Modifications to secondary gravitational waves due to scalar non-Gaussianity

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Talk based on
Accounting for scalar non-Gaussianity in secondary gravitational waves,
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Overview

- Introduction
- Modification to $\mathcal{P}_{\mathrm{S}}(k)$ due to f_{NL}
- Contributions to secondary Ω_{GW} due to f_{NL}
- Models and Results
- Summary

Secondary gravitational waves

- Models of inflation with enhanced scalar power on small scales are known to generate secondary gravitational waves[1].
- These gravitational waves are tensor perturbations sourced by the scalar perturbations at the second order.
- Models in this context are also known to produce non-trivial amplitudes and shapes of scalar non-Gaussianity[2].
- Therefore it is important to account for scalar non-Gaussianity in the computation of secondary gravitational waves[3].

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[1] See, for a recent review, *G. Domènech, Universe,* **7**, *398 (2021), arXiv:2109.01398 [gr-qc]*

[2] See, for example *J. M. Ezquiaga, J. Garca-Bellido, and V. Vennin, JCAP* **03** *029 (2020)*

[3] See, for an early effort, *R.-g. Cai, S. Pi, and M. Sasaki, Phys. Rev. Lett.* **122**, *201101 (2019)*

$f_{\rm NL}$ - definition and extension

The parameter quantifying the primordial scalar non-Gaussianity, $f_{\rm NL}$, is conventionally defined through the relation[4]

$$\mathcal{R}(\eta, \boldsymbol{x}) = \mathcal{R}^{\rm G}(\eta, \boldsymbol{x}) - \frac{3}{5} f_{\rm NL} [\mathcal{R}^{\rm G}(\eta, \boldsymbol{x})]^2.$$

[4] *J. Maldacena, JHEP* **05***, 013 (2003)*

[5] *F. Schmidt and M. Kamionkowski, Phys. Rev. D* **82***, 103002 (2010)*

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In our work, we shall extend this definition to be[5]

$$\mathcal{R}_{\boldsymbol{k}}(\eta) = \mathcal{R}^{\rm G}_{\boldsymbol{k}}(\eta) - \frac{3}{5} \int \frac{{\rm d}^3 \boldsymbol{k}_1}{(2\,\pi)^{3/2}} \mathcal{R}^{\rm G}_{\boldsymbol{k_1}}(\eta) \mathcal{R}^{\rm G}_{\boldsymbol{k}-\boldsymbol{k_1}}(\eta)\, f_{\rm NL}[\boldsymbol{k}, (\boldsymbol{k}_1 - \boldsymbol{k}), -\boldsymbol{k}_1].$$

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We can write $f_{\rm NL}$ in terms of the scalar bispectrum $G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ and the power spectrum $\mathcal{P}_{\rm S}(k)$ as

$$f_{\rm NL}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -\frac{10}{3} \frac{(k_1 k_2 k_3)^3}{(2\pi)^4} \frac{G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)}{\left[k_1^3 \mathcal{P}_{\rm S}(k_2) \mathcal{P}_{\rm S}(k_3) + \text{ two permutations}\right]}$$

[4] *J. Maldacena, JHEP* **05***, 013 (2003)*

[5] *F. Schmidt and M. Kamionkowski, Phys. Rev. D* **82***, 103002 (2010)*

Modification to $\mathcal{P}_{\rm S}(k)$ due to $f_{\rm NL}$

If we compute the two-point correlation of $\mathcal{R}_{\boldsymbol{k}}$ with $f_{\rm NL}$ introduced as shown, then we obtain the modified scalar power spectrum to be

$$\mathcal{P}_{\rm S}^{\rm M}(k) = \mathcal{P}_{\rm S}(k) + \underbrace{\frac{9}{25}\int_0^\infty {\rm d}x \int_{|1-x|}^{|1+x|} {\rm d}y \, \frac{\mathcal{P}_{\rm S}(kx)}{x^2}\,\frac{\mathcal{P}_{\rm S}(ky)}{y^2}\, f_{\rm NL}^2[k,kx,ky]}_{\mathcal{P}_{\rm C}(k)}$$

Modification to $\mathcal{P}_{\mathrm{S}}(k)$ due to f_{NL}

If we compute the two-point correlation of $\mathcal{R}_{\boldsymbol{k}}$ with f_{NL} introduced as shown, then we obtain the modified scalar power spectrum to be

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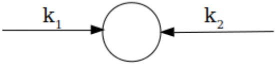
In terms of Feynman diagrams, we can understand these terms as



If $\mathcal{P}_{\mathrm{C}}(k) > \mathcal{P}_{\mathrm{S}}(k)$, it implies large non-Gaussian modification to the power spectrum.

