Causality Constraints in Quadratic Gravity

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Brief Outline

- Motivation for our work
  — “Causality constraints in Quadratic Gravity”, *JHEP* 09 (2021) 150; J. D. Edelstein, RG, A. Laddha, S. Sarkar

- Review of Shapiro time shift

- Shock wave in GR and Gauss-Bonnet gravity
  - Shapiro time advancement
  - Issues with Causality

- Causality Constraints in QG
  - No Shapiro time advancement, only delay

- Conclusion

Motivation

- Let's start with the Einstein-Hilbert action in GR, variation of which gives Einstein's field equations:

$$\mathscr{A}_{EH} = \int d^4x \sqrt{-g}\, R\,.$$

- This action is invariant under general coordinate transformations, $x^\mu \to x^\mu + \xi^\mu(x)$: Diffeomorphism invariance. This has an important consequence of local energy-momentum conservation, when matter Lagrangian is added.

- Theoretically, one may add terms, such as R^2, $R^{ab}R_{ab}$ etc. maintaining diffeomorphism invariance. For example,

  QG: $L = R + \alpha R^2 + \beta R^2_{ab}$; GB gravity: $L = R + \lambda \left(R^2 - 4R^2_{ab} + R^2_{abcd}\right)$.

- We have an infinite number of theories to consider. But, not all diffeomorphism invariant gravitational theories are "healthy".

- Some of them posses theoretical issues regarding causality might be ruled out!

  - Problem with Shapiro time advancement and causality, e.g.- Gauss-Bonnet gravity as shown by **C**amanho-**E**delstein-**M**aldacena-**Z**hiboedov (CEMZ) [JHEP 02(2016) 020].

- Goal: Can we construct a theory of gravity which doesn't possess causality issue? Is there a general class?

- A toy model: We emphasis that our interest in QG is not to consider it as a viable theory of gravity but to use it as the simplest non-GR model which may offer interesting perspective on causality constraints causality constraint.

- These consistency criteria may help us in a finer classification of all classical theories of gravity. We could also be able to make some general statement on a family of higher curvature theory.

