Deviations from slow roll inflation and implications for magnetogenesis

Sagarika Tripathy

Department of Physics, IIT Madras

Talk based on

On the challenges in the choice of the non-conformal coupling function in inflationary magnetogenesis
arXiv:2111.01478 [astro-ph.CO]

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Outline

- Introduction
- Choice of coupling function for slow-roll inflationary models
- Construction of coupling function for models generating features in scalar power spectrum
  - Models with feature over large scale
  - Models with feature over small scales
- Conclusion

Observational evidence

- In galaxies, magnetic field of strength $\sim 10^{-6}$ G on scales of $1 - 10$ Kpc [1]
- In clusters of galaxies, the strength is $\sim 10^{-7} - 10^{-6}$ G on scales of 10Kpc $-$ 1 Mpc[2]
- In intergalactic medium(IGM) voids $\geq 10^{-16}$ G on scales above 1 Mpc [3]

The origin of the seed magnetic field could be astrophysical or cosmological.

The origin of these large scale magnetic fields can be assigned to the processes during inflation in the early Universe.

[1] Beck R 2001 Space Sci. Rev. 99 243–60,Beck R and Wielebinski R 2013 Planets, Stars and Stellar Systems vol 5, ed T D Oswalt and G Gilmore (Dordrecht: Springer) p 641

[2] Clarke T E, Kronberg P P and Böhringer H 2001 Astrophys. J. 547 L111–4,Govoni F and Feretti L 2004 Int. J. Mod. Phys. D 13 1549–94

[3] Neronov A and Vovk I 2010 Science 328 73

Generation of primordial magnetic field (PMF)

$$S[A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\sqrt{-g}J(\phi)^2 F_{\mu\nu}F^{\mu\nu}$$

The corresponding Mukhanov-Sasaki equation is given by[4]

$$\mathcal{A}_k'' + \left(k^2 - \frac{J''}{J}\right)\mathcal{A}_k = 0.$$

The power spectra associated with the magnetic and electric fields are defined to be

$$\begin{aligned}\mathcal{P}_{\rm B}(k) &= \frac{k^5}{2\,\pi^2\,a^4}\,|\mathcal{A}_k|^2,\\ \mathcal{P}_{\rm E}(k) &= \frac{k^3}{2\,\pi^2\,a^4}\left|\mathcal{A}_k' - \frac{J'}{J}\,\mathcal{A}_k\right|^2.\end{aligned}$$

[4] Jerome Martin, Jun'ichi Yokoyama, JCAP 01 (2008) 025

EM power spectra

For the choice of coupling function $J(\eta) \propto a(\eta)^n$(in de-Sitter $a = -1/H_I \eta$)

$$n_{\rm B} = \begin{cases} 2\,n+6, & \text{for } n < -\frac{1}{2}, \\ 4-2\,n, & \text{for } n > -\frac{1}{2}, \end{cases} \qquad n_{\rm E} = \begin{cases} 2\,n+4, & \text{for } n < \frac{1}{2}, \\ 6-2\,n, & \text{for } n > \frac{1}{2}. \end{cases}$$

For $n = 2$

$$\begin{aligned} \mathcal{P}_{\rm B}(k) &= \frac{9\,H_{\rm I}^4}{4\,\pi^2} \simeq \frac{9\,\pi^2}{16}\,(r\,A_{\rm s})^2\,, \\ \mathcal{P}_{\rm E}(k) &= \frac{H_{\rm I}^4}{4\,\pi^2}\,(-k\,\eta_{\rm e})^2 \simeq \frac{\mathcal{P}_{\rm B}(k)}{9\,M_{\rm Pl}^4}\left(\frac{k_*}{k_e}\right)^2. \end{aligned}$$