Non-Gaussian contributions to $\Omega_{\rm GW}$

The scalar perturbations source the tensors at the second order and this leads to detectable strengths of secondary gravitational waves[6].
The power spectrum of such secondary tensor perturbations is given by

$$\frac{2\pi^2}{k^3}\delta^{(3)}(\boldsymbol{k}+\boldsymbol{k}')\overline{\mathcal{P}_h(k,\eta)} = \frac{16}{81}\frac{1}{kk'\eta^2}\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^{3/2}}\int\frac{\mathrm{d}^3\boldsymbol{p}'}{(2\pi)^{3/2}}\,Q^\lambda(k,p)Q_\lambda(k',p') \times\mathcal{I}(k,p)\mathcal{I}(k',p')\,\langle\mathcal{R}_{\boldsymbol{p}}\mathcal{R}_{\boldsymbol{k}-\boldsymbol{p}}\mathcal{R}_{\boldsymbol{p}'}\mathcal{R}_{\boldsymbol{k}'-\boldsymbol{p}'}\rangle.$$

The dimensionless energy density of corresponding secondary GWs is

$$\mathrm{h}^2\,\Omega_{\rm GW}(k) = \frac{1}{24}\left(\frac{g_{*,k}}{g_{*,0}}\right)^{-1/3}\Omega_{\rm r}\,\mathrm{h}^2\,(k^2\eta^2)\,\overline{\mathcal{P}_h(k,\eta)}\ ,$$

where $\Omega_{\rm r}$ denotes the fraction of relativistic matter in the current universe.

[6] See, for instance, *K. Kohri, and T. Terada, Phys. Rev.* **D 97**, *123532 (2018)*.

Non-Gaussian contributions to $\Omega_{\rm GW}$

To estimate the non-Gaussian contributions to $\Omega_{\rm GW}$, we may write[7]

$$\Omega_{\rm GW}(f) \sim \mathcal{P}_h(k) + \mathcal{P}_h^{(2)}(k) + \mathcal{P}_h^{(4)}(k)$$

where

$$\mathcal{P}_h(k) \sim \int \mathrm{d}^3\boldsymbol{k}_1 \mathcal{P}_{\rm S}(k_1)\mathcal{P}_{\rm S}(k-k_1)$$

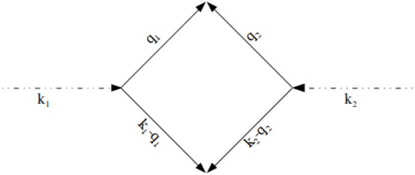
$$\mathcal{P}_h^{(2)}(k) \sim \int \mathrm{d}^3\boldsymbol{k}_1 \int \mathrm{d}^3\boldsymbol{k}_2\, \mathcal{P}_{\rm S}(k_1+k_2)\mathcal{P}_{\rm S}(k-k_2)\mathcal{P}_{\rm S}(k+k_1) \times\left[f_{\rm NL}(k,k_1,k_2) f_{\rm NL}(k,k_1+k_2,k-k_2)\right]$$

$$\mathcal{P}_h^{(4)}(k) \sim \int \mathrm{d}^3\boldsymbol{k}_1 \int \mathrm{d}^3\boldsymbol{k}_2 \int \mathrm{d}^3\boldsymbol{k}_3\, \mathcal{P}_{\rm S}(k_3)\mathcal{P}_{\rm S}(k_1-k_3)\mathcal{P}_{\rm S}(k-k_1+k_3) \times\mathcal{P}_{\rm S}(k_1+k_2-k_3)\Big[f_{\rm NL}(k_1,k_1-k_3,k_3) f_{\rm NL}(k-k_1,k_3,k-k_1+k_3) f_{\rm NL}(k_2,k_1+k_2-k_3,k_3-k_1) f_{\rm NL}(k+k_2,k_1+k_2-k_3,k-k_1+k_3)\Big]$$

