Tidal deformability of dynamical horizons in binary black hole mergers

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Relevant works

Based on

1. TIDAL DEFORMABILITY OF DYNAMICAL HORIZONS IN BINARY BLACK HOLE MERGERS.
   Vaishak Prasad, *Anshu Gupta, Sukanta Bose, Badri Krishnan.*
   Phys. Rev. D 105, 044019
   Pre-print: arXiv:2106.02595

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The physical system

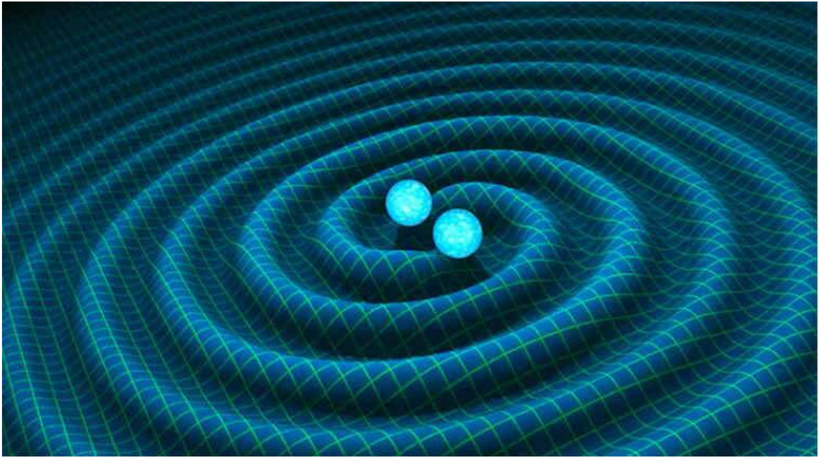

Figure: Merging compact objects (Credit: R. Hurt/Caltech-JPL)

The non-relativistic treatment

- Multipole moments of the source distribution:

$$Q^L = \int \rho x^{<L>} d^3x$$

- External tidal field:

$$\mathcal{E}_L = -\frac{1}{(\ell-2)!}\partial_L U_{ext}(\mathbf{0}),$$

- Tidal Love number:

$$\delta Q_L = -\frac{2(\ell-2)!}{(2\ell-1)!!} k_\ell R^{2\ell+1} \mathcal{E}_L$$

- No spin effects.

The relativistic treatment

- Multipole moments of the asymptotic field $\mathcal{M}_\ell, \mathcal{S}_\ell$ (Gerosch-Hansen).
- Metric perturbation:

$$
\begin{aligned}
g_{ab} =& g_{ab}^{(0)} + h_{ab} \\
h_{ab} =& h_{ab}^{ext} + h_{ab}^{resp}
\end{aligned}
$$

- Tidal Love numbers (field):

$$
\begin{aligned}
\delta\mathcal{M}_\ell =& \lambda_\ell^{el} \mathcal{E}_\ell \\
\delta\mathcal{S}_\ell =& \lambda_\ell^{mag} \mathcal{E}_\ell
\end{aligned}
$$

Limitations

1. Valid for quasi-static/stationary processes:
   1.1 Static tides.
   1.2 Slow/linear time evolution.
2. Weak external tidal field.
3. Asymptotic regions of each black hole.

In practice:

1. Black holes get close:
   1.1 Cannot isolate the asymptotic fields of each black hole.
   1.2 Strong external tidal fields.
   1.3 Highly dynamical.
2. **Validity of perturbation theory?**
3. **Deformation of black holes in the strong field regime?**

Requires a different treatment of the strong field tidal deformability of black holes:
Dynamical Horizons

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Black hole horizons

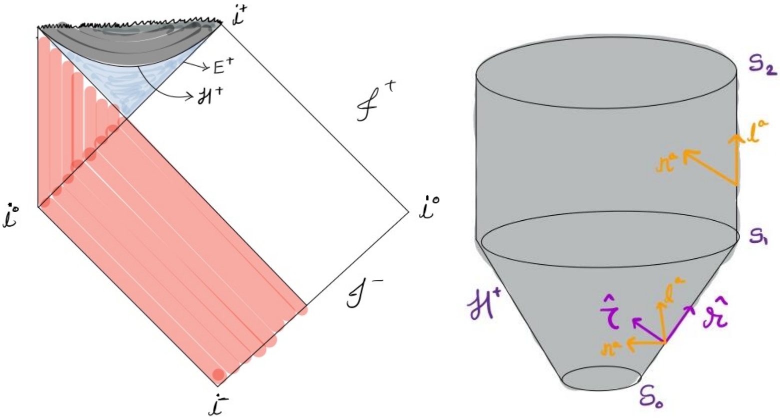

Figure: Left: Collapse of a null fluid to form a black hole, followed by its growth. Right: A dynamical horizon settling down to an isolated horizon

Foliations of a Dynamical Horizon

Marginally Outer Trapped Surfaces (MOTS):

$$\theta_\ell = \tilde{q}^{ab}\nabla_a \ell_b = 0 \qquad \theta_n = \tilde{q}^{ab}\nabla_a n_b < 0$$

Salient features[2]:

1. Definitions of Mass, Angular momentum (Hayward, Ashtekar, Krishnan +).
2. No teleological properties.
3. Generalized laws of black hole mechanics (Hayward, Ashtekar, Krishnan +).
4. Uniqueness (Ashtekar, Galloway +).
5. Existence, Stability, evolutionary properties (Kolb, Krishnan +).
6. Physics, properties of black holes (Booth, Jaramillo, Gupta, Prasad +).

