^{2\gamma}$$

Magnetic field spectrum

$$\frac{d\rho_B}{d\,\mathrm{lnk}}(\eta, k) \propto \left(\frac{k}{aH}\right)^{4+2\delta}$$

where $\delta = \gamma$ if $\gamma \leq 1/2$ and $\delta = 1 - \gamma$ if $\gamma \geq 1/2$.

The tilt of the spectrum is $n_B = 4 + 2\delta$ and $n_B = 0$ for $\alpha = 2$ or $\gamma = -2$. However, $n_B = 0$ also for $\gamma = 3$ but then the electric field vary strongly and so not interesting.

Martin & Yokoyama, JCAP 0801, 025 (2008)
Subramanian, 0911.4771 (2009)
Ferreira, **RKJ** & Sloth, JCAP 1310, 004 (2013)

Constraints for successful magnetogenesis

- Background
  - Strong coupling
  - Backreaction
- Perturbations
  - Power spectrum
  - Induced bispectrum
- Energy scale of inflation (from tensor modes)
- Schwinger effect — strong E field induces charged particle production

Constraints from strong coupling

- Adding the EM coupling to the SM fermions

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{4}\lambda(\phi)F_{\mu\nu}F^{\mu\nu} - \bar{\psi}\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\psi\right]$$

- The physical EM coupling now is

$$e_{\rm phys} = e/\sqrt{\lambda(\phi)}$$

- Since $\sqrt{\lambda} \propto a^{\alpha}$ then for $\alpha > 0$, the physical coupling decreases by a large factor during inflation, and must have been very large at the beginning of inflation.

- Strongly coupled regime at the beginning — problematic — perturbative calculations not reliable.

Demozzi & Mukhanov, 2009

Constraints from backreaction

- The produced magnetic fields should not backreact on the background dynamics of the universe i.e.

$$\rho_{\rm em} < \rho_{\rm inf}$$

- Backreaction + strong coupling constraints at most lead to B $\sim 10^{-32}$ G today. (Demozzi & Mukhanov, 2009)

- Very weak strength — not even enough as seed field for dynamo to work!

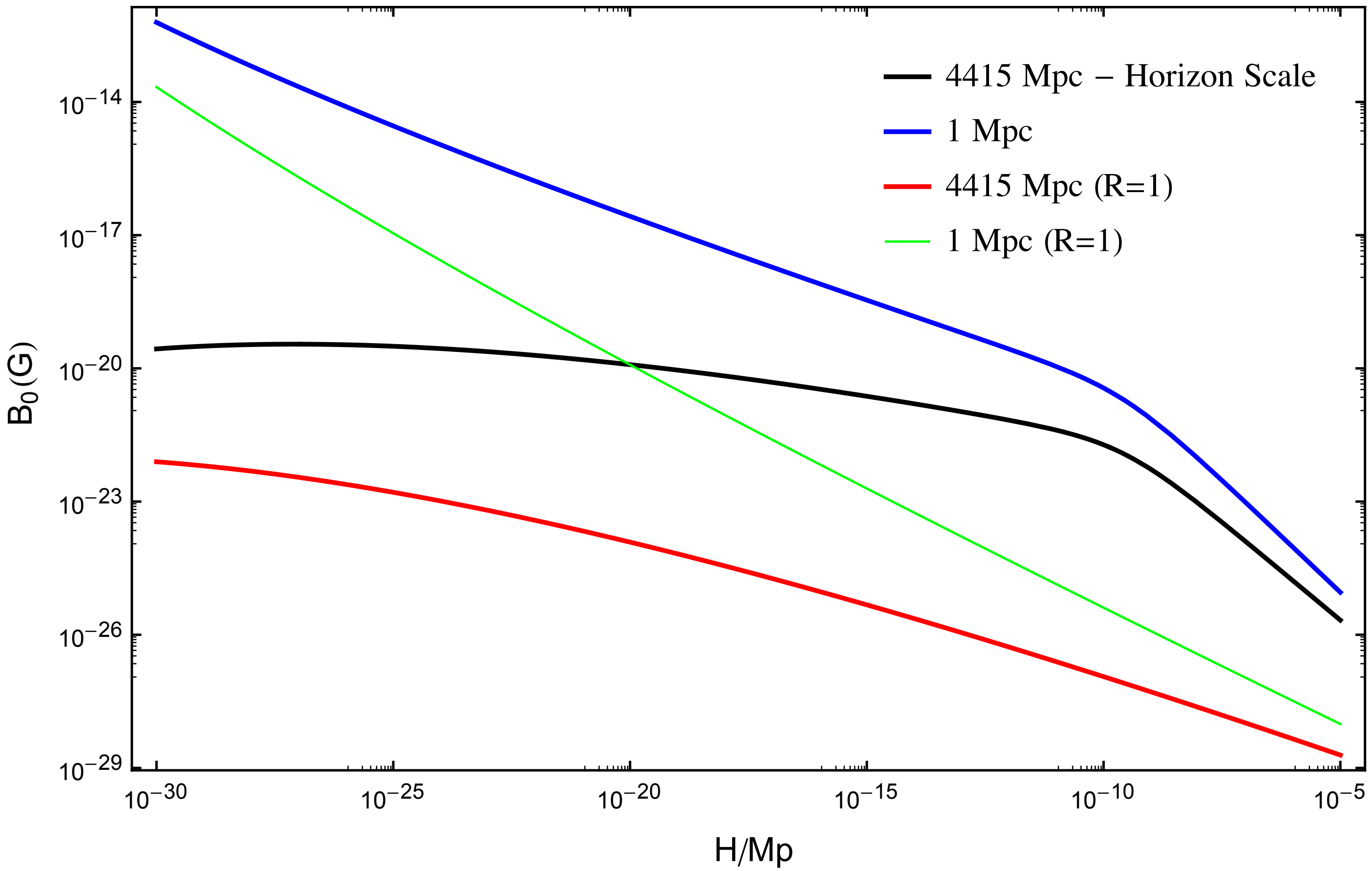
Is it possible to overcome this result ?