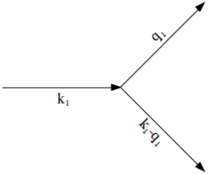
[7] For instance, *C. Unal, Phys. Rev. D 99, 041301 (2019); V. Atal and G. Domènech, JCAP,* **06**, *001 (2021); P. Adshead, K. D. Lozanov and J. Weiner, JCAP* **10**, *080 (2021).*

Loop diagrams

We can understand these contributions using Feynman diagrams constructed accordingly[8].



$$\mathcal{P}_h(k_1) \sim \mathcal{P}_{\rm S}^2$$

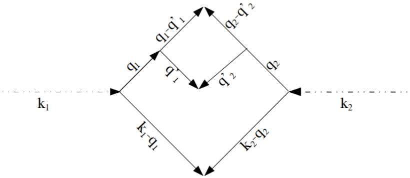


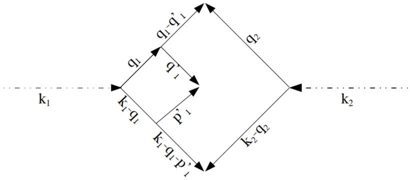
$$f_{\rm NL}\, \mathcal{R}_k^2$$

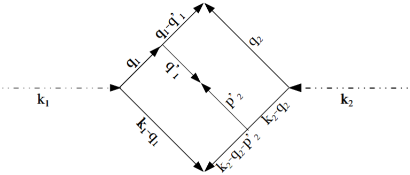
[8] *P. Adshead, K. D. Lozanov, and Z. J. Weiner, JCAP* **10**, *080 (2021)*

Loop diagrams

We can present the contributions of order $f_{\rm NL}^2$ as



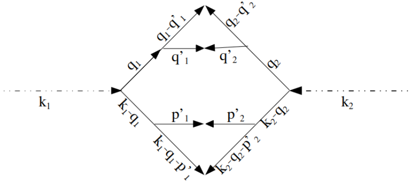


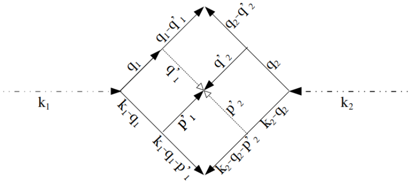


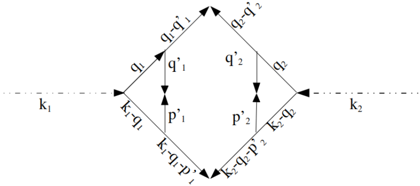
$$\mathcal{P}_h^{(2)} \sim \int \mathcal{P}_{\rm S}^3\, f_{\rm NL}^2$$

Loop diagrams

Similarly we also represent contributions proportional to $f_{\rm NL}^4$ as [9]







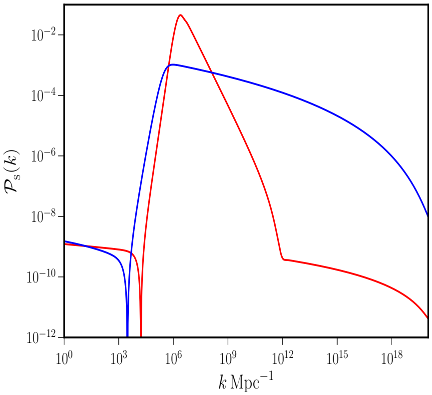
$$\mathcal{P}_h^{(4)} \sim \int \mathcal{P}_{\rm S}^4 \, f_{\rm NL}^4$$

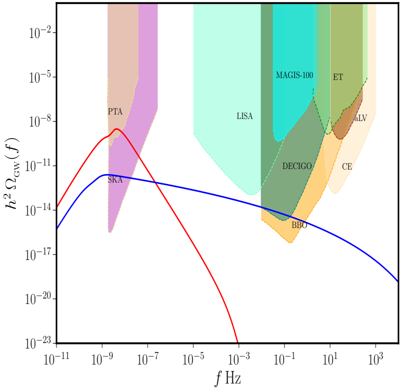
[9] For comments about these contributions, see, *R.-G. Cai, S. Pi, S.-J. Wang and X.-Y. Yang, JCAP* **05**, *013 (2019); R.-g. Cai, S. Pi and M. Sasaki, Phys. Rev. Lett.* **122** *201101 (2019)*

