Black hole hair removal for $\mathcal{N} = 4$ CHL models

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Outline

- ✂ Puzzle
- ✂ Model of interest
- ✂ Bosonic & Fermionic hair candidates
- ✂ Hair modes
- ✂ Resolution of the puzzle - hair removal
- ✂ Summary and ongoing work

Outline

Black hole hair

- John Wheeler famously made the statement : “Black holes have no hair."
- No hair theorem ⟶ in the presence of no matter content other than Maxwell’s field, a black hole is completely described by its mass, electric and magnetic charge and spin.
- For more complicated set-ups (higher dimensions, form-fields, fermions), black hole hair is possible.
- In our context, black hole hair is defined as smooth, normalisable, bosonic and fermionic degrees of freedom that live entirely outside the horizon.

Jatkar, Sen, Srivastava - 2009

Why black hole hair?

- Black hole entropy in terms of microscopic degrees of freedom - insensitive to nature of solution away from the horizon.
  - $\Longrightarrow$ Two different black holes with identical near horizon geometries have the same microscopic indices.
- Counterexample:
  BMPV black hole in flat space and in Taub-NUT space - identical near horizon geometries, but microscopic indices mismatch.
- Proposed resolution for the above conundrum - hair removal.

  Banerjee, Mandal, Sen - 2009 ; Jatkar, Sen, Srivastava - 2009
- Motivated by this: hair removal for (general) Z_N CHL models.

Outline

CHL Models

- Exact calculation of microscopic indices with $\mathcal{N}=4$ supersymmetry has seen enormous progress over the years.
- CHL models provide a rich arena for such discussions.
- Chaudhuri, Hockney, Lykken models are obtained as Z_N orbifolds of IIB theory on $K3 \times S^1 \times \tilde{S}^1$.
- For a class of CHL models, in the lower dimensional viewpoint, black holes can be described in terms of six dimensional $(2,0)$ supergravity coupled to n_t tensor multiplets.

Field content

Six dimensional $(2,0)$ supergravity coupled to n_t tensor multiplets - many fields.

- Bosonic Sector: metric, self-dual and anti-self-dual 2-form fields, scalars.
- Fermionic sector: gravitinos, other Majorana fermions.

Solutions we consider

- scalars are set to constant and spin $1/2$ fermions and anti-self-dual 2-forms are set to zero.
- preserve 4 of the 16 supersymmetries.

BMPV black hole

For simplicity - Non-rotating BMPV black hole:

- BMPV black hole : 5D black hole (in 6 dim.) from now on.

$$ds^2 = \psi^{-1}(r)\left[dudv + (\psi(r)-1)dv^2\right] + \psi(r)dw^i dw^i,$$

$$u = x^5 - t, \quad v = x^5 + t.$$

- BMPV black hole in Taub-NUT: 4D black hole (in 6 dim.) from now on. Replace ds^2_{flat} with

$$ds^2_{TN} = \chi(r)^{-1}(dx^4 + \cos\theta d\phi)^2 + \chi(r)d\Omega_3^2,$$

$$\psi(r) = \left(1 + \frac{r_0}{r}\right), \qquad \chi(r) = \left(\frac{4}{R_4^2} + \frac{1}{r}\right).$$

- The black holes have identical near horizon geometry.

Outline

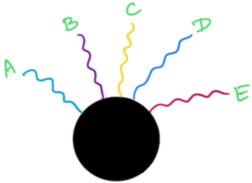
Which hair to cut?

Aim: Construct black hole hair - what kinds do we have and how many of each?

- Analysing all possible hair is hard - no proof that only these hair exist.
- Even if all hair candidates are not known, the mismatch of indices may be explained with the knowledge of just a few - there could be other hair, equal in number for both black holes.

For these two black holes, we will study only these three:

- ✓ Garfinkle-Vachaspati hair
- ✓ Form field hair
- ✓ Fermion hair



Bosonic hair candidates

↪ Garfinkle-Vachaspati transform: Solution generating technique to add wave like deformations on a metric.

- Known solution $\xrightarrow[\text{transform}]{\text{Garfinkle-Vachaspati}}$ New exact solutions
- Requirements:
  - a vector field k^M: null, hypersurface orthogonal and Killing.
  - matter fields - mild conditions (do not transform).

Transform:

$$G'_{MN} = G_{MN} + e^A T k_M k_N,$$

for some scalars A and T such that $\nabla^2 T = 0$ and $k^M \partial_M T = 0$.