Post-inflationary evolution

- Flux conservation leads to adiabatic decay of magnetic fields after inflation.
- Problem with modifying the inflationary part to generate even larger field strength during inflation.
- Modify the post-inflationary evolution of magnetic fields until today.
- Consider prolonged reheating rather than instantaneous reheating.
- Effects of lowering Hubble scale during inflation — low scale inflationary models.

Ferreira, **RKJ** & Sloth, JCAP 1310, 004 (2013)

Final magnetic field strength



Ferreira, **RKJ** & Sloth, JCAP 1310, 004 (2013)

Constraints from perturbations

- Anisotropic constraints
  - Amplitude of induced curvature perturbations due to the EM field must be smaller than the observed power spectrum:

$$\mathcal{P}_{\zeta_{\rm em}}^{\rm max} < \mathcal{P}_{\zeta}^{\rm obs} \quad , \quad \zeta_{\rm em}(\tau) = \int_{\tau_0}^{\tau} d\ln\tilde{\tau}\lambda(\tilde{\tau})\frac{B_i B^i}{3H^2\epsilon}$$

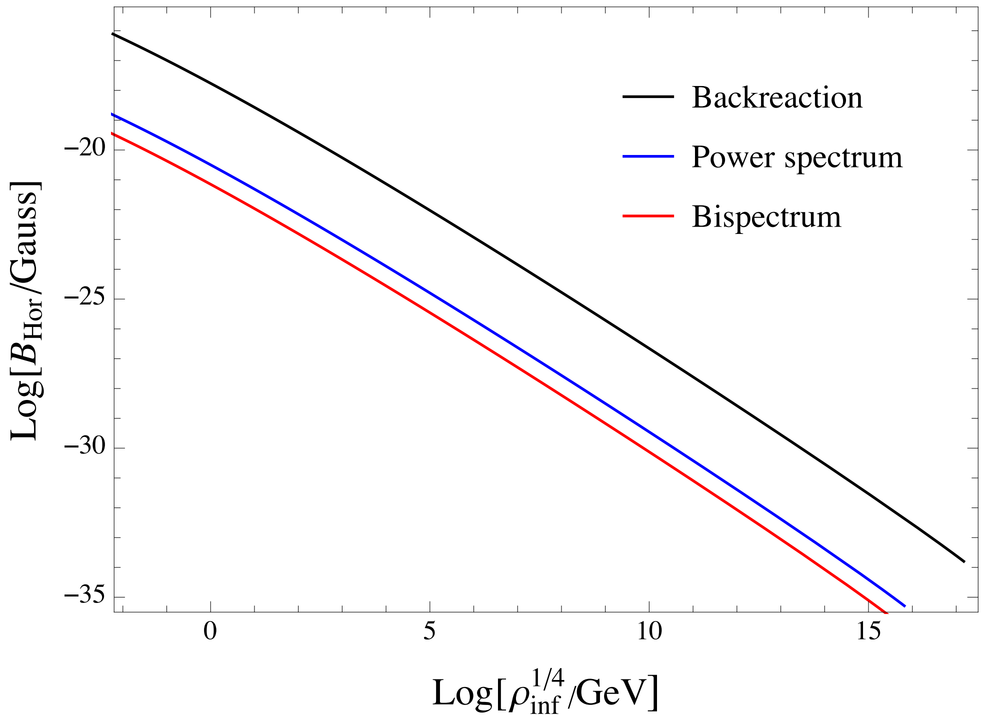
  - Non-gaussianity must be in agreement with Planck.
- Primordial gravitational waves
  - Tensor modes fix the energy scale of inflation.