Helical magnetic field

$$S[A^\mu] \;=\; -\frac{1}{16\,\pi}\int {\rm d}^4x\,\sqrt{-g}\left[J^2(\phi)\,F_{\mu\nu}\,F^{\mu\nu} - \frac{\gamma}{2}\,I^2(\phi)\,F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right],$$

where $\widetilde{F}^{\mu\nu} = (\epsilon^{\mu\nu\alpha\beta}/\sqrt{-g})\,F_{\alpha\beta}$
The equation of motion

$$\mathcal{A}_k^{\sigma\,\prime\prime} + \left(k^2 + \frac{2\,\sigma\,\gamma\,k\,J'}{J} - \frac{J''}{J}\right)\mathcal{A}_k^\sigma = 0.$$

where $\sigma = \pm 1$ represents positive and negative helicity.
The power spectra of the magnetic and electric fields

$$\mathcal{P}_{\rm B}(k) \;=\; \frac{k^5}{4\,\pi^2\,a^4}\left[\left|\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^-\right|^2\right],$$

$$\mathcal{P}_{\rm E}(k) \;=\; \frac{k^3}{4\,\pi^2\,a^4}\left[\left|\mathcal{A}_k^{+\prime} - \frac{J'}{J}\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^{-\prime} - \frac{J'}{J}\mathcal{A}_k^-\right|^2\right].$$

For $n = 2$,

$$\begin{aligned}\mathcal{P}_{_{\rm B}}(k) &= \frac{9\,H_{_{\rm I}}^4}{4\,\pi^2}\,f(\gamma),\\ \mathcal{P}_{_{\rm E}}(k) &= \frac{9\,H_{_{\rm I}}^4}{4\,\pi^2}\,f(\gamma)\left[\gamma^2 - \frac{\sinh^2(2\,\pi\,\gamma)}{3\,\pi\,\left(1+\gamma^2\right)\,f(\gamma)}\,(-k\,\eta_{\rm e})\right.\\ &\left.+\frac{1}{9}\,\left(1+23\,\gamma^2+40\,\gamma^4\right)\,(-k\,\eta_{\rm e})^2\right],\end{aligned}$$

where the function $f(\gamma)$ is given by

$$f(\gamma) = \frac{\sinh\left(4\,\pi\,\gamma\right)}{4\,\pi\,\gamma\,\left(1+5\,\gamma^2+4\,\gamma^4\right)}.$$

$$\frac{\mathcal{P}_{_{\rm B}}(k)}{M_{_{\rm Pl}}^4} \simeq \frac{9\,\pi^2}{16}\,\left(r\,A_{\rm s}\right)^2 f(\gamma), \quad \frac{\mathcal{P}_{_{\rm E}}(k)}{M_{_{\rm Pl}}^4} \simeq \frac{\mathcal{P}_{_{\rm B}}(k)}{M_{_{\rm Pl}}^4}\,\gamma^2.$$

For $\gamma = 1, \quad f(\gamma) \simeq 10^3$

Power spectrum in slow roll inflationary models

- The Klein-Gordon equation for inflaton field is

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0.$$

In terms of e-folds, the coupling function is given by

$$J(N) = \left(\frac{a}{a_e}\right)^n = \exp\left[n\left(N - N_{\mathrm{e}}\right)\right]$$

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- **The quadratic potential**

$$V(\phi) = \frac{m^2}{2}\,\phi^2.$$

we can arrive at the form of $J(N)$ that we desire if we choose $J(\phi)$ to be

$$J(\phi) = \exp\left[-\frac{n}{4\,M_{_{\rm Pl}}^2}\,(\phi^2 - \phi_{\rm e}^2)\right].$$

- **The small field model** described by the potential

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^q\right]$$

and we shall focus on the case wherein $q = 2$.
We choose the coupling function $J(\phi)$ to be

$$J(\phi) \simeq \left(\frac{\phi}{\phi_{\rm e}}\right)^{n\,\mu^2/2M_{_{\rm Pl}}^2} \exp\left[-\frac{n}{4\,M_{_{\rm Pl}}^2}\,(\phi^2 - \phi_{\rm e}^2)\right].$$

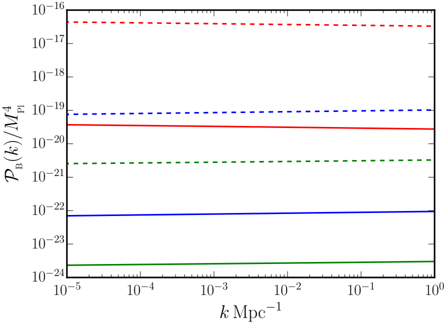
- **The first Starobinsky model** described by the potential