Models for illustration

SM with a dip: $V(\phi) = V_0 \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}\right)\right]^2 \left\{1 - \lambda \exp\left[-\frac{1}{2}\left(\frac{\phi-\phi_0}{\Delta\phi}\right)^2\right]\right\}$

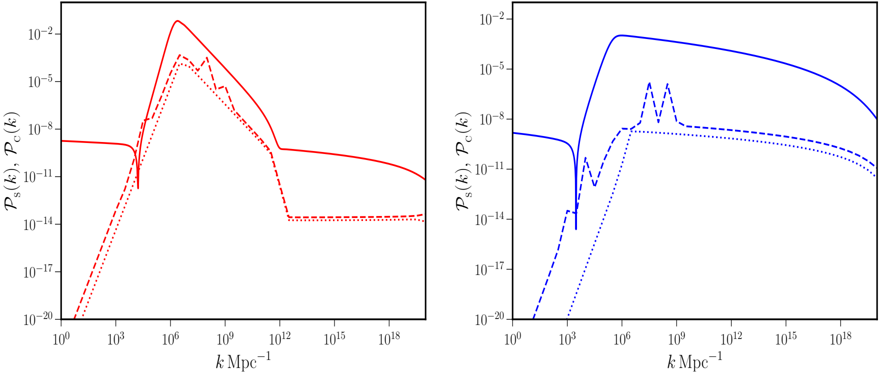
Critical-Higgs model: $V(\phi) = V_0 \frac{\left[1+a\,\ln^2\left(\frac{\phi}{\mu}\right)\right]\left(\frac{\phi}{\mu}\right)^4}{\left\{1+c\left[1+b\ln\left(\frac{\phi}{\mu}\right)\right]\left(\frac{\phi}{\mu}\right)^2\right\}^2}$





Non-Gaussian correction against the original spectrum

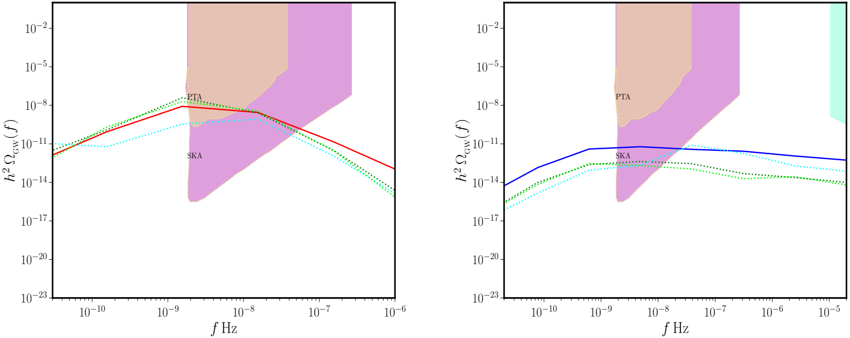
$$\mathcal{P}_{\mathrm{C}}(k) = \begin{cases} \frac{1}{8}\left(\frac{k}{k_f}\right)^3 \{\mathcal{P}_{\mathrm{S}}(k_f)[n_{\mathrm{S}}(k_f)-1]\}^2 , & \text{for } k < k_f , \\ \frac{1}{4}\,\mathcal{P}_{\mathrm{S}}(k_f)\mathcal{P}_{\mathrm{S}}(k)\,[n_{\mathrm{S}}(k)-1]^2 , & \text{for } k > k_f . \end{cases}$$



$\mathcal{P}_{\mathrm{C}}(k)$ (dashed lines) against $\mathcal{P}_{\mathrm{S}}(k)$ (solid lines) [10]

[10] *H. V. Ragavendra, arXiv:2108.04193 [astro-ph.CO];* $n_{\mathrm{S}}(k) - 1 = \mathrm{d}\ln\mathcal{P}_{\mathrm{S}}/\mathrm{d}\ln k$

Non-Gaussian contributions to $\Omega_{\rm GW}$



Gaussian contribution to $\Omega_{\rm GW}$ is plotted as solid lines against the non-Gaussian contributions due to $\mathcal{P}_h^{(2-1)}(k)$ (in light green), $\mathcal{P}_h^{(2-2)}(k)$ (in cyan) and $\mathcal{P}_h^{(2-3)}(k)$ (in dark green) as dotted lines.