Backreaction vs. anisotropic constraints

- Long enough inflation implies that backreaction is the strongest constraint.

  Fujita & Yokoyama, 2013

- If inflation lasts closer to the minimum duration required, the hierarchy of constraints is reversed — an interesting result !!



Ferreira, **RKJ** & Sloth, JCAP 1406, 053 (2014)

Deviations from slow roll !!

All the discussions so far apply to single field slow roll scenarios of inflation

What happens to the magnetic field spectrum in case of deviations from slow roll ?

Interesting signatures and constraints arise on the magnetic field spectrum in such cases

Look out for S. Tripathy's talk on Friday !

Tripathy, Chowdhury, **RKJ** & Sriramkumar, 2111.01478

Helical magnetic fields from inflation

- Axial coupling: $f(\phi, \mathcal{R}) F_{\mu\nu} \tilde{F}^{\mu\nu}$
- Parity violation implies helical magnetic fields.
- No strong coupling problem but stringent constraints from backreaction.
- Always a blue spectrum — a no go result.
- Final field strength not enough even after inverse cascade — transfer of power from small scales to large scales.
- Except for low scale inflation, field strength not enough as seeds for galactic dynamo.

Durrer, Hollenstein & **RKJ**, JCAP 1103, 037 (2011)

Resonant magnetic fields from inflation

Axial coupling $f(\phi, \mathcal{R}) F_{\mu\nu} \tilde{F}^{\mu\nu}$

- Novel generation scenario with an oscillating coupling function.
- Magnetic field amplification by parametric resonance.
- Very strong constraints from backreaction — problematic for inflation.
- A general no-go result for any scenario with resonant production of magnetic fields.
- Minimal required duration of inflation produces only crumbs of magnetic fields today, $B \ll 10^{-47}\,\mathrm{G}$.

Byrnes, Hollenstein, **RKJ** & Urban, JCAP 1203, 009 (2012)

Non-Gaussian imprints of inflationary magnetic fields

Non-Gaussian imprints of inflationary magnetic fields

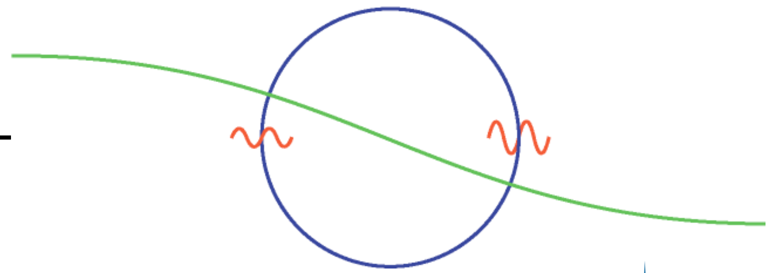
- An interesting consequence of inflationary magnetogenesis is a non-trivial correlation of primordial curvature perturbation with magnetic fields.

- Such cross-correlations are non-Gaussian in nature and it is important to understand their strength in a given scenario.

- A model-independent calculation can not be done as these correlations depend on the coupling function.

