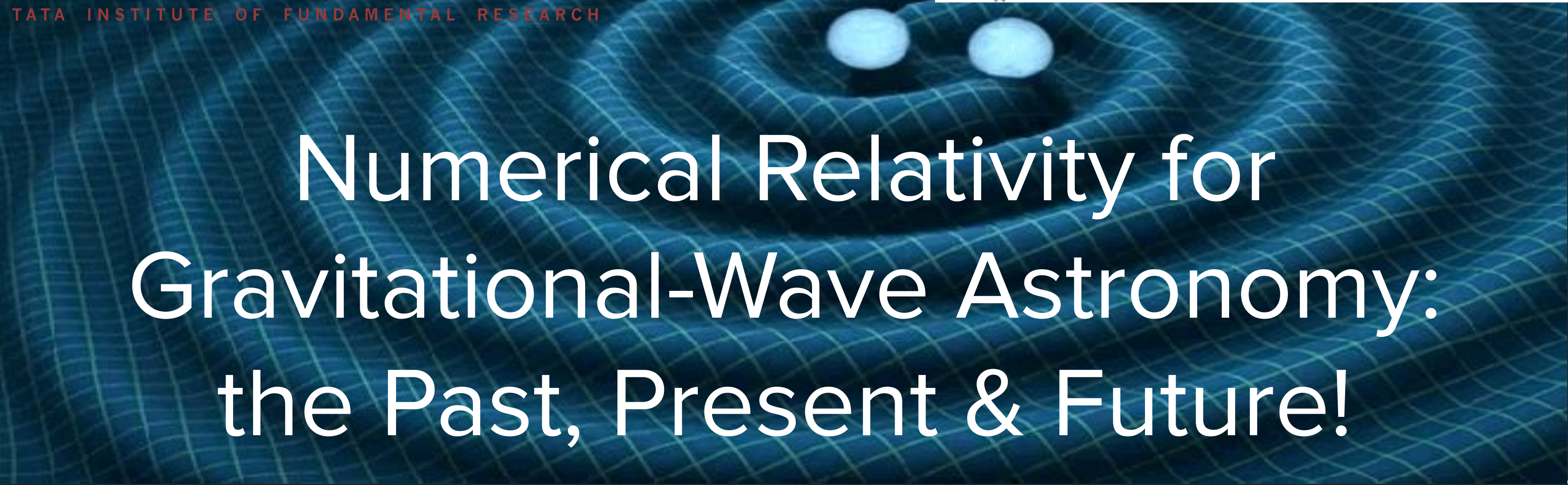
ICTS
INTERNATIONAL CENTRE *for* THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

SIMULATING EXTREME SPACETIMES
Black holes, neutron stars, and beyond...



Numerical Relativity for Gravitational-Wave Astronomy: the Past, Present & Future!

Prayush Kumar
International Center for Theoretical Sciences

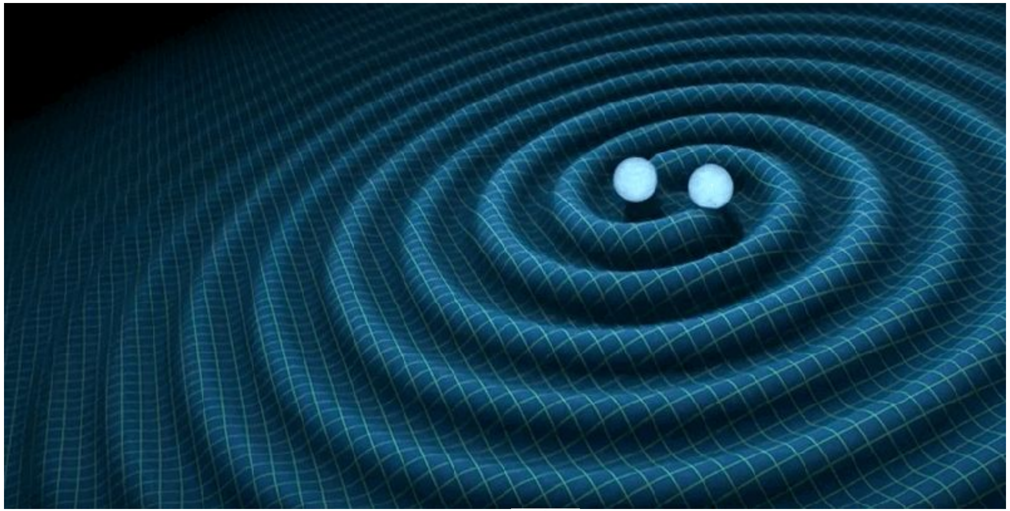
Chennai Symposium on Gravitation and Cosmology
February 4, 2022

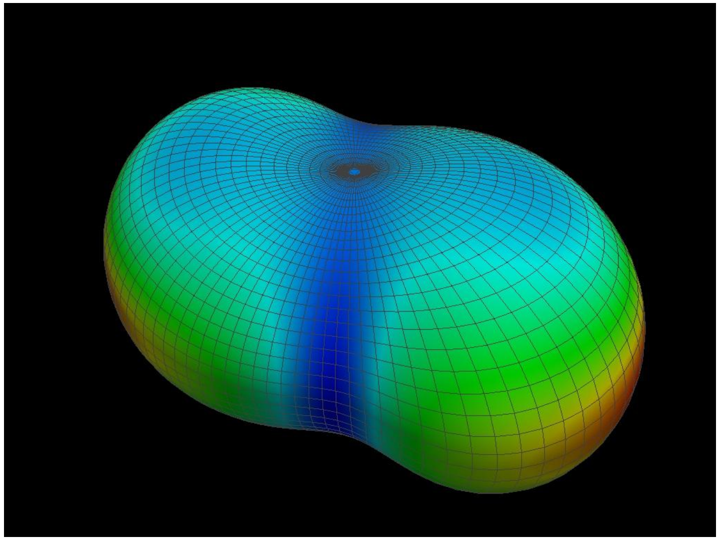
Centre for Strings, Gravitation and Cosmology, Department of Physics,
Indian Institute of Technology Madras, Chennai
Chennai Mathematical Institute, Chennai
The Institute of Mathematical Sciences, Chennai

I. Why Numerical Relativity at all?

Gravitational Waves!

- Orbiting systems of stars evolve into binary black holes. They emit gravitational waves and lose orbital energy.
- Orbits keeps tightening till the black holes collide. Remnant is also a black hole.
- Remnant black hole is very distorted at birth. It emits gravitational waves and settles down to a quiescent state.





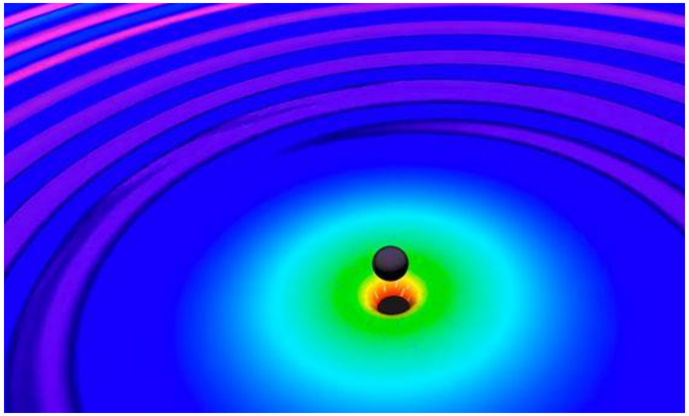
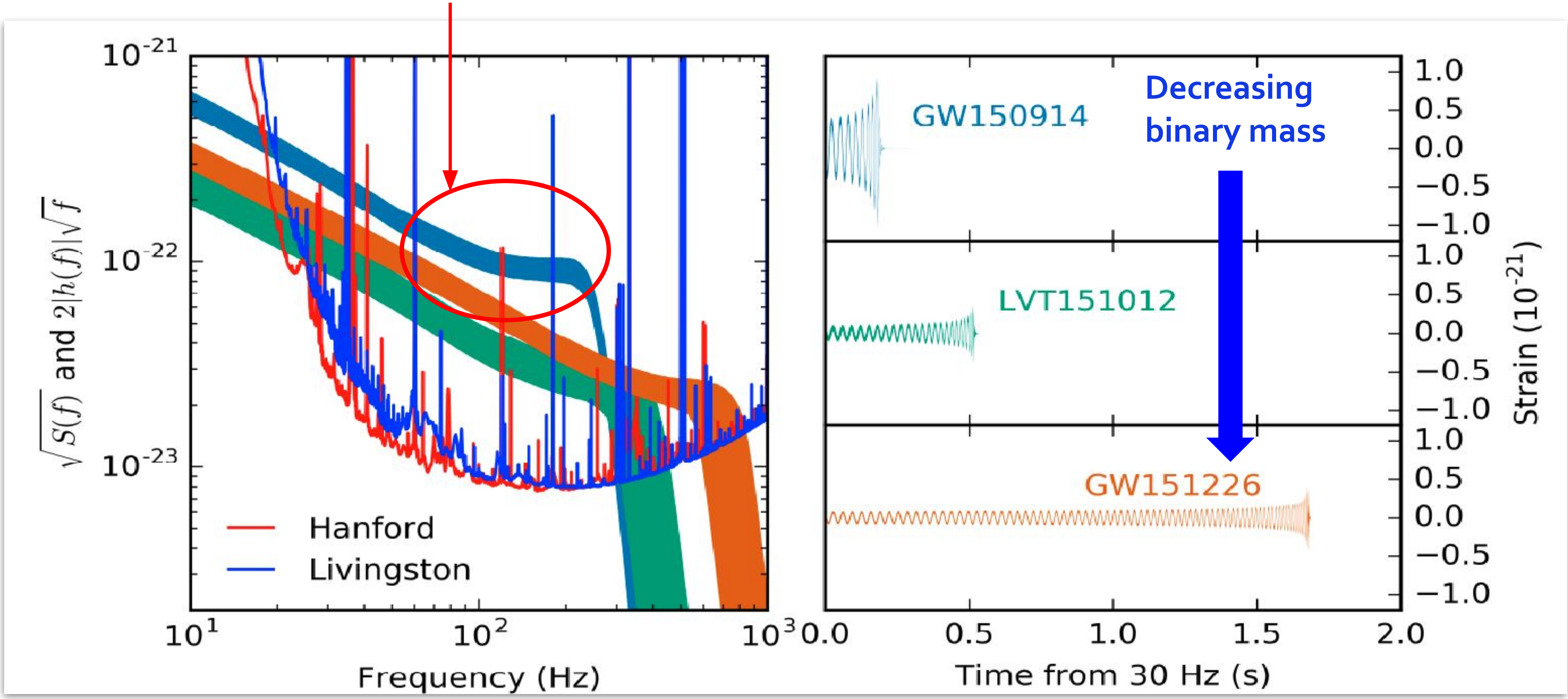


Image credit: Kavli Foundation, LSC; https://cqgplus.files.wordpress.com;

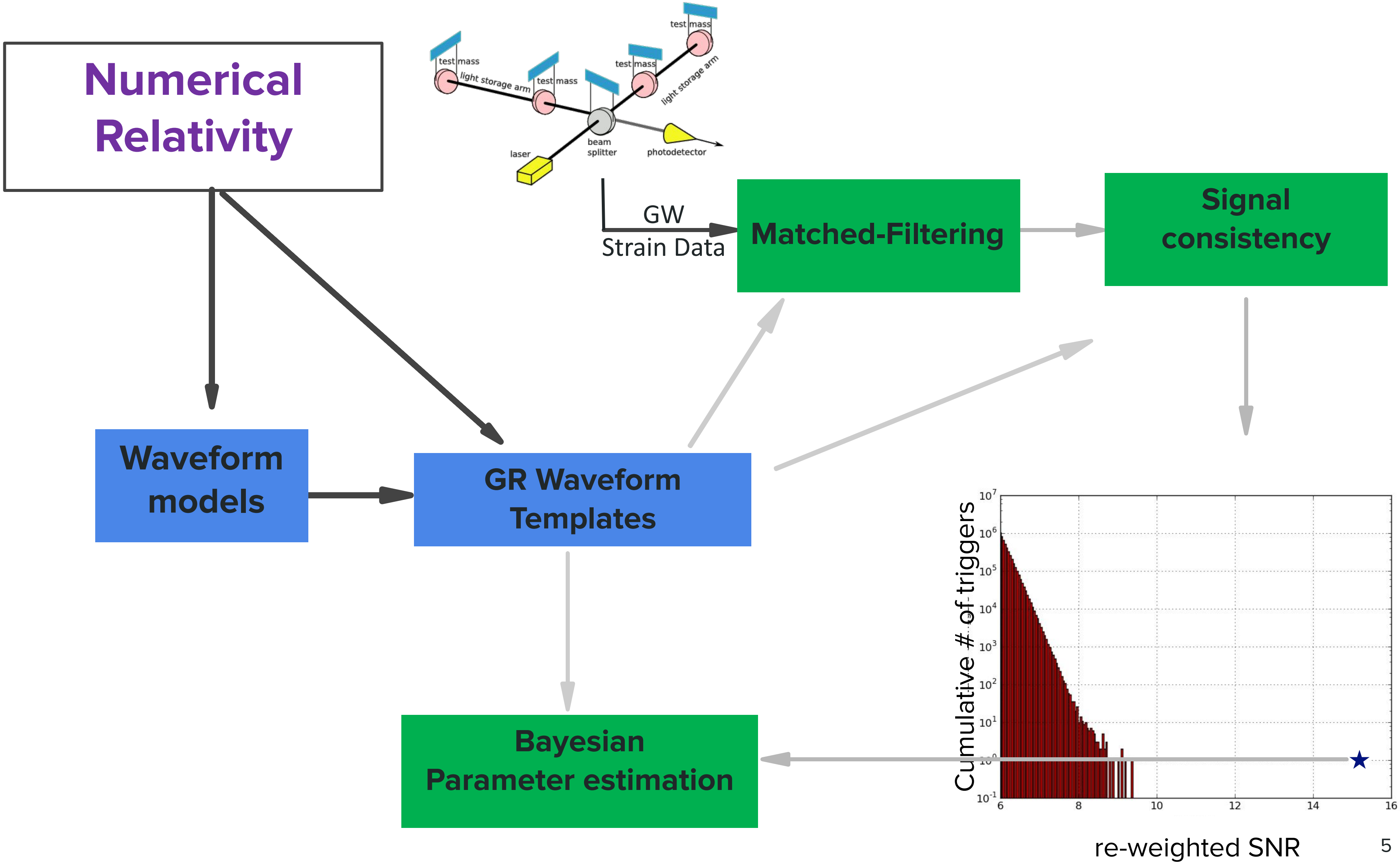
GW observations: these black holes are **heavy**!

Massive binaries ➔ Strong-field nonlinear general relativistic dynamics becomes **measurable**!