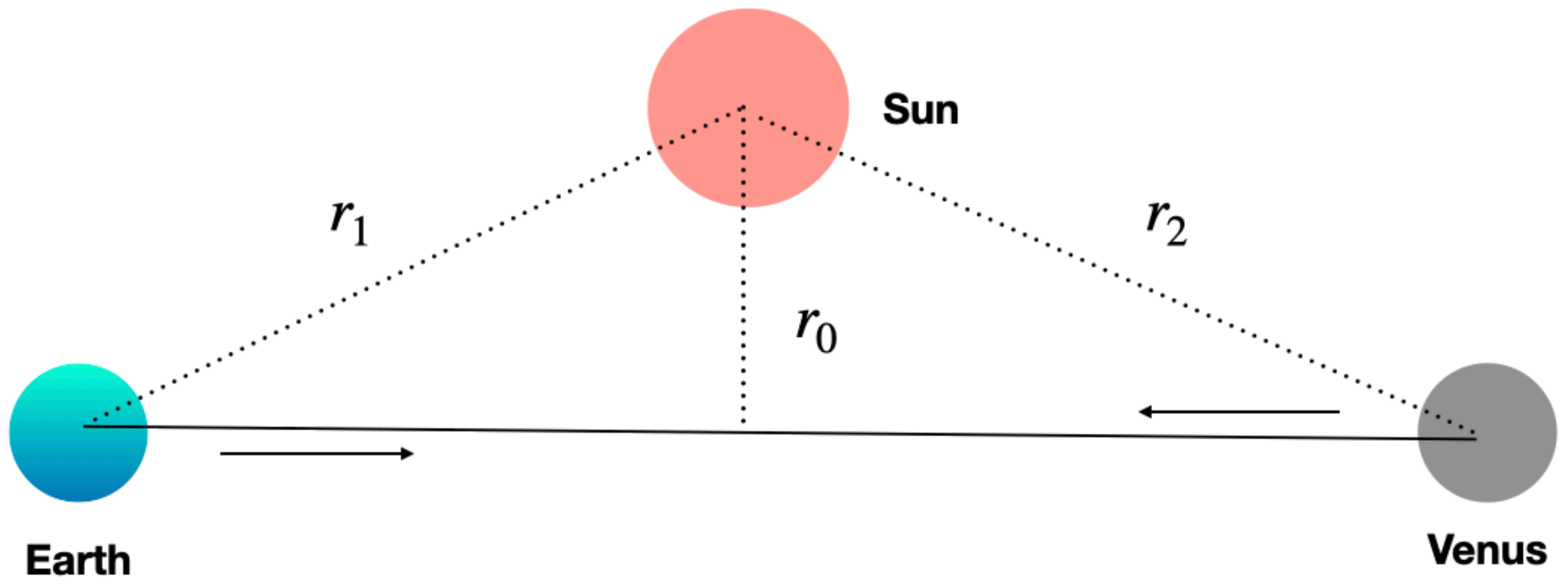
Review of Shapiro time shift

- Consider a light beam travelling back and forth from Earth to Venus, influenced by the gravitational field of the sun.

- In the absence of sun, total time needed to complete the journey is,

$$T_M = 2\left(\sqrt{r_1^2 - r_0^2} + \sqrt{r_2^2 - r_0^2}\right).$$

r_0 : closest approach of the light beam to the sun.



- In the presence of the sun, the spacetime outside is given by the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}} + r^2 \left(d\theta^2 + sin^2\theta \, d\phi^2\right) .$$

- Using this modified geometry, one can calculate the light travel time $T_S > T_M$.

- The Shapiro time shift is defined as, $T_{Shapiro} = T_S - T_M$. If this is positive (negative), we call it Shapiro time delay (advancement).

- Advancement is not, in general, acausal, e.g.- timelike propagation between two Schwarzschild BHs. However, $T_S < 0$ is acausal.

- We want to calculate the Shapiro time shift in the shock wave spacetime.

Shock wave in GR

- A shock wave geometry is sourced by a particle moving very fast, say along v-direction (localized in u=0 plane) with momentum $P_u < 0$. The stress-energy tensor can be written as: $T_{uu} = -P_u \delta(u) \delta^{D-2}(\vec{x})$.

- With this, we can solve Einstein's field equations to obtain the metric in double-null coordinates ,

$$ds^2 = -du\,dv + h_0(u, x^i)\,du^2 + (dx^i)^2 \ .$$

Here, we have (for D > 4)

$$h_0(u, x^i) = f_0(r)\,\delta(u) = \frac{4\Gamma(D/2-2)}{\pi^{D/2-2}} \cdot \frac{G|P_u|}{r^{D-4}}\ \delta(u) \ .$$