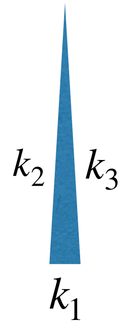
$$\langle \zeta(k_1)\mathbf{B}(k_2).\mathbf{B}(k_3)\rangle$$

Semi-classical estimate in the squeezed limit



- Squeezed limit: $k_1 \ll k_2 \sim k_3$

- Consider $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$ in the squeezed limit i.e. $k_1 \to 0$



  - The long wavelength mode rescales the background for short wavelength modes

$$ds^2 = -dt^2 + a^2(t)\, e^{2\zeta(t,\mathbf{x})} d\mathbf{x}^2$$

  - Taylor expand in the rescaled background

$$\langle \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} = \langle \zeta_{k_2} \zeta_{k_3} \rangle + \zeta_1 \frac{\partial}{\partial \zeta_1} \langle \zeta_{k_2} \zeta_{k_3} \rangle + \dots$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} \approx \left\langle \zeta_{k_1} \langle \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} \right\rangle \sim \langle \zeta_{k_1} \zeta_{k_1} \rangle\, k \frac{d}{dk} \langle \zeta_{k_2} \zeta_{k_3} \rangle$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \sim -(n_s - 1) \langle \zeta_{k_1} \zeta_{k_1} \rangle \langle \zeta_{k_2} \zeta_{k_3} \rangle$$

$$f_{NL}^{\rm local} = -(n_s - 1)$$

Maldacena, JHEP 0305, 013 (2002)

Non-Gaussian cross-correlation

- Define the cross-correlation bispectrum of the curvature perturbation with magnetic fields as

$$\langle\zeta(\mathbf{k}_1)\mathbf{B}(\mathbf{k}_2)\cdot\mathbf{B}(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

$$B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv b_{NL}P_\zeta(k_1)P_B(k_2)$$

- $\mathcal{L}ocal$ resemblance between f_{NL} and b_{NL}

$$\zeta = \zeta^{(G)} + \frac{3}{5}f_{NL}^{local}\left(\zeta^{(G)}\right)^2$$

$$\mathbf{B} = \mathbf{B}^{(G)} + \frac{1}{2}b_{NL}^{local}\zeta^{(G)}\mathbf{B}^{(G)}$$

RKJ & Sloth, Phys. Rev. D 86, 123528 (2012)

A novel magnetic consistency relation

- Using Maldacena's approach, the cross-correlation becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1)\mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3)\rangle$$
$$= -\frac{1}{H}\frac{\dot{\lambda}}{\lambda}(2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)P_\zeta(k_1)P_B(k_2)$$

- With the coupling $\lambda(\phi(\tau)) = \lambda_I(\tau/\tau_I)^{-2n}$, we obtain

$$\boxed{b_{NL} = n_B - 4}$$

- For scale-invariant magnetic field spectrum, $n_B = 0$ and therefore,

$$b_{NL} = -4$$

RKJ & Sloth, Phys. Rev. D 86, 123528 (2012)

A novel magnetic consistency relation

- In the squeezed limit $k_1 \ll k_2, k_3 = k$, we obtain a *new* magnetic consistency relation

$$\langle \zeta(k_1) \mathbf{B}(k_2) \cdot \mathbf{B}(\mathbf{k_3}) \rangle = (n_B - 4)(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k)$$

with $b_{NL}^{\text{local}} = (n_B - 4)$

- Compare with Maldacena's consistency relation

$$\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = -(n_s - 1)(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_\zeta(k)$$

with $f_{NL}^{\text{local}} = -(n_s - 1)$

RKJ & Sloth, Phys. Rev. D 86, 123528 (2012)

Full in-in calculation

- One has to cross-check the consistency relation by doing a complete in-in calculation.

- The final result is

$$\langle\zeta(\tau_I,\mathbf{k}_1)\mathbf{B}(\tau_I,\mathbf{k}_2)\cdot\mathbf{B}(\tau_I,\mathbf{k}_3)\rangle = -\frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I}(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)|\zeta_{k_1}^{(0)}(\tau_I)|^2|A_{k_2}^{(0)}(\tau_I)||A_{k_3}^{(0)}(\tau_I)| \times\left[\left(\mathbf{k}_2\cdot\mathbf{k}_3+\frac{(\mathbf{k}_2\cdot\mathbf{k}_3)^3}{k_2^2k_3^2}\right)k_2k_3\tilde{\mathcal{I}}_n^{(1)}+2(\mathbf{k}_2\cdot\mathbf{k}_3)^2\tilde{\mathcal{I}}_n^{(2)}\right] .$$

- The two integrals can be solved exactly for different values of n.