Decreasing binary mass

Role of Numerical Relativity



Numerical Relativity

Waveform models

GR Waveform Templates

GW Strain Data

Matched-Filtering

Signal consistency

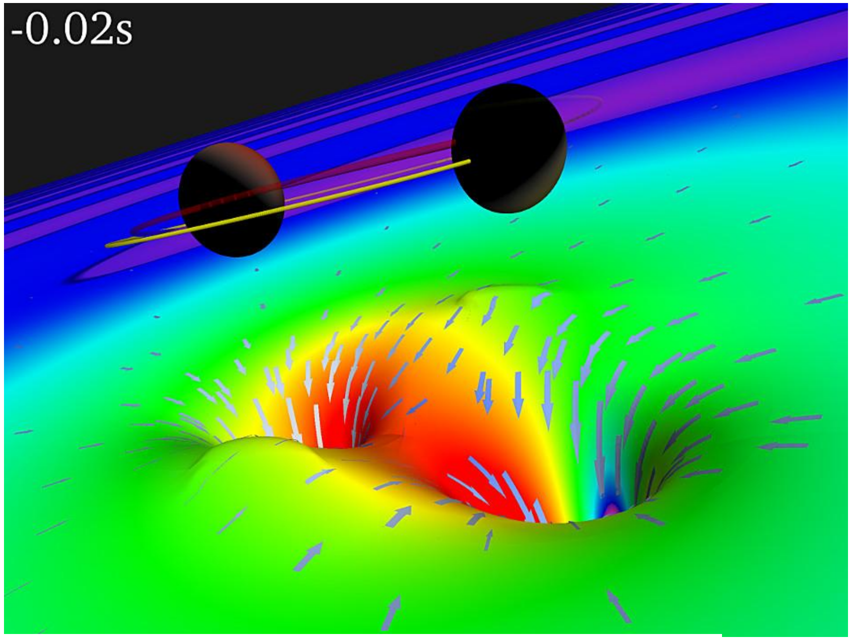
Bayesian Parameter estimation

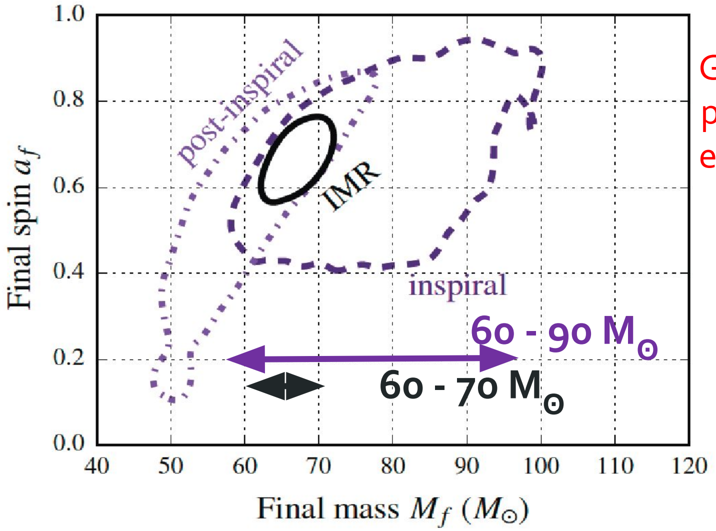
Numerical simulations are **necessary** for BBH science

For BBH, last ~10 orbits, merger and ringdown, can only be computed with full numerical solutions of Einstein's equations.

Without Numerical Relativity:

- GW events like GW150914,GW151226, GW170104 - would have had much lower significance ("probable" vs "confident" detection)
- If GW150914's source merged 25% further away, it would not even have been detected in Livingston
- We would only very approximately determine black hole characteristics from the GW signal →
- We could not have tested GR





GW150914 parameter estimation

Image credit: Harald Pfeiffer, SXS Collaboration;
Abbott ..PK..et al (2016), Phys. Rev. Lett. **116**, 061102;

II. Simulating Compact Binary Coalescence

GR and Einstein’s Equations

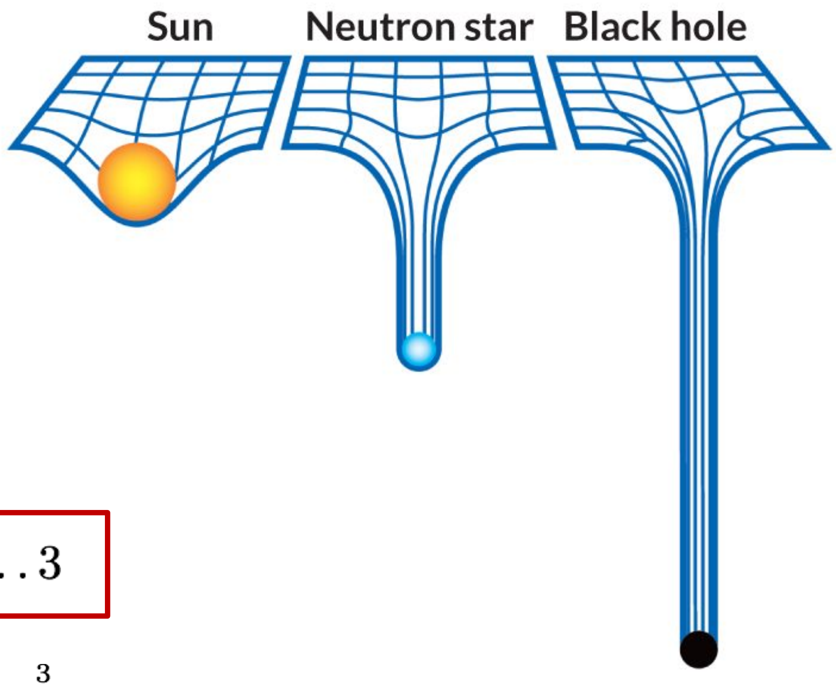
- Newtonian gravity:
  Flat Space-time

$$\vec{\nabla}^2\Phi = 4\pi G\rho \qquad \vec{a} = -\vec{\nabla}\Phi$$

- Einsteinian gravity:

  (i) Curved space-time

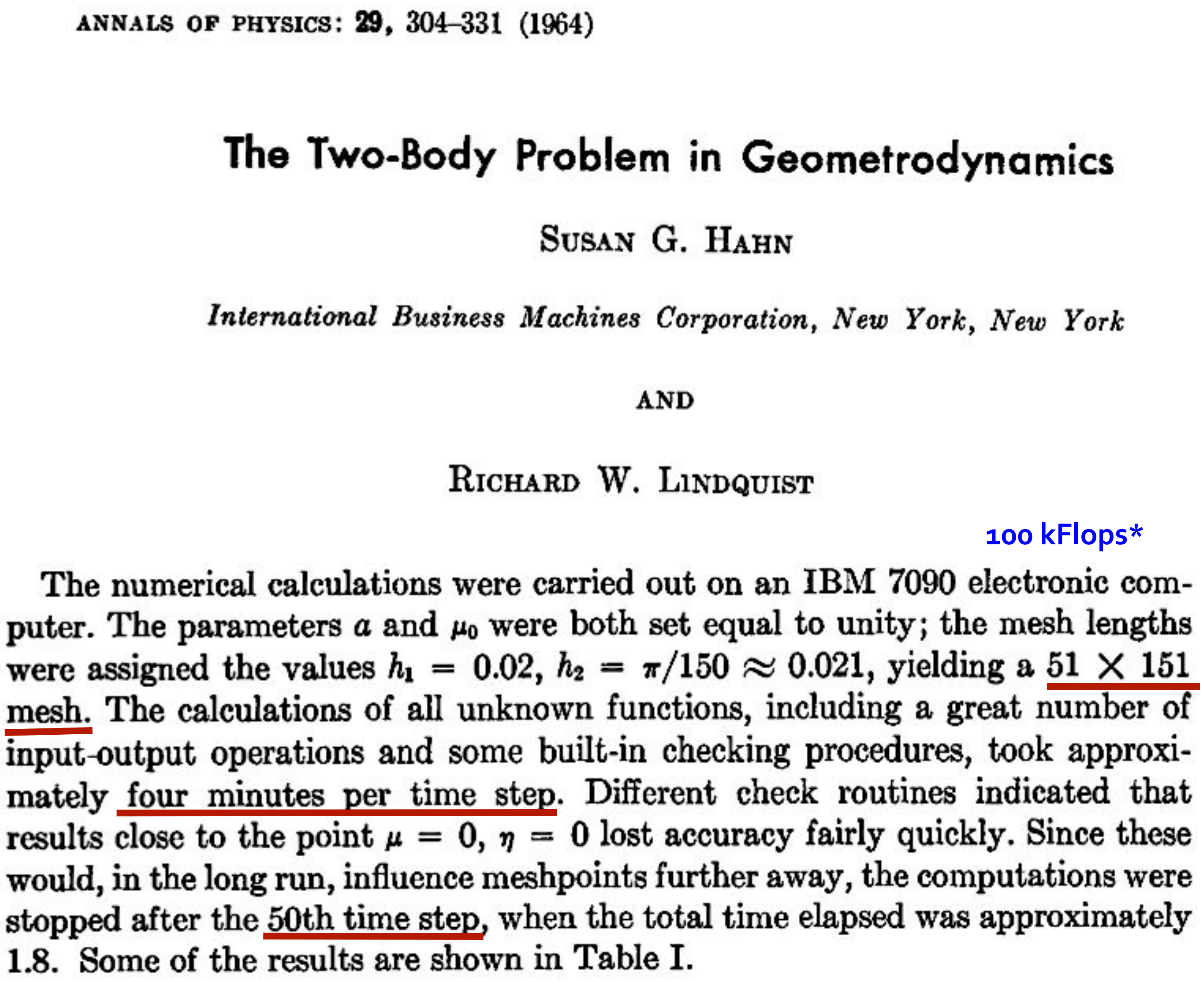
  (ii) Geometry represented by the space-time metric $g_{ab}(\vec{x},t)$, a,b = { x,y,z,t }.
  Metric is determined by solving Einstein Field Equations



$$\boxed{R_{ab}[g_{ab}(\vec{x},t)] = 0, \qquad a,b = 0,\dots 3}$$

$$R_{ab} = \sum_{d=0}^{3}\partial_d\Gamma^d_{ab} - \sum_{d=0}^{3}\partial_b\Gamma^d_{da} + \sum_{c,d=0}^{3}\Gamma^c_{cd}\Gamma^d_{ab} - \sum_{c,d=0}^{3}\Gamma^d_{bc}\Gamma^c_{da}$$

$$\Gamma^a_{bc} = \sum_{d=1}^{4}(g_{ad})^{-1}\left(\partial_b g_{db} + \partial_c g_{bd} - \partial_d g_{bc}\right)$$



ANNALS OF PHYSICS: **29**, 304–331 (1964)

The Two-Body Problem in Geometrodynamics

SUSAN G. HAHN

International Business Machines Corporation, New York, New York

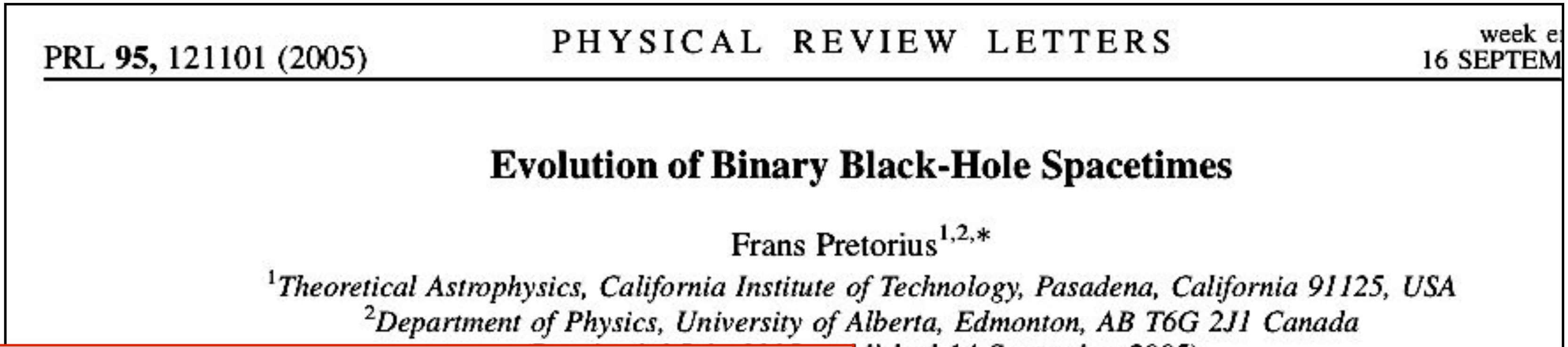
AND

RICHARD W. LINDQUIST

100 kFlops*

The numerical calculations were carried out on an IBM 7090 electronic computer. The parameters a and μ_0 were both set equal to unity; the mesh lengths were assigned the values $h_1 = 0.02$, $h_2 = \pi/150 \approx 0.021$, yielding a 51×151 mesh. The calculations of all unknown functions, including a great number of input-output operations and some built-in checking procedures, took approximately four minutes per time step. Different check routines indicated that results close to the point $\mu = 0$, $\eta = 0$ lost accuracy fairly quickly. Since these would, in the long run, influence meshpoints further away, the computations were stopped after the 50th time step, when the total time elapsed was approximately 1.8. Some of the results are shown in Table I.

* https://en.wikipedia.org/wiki/IBM_7090



PRL **95,** 121101 (2005)