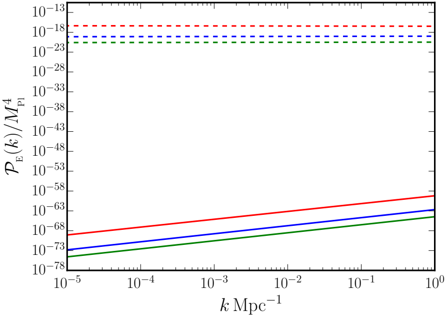
$$V(\phi) = V_0 \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{\rm Pl}}\right)\right]^2.$$

We can choose $J(\phi)$ in the model to be

$$J(\phi) = \exp\left\{-\frac{3\,n}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{\rm Pl}}\right) - \exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi_{\rm e}}{M_{\rm Pl}}\right) - \sqrt{\frac{2}{3}}\left(\frac{\phi}{M_{\rm Pl}} - \frac{\phi_{\rm e}}{M_{\rm Pl}}\right)\right]\right\}.$$

EM power spectra





The spectra of the magnetic (on the left) and electric (on the right) for the quadratic potential (in red), the small field model (in blue) and the first Starobinsky model (in green) in both the non-helical (as solid lines) and helical (as dashed lines) cases.[5]

[5] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

Coupling function for models generating features in scalar power spectrum

Features over large scales

- **Introducing a step by hand in the slow roll potential**[6]

$$V_{\rm step}(\phi) = V(\phi)\,\left[1 + \alpha\tanh\left(\frac{\phi - \phi_0}{\Delta\phi}\right)\right]$$

where, ϕ_0, α and $\Delta\phi$ denote the location, the height and the width of the step .

[6] J. A. Adams, B. Cresswell, and R. Easther, Phys. Rev. D 64, 123514 (2001).

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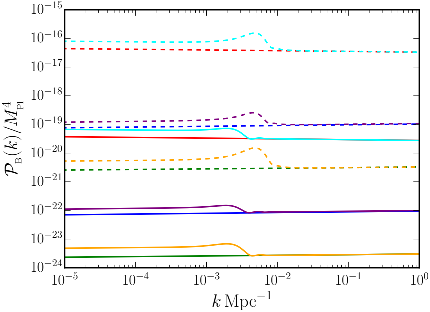
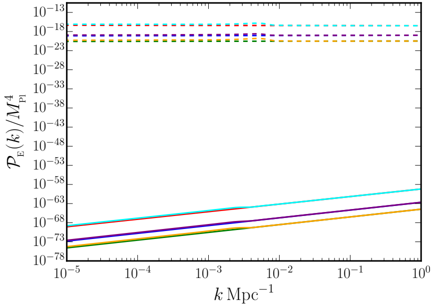


The spectra of the magnetic field for potential with step for (in cyan), the small field model (in purple) and the first Starobinsky model (in orange) in both the non-helical (as solid lines) and helical (as dashed lines) cases.[7]

[6] J. A. Adams, B. Cresswell, and R. Easther, Phys. Rev. D 64, 123514 (2001).



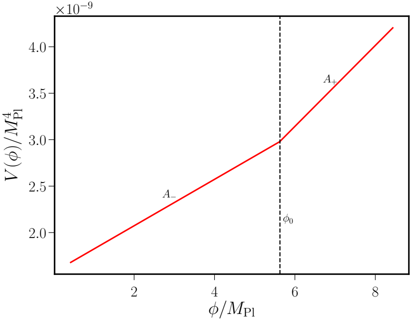
The spectra of the Electric field for potential with step for (in cyan), the small field model (in purple) and the first Starobinsky model (in orange) in both the non-helical (as solid lines) and helical (as dashed lines) cases.[7]

[7] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

Model with change in slope in potential

Second Starobinsky model[8],

$$V(\phi) = \begin{cases} V_0 + A_+ (\phi - \phi_0), & \text{for } \phi > \phi_0, \\ V_0 + A_- (\phi - \phi_0), & \text{for } \phi < \phi_0. \end{cases}$$

