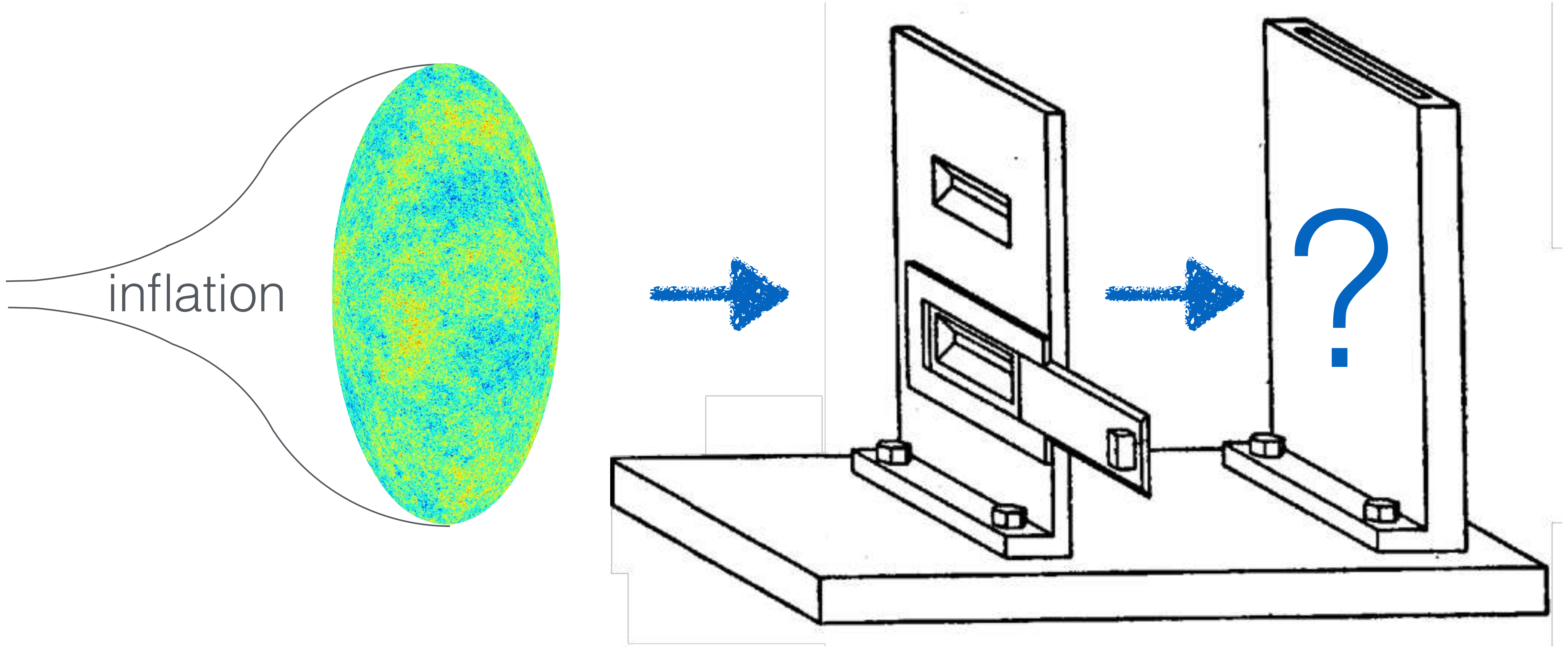
Can we prove that cosmic structures are of quantum-mechanical origins?