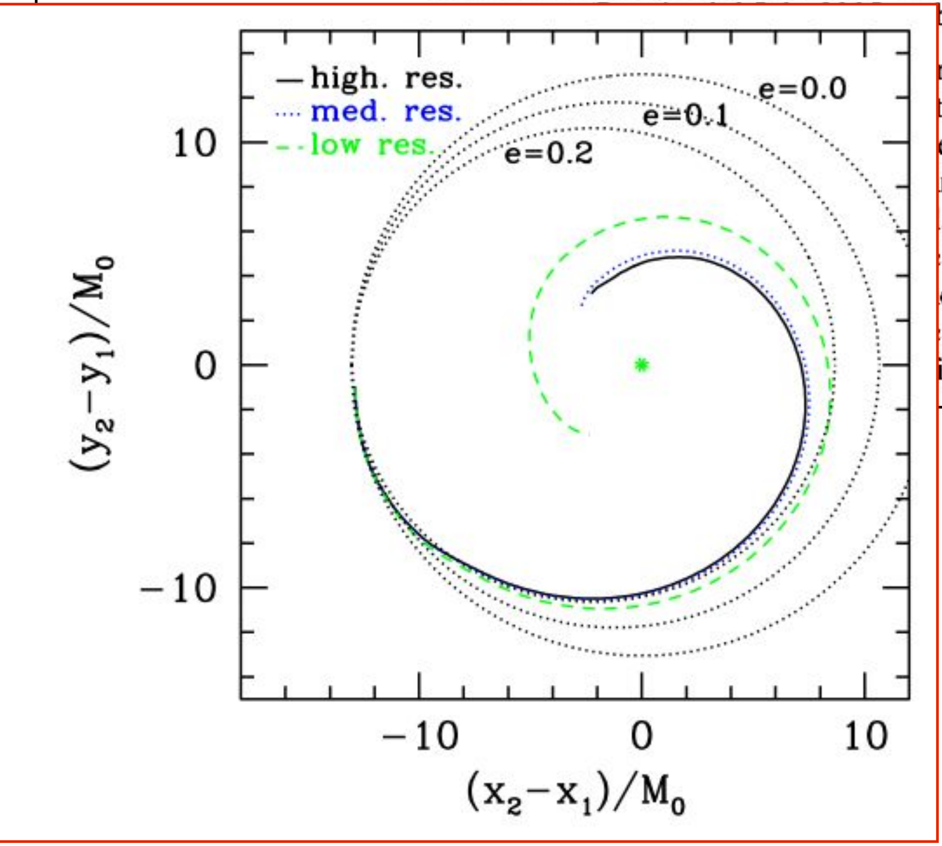
PHYSICAL REVIEW LETTERS

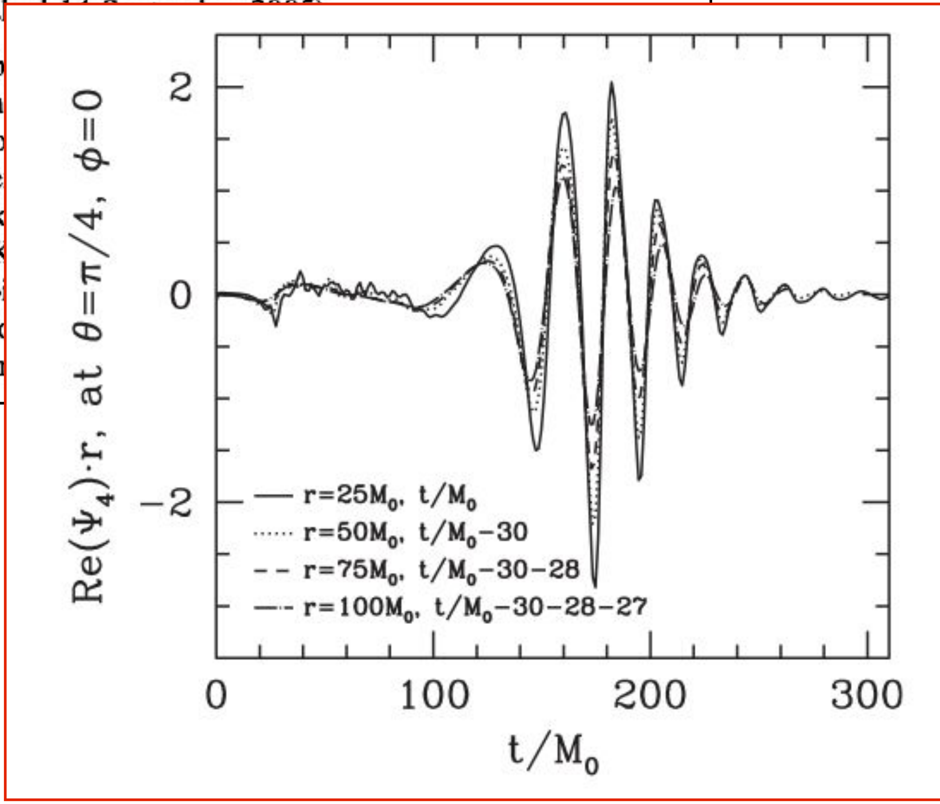
week e
16 SEPTEM

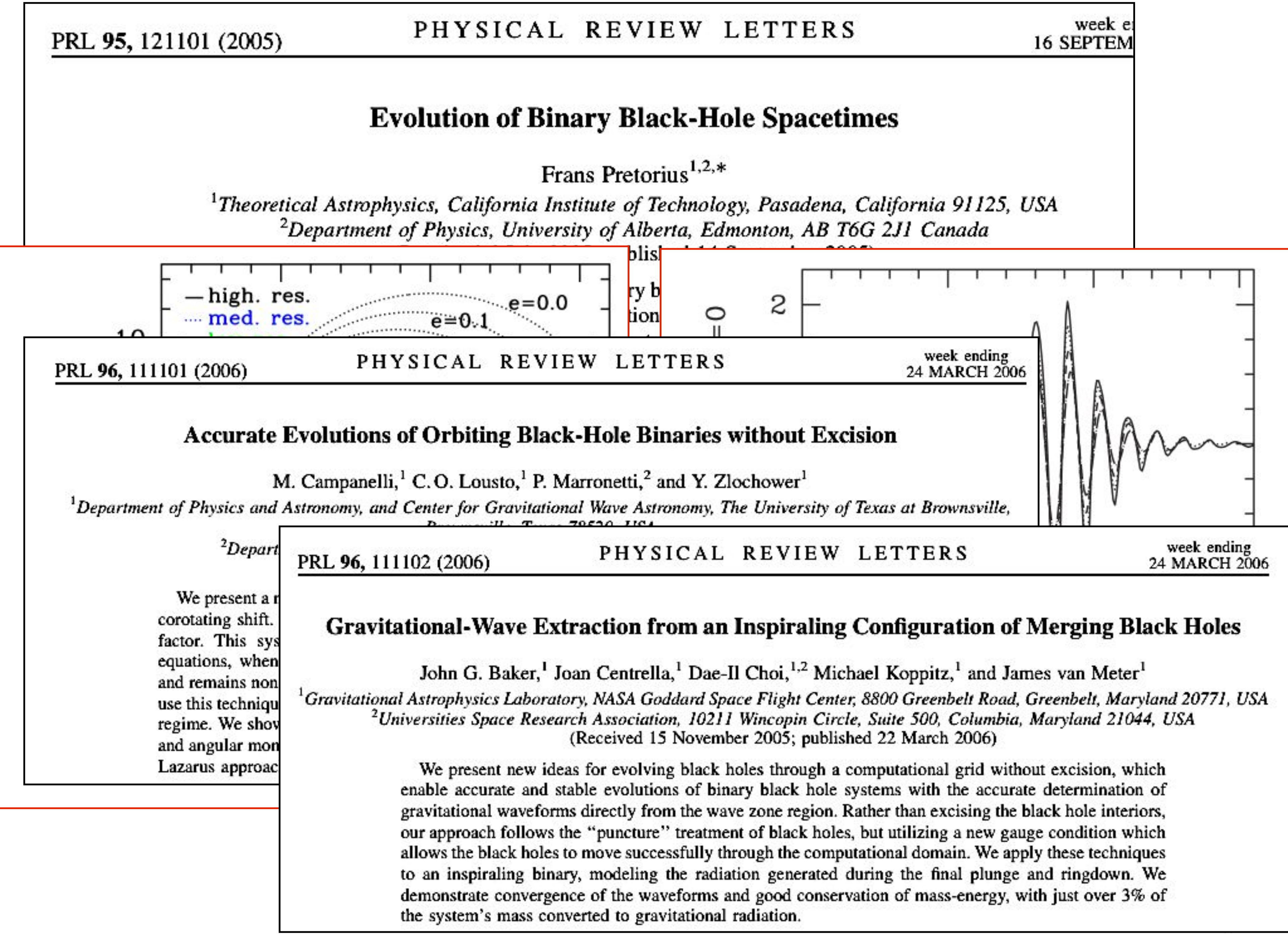
Evolution of Binary Black-Hole Spacetimes

Frans Pretorius[1,2,*]

[1] *Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA*
[2] *Department of Physics, University of Alberta, Edmonton, AB T6G 2J1 Canada*







PRL **95,** 121101 (2005) PHYSICAL REVIEW LETTERS week e
16 SEPTEM

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[1]*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA*
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PRL **96,** 111101 (2006) PHYSICAL REVIEW LETTERS week ending
24 MARCH 2006

Accurate Evolutions of Orbiting Black-Hole Binaries without Excision

M. Campanelli,[1] C. O. Lousto,[1] P. Marronetti,[2] and Y. Zlochower[1]
[1]*Department of Physics and Astronomy, and Center for Gravitational Wave Astronomy, The University of Texas at Brownsville,*
[2]*Depart*

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PRL **96,** 111102 (2006) PHYSICAL REVIEW LETTERS week ending
24 MARCH 2006

Gravitational-Wave Extraction from an Inspiraling Configuration of Merging Black Holes

John G. Baker,[1] Joan Centrella,[1] Dae-Il Choi,[1,2] Michael Koppitz,[1] and James van Meter[1]
[1]*Gravitational Astrophysics Laboratory, NASA Goddard Space Flight Center, 8800 Greenbelt Road, Greenbelt, Maryland 20771, USA*
[2]*Universities Space Research Association, 10211 Wincopin Circle, Suite 500, Columbia, Maryland 21044, USA*

(Received 15 November 2005; published 22 March 2006)

We present new ideas for evolving black holes through a computational grid without excision, which enable accurate and stable evolutions of binary black hole systems with the accurate determination of gravitational waveforms directly from the wave zone region. Rather than excising the black hole interiors, our approach follows the "puncture" treatment of black holes, but utilizing a new gauge condition which allows the black holes to move successfully through the computational domain. We apply these techniques to an inspiraling binary, modeling the radiation generated during the final plunge and ringdown. We demonstrate convergence of the waveforms and good conservation of mass-energy, with just over 3% of the system's mass converted to gravitational radiation.

Solving Einstein Equations: 3+1 split

- Goal: Space-time metric g_{ab} satisfying

$$R_{ab}[g_{ab}] = 0$$

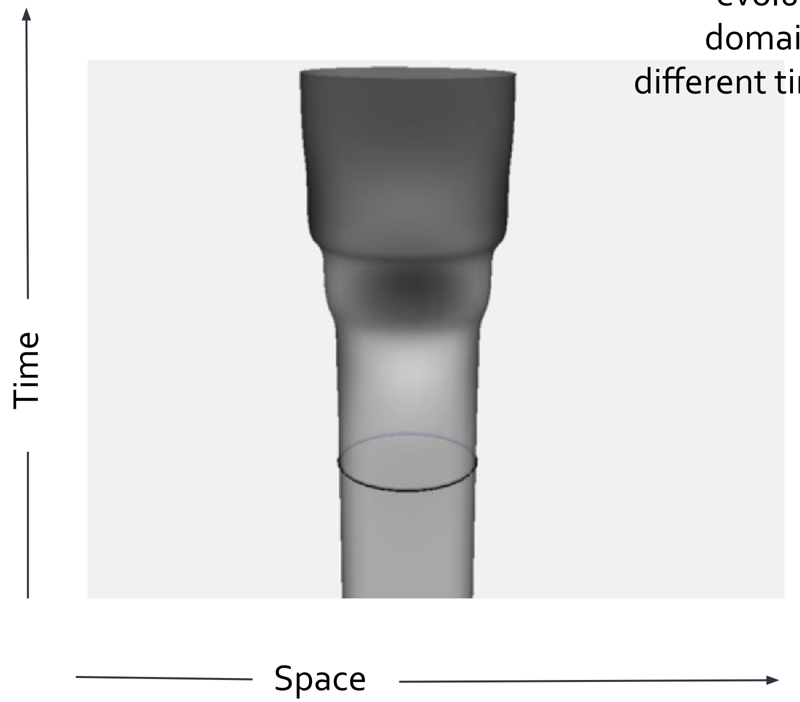
- Split space-time into space *and* time

Evolution equations

$$\partial_t g_{ij} = \ldots$$
$$\partial_t K_{ij} = \ldots$$

Constraints

$$R[g_{ij}] + K^2 - K_{ij}K^{ij} = 0$$
$$\nabla_j \left(K^{ij} - g^{ij}K\right) = 0$$



Snapshots of evolution domain at different times

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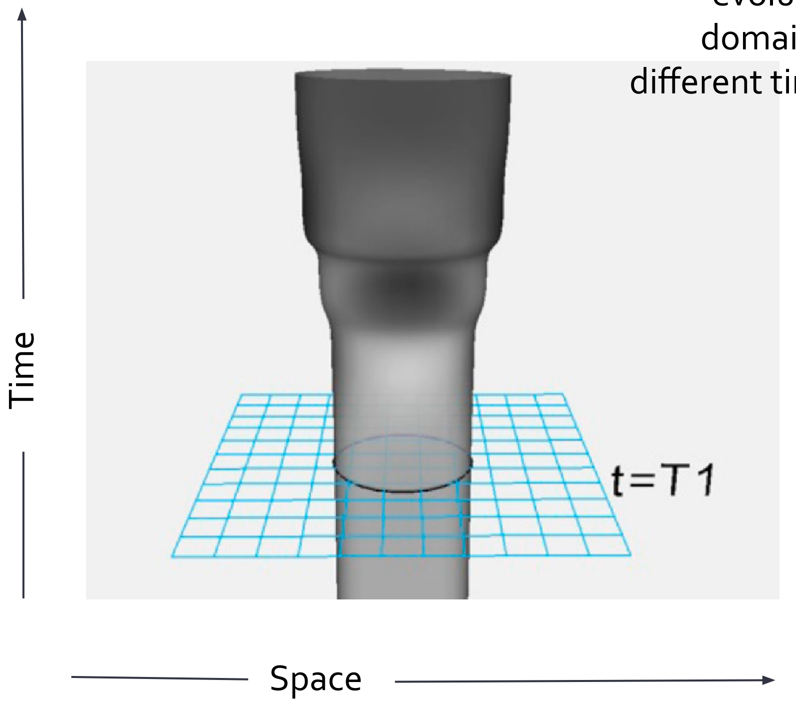

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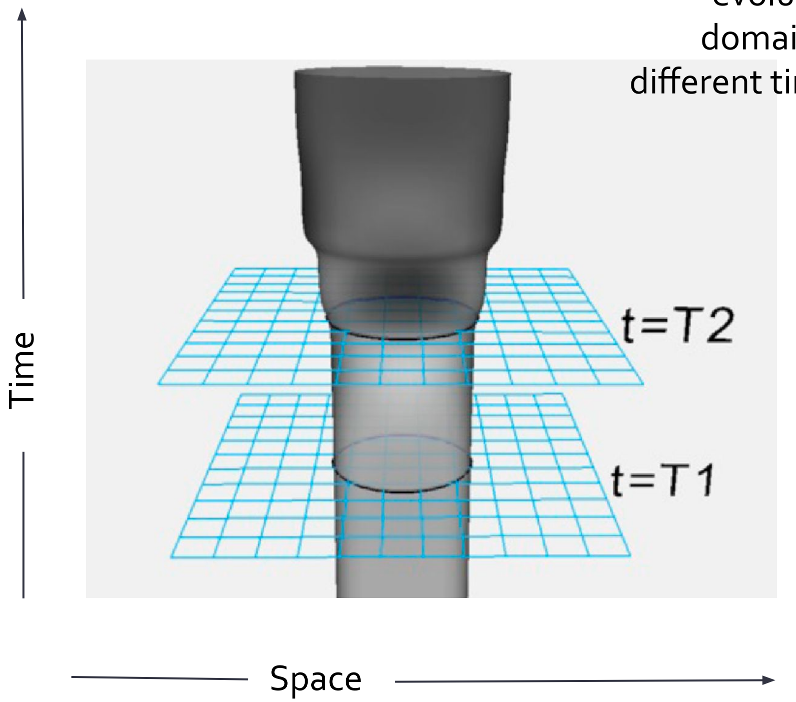

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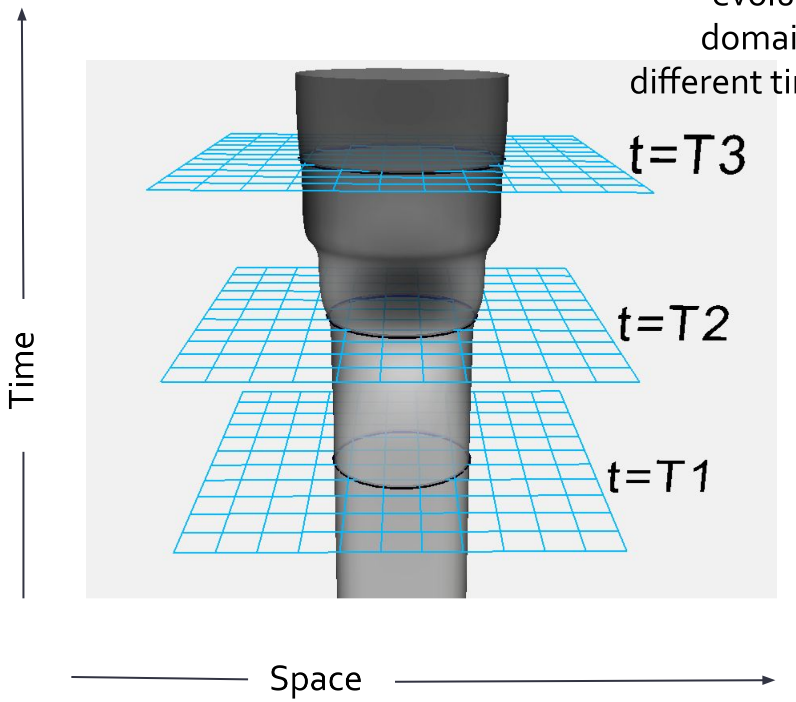

Snapshots of evolution domain at different times

Maxwell's equations

$$\boxed{\begin{aligned} \partial_t \vec{E} &= \nabla \times \vec{B} \\ \partial_t \vec{B} &= -\nabla \times \vec{E} \\ \nabla \cdot \vec{E} &= 0 \\ \nabla \cdot \vec{B} &= 0 \end{aligned}}$$

Goals: What makes it challenging

Multiple length/time scales, Courant limit, Accuracy required

1. Multiple length scales:

- Size of BH ~ **O(1M)**
- Separation ~ **O(10M)**
- Wavelength λ_{GW} ~ **O(100M)**
- Wave extraction ~ **several λ_{GW}**
- GW flux, that drives the inspiral, is small:

$$\dot{E}/E \sim 10^{-5}$$

Goals: What makes it challenging

Multiple length/time scales, Courant limit, Accuracy required

1. Multiple length/time scales

2. Which coordinates to use (for a spacetime one doesn't know yet)?

3. Putting Black holes (singularity) on a grid

4. Einstein constraints grew exponentially: for many ~~years~~ decades

5. Resolving shocks (discontinuities)

6. Computational Challenges:

- 20–50 variables
- Global timestep too small
- Computing efficiency

7. High accuracy required by LIGO:

- Absolute phase error << 1 rad / 20+ orbits

Goals: What makes it challenging

Multiple length/time scales, Courant limit, Accuracy required

1. **Multiple length/time scales**

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3. **Putting Black holes (singularity) on a grid**

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6. **Computational Challenges:**

- 20–50 variables
- Global timestep too small
- Computing efficiency

But, in vacuum, solutions are smooth

$\Rightarrow$ **Spectral methods**

7. **High accuracy required by LIGO:**

- Absolute phase error << 1 rad / 20+ orbits

Spectral Einstein Code (**SpEC***)

Goal: Solve Einstein's equations to enable robust gravitational-wave science

In development since 2002

650,000 lines, 130 publications

Simulations of Extreme Spacetimes (SXS) collaboration

Brief timeline of developments:

2005, Pretorius: First BBH merger

2006, Goddard group & UBT group: BBH mergers with different formulation

2007, BBH mergers with SpEC code: Now leading code to provide waveforms for LIGO

* http://www.black-holes.org/

SpEC: (non-local) Spectral discretization

Evolution quantities are smoothly varying.