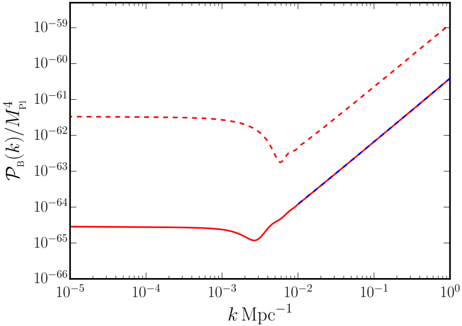


For numerical analysis

$$\begin{aligned} V(\phi) &= V_0 + \frac{1}{2}(A_+ + A_-)(\phi - \phi_0) \\ &+ \frac{1}{2}(A_+ + A_-)(\phi - \phi_0)\tanh\left(\frac{\phi - \phi_0}{\Delta\phi}\right), \end{aligned}$$

[8] A. A. Starobinsky, JETP Lett. 55, 489 (1992)

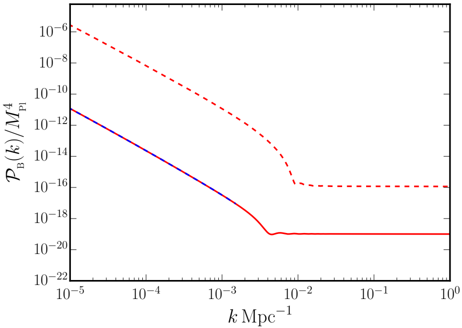
$$J_+(\phi) \quad = \quad J_{0+} \exp\left\{-\frac{n}{2\,M_{\rm Pl}^2}\left[\left(\phi_+ - \phi_0 + \frac{V_0}{A_+}\right)^2 - \left(\phi_{\rm i} - \phi_0 + \frac{V_0}{A_+}\right)^2\right]\right\}$$ [9]



A linear fit (indicated in dashed blue) to the non-helical power spectra over the small scales lead to the spectral indices $n_{\rm B} = 1.75$. For the values of the parameters we have worked with, the analytical estimates ($n_{\rm B} = -4\,(A_- - A_+)/A_+$) for these indices prove to be $n_{\rm B} = 1.71$.

[9] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

$$J_-(\phi) \;=\; J_{0-}\exp\left\{-\frac{n}{2\,M_{_{\rm Pl}}^2}\left[\left(\phi_- - \phi_0 + \frac{V_0}{A_-}\right)^2 - \left(\frac{V_0}{A_-}\right)^2 - 2\,N_0\,M_{_{\rm Pl}}^2\right]\right\}^{10},$$

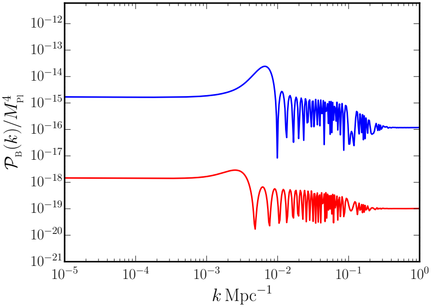


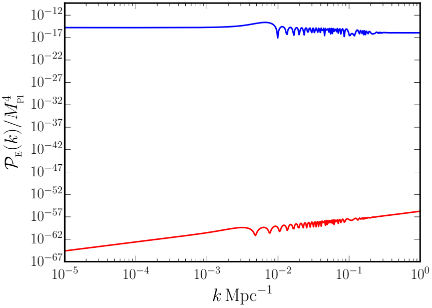
A linear fit (indicated in dashed blue) to the non-helical power spectra over the large scales lead to the spectral indices $n_{_{\rm B}} = -2.78$. For the values of the parameters we have worked with, the analytical estimates $(n_{_{\rm B}} = 4\,(A_- - A_+)/A_-)$ for these indices prove to be $n_{_{\rm B}} = -2.92$.