RKJ & Sloth, JCAP 1302, 003 (2013)

Full in-in calculation

- **The flattened shape:** In this limit, $k_1 = 2k_2 = 2k_3$, the cross-correlation becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1)\mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3)\rangle \simeq 96 \ln(-k_t\tau_I) P_\zeta(k_1) P_B(k_2)$$

- For the largest observable scale today, $\ln(-k_t\tau_I) \sim -60$,

$$\left| b_{NL}^{flat} \right| \sim 5760$$

- **The squeezed limit:** In this limit, $k_1 \to 0$ and

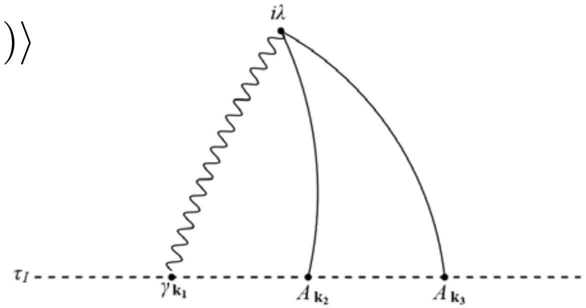
$$\langle \zeta(\tau_I, \mathbf{k}_1)\mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3)\rangle = -\frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I}(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

with $b_{NL} = -\frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I} = n_B - 4$ in agreement with the consistency relation.

RKJ & Sloth, JCAP 1302, 003 (2013)

Cross-correlations with gravitons

$$\langle \gamma(\mathbf{k_1})\mathbf{A}(\mathbf{k_2})\cdot\mathbf{A}(\mathbf{k_3})\rangle\ , \qquad \langle \gamma(\mathbf{k_1})\mathbf{B}(\mathbf{k_2})\cdot\mathbf{B}(\mathbf{k_3})\rangle\ , \qquad \langle \gamma(\mathbf{k_1})\mathbf{E}(\mathbf{k_2})\cdot\mathbf{E}(\mathbf{k_3})\rangle$$



$$ds^2 = -dt^2 + a^2(t)\left[e^{\gamma}\right]_{ij} dx^i dx^j \approx -dt^2 + a^2(t)\left[\delta_{ij} + \gamma_{ij}\right] dx^i dx^j$$