- Expand them in basis-functions, solve for coefficients

$$u(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi_k(x)$$

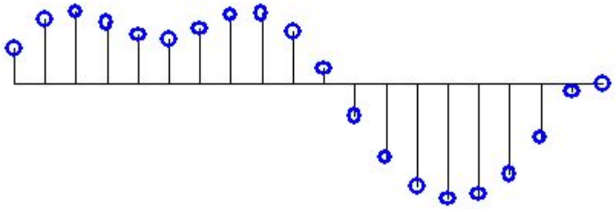
- Compute spatial derivatives *exactly*

$$u'(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi'_k(x)$$

- Compute nonlinearities in physical space

Spectral

Finite differences



Goals: What makes it challenging

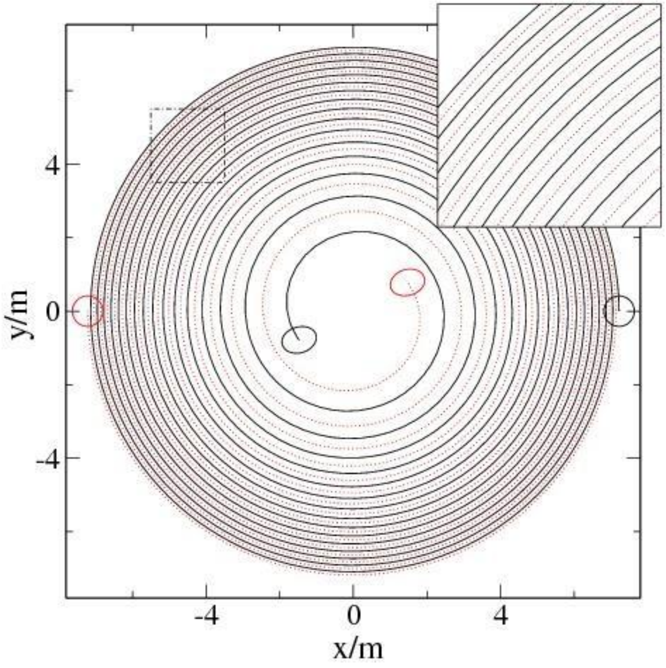
Multiple length/time scales, Courant limit, Accuracy required

1. **Multiple length/time scales** ⇒ **Adaptive Meshes**
2. **Which coordinates to use (for a spacetime one doesn't know yet)?** ⇒ **Gen. Harmonic**
3. **Putting Black holes (singularity) on a grid** ⇒ **Excision**
4. **Einstein constraints grew exponentially** ⇒ **Modified evolution system**
5. **Resolving shocks (discontinuities)** ⇒ **Duplicate Mesh**
6. **Computational Challenges** ⇒ **Spectral methods**
7. **High accuracy required by LIGO** ⇒ **Spectral methods, Optimizations**

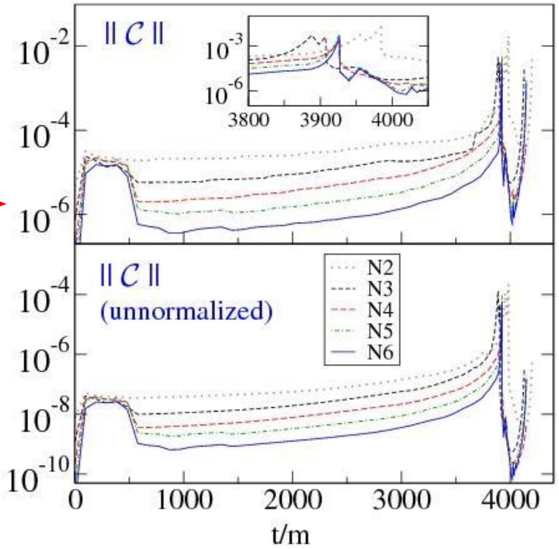
III. Results

Early Waveforms with SpEC: 2007

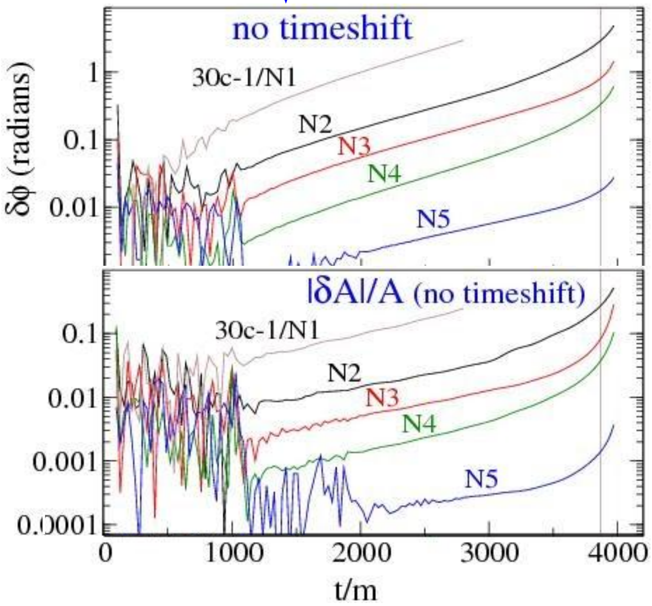
Equal-mass, non-spinning, 15 orbits



Einstein constraints under control



Simulation Accuracy



Boyle et al, Phys.Rev.D76:124038, (2007)

First open-access catalog of simulations: 2013

In 2013, first catalog of 174 SpEC simulations was made open-access: https://www.black-holes.org/waveforms/

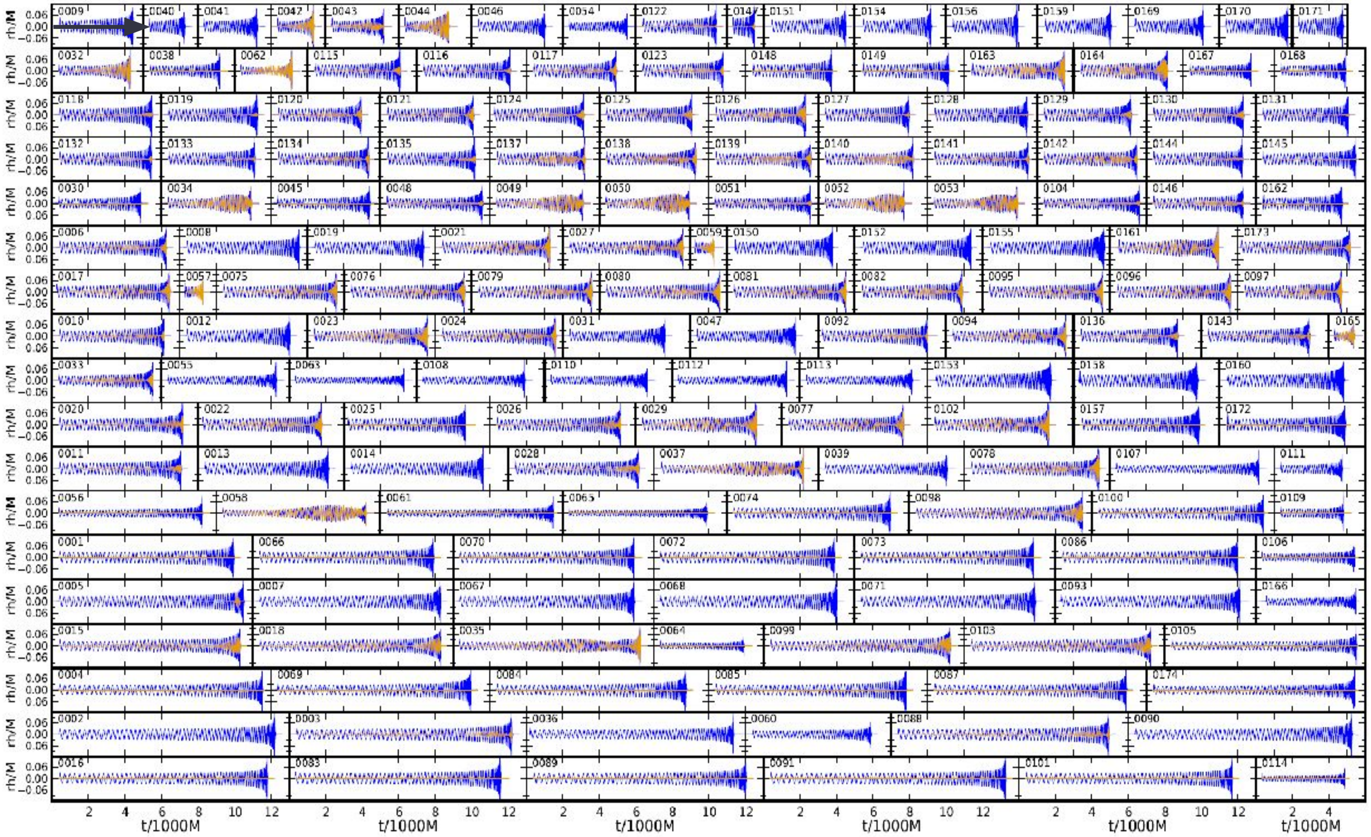
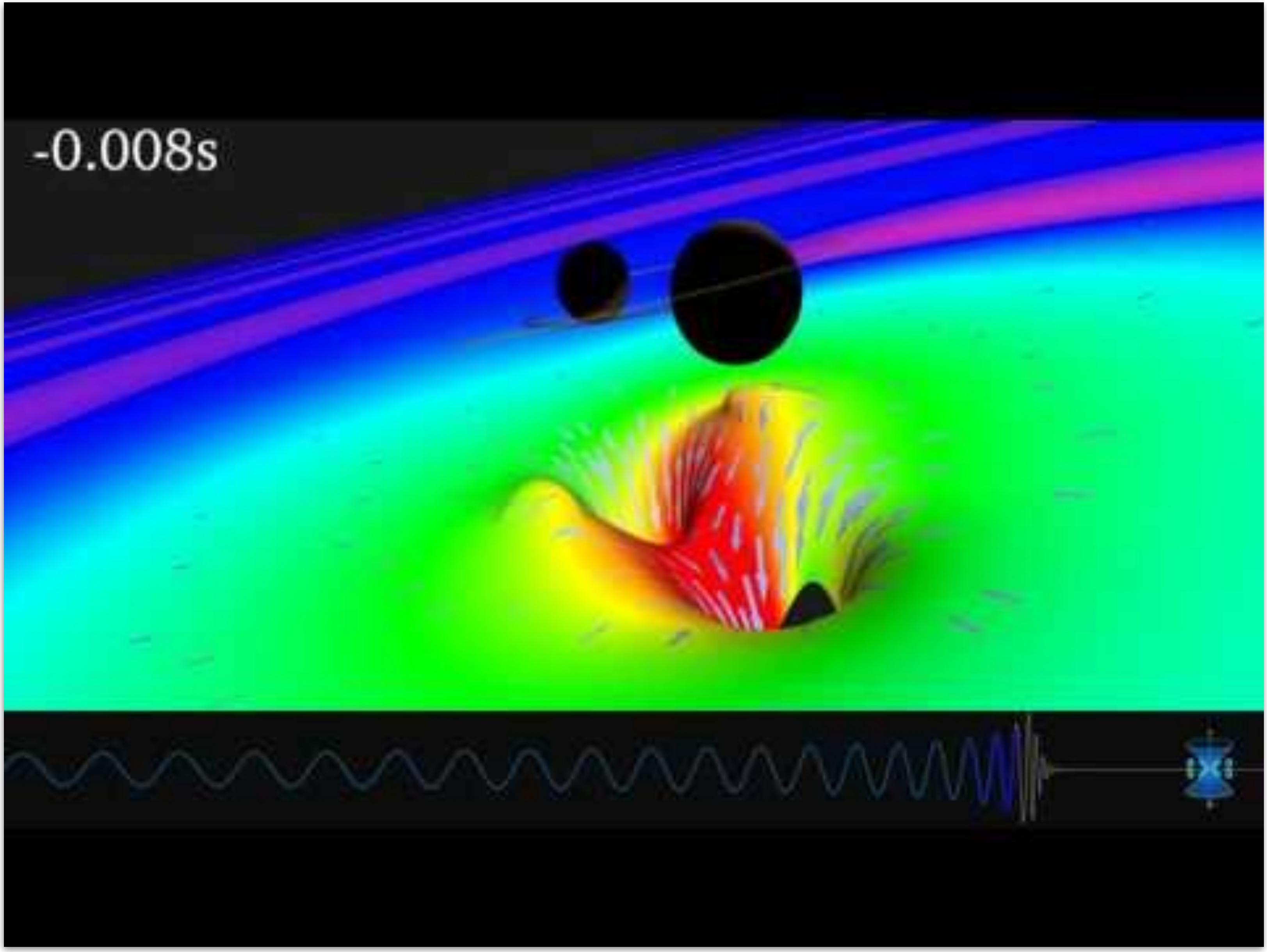


FIG. 3: Waveform polarizations $(r/M)h_+$ (blue) and $(r/M)h_\times$ (orange) in a sky direction parallel to the initial orbital plane of each simulation. The unit of the time axis corresponds to $1000M = 0.1s$ for binaries with total mass $M = 20M_\odot$.

Mroue et al, Phys. Rev. Lett. 111, 241104 (2013)

BBH Simulation: GW150914



And more ...: 2016

In 2016, another catalog of 95 simulations was made open-access: https://www.black-holes.org/waveforms/

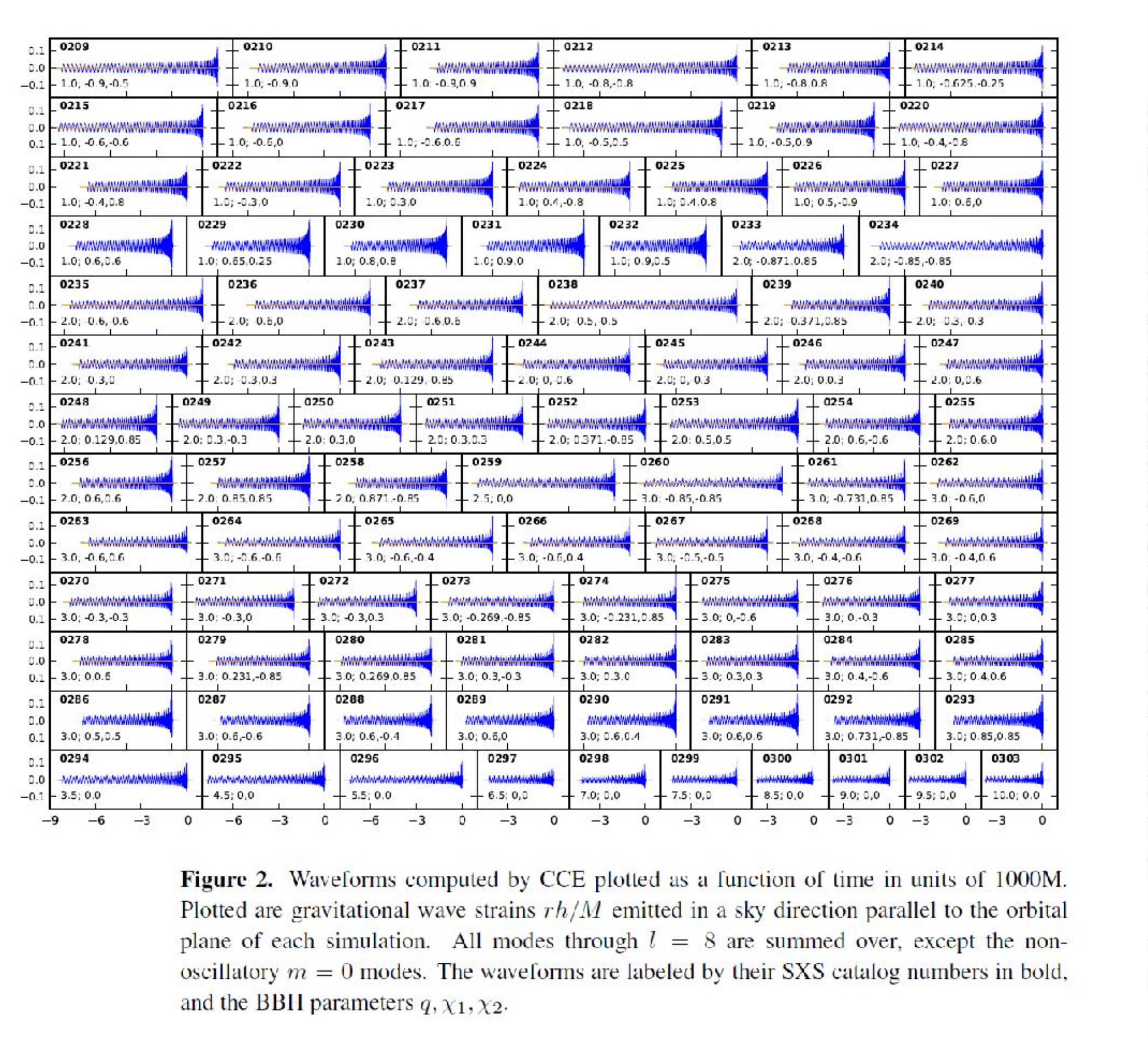
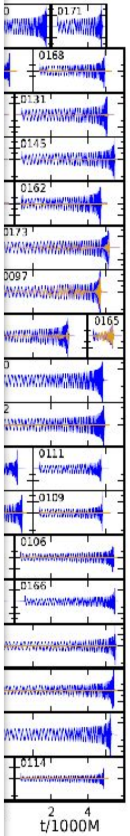


Figure 2. Waveforms computed by CCE plotted as a function of time in units of 1000M. Plotted are gravitational wave strains rh/M emitted in a sky direction parallel to the orbital plane of each simulation. All modes through $l = 8$ are summed over, except the non-oscillatory $m = 0$ modes. The waveforms are labeled by their SXS catalog numbers in bold, and the BBH parameters q, χ_1, χ_2.