Vincent Vennin

Second Chennai Symposium on Gravitation and Cosmology
5 February 2022

The universe as a quantum lab

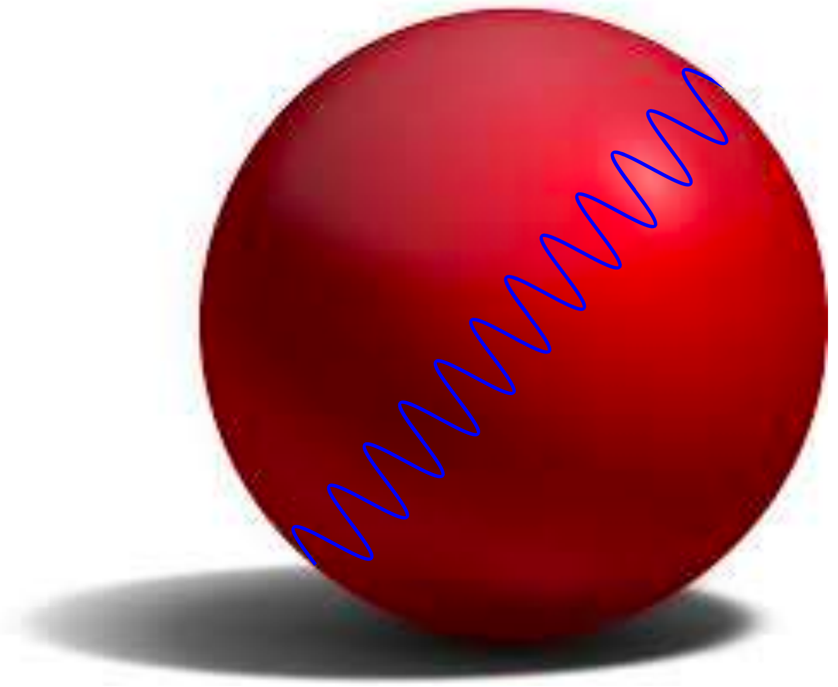
Credit: ESO/M. Kornmesser

Cosmic Inflation

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2$$

Hubble parameter $H = \dot{a}/a$

H^{-1} : characteristic time scale, or length scale ($c = 1$), of the expansion



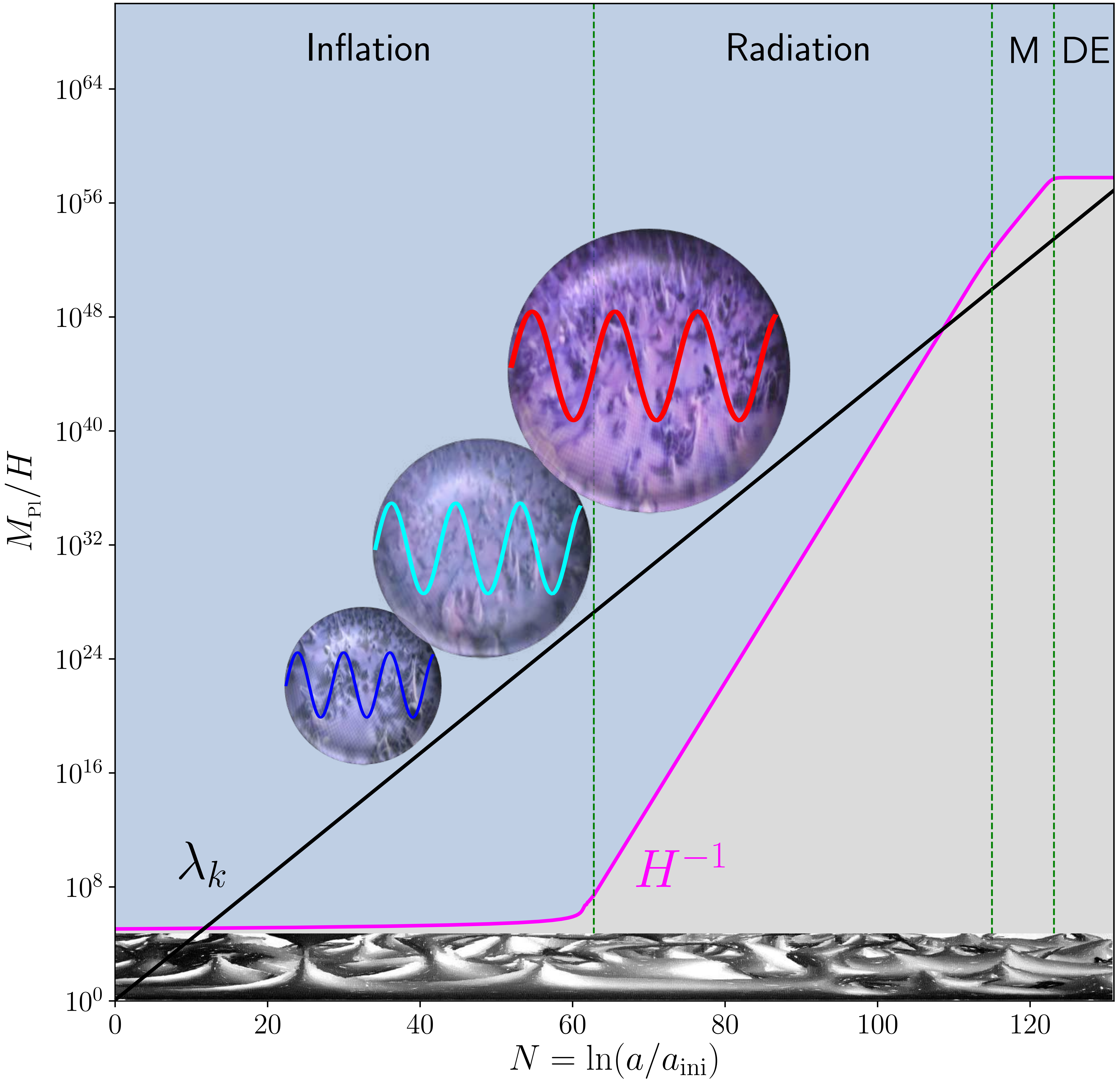
$\lambda \ll H^{-1}$

Insensitive to space-time curvature

$\lambda \gtrsim H^{-1}$

Feels space-time curvature

Cosmic Inflation



Structure formation by the gravitational amplification of quantum fluctuations

Quantum mechanics on cosmological scales!

- Can we trust QM at those scales?
- Is it legit to quantise metric fluctuations?
- What about the QM measurement problem?