[10] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

Ironing out the feature

$$
\begin{aligned}
J(\phi) &= \frac{J_1}{2\,J_{0+}}\left[1+\tanh\left(\frac{\phi-\phi_0}{\Delta\phi_1}\right)\right] J_+(\phi) \\
&\quad + \frac{J_1}{2\,J_{0-}}\left[1-\tanh\left(\frac{\phi-\phi_0}{\Delta\phi_1}\right)\right] J_-(\phi),
\end{aligned}
$$



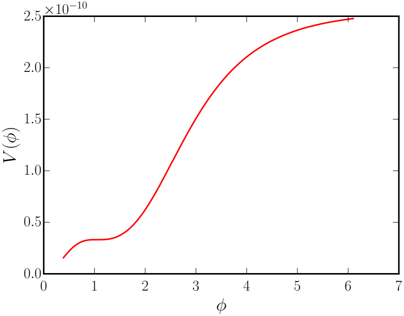


Features due to models with inflection point in the potential

Features over large scales

- **First punctuation inflation model**[11]

$$V(\phi) = \frac{m^2}{2}\,\phi^2 - \frac{2\,m^2}{3\,\phi_0}\,\phi^3 + \frac{m^2}{4\,\phi_0^2}\,\phi^4.$$



Coupling function

$$J(\phi) = \exp\left\{n\left[a_1\left(\frac{\phi^2-\phi_{\mathrm{e}}^2}{M_{_{\mathrm{Pl}}}^2}\right) + b_1\left(\frac{\phi-\phi_{\mathrm{e}}}{M_{_{\mathrm{Pl}}}}\right)\right]\right\}$$

[11] R. K. Jain, P. Chingangbam, L. Sriramkumar, and T. Souradeep, Phys. Rev. D 82, 023509 (2010), H. Ragavendra, D. Chowdhury, and L. Sriramkumar, (2020), arXiv:2003.01099 .

Features over small scales

- **Ultra slow roll model**[12]

$$V(\phi) = V_0 \left\{\tanh\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right) + A\,\sin\left[\frac{1}{f_\phi}\tanh\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right)\right]\right\}^2.$$

Coupling function

$$\begin{aligned} J(\phi) \quad = \quad & \exp\left\{n\left[a_2\left(\frac{\phi^4-\phi_{\rm e}^4}{M_{_{\rm Pl}}^4}\right) + b_2\left(\frac{\phi^3-\phi_{\rm e}^3}{M_{_{\rm Pl}}^3}\right)\right.\right. \\ & \left.\left. + c_2\left(\frac{\phi^2-\phi_{\rm e}^2}{M_{_{\rm Pl}}^2}\right) + d_2\left(\frac{\phi-\phi_{\rm e}}{M_{_{\rm Pl}}}\right)\right]\right\}, \end{aligned}$$

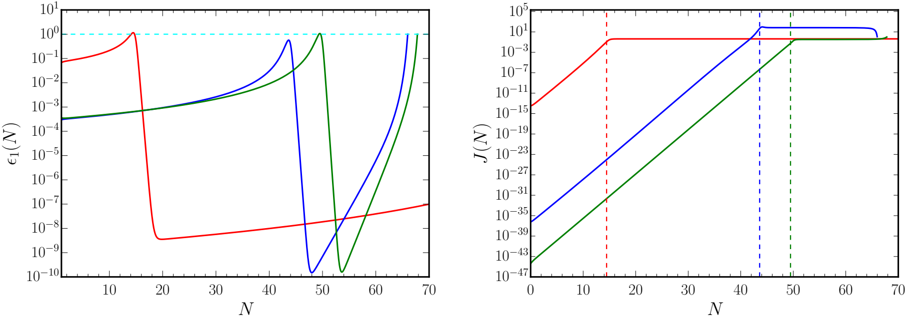
[12] I. Dalianis, A. Kehagias, and G. Tringas, JCAP 01, 037