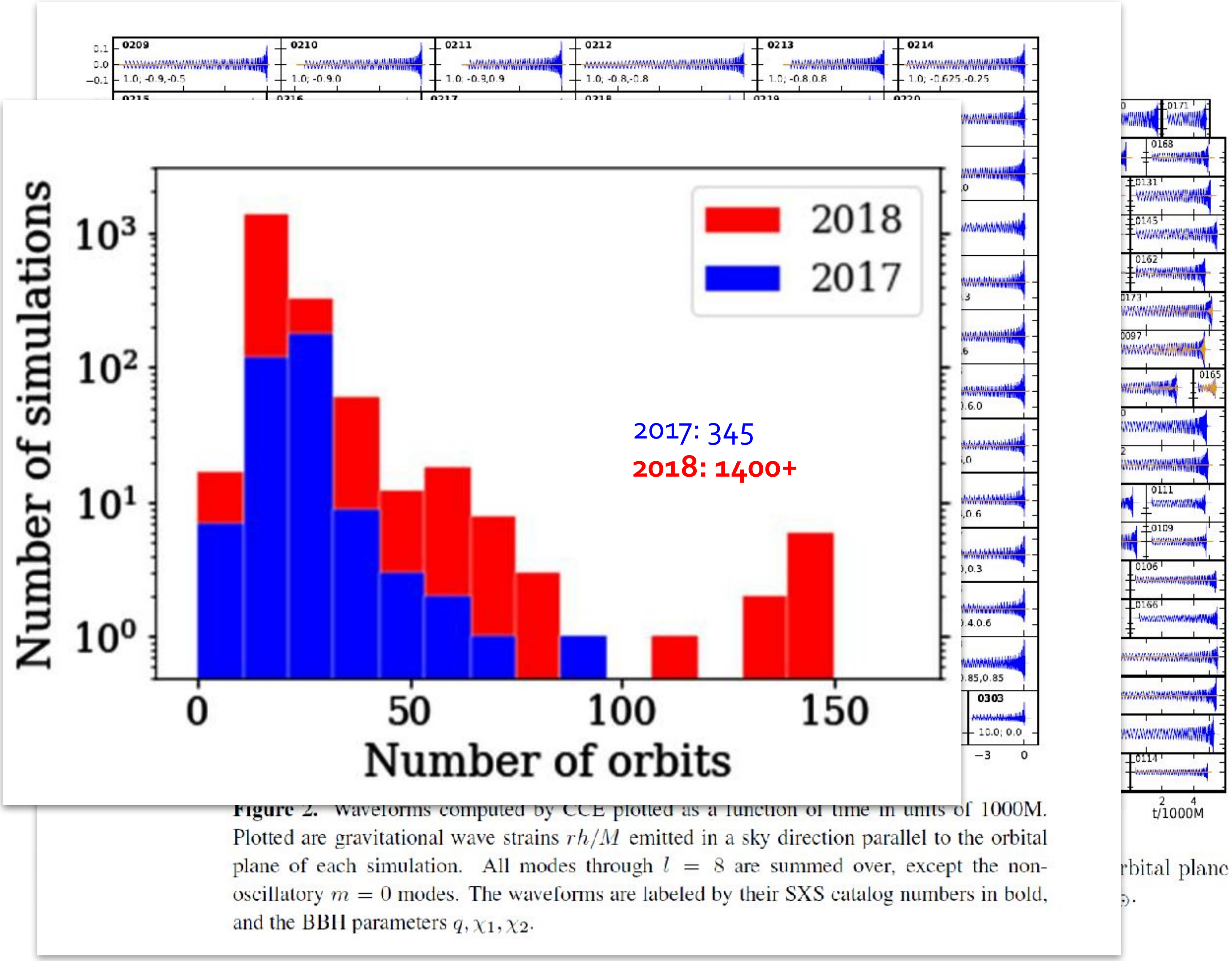
rbital plane

).

Chu, Fong, **PK**, et al, Class. Quant. Grav. Vol 33, No 16 (2016)

And more ...: 2018

Shortly, another major release of ~**1000** simulations was made: https://www.black-holes.org/waveforms/



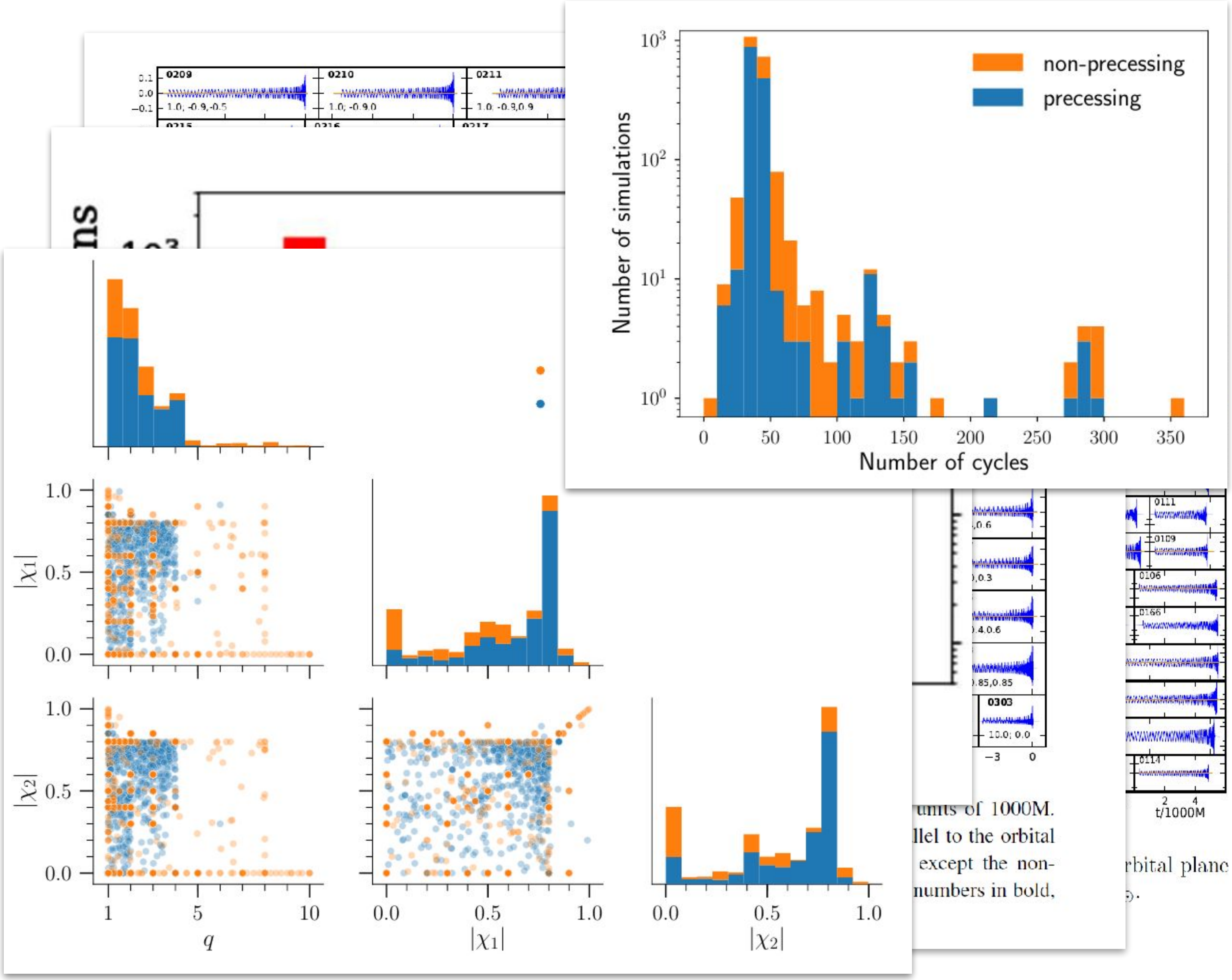
2017: 345

2018: 1400+

Figure 2. Waveforms computed by CCE plotted as a function of time in units of 1000M. Plotted are gravitational wave strains rh/M emitted in a sky direction parallel to the orbital plane of each simulation. All modes through $l = 8$ are summed over, except the non-oscillatory $m = 0$ modes. The waveforms are labeled by their SXS catalog numbers in bold, and the BBH parameters q, χ_1, χ_2.

And more ...: 2019

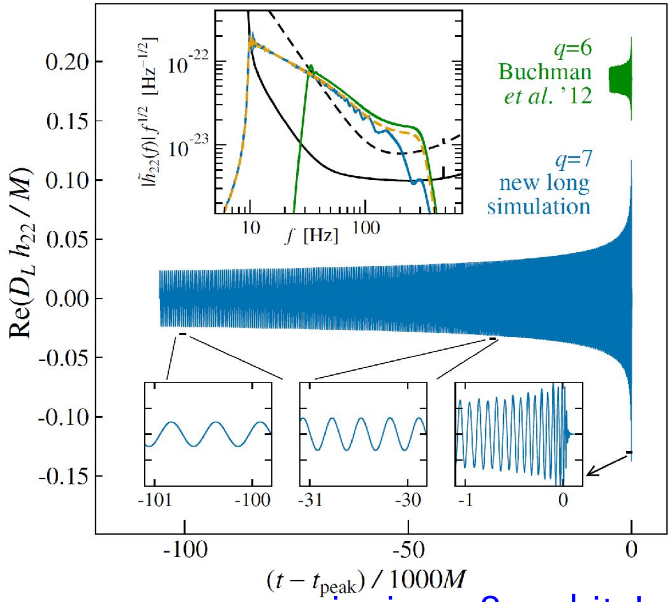
Shortly, another major release of ~**2000** simulations was made: https://www.black-holes.org/waveforms/



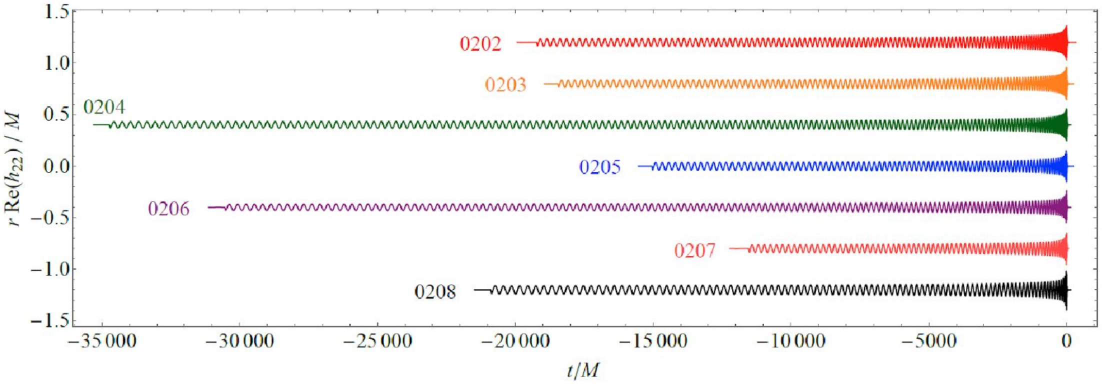
Class.Quant.Grav. 36 (2019) 19, 195006

Broken one barrier: Stability

Due to AMR, and better control systems, we can now perform longer stable simulations: **O($10^{1\text{-}2}$) orbits.**

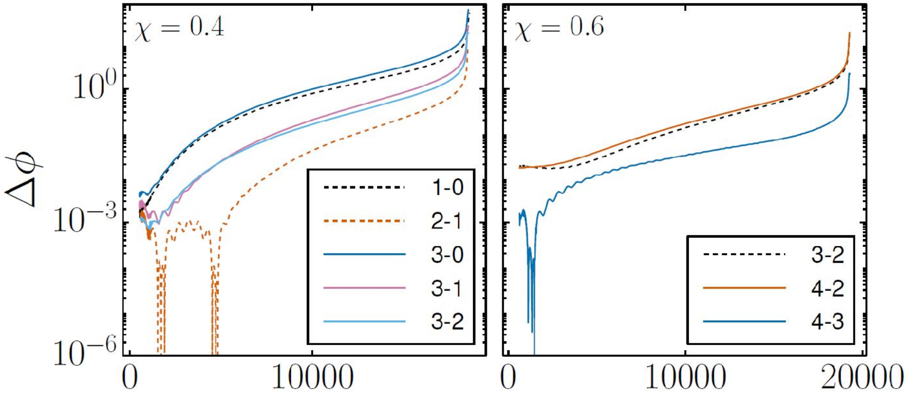


q = 7, non-spinning, 180 orbits!



q = 7,
aligned-spins: $\chi_1 = \pm 0.6, \pm 0.4$,
45-90 orbits!