Summary

- Scalar non-Gaussianity, $f_{\rm NL}$, arising in models of inflation leading to enhanced power on small scales has non-trivial scale dependence.
- The corrections to the scalar power spectrum due to $f_{\rm NL}$ seem largely negligible in the models of interest.
- However, accounting for $f_{\rm NL}$ in the calculation of secondary GWs leads to significant non-Gaussian contributions. Hence, they have to be consistently taken into account while comparing $\Omega_{\rm GW}$ against observational data.
- This opens up the possibility of arriving at indirect constraints on large amplitudes of $f_{\rm NL}$ by studying their effects on $\Omega_{\rm GW}$.

Thank you for your attention.

Talk was based on **H. V. Ragavendra**, *Accounting for scalar non-Gaussianity in secondary gravitational waves, arXiv:2108.04193 [astro-ph.CO]*

Secondary gravitational waves

The equation of motion governing $h_{\boldsymbol{k}}$ at the second order relating it to the scalar perturbation is given by[11]

$$h_k^{\lambda\,\prime\prime} + 2\frac{a'}{a}h_k^{\lambda\,\prime} + k^2 h_k^\lambda = S_k^\lambda(\eta)\,,$$

where the source term $S_k^\lambda(\eta)$ contains terms quadratic in $\mathcal{R}_k$.
The time averaged power spectrum of the secondary tensor perturbations is given by

$$\overline{\mathcal{P}_h(k,\eta)} = \frac{2}{81\,k^2\,\eta^2}\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left[\frac{4\,v^2-(1+v^2-u^2)^2}{4\,u\,v}\right]^2 \mathcal{P}_{\mathrm{S}}(k\,v)\,\mathcal{P}_{\mathrm{S}}(k\,u) \times \left[\mathcal{I}_c^2(u,v)+\mathcal{I}_s^2(u,v)\right],$$

where $\mathcal{I}_c$ and $\mathcal{I}_s$ arise from the transfer functions.

The dimensionless energy density of the secondary GWs today is

$$\mathrm{h}^2\,\Omega_{\mathrm{GW}}(k) = \frac{1}{24}\left(\frac{g_{*,k}}{g_{*,0}}\right)^{-1/3}\Omega_{\mathrm{r}}\,\mathrm{h}^2\left(\frac{k}{aH}\right)^2\overline{\mathcal{P}_h(k,\eta)}\,.$$

[11] See, for instance, *K. Kohri and T. Terada, Phys. Rev.* **D 97**, *123532 (2018)*.

Secondary gravitational waves

The source function in the evolution of mode function of the secondary tensor perturbation is given by

$$S_{\boldsymbol{k}}^{\lambda}(\eta) = 4\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\,\pi)^{3/2}}\, e^{\lambda}(\boldsymbol{k},\boldsymbol{p}) \Bigg\{ 2\,\Psi_{\boldsymbol{p}}(\eta)\,\Psi_{\boldsymbol{k}-\boldsymbol{p}}(\eta) + \frac{4}{3\,(1+w)\,\mathcal{H}^2}\,\left[\Psi'_{\boldsymbol{p}}(\eta)+\mathcal{H}\,\Psi_{\boldsymbol{p}}(\eta)\right]\,\left[\Psi'_{\boldsymbol{k}-\boldsymbol{p}}(\eta)+\mathcal{H}\,\Psi_{\boldsymbol{k}-\boldsymbol{p}}(\eta)\right] \Bigg\}^{12} .$$

The Bardeen potential is related to the primordial scalar perturbation as

$$\Psi_{\boldsymbol{k}}(\eta) = \frac{2}{3}\,\mathcal{T}(k\,\eta)\,\mathcal{R}_{\boldsymbol{k}}\,,$$

where $\mathcal{T}(k\,\eta)$ is the transfer function

$$\mathcal{T}(k\,\eta) = \frac{9}{(k\,\eta)^2}\,\left[\frac{\sin\left(k\,\eta/\sqrt{3}\right)}{k\,\eta/\sqrt{3}} - \cos\left(k\,\eta/\sqrt{3}\right)\right].$$

[12] $\mathcal{H} = aH$