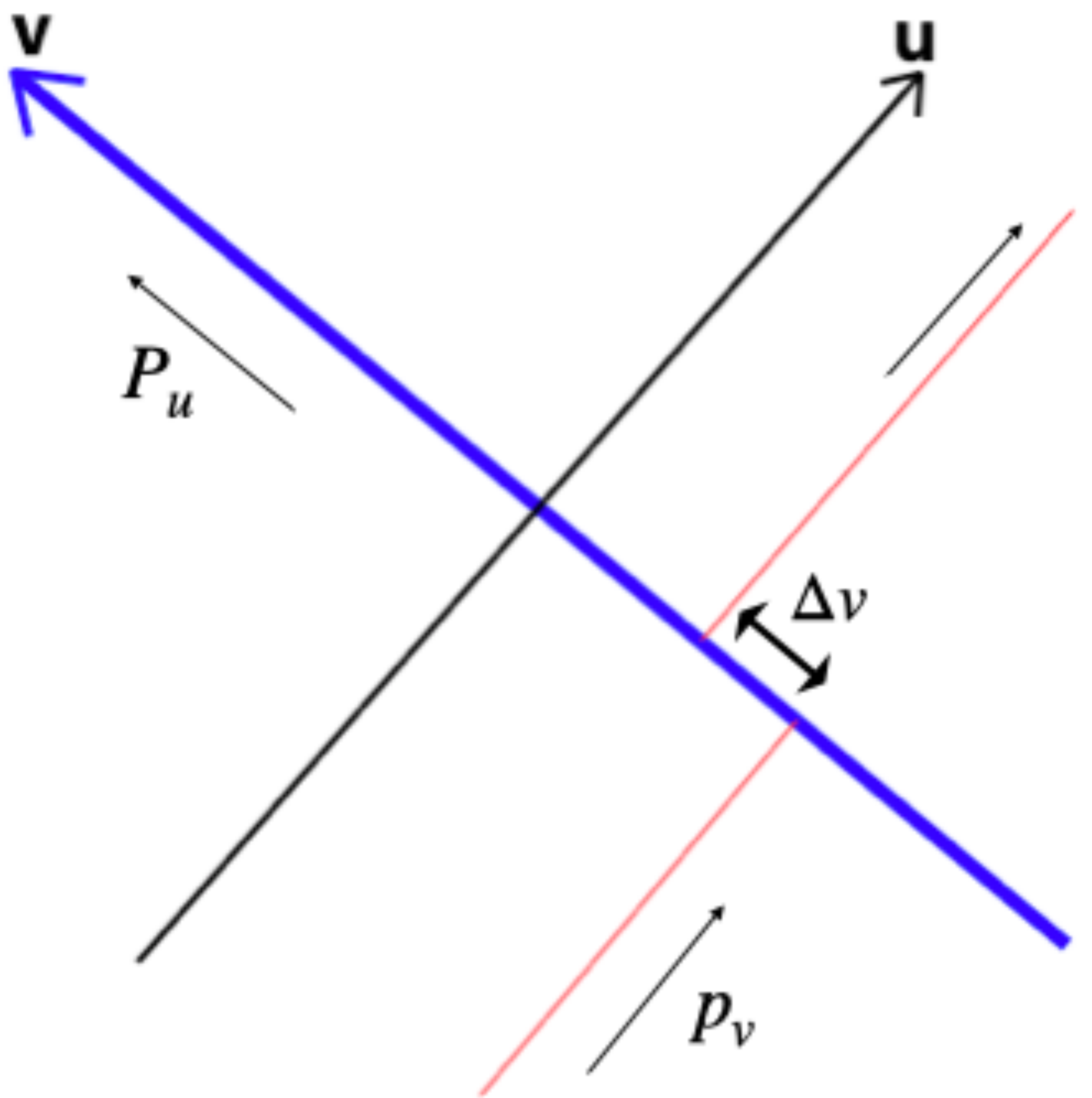
- We denote by 'r' the transverse distance from the uv-plane.

- The spacetime is everywhere flat except in u=0 plane. Curvature $\propto \delta(u)$.

Particle crossing the shock

- Consider a probe particle moving along u-direction crosses the shock with an impact parameter r = b.
- We can remove the delta-function from the metric, locally at r = b by the following coordinate transformation, $v = v_{new} + f(b)\,\theta(u)$.
- The geodesic that goes through r = b is continuous in the new coordinates, which implies in the old frame it suffers a delay: $(\Delta v)_{GR} = f(b) > 0$.
- This shift is with respect to the fastest mode of propagation along the boundary, $b \to \infty$.

[Gao and Wald, Class. Quant. Grav. 17, 4999 (2000)]



Scalar field crossing the shock

- Now, consider a massless scalar field propagating in the shock wave background. The scalar wave equation takes the form,

$$\left[\partial_u \partial_v + h_0(u, x^i)\, \partial_v^2\right]\ \phi = 0\ .$$

- This equation can be integrated in the Fourier-space of v-coordinate, and we get, $\phi(u = 0^+, v, x^i) = \phi\left(u = 0^-, v - \int_{0^-}^{0^+} h_0(u, x^i)\, du, x^i\right)$.

- The scalar field suffers a Shapiro time delay, $(\Delta v)_{GR} = f(b)$, same as previous result.

- A similar calculation can be repeated for the metric perturbation. In this case also, we obtain the same time delay.

Shock wave in GB gravity

- Lagrangian: $L = R + \lambda\left(R^2 - 4R_{ab}^2 + R_{abcd}^2\right)$, λ is the GB coupling constant different from zero.
- It can be shown that GB gravity supports the same shock wave solution as that of GR in D > 4.
- However, the metric perturbation $\left(g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}\right)$ obeys an equation different from GR

$$\partial_u \partial_v h_{ij} + \left(\delta_{ik} + 4\lambda\, \partial_i \partial_k h_0\right) \partial_v^2 h_{kj} = 0 \ .$$

[Camanho et al, JHEP 02(2016) 020]

- It will suffer a different Shapiro time shift: $\Delta v = (\Delta v)_{GR}\left[1 + \lambda \times \text{correction term}\right]$.
- This correction term depends on the polarization direction and doesn't have any particular sign.
- Therefore, it is always possible to choose the direction of polarization so that the perturbation suffers a time advancement and Δv changes sign.