Orbital phase tracking :
better than 1 rad over
1000+ rads of evolution

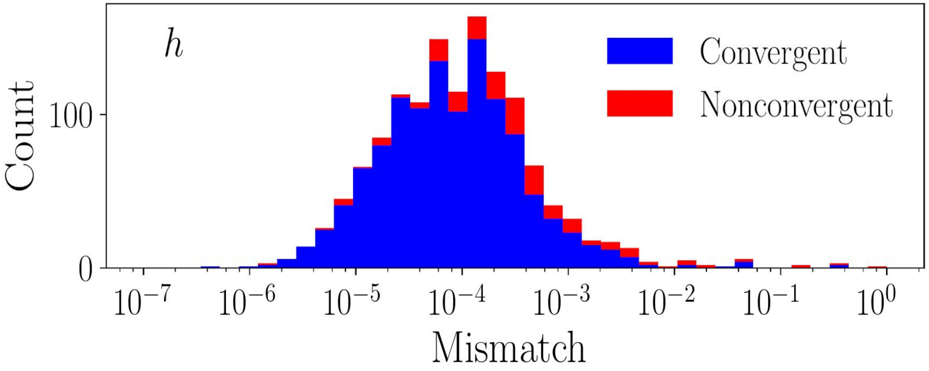


Szilagyi et al, PRL 115, 031102 (2015);
PK, et al, Phys. Rev. D 92, 102001 (2015)

Accuracy & Cost

Simulation Accuracy:

NR errors at 0.1% level

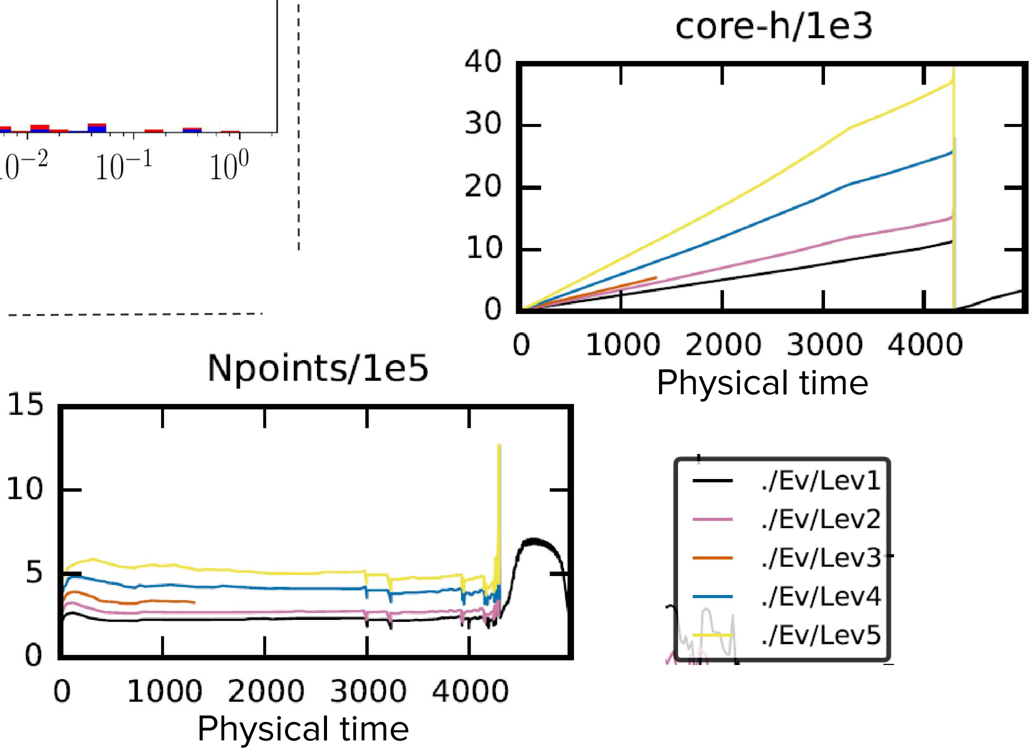


⇒ Simulation errors small enough for LIGO Science

Computational cost:

(1-2) x 20,000 CPU-hours (~20 orbits)

Cost can vary depending on length/accuracy requirement & black hole parameters



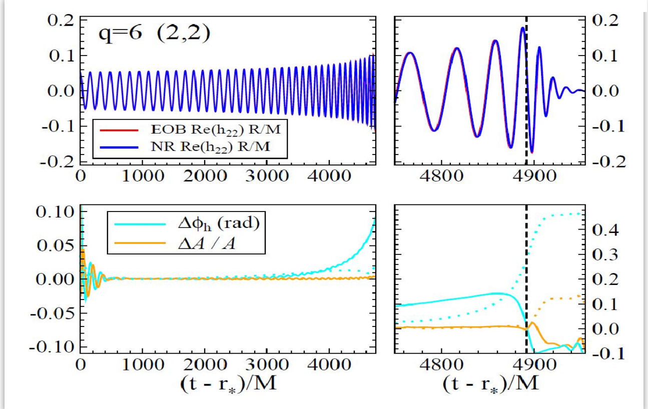
$M_1 / M_2 = 1.2$; $|S_1| = |S_2| = 0.85$

Figure: APS 2018 Talk by Catherine Woodford for SXS; **PK** (unpublished)

IV. Applications to GW Astronomy

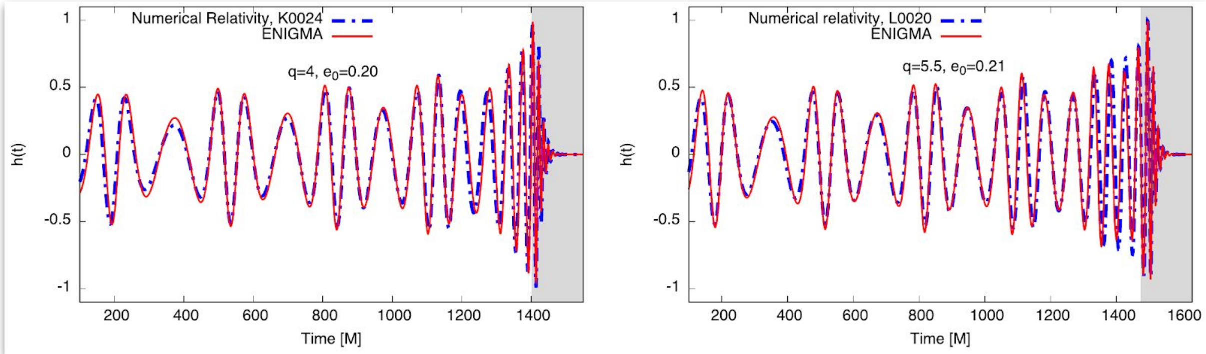
NR-based **Waveform Models**

Non-Spinning



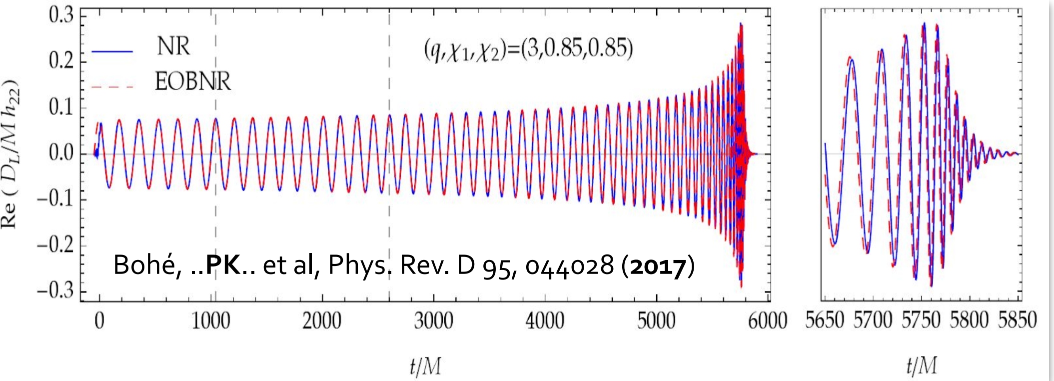
Pan et al, Phys. Rev. D 84, 124052 (**2011**)

Eccentric



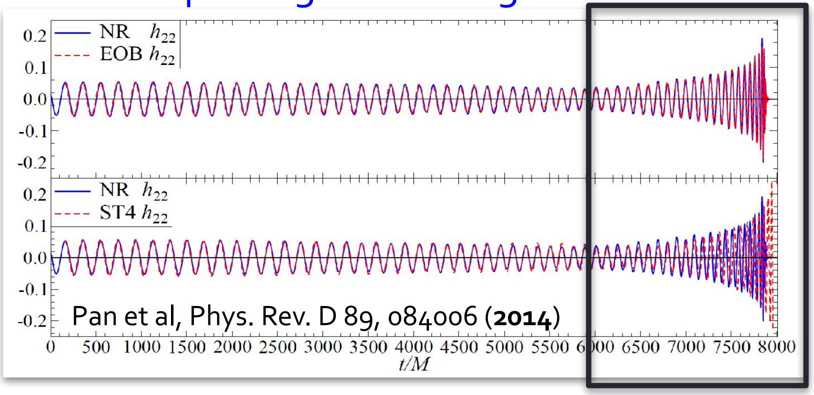
Huerta, ..**PK**.. et al, Phys. Rev. D 97, 024031 (**2018**)

Aligned-Spinning



Bohé, ..**PK**.. et al, Phys. Rev. D 95, 044028 (**2017**)

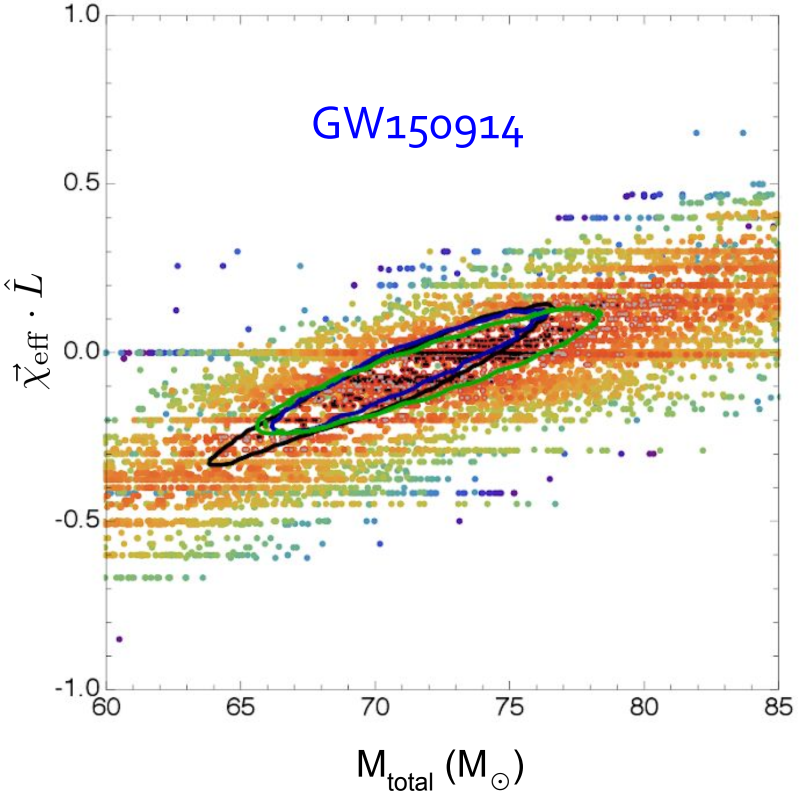
Spinning-Precessing

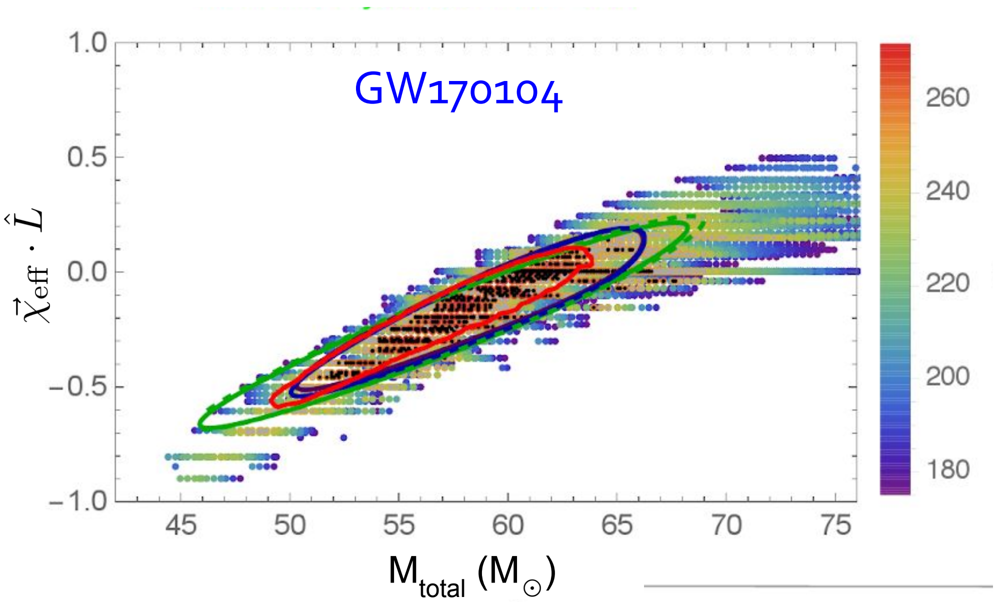


Pan et al, Phys. Rev. D 89, 084006 (**2014**)

<u>All</u> waveform models used to infer astrophysical information from LIGO-Virgo's observations are calibrated to SpEC simulations, in addition to simulations from other NR groups

Parameter estimation with NR





Key:

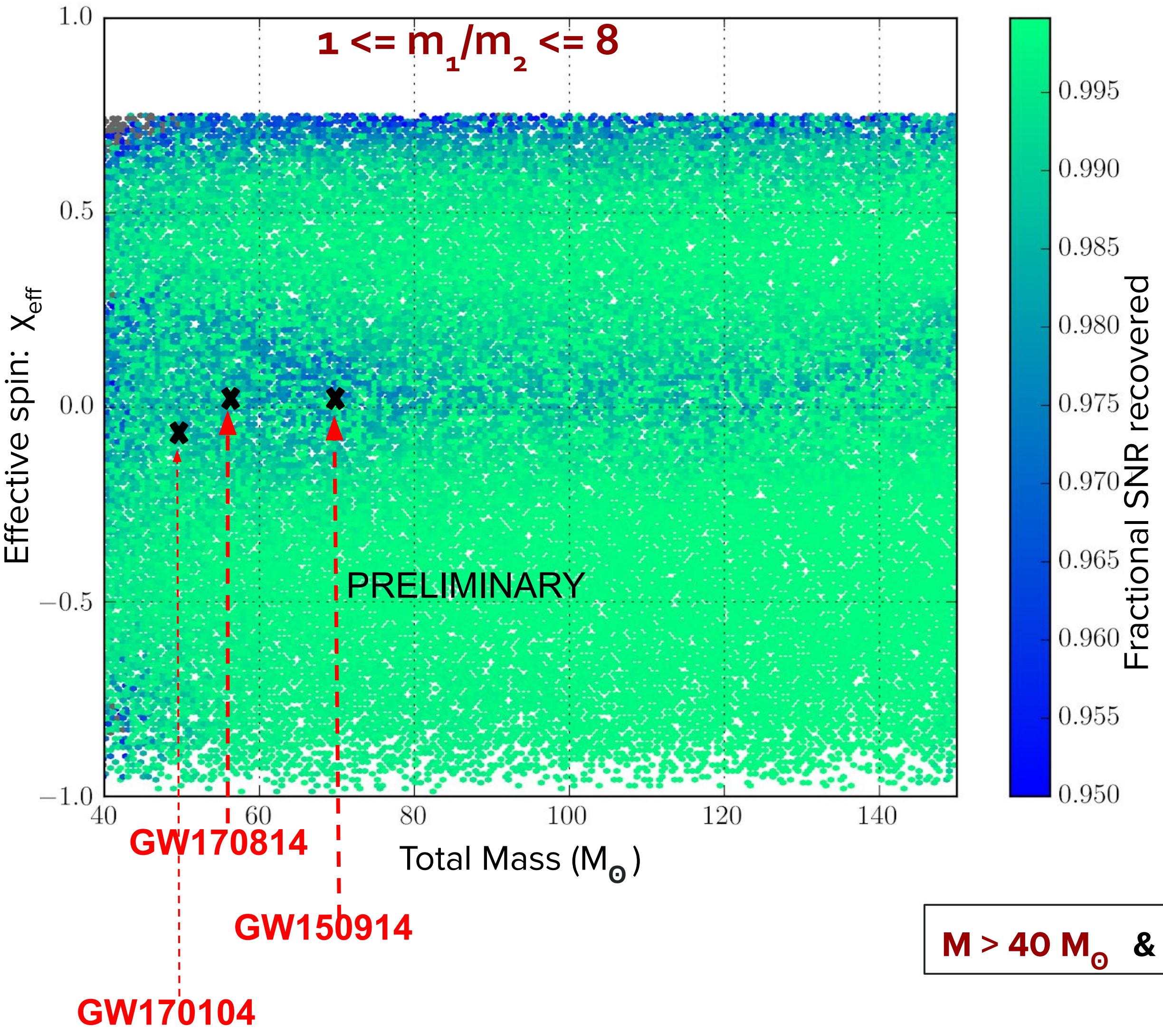
- **Green solid [dashed] contour - NR with l = 2 [l <= 3] modes**
- **Black / Blue / Red: Models**

Large heterogeneous set of 1100+ numerical simulations from SXS/RIT/GT/BAM.

