Systematic biases on parameterized test of GR due to neglect of orbital eccentricity

Pankaj Saini

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Chennai Mathematical Institute

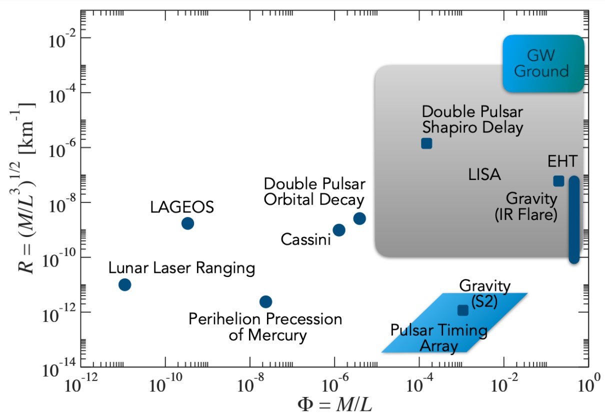
With Marc Favata and K.G.Arun

CSGC-2022

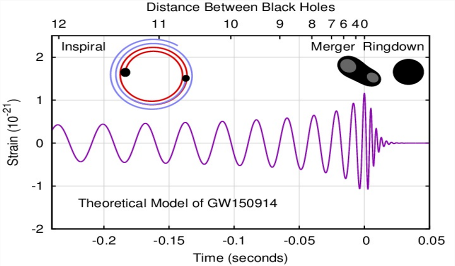
- Gravitational waves probe the dynamical, strong-field regime of gravity.

- Curvature is at least four orders of magnitude larger for GW events than for binary pulsar.

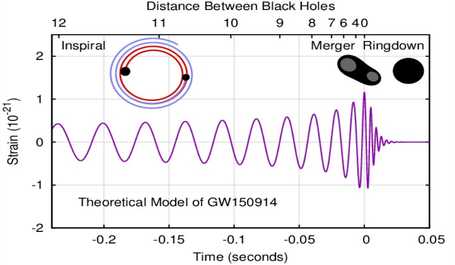
Sathyaprakash+2019



parameterized test of GR



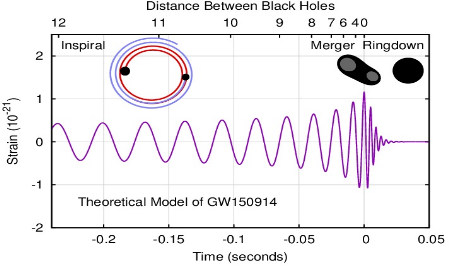
Gerosa+2018



Gerosa+2018

- In post-Newtonian theory, inspiral phase is expressed as a series in v/c

$$\Phi(\mathrm{v}) = \left(\frac{\mathrm{v}}{\mathrm{c}}\right)^{-5}\left[\varphi_{-2}\left(\frac{\mathrm{v}}{\mathrm{c}}\right)^{-2} + \varphi_0 + \varphi_2\left(\frac{\mathrm{v}}{\mathrm{c}}\right)^{2} + \cdots + \varphi_{5l}\ \ln\left(\frac{\mathrm{v}}{\mathrm{c}}\right)\left(\frac{\mathrm{v}}{\mathrm{c}}\right)^{5} + \cdots + \varphi_7\left(\frac{\mathrm{v}}{\mathrm{c}}\right)^{7}\right] \tag{1}$$



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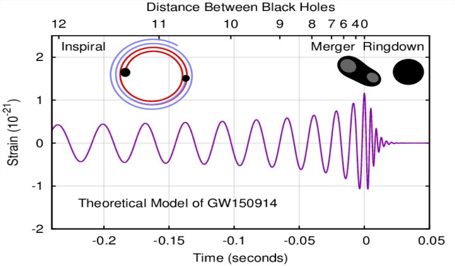
$$\varphi_i = \varphi_i(m_1, m_2, \chi_1, \chi_2)$$



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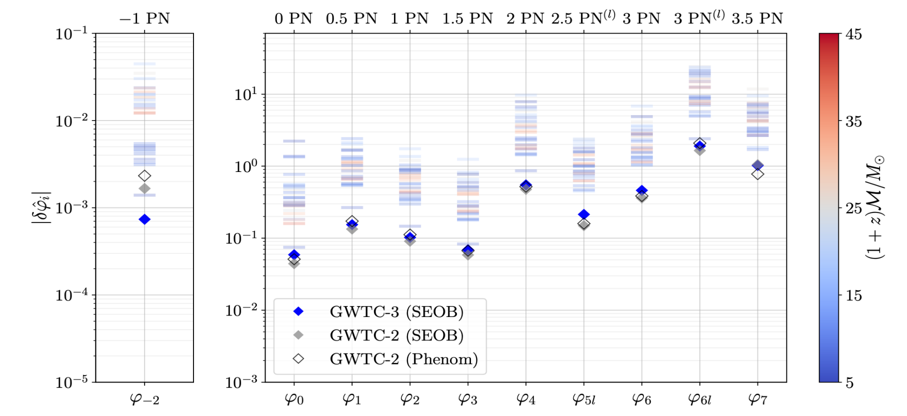
- In parameterized test, non-GR deviations are introduced at each PN order.