Construction of time machine

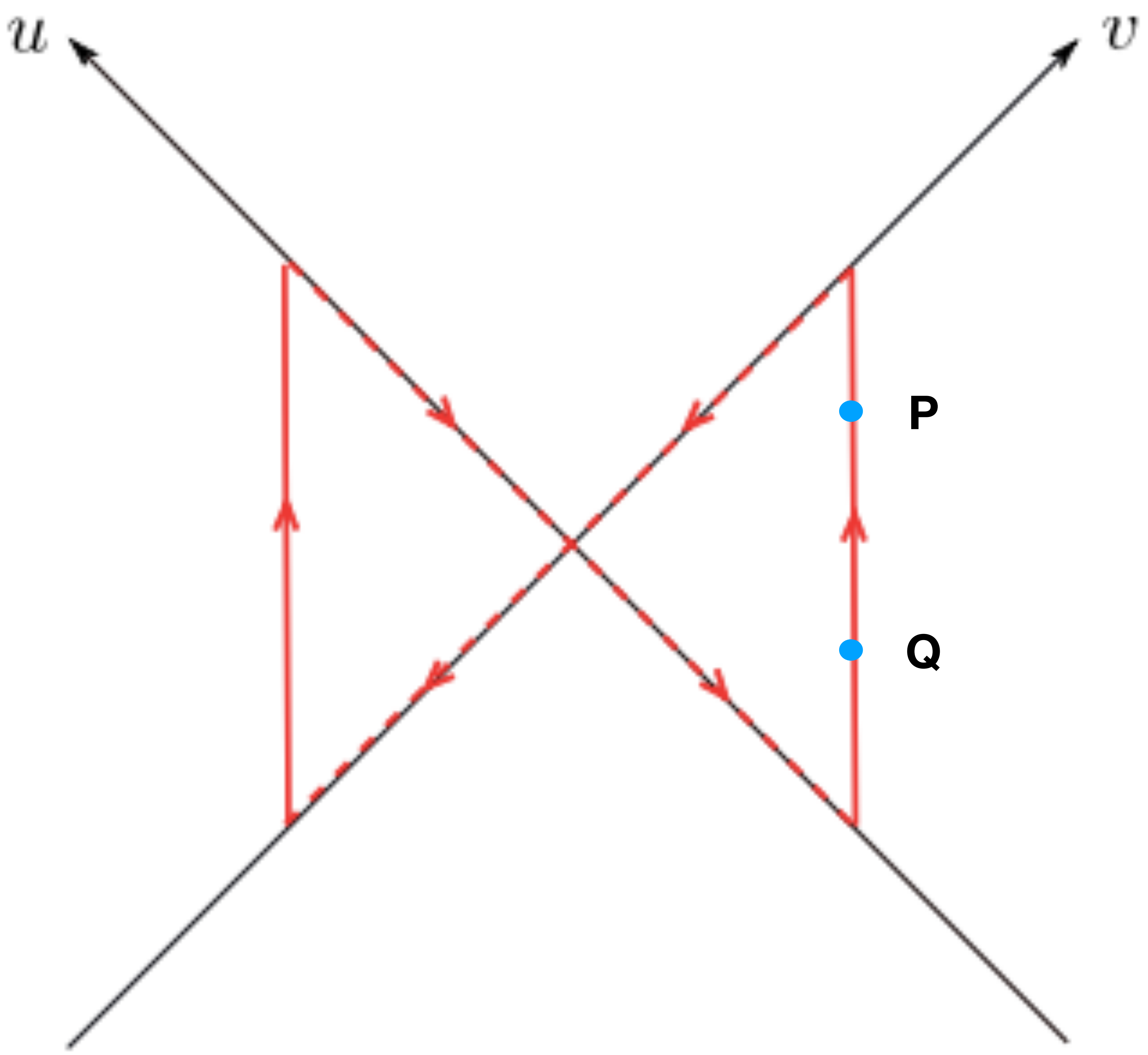
- Using the time advancements, we can arrange a time machine in the background of two shocks.