Both aligned-spin & precessing-spin

Phys. Rev. D 94, 064035 (2016); Phys. Rev. D 96, 104041 (2017)

Searches with NR: large BBH spins



This shows the performance of an ***NR-only*** spinning bank for the **O1** noise curve

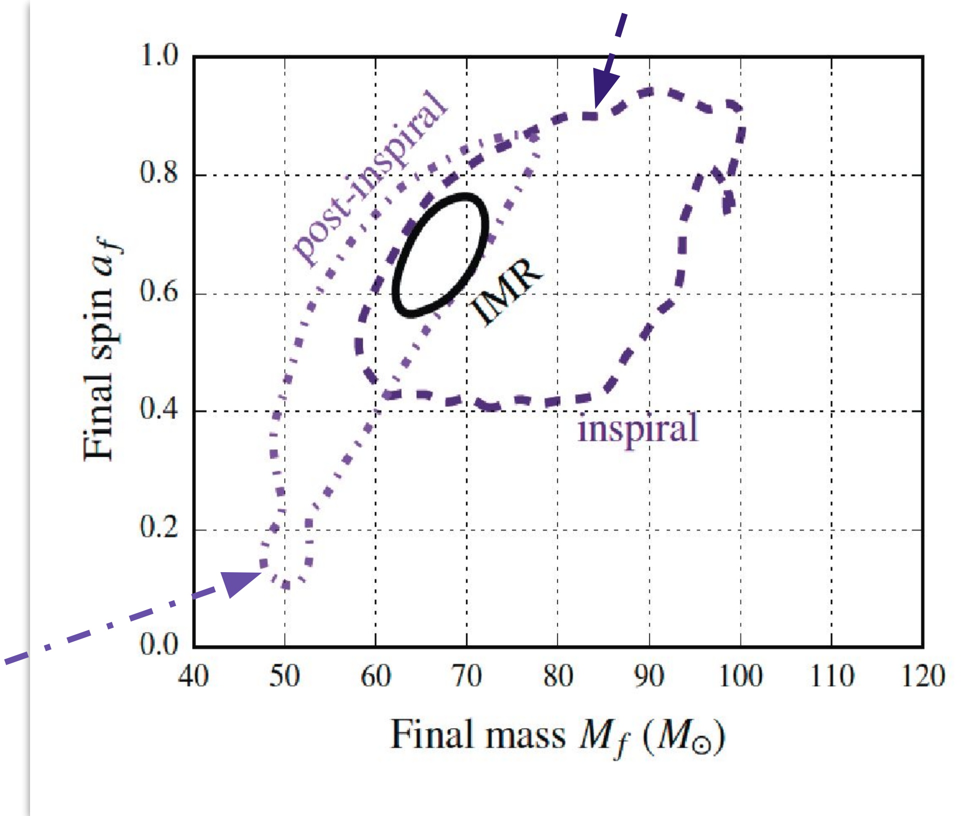
$M > 40\ M_\odot$ & $q <= 8$ & $-1 <= \chi_{eff} <= +0.75$

PK et al (unpublished)

Testing General Relativity

Q. Is the inspiral portion of a GW signal consistent with its merger, as predicted by GR?

⇒ We compare the mass $\mathbf{M_f}$ & spin $\mathbf{a_f}$ of the post-merger BH computed from either portion, *for consistency*



(m_1 , m_2) from inspiral **mapped** to (M_f , a_f). **This mapping comes from NR catalogs.**

(M_f , a_f) from post-inspiral signal

LIGO-Virgo Collaborations, arxiv:1602.03841 (2016)

V. Future: Scaling up!

Goals: What made it challenging

Multiple length/time scales, Courant limit, Accuracy required

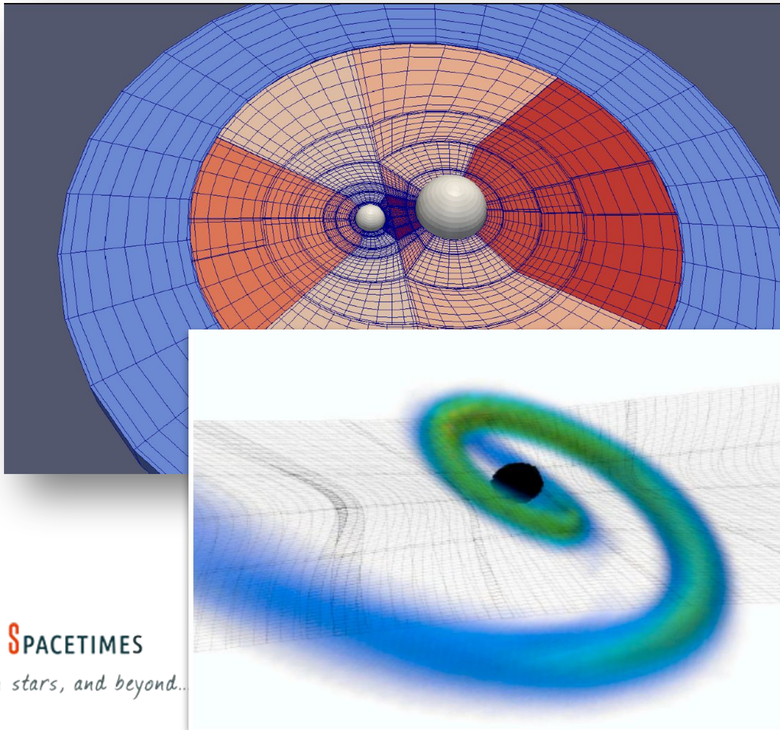
1. **Multiple length/time scales**
2. **Which coordinates to use (for a spacetime one doesn't know yet)?**
3. **Putting Black holes (singularity) on a grid**
4. **Einstein constraints grew exponentially**
5. **Resolving shocks (discontinuities)**
6. **Computational Challenges**
7. **High accuracy required by LIGO**

What still makes it challenging

1. **Multiple length/time scales**
2. ~~Which coordinates to use (for a spacetime one doesn't know yet)?~~
3. ~~Putting Black holes (singularity) on a grid~~
4. ~~Einstein constraints grew exponentially~~
5. **Resolving shocks (discontinuities)**
6. **Computational Challenges**

↕

7. **High accuracy required by LIGO**



SIMULATING EXTREME SPACETIMES

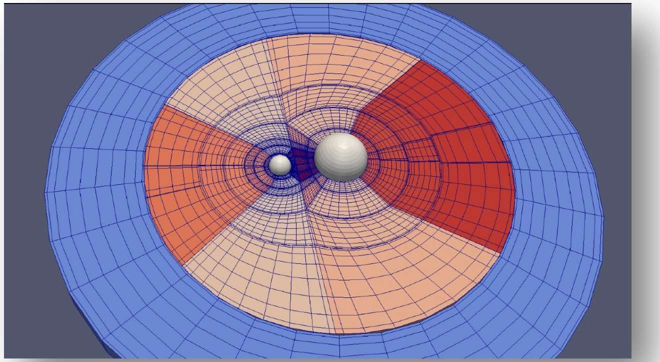
Black holes, neutron stars, and beyond...

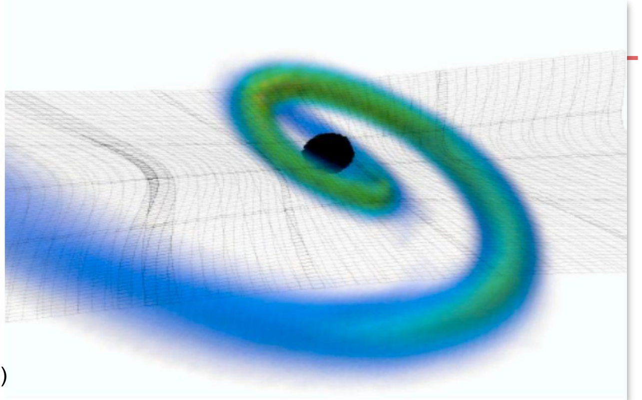
Szilagyi et al (2014); Fou

Back to the drawing board

1. Multiple scales
2. Computational Challenges
3. Shocks
4. High accuracy

1. Discretization scheme that:
   a. is local at high order
   b. can handle discontinuities
   c. amenable to inhomogeneous grid
2. Parallelization scheme that can scale, and use all computing available
3. Local time-stepping to handle multiple time scales





Szilagyi et al (2014); Foucart et al (2016)

Discretization: Discontinuous Galerkin (DG)

DG-FEM

Local at low-order	↑
Local at high-order	↑
Handle discontinuities	↑
Inhomogeneous grids	↑

Finite Difference

Local at low-order	↑
Local at high-order	↓
Handle discontinuities	↑
Inhomogeneous grids	↑

Spectral Methods

Local at low-order	↑
Local at high-order	↑
Handle discontinuities	↓
Inhomogeneous grids	↑

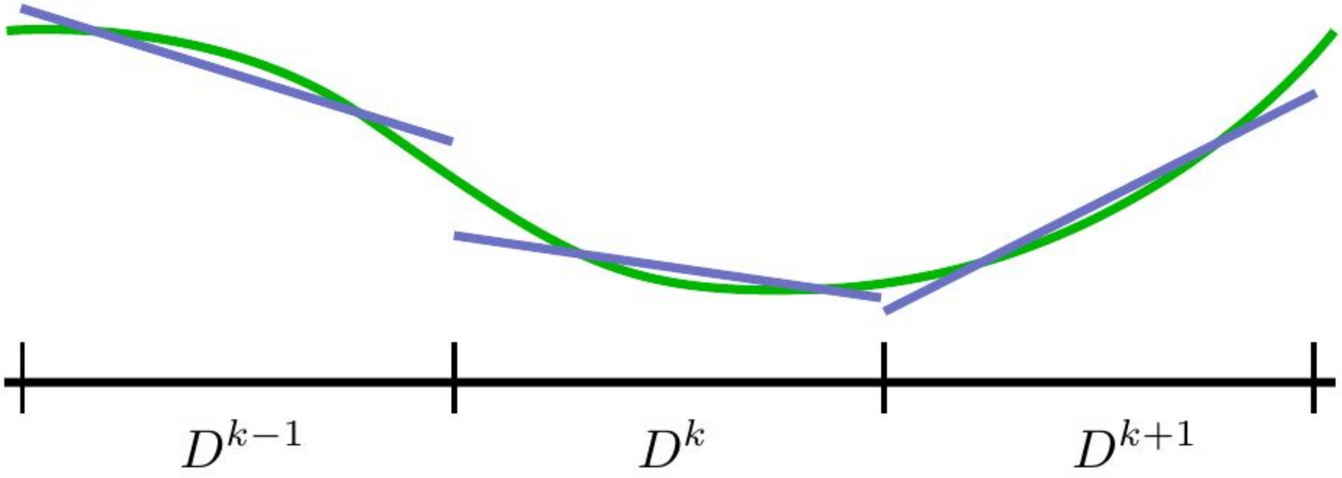
Finite Volume

Local at low-order	↑
Local at high-order	↓
Handle discontinuities	↑
Inhomogeneous grids	↑

Images adapted from: Francois Hebert (Caltech), Allan P. Engsig-Karup (TU Dresden)

Discretization: Discontinuous Galerkin (DG)

- Solution expanded on a **local basis**

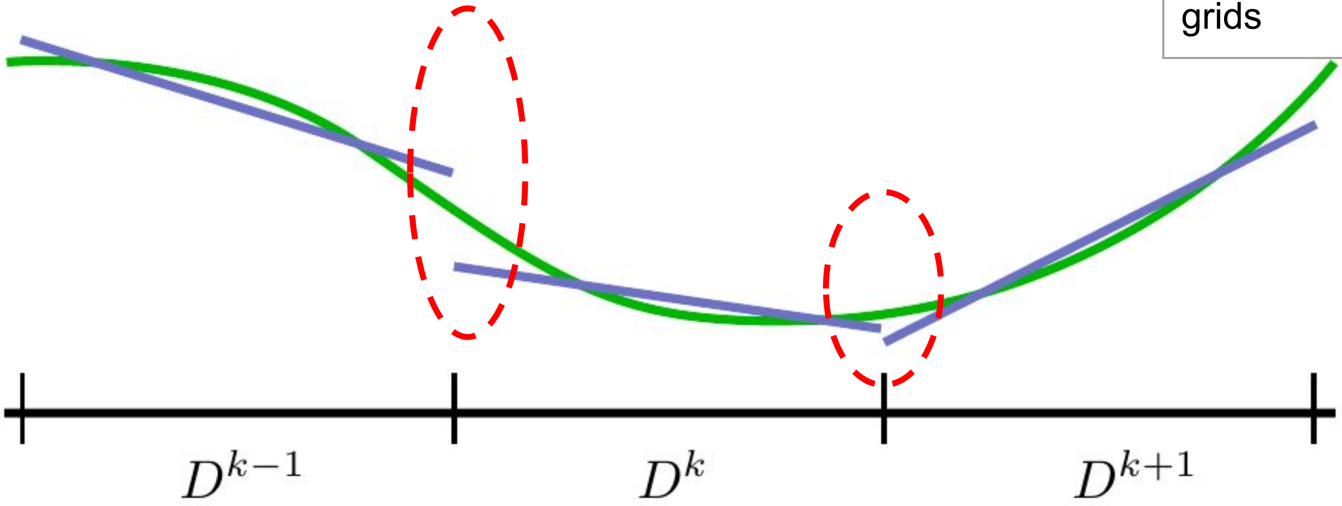


Images adapted from: Francois Hebert (Caltech), Allan P. Engsig-Karup (TU Dresden)

Discretization: Discontinuous Galerkin (DG)

- Solution expanded on a **local basis**
- Exponential convergence in smooth regions
- … **and formulation allows “arbitrary” fluxes ⇒ can handle shocks!**

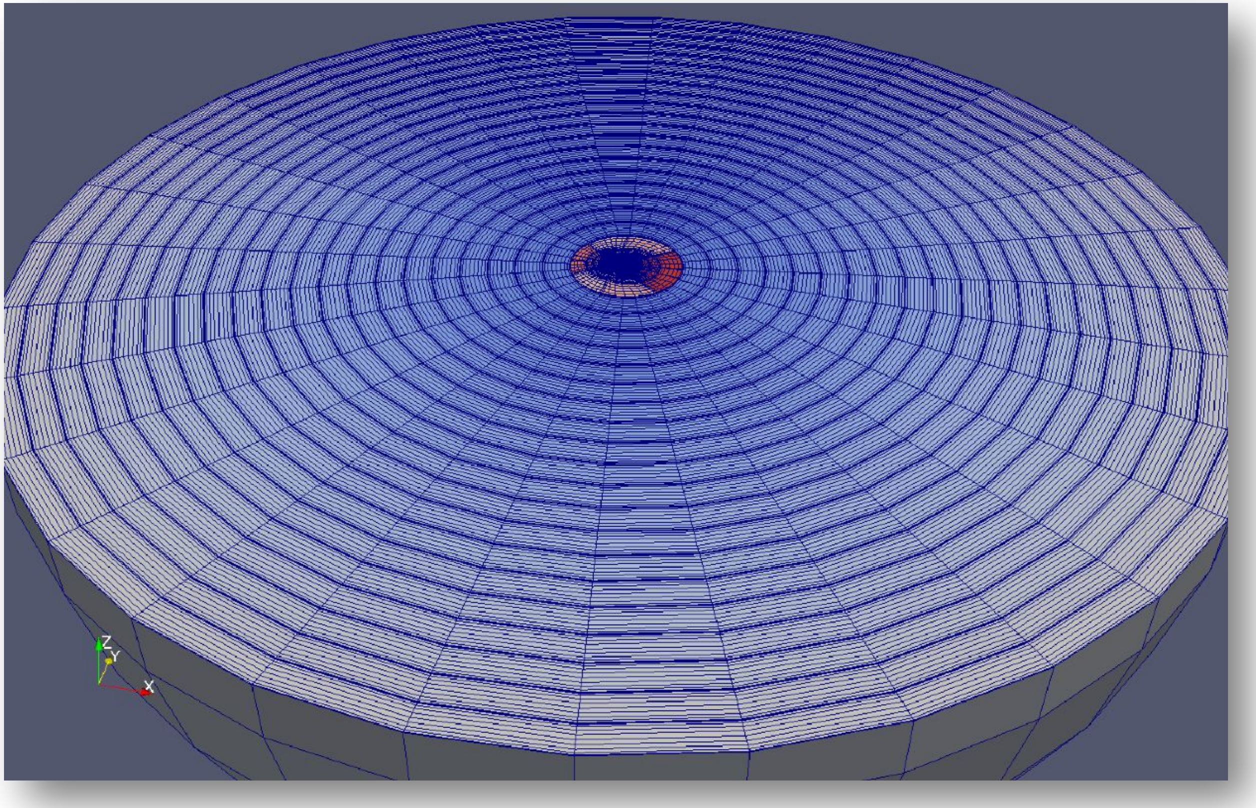
Local at low-order	↑
Local at high-order	↑
Handle discontinuities	↑
Inhomogeneous grids	↑