GV hair Candidates

- Deformation T is independent of u - new solution also possesses the same null, Killing vector field - and v dependence is unconstrained.
- It adds deformation only to the G_{vv} component of the metric:

$$\delta(ds^2) = \psi(r)^{-1}\, T(v,\vec{x})\, dv^2.$$

	5D black hole	4D black hole
GV deformation	$T(v,\vec{w}) = f_i(v)\, w^i$	$\widetilde{T}(v,\vec{y}) = g_i(v)\, y^i$
Hair candidates	Four arbitrary scalar functions f_i, periodic in v.	Three arbitrary scalar functions g_i, periodic in v.

↬ Deformations of the form field of the Taub-NUT space

- Taub-NUT space is equipped with a self-dual, harmonic 2-form field ω_{TN}.
  - An anti-self-dual 3-form field can be constructed

$$\delta H^s_{MNP} = h^s(v)\, dv \wedge \omega_{TN}, \qquad 1 \leq s \leq n_t.$$

    (one deformation for each multiplet)
  - Collateral damage: metric deformations - in order to satisfy the bosonic sector equations:

$$\delta(ds^2) = \psi^{-1}(r)\tilde{S}(v,r)dv^2,$$
$$\tilde{S} \to \text{quadratic in } h^s(v).$$

- Form field deformations are characterized by the n_t arbitrary scalar functions $h^s(v)$.

Fermionic hair candidates

- Solve fermionic sector equations, obtain the deformation Ψ^{α}_{M}.

  Our fermionic solution is
  - $\Psi^{\alpha}_{M} = 0, \quad \text{if} \quad M \neq v,$
  - Ψ^{α}_{v} independent of u.

  Possible fermionic hair:

$$\Psi^{\alpha\,(1)}_{v} = \psi(r)^{-3/2}\eta^{(1)}(v,\theta,\phi) \qquad \text{or} \qquad \Psi^{\alpha\,(2)}_{v} = \psi(r)^{-1/2}\eta^{(2)}(v,\theta,\phi).$$

- These Ψ^{α}_{v} remain valid solutions even in the presence of GV deformations.
- Taub-NUT space does not have any effect on the fermionic deformations obtained.
- For both black holes, independent arbitrary functions of v - four each for $\Psi^{\alpha\,(1)}_{v}$ and $\Psi^{\alpha\,(2)}_{v}$.

Outline

To hair or not to hair?!

Need to pass three tests for a deformation to be a hair:

- ✓ Smoothness checks - are the deformations smooth near the horizon? Define suitable coordinates that are smooth across the horizon.
- ✓ SUSY preservation checks - no additional SUSY is broken by the deformations. Is the Killing spinor analysis affected by the deformed solutions?
- ✓ Support is only outside the horizon - each of the smooth deformations need to vanish at the horizon.

And the culprits are...

Deformation	5D black hole	4D black hole
Garfinkle-Vachaspati	✗	✓
Form field	NA	✓
Fermion	✓	✓

➢ Spectrum of hair modes:

Hair	5D black hole	4D black hole
Bosonic	0	$n_t + 3$
Fermionic	4	4

Outline

Snip Snip ✂ ✂

- Question: Do the hair modes correctly account for the mismatch factors for each N?
- Z_N group action is on the form fields; generates twisted sectors with twist $e^{2\pi is/N}$, for $1 \leq s \leq N-1$. Untwisted sector: $s = 0$.
- Split the form fields according to their twist.
  - Example: N = 2
    Splitting is $19 = 11 + 8$ and this reflects as

$$f(\rho) \sim \prod_{k=1}^{\infty}(1-q^{2k})^{11+5}\prod_{l=1}^{\infty}(1-q^{2l-1})^{8}$$
$$\sim \eta(\rho)^8\eta(2\rho)^8.$$

  - Example: N = 3
    Splitting of $19 = 7 + 6 + 6$ gives the correct factor

$$f(\rho) \sim \eta(\rho)^6\eta(3\rho)^6.$$

Outline

What have we done so far?

- Explicit construction of hair modes for the 4D and 5D black holes.
- Takeaway: There are $n_t + 3$ additional bosonic hair modes for the 4D black hole than the 5D black hole.
- For each N, these hair modes correctly account for the extra factors and solve the puzzle.

What is the path forward?

- $Z_M \times Z_N$ models involving twisting and twining.

A. Sen - 2010, S. Govindarajan - 2010.

- Hair removal story for the above - A. Chowdhury, S. Govindarajan, S. Samanta, P. S, A.Virmani - Coming soon!

Thank you for your attention.

For comments and feedback:
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