[2] The above reference list is not complete

Source Multipole Moments of a Dynamical Horizon

$$\mathcal{M}_n = \frac{M_{\mathcal{S}} R_{\mathcal{S}}^n}{8\pi} \oint_S \tilde{\mathcal{R}} P_n(\zeta) d^2 S \,,$$

On each slice $\mathcal{S}$ of $\mathcal{H}^+$,
$\tilde{\mathcal{R}}$ The 2D Ricci scalar of $\mathcal{S}$, ζ: analogous to $cos\theta$, P_n: Legendre polynomials.

1. Are different from the GH field multipoles.
2. Characterize the intrinsic geometry of $\mathcal{S}$.
3. Isolated Kerr black holes: Odd $\mathcal{M}_n$ zero, $\mathcal{M}_0$ is the Mass.
4. Evolve with time.

Source Love numbers

1. Change in horizon geometry $\to \delta\mathcal{M}_n$.
2. Expand $\delta\mathcal{M}_n$ in Taylor Series in the strength of the external tidal field (Caberro, Krishnan 2014):

$$\frac{\delta\mathcal{M}_n}{M_{td}^{n+1}} = \sum_{i,j=1}^{\infty} \alpha_{ij}^{(n)} \frac{M_{td}^{i} M_{tf}^{j}}{d^{i+j}} \, .$$

- d : Distance of separation between the holes.
- M_{td} : Mass of the tidally deformed black hole horizon.
- M_{tf} : Horizon mass of the source of external tidal field.
- Newtonian tidal interaction starts at $1/d^3 \implies i+j \geq 3$.
- α_{ij}^{n} : Dimensionless, scale-free tidal coefficients: **Love Numbers**.

Strategy

$$\textbf{Model} \quad \textbf{A}: \quad \frac{\mathcal{M}_2}{M_{td}^3} = \frac{a_3^{(2)}}{d^3} + \mathrm{const.} \tag{1}$$

and

$$\textbf{Model} \quad \textbf{B}: \quad \frac{\mathcal{M}_2}{M_{td}^3} = \frac{a_3^{(2)}}{d^3} + \frac{a_4^{(2)}}{d^4} + \mathrm{const.} \tag{2}$$

$$a_3^{(2)} = M_{td} M_{tf}^2 \alpha_{12}^{(2)} + M_{td}^2 M_{tf} \alpha_{21}^{(2)} \,, \tag{3}$$

$$a_4^{(2)} = \alpha_{13}^{(2)} M_{td} M_{tf}^3 + \alpha_{22}^{(2)} M_{td}^2 M_{tf}^2 + \alpha_{31}^{(2)} M_{td}^3 M_{tf} \,. \tag{4}$$

1. Simulate binary black hole mergers.
2. Compute the source moments.
3. Analyzed the quadrupolar deformations.
4. Applications.

Numerical Relativity Simulations

1. **The Einstein Toolkit**: Public, open source NR code.
2. Gauge, slicing: Punctures, BSSN, 1+log slicing, Gamma driver shift.
3. **Quasi-local computations**: for DH related quantities.

Quasi-local calculations: Provided by **QuasiLocalMeasures**.
Fitting : **An MPI based least squares finder**.
Library : **waveformtools**.
Grid on the horizon: 37×76

Non-spinning BBH Simulations					
Mass ratio	d	M_1	M_2	p_r	p_t
1.0	11.0	0.5	0.5	-7.220e-04	0.09019
0.85	12.0	0.54051	0.4595	-5.290e-04	0.08448
0.75	11.0	0.5714	0.4286	-6.860e-04	0.08828
0.6667	11.75	0.6	0.4000	-5.290e-04	0.08281
0.50	11.0	0.6667	0.3333	-5.720e-04	0.0802
0.40	11.25	0.7143	0.2857	-4.500e-04	0.07262

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Fits

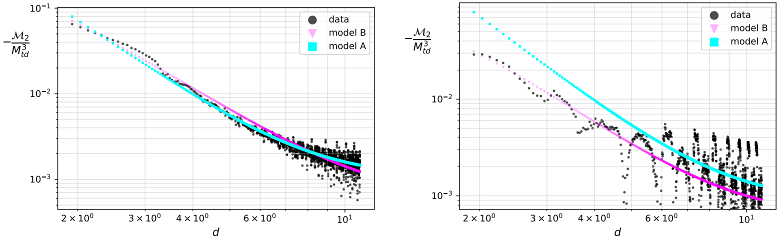

Figure: The multipole moment data and the distance d are plotted on a logarithmic scale.

Tidal coefficients

Tidal coefficients: third order		
Model	α_{12}	α_{21}
A	-0.15 ± 0.21	-3.43 ± 0.21
B	-0.89 ± 0.21	-5.45 ± 0.21

Table: Tidal coefficient values estimated from a re-fit of the fit coefficients a_3, a_4.

Tidal coefficients: fourth order			
Model	α_{13}	α_{22}	α_{31}
B	1.79 ± 1.72	5.3 ± 3.65	4.68 ± 1.72

Table: Tidal coefficient values estimated from a re-fit of the fit coefficients a_3, a_4.

Conclusions

Brief summary:

- Tidal deformation of non-spinning black hole dynamical horizons.
- Strong field, dynamical regime all the way upto merger.
- Numerical relativity: The Einstein toolkit.
- Computed tidal Love numbers (incorporating upto equivalent 1PN order).
- May be more important in the merger phase.

Several questions:

- Relationship with field Love numbers.
- Observational implications.

Thankyou!

References

- Isolated and Dynamical Horizons and their applications
- Work on which this presentation was based: Phys. Rev. D 105, 044019 Pre-print: arXiv:2106.02595.
- The open source numerical relativity infrastructure: The Einstein Toolkit.