Images adapted from: Francois Hebert (Caltech), Allan P. Engsig-Karup (TU Dresden)

Local time-stepping

- Evolve the solution in time depending on the local needs
- No wastage of computing due to one corner with high-frequency activity



Szilagyi et al (2014)

Local time-stepping

- Evolve the solution in time depending on the local needs
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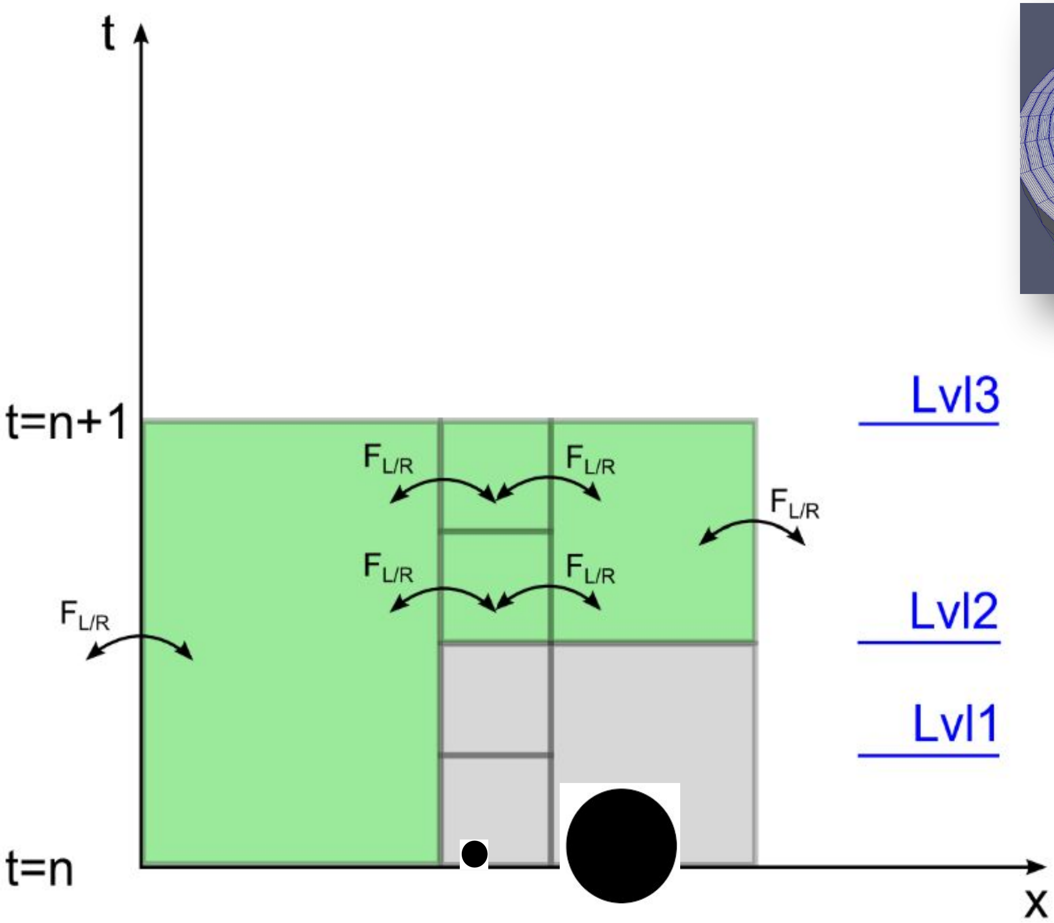
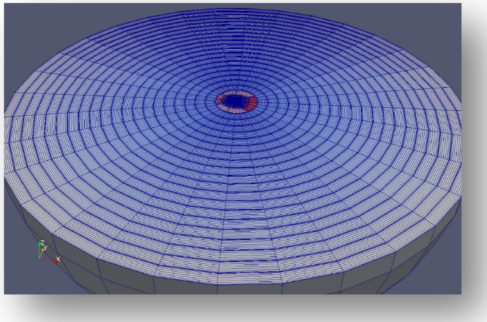
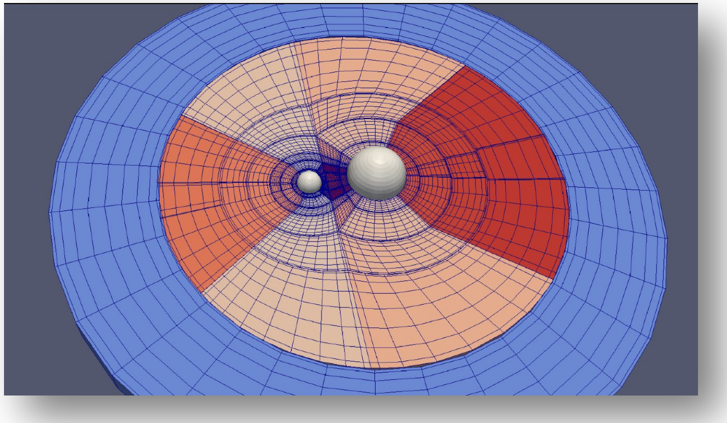
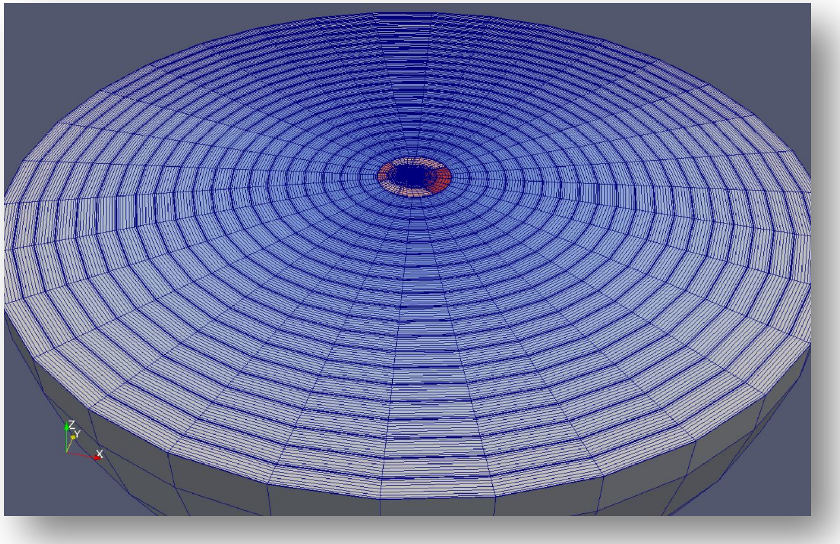




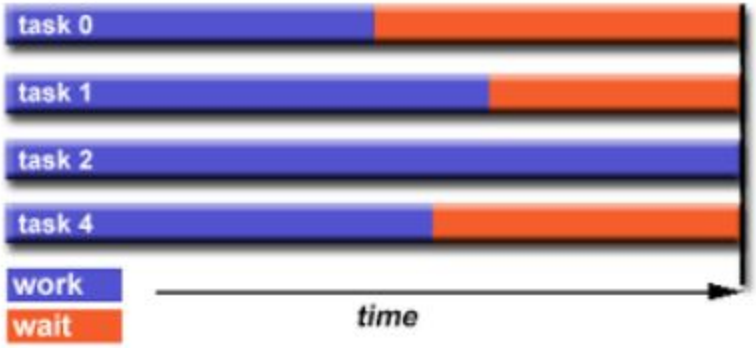
PC: Throwe and Teukolsky

Parallelization scheme: MPI Domain based





- Allocate one domain element per core
- Use MPI

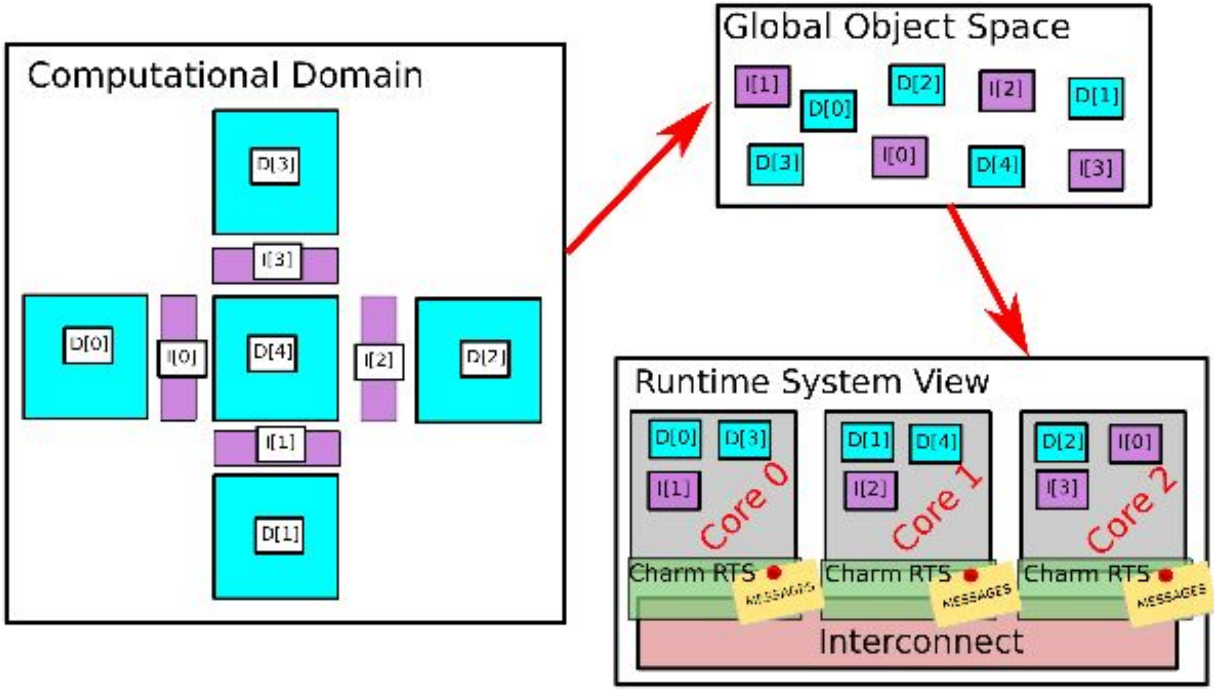


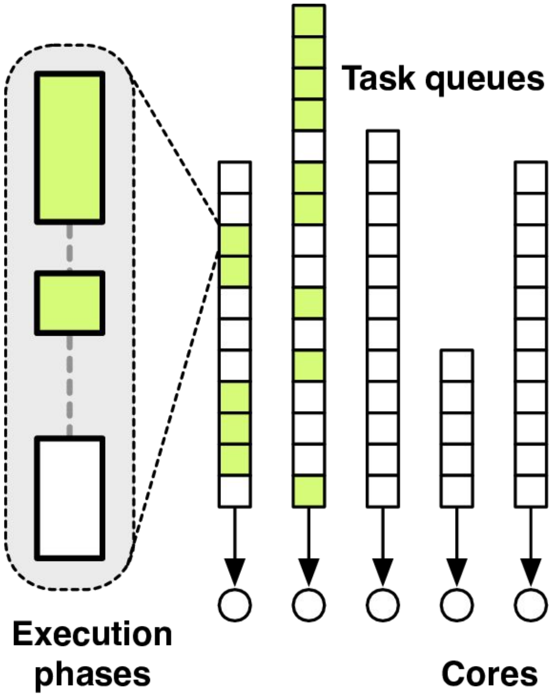
⇒ …terrible terrible idea for systems with length scales that span several orders of magnitude!

Szilagyi et al (2014).

Parallelization scheme: Task-based

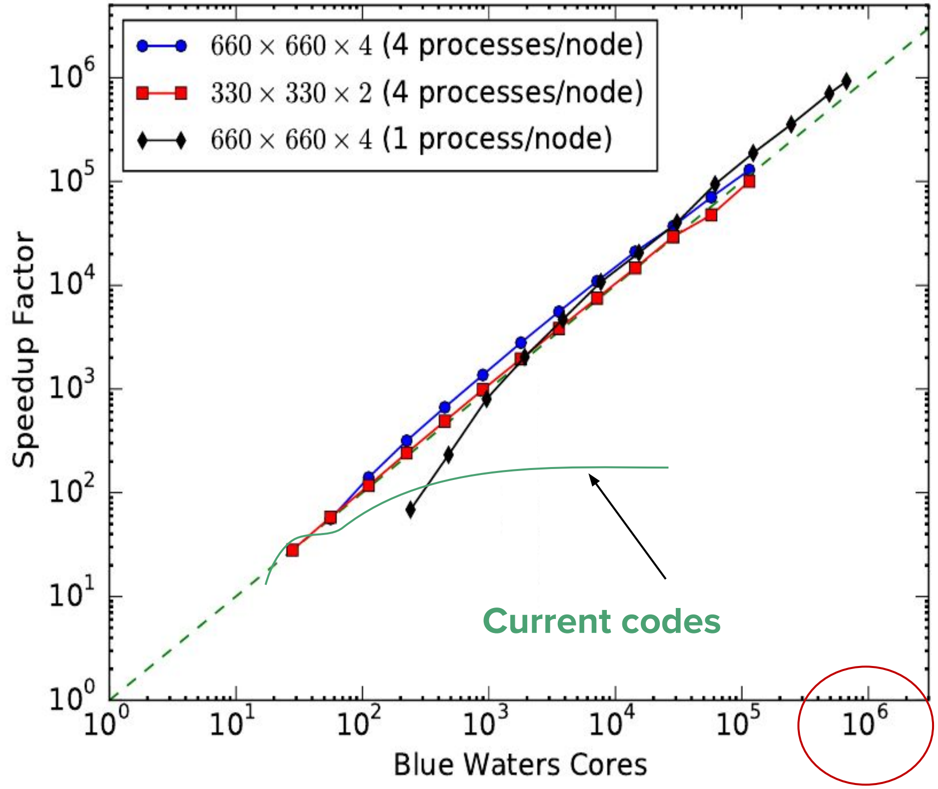
- Divide computation by tasks, not physical domain
- Make communication of data between elements also a task
- Communication-cost hidden behind computation

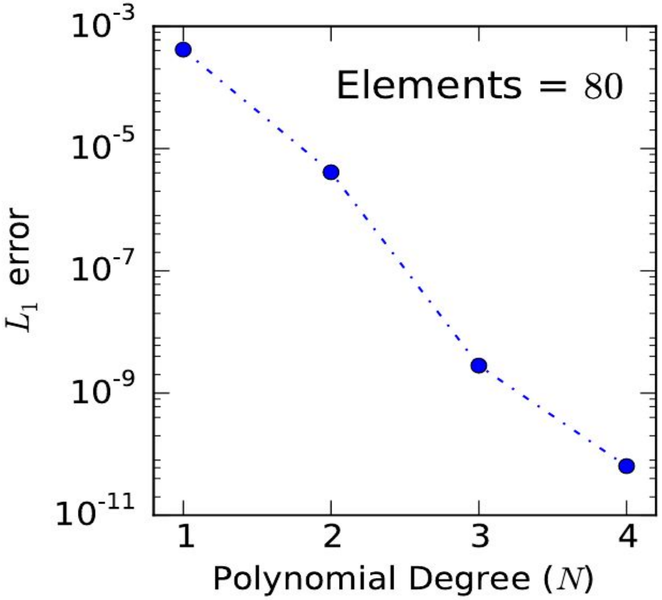




Kidder et al (2016); Stark et al (2014).

SpECTRE: scaling





- SpECTRE aims to **combine the high-order accuracy of spectral methods with the local nature of finite-volume/element methods**

- **Future proof:** Computing efficiently scales to ~600, 000 cores. Future proof: exascale computing!

Kidder et al (2016);

Summary

- Numerical simulations of black hole binaries’ coalescence is key to extracting scientific information from LIGO’s observation of black hole binaries

- NR simulations contribute to GW science through waveform models. More recently, they have also been used directly to analyze GW data.

- **SpEC** is a flexible infrastructure for solving partial differential equations using multi-domain spectral methods

- **Spectre** is a radically forward-looking computational (astro)physics code that adopts cutting-edge computing paradigms that will enable exascale computing:
  - **DG-FEM discretization**
  - **Local time-stepping**
  - **Task-based parallelism**

- Einstein/MHD equations implemented. Boundary treatment nearly complete. Working on control systems!

- **Spectre is open-source** → https://github.com/sxs-collaboration/spectre

Thank You for Listening!

Questions?