In the squeezed limit

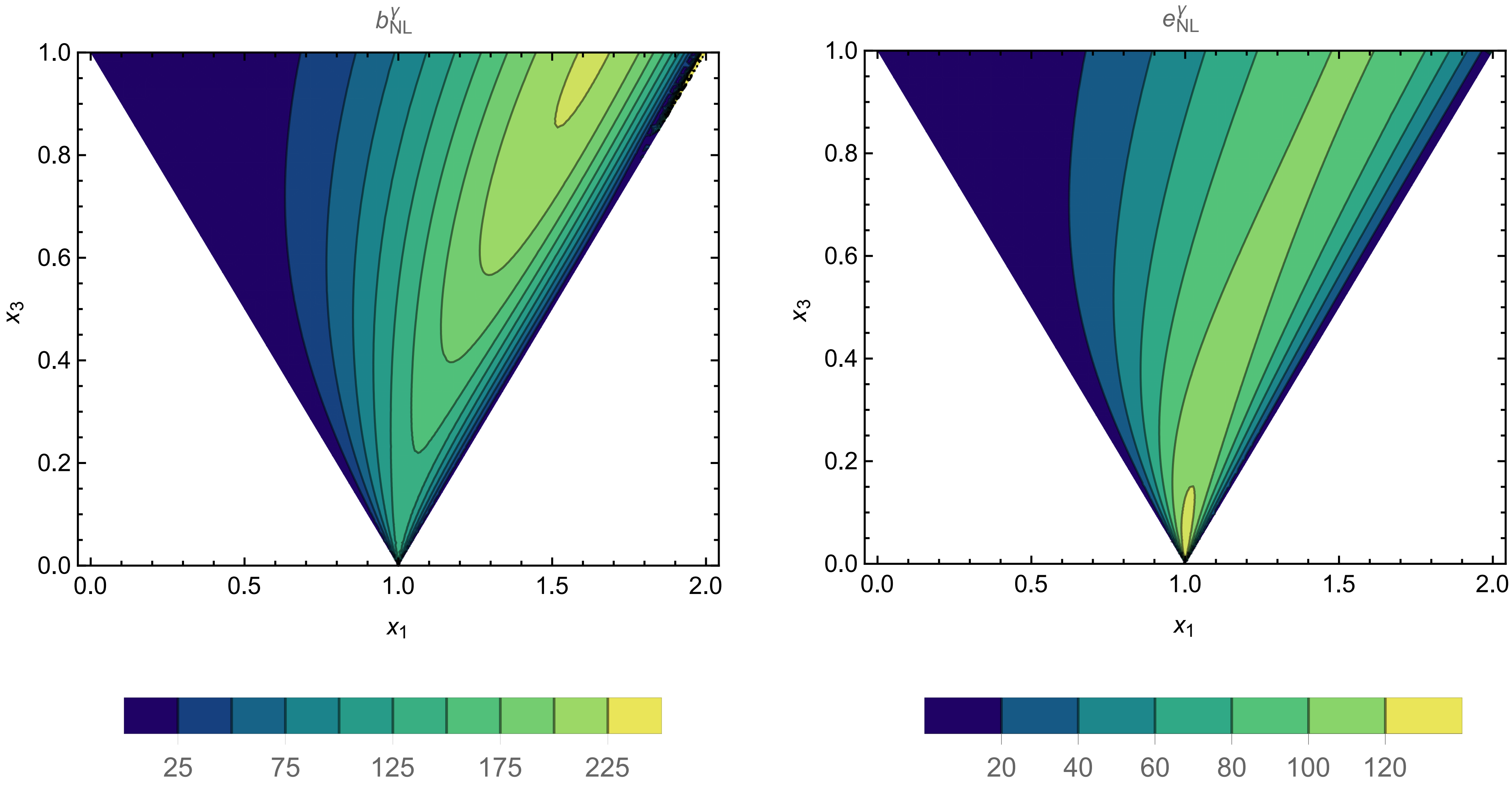
$$\lim_{k_1\to 0} \langle \gamma(\tau_I, \mathbf{k}_1) B_\mu(\tau_I, \mathbf{k}_2) B^\mu(\tau_I, \mathbf{k}_3)\rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\left(n - \frac{1}{2}\right)\epsilon_{ij}\frac{k_{2i}k_{2j}}{k_2^2} P_\gamma(k_1) P_B(k_2)$$

$$\lim_{k_1\to 0} \langle \gamma(\tau_I, \mathbf{k}_1) E_\mu(\tau_I, \mathbf{k}_2) E^\mu(\tau_I, \mathbf{k}_3)\rangle = -(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\left(n + \frac{1}{2}\right)\epsilon_{ij}\frac{k_{2i}k_{2j}}{k_2^2} P_\gamma(k_1) P_E(k_2)$$

$$\begin{aligned} b_{NL}^{\gamma} &= \left(n - \frac{1}{2}\right)\epsilon_{ij}\frac{k_{2i}k_{2j}}{k_2^2}, \quad n > -1/2 \\ e_{NL}^{\gamma} &= -\left(n + \frac{1}{2}\right)\epsilon_{ij}\frac{k_{2i}k_{2j}}{k_2^2}, \quad n < 1/2 \end{aligned}$$

RKJ, Sai & Sloth, 2108.10887

Cross-correlations with gravitons



RKJ, Sai & Sloth, 2108.10887

Conclusions

- Dynamical gauge fields during inflation lead to a very rich phenomenology with interesting cosmological imprints.
- Inflationary magnetogenesis — required to generate seed fields — to explain the weak coherent fields in the intergalactic medium.
- Interesting cross-correlations with curvature and tensor perturbations — new consistency relations — large contributions in specific shapes.
- Other interesting signatures — induced gravitational waves on interferometer scales — probed by LISA, DECIGO or BBO.
- Cosmological observations have enormous potential to constrain the rich fundamental physics of the early universe.

Thank you.