Second punctuated inflation model[13]

$$V(\phi) = V_0 \left[c_0 + c_1 \tanh\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right) + c_2 \tanh^2\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right) + c_3 \tanh^3\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right)\right]^2.$$

$$J(\phi) = \exp\left\{n\left[a_3\left(\frac{\phi^6-\phi_{\rm e}^6}{M_{\rm Pl}^6}\right) + b_3\left(\frac{\phi^5-\phi_{\rm e}^5}{M_{\rm Pl}^5}\right) + c_3\left(\frac{\phi^4-\phi_{\rm e}^4}{M_{\rm Pl}^4}\right) + d_3\left(\frac{\phi^3-\phi_{\rm e}^3}{M_{\rm Pl}^3}\right) + e_3\left(\frac{\phi^2-\phi_{\rm e}^2}{M_{\rm Pl}^2}\right) + f_3\left(\frac{\phi-\phi_{\rm e}}{M_{\rm Pl}}\right)\right]\right\},$$

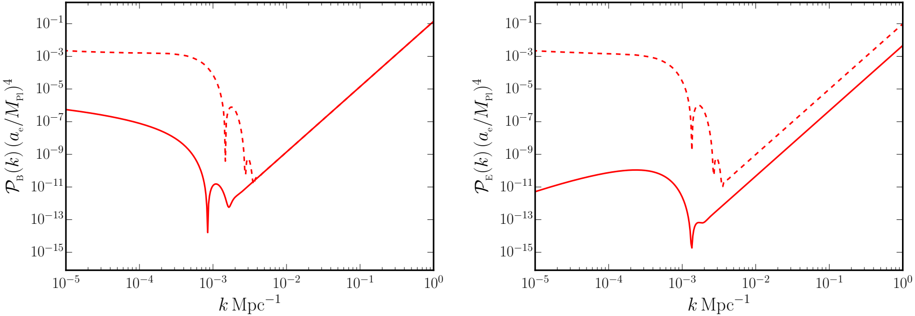
[13] I. Dalianis and K. Kritos, Phys. Rev. D 103, 023505 (2021)



The evolution of ϵ_1 and $J(N)$ for the first and second punctuated inflation model and ultra slow model(in solid red, green and blue, respectively) as a function of the e-fold N. [14]

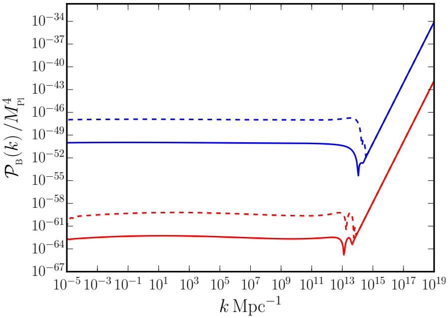
[14] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

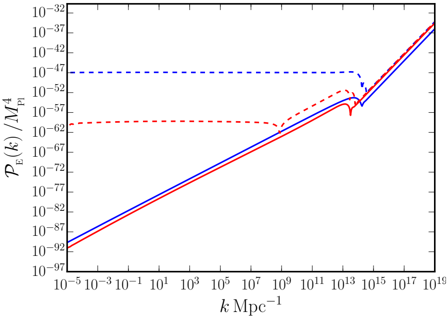
EM power spectra for first punctuated inflation model[15]



[15] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

EM power spectra for Ultra slow roll and second punctuated inflation model[16]





The spectra of the magnetic (on the left) and electric (on the right)fields in the ultra slow roll inflationary model (in red) and the second punctuated inflationary model(in blue) in the non-helical (as solid lines) and helical (as dashed lines)cases.

[16] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, L. Sriramkumar, arXiv:2111.01478 [astro-ph.CO]

Conclusion

- Although a nearly scale invariant primordial scalar power spectrum generated in slow roll inflationary models, is remarkably consistent with the CMB data, it has been repeatedly noticed that certain features in the scalar power spectrum can improve the fit to the data.
- When strong departures from slow roll arise, these deviations also led to features in the spectra of electromagnetic fields.
- In certain scenarios, it is also possible that the strengths of the magnetic fields are considerably suppressed on large scales.
- While it seems possible to remove the strong features in the spectra of the electromagnetic fields allowing us to arrive at nearly scale invariant spectra of required strengths, it is achieved at the terrible cost of extreme fine-tuning.

Thank You