$$\boxed{\varphi_i \rightarrow \varphi_i \left(1 + \delta\hat{\varphi}_i\right)} \quad (2)$$

 Arun+2006, Yunes+2009, Li+2011

Current status of parameterized test

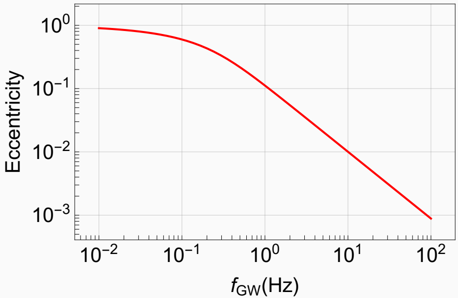


GWTC-3: 90% upper bounds on the fractional deviations with respect to their GR value.

Abbott+2021

Eccentricity as a function of GW frequency

- Eccentricity is a sharply decreasing function of gravitational wave frequency
- $e_t \, / \, e_0 \sim (f \, / \, f_0)^{-19/18}$.
- Current gravitational searches employ quasi-circular waveform



Astrophysical expectations of eccentric BBHs

- Dynamical formation of compact binaries can lead to non-negligible eccentricity when they enter the frequency band of ground based detectors (Samsing+2018, Rodriguez+2018, Antonini+2014).
- Using circular waveform for parameter estimation, may lead to significant systematic bias.
- Systematic bias on deformation parameters may mimic deviations from GR.

Credit: NASA,ESA

Star cluster Messier 15

Systematic bias

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$$\Delta\theta_{\mathrm{a}} = |\theta_{\mathrm{T}} - \theta_{\mathrm{bf}}| \tag{3}$$

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- $\Delta\theta_a$ can be approximated by the following expression (Cutler and Vallisneri (2007))

$$\boxed{\Delta\theta_a \;\;\approx \Sigma^{ab}\left[\left(\Delta\mathcal{A}^{ecc} + i\;\;\mathcal{A}_{AP}\;\;\Delta\Psi^{ecc}\right)\; e^{i\Psi_{AP}}\;\;\middle|\;\;\partial_b\tilde{h}_{AP}\right]} \tag{6}$$

- We use TaylorF2Ecc waveform model.

$$\Psi(f) = 2\pi f t_c + \phi_c + \frac{3}{128\eta v^5}\left(\Delta\Psi^{\mathrm{circ}}_{3.5\mathrm{PN}} + \Delta\Psi^{\mathrm{ecc}}_{3\mathrm{PN}}\right) \tag{7}$$

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- The structure of $\Delta\Psi^{\rm ecc}_{\rm 3PN}$ can be seen from the following expression (Blake Moore et al. (2016))

$$\Delta\Psi^{\rm ecc}_{\rm 3PN} = -\frac{2355}{1462}\, e_0^2 \left(\frac{v_0}{v}\right)^{19/3}\left[1 + \left(\frac{299076223}{81976608} + \frac{18766963}{2927736}\eta\right)v^2 + \left(\frac{2833}{1008} - \frac{197}{36}\eta\right)v_0^2 - \frac{2819123}{282600}\pi v^3 + \frac{377}{72}\pi v_0^3 + \ldots O(v^6)\right]$$

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- The waveform is valid in the small eccentricity limit $e_0 \sim 0.2$.

- We use Fisher matrix framework to get the bounds on $\delta\hat{\varphi}_{\mathrm{k}}$.

Parameter estimation

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- Parameters of the waveform are

$$\theta_{\rm a} = \left\{ {\rm t_c},\ \phi_{\rm c},\ {\rm M},\ \eta,\ \chi_1,\ \chi_2,\ \delta\hat{\varphi}_{\rm k} \right\} \tag{8}$$