- Thus, to respect causality we must set the GB coupling constant to zero, reducing the theory to GR. [Camanho et al, JHEP 02(2016) 020]

- The same is true for all higher order Lovelock-Lanczos theory also.

- Is there a higher curvature theory which does not have this problem?

  Ans: Quadratic Gravity (QG).



Shock wave in QG

- Quadratic Gravity: $L = R + \alpha R^2 + \beta R_{ab}^2$. No tachyon: $\beta \leq 0$, etc.

- There is an exact shock wave solution in QG with the profile function (D > 4),

$$f(r) = -\frac{8\pi G\,|P_u|\,\Gamma(D/2-1)}{\pi^{D/2-1}}\left[\frac{(-2\beta)^{2-D/2}}{\Gamma\left(\frac{D}{2}-1\right)}\left(\frac{r}{\sqrt{-\beta}}\right)^{2-D/2} K_{2-D/2}\left(\frac{r}{\sqrt{-\beta}}\right)-\frac{1}{D-4}\left(\frac{1}{r}\right)^{D-4}\right].$$

[Campanelli et al, PRD.54.3854 (1996)]

- We want to study the metric perturbation in the background of this exact shock wave solution of QG.

- First-order metric perturbation equation: $\Box\left(1+\beta\Box\right)h_{ij}=0$. Whereas in GR, $\bar{\Box}\,h_{ij}=0$.

- Interestingly, this equation can be solved by using Fourier transformation in v-coordinate.

Time shift in QG

- Again, considering the perturbation crossing the shock, we get the Shapiro time shift as:

$$\Delta v = (\Delta v)_{\text{GR}} \times \left(1 - \frac{1}{2^{n-1}\Gamma(n)} x^n K_{-n}(x)\right),$$

  where $x = b/\sqrt{-\beta}$, and $n = D/2 - 2$.

- It can be shown that the second term in the parentheses is bounded above by unity. Time shift vanishes as $x \to 0^+$ and reduces to the GR-value as $x \to +\infty$.

- It implies the time shift is in fact a delay and causality is respected.

Subtleties with field redefinition

- The Shapiro time shift in QG has a similar properties to that in GR.
- Is there a field redefinition of the metric which will map the shock wave solution of QG to that of GR?
- Under this field redefinition, does the time delay in QG also map to that in GR? If true, QG is equivalent to GR for the study of causality issue.
- Answer to the 1st question is YES: $g^{\mu\nu}_{GR} = g^{\mu\nu}_{QG} + 2\beta\, R^{\mu\nu}_{QG}$.

[Mozaffar et al, arXiv:1603.05713]

- Answer to the 2nd question is NO:
$\bar{\Box}\, \bar{h}_{ij} = 4\left[h_0 - \bar{h}_0\right] \partial_v^2 \left(1 + \beta \Box\right) h_{ij} \neq 0$, in general.

Generalizations

- Consider a special class of gravitational theories classified by the perturbation equation of the form: $\left(1+\gamma\,\Box^{n}\right)\Box\, h_{ij}=0$.

- This structure is special for two reasons-

  1. Differential operator acting on h_{ij} factories into the GR part and another part coming from higher curvature terms, like in the case of QG for which n=1.

  2. It does not contain any transverse derivative of the profile function unlike GB case.

- Such equation can be solved exactly by Fourier transform and time shift can be calculated as, $\Delta v = f(b)$. However, the form of the profile will be different for different theories.

- Example: n=2 for the theory, $L=\left(R+\alpha\,R^{2}+\gamma\,R_{ab}\,\Box\,R^{ab}\right)$. Although we don't know the shock wave solution yet.

Takeaway

- We find out a general class of gravitational theory which is free from causality issue as defined by CEMZ. QG is a member of this class.
- QG is well behaved than naively expected- it supports ghost/tachyonic propagation modes, though free from causality problem unlike the GB gravity.
- Time delay in QG cannot be mapped back to GR by a field-redefinition.
- Analysing such theoretical consistencies may help us in classifying all gravitational theories.

Thank You!