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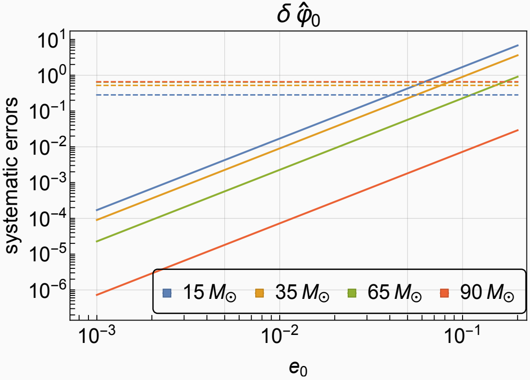
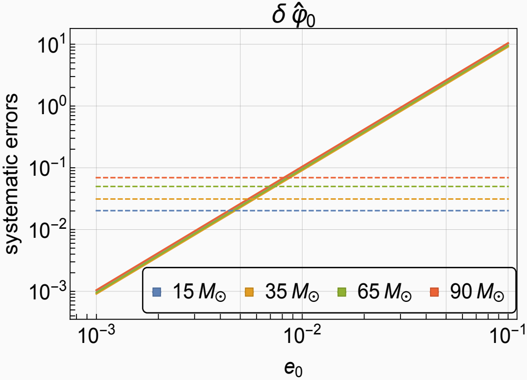
- Sources are fixed at a luminosity distance of 500 Mpc with mass ratio 1:2.
- Initial eccentricity is defined at a reference frequency of 10 Hz.



$\delta\,\hat{\varphi}_0$
systematic errors
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-3}
10^{-2}
10^{-1}
e_0
15 $M_\odot$
35 $M_\odot$
65 $M_\odot$
90 $M_\odot$

(aLIGO)



$\delta\,\hat{\varphi}_0$
systematic errors
10^{1}
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10^{-3}
10^{-2}
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35 $M_\odot$
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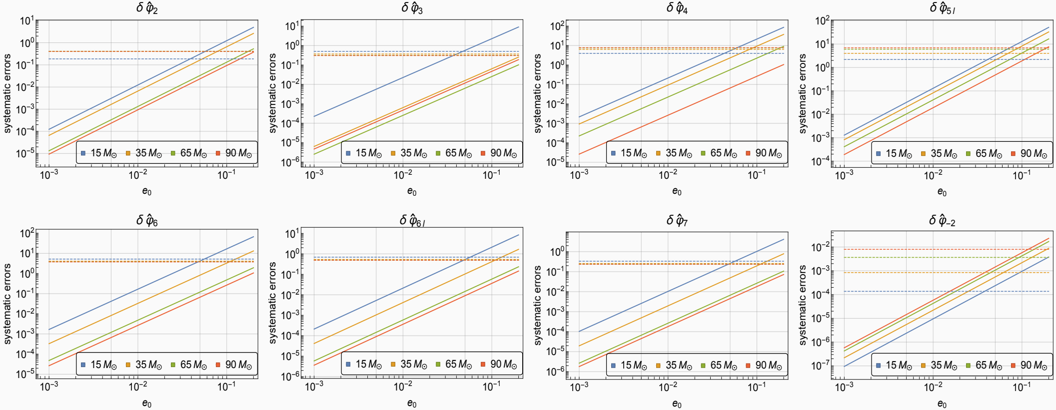
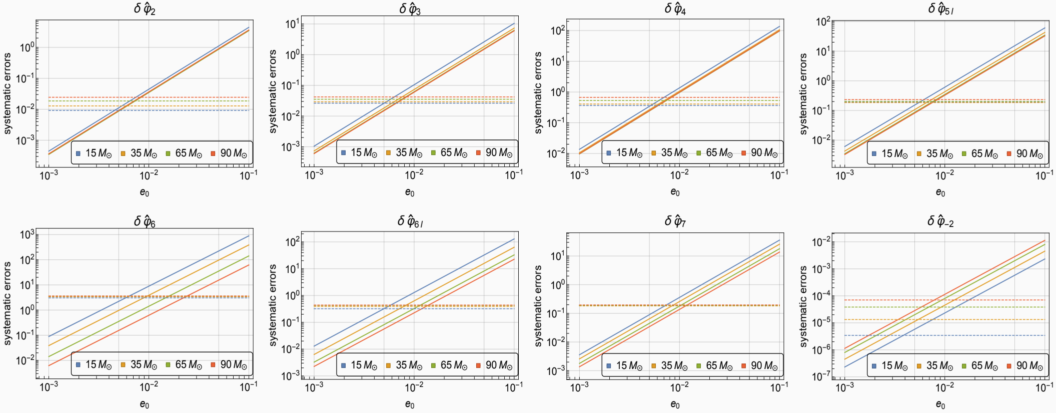
(CE)

Results: Systematic and statistical errors in aLIGO



Results: Systematic and statistical errors in Cosmic Explorer



Conclusions

- A fraction of the BBHs may have non-negligible eccentricity while observed by GW detectors.
- Systematic bias on non-GR deviations exceeds statistical errors at $e_o \sim 4 \times 10^{-2}$ at 10Hz for $M = 15\ M_\odot$ in aLIGO band.
- For CE, systematic bias dominates at very low eccentricity $\sim 5\times 10^{-3}$ at 10 Hz.
- Eccentricity induced systematic bias on non-GR modifications may mimic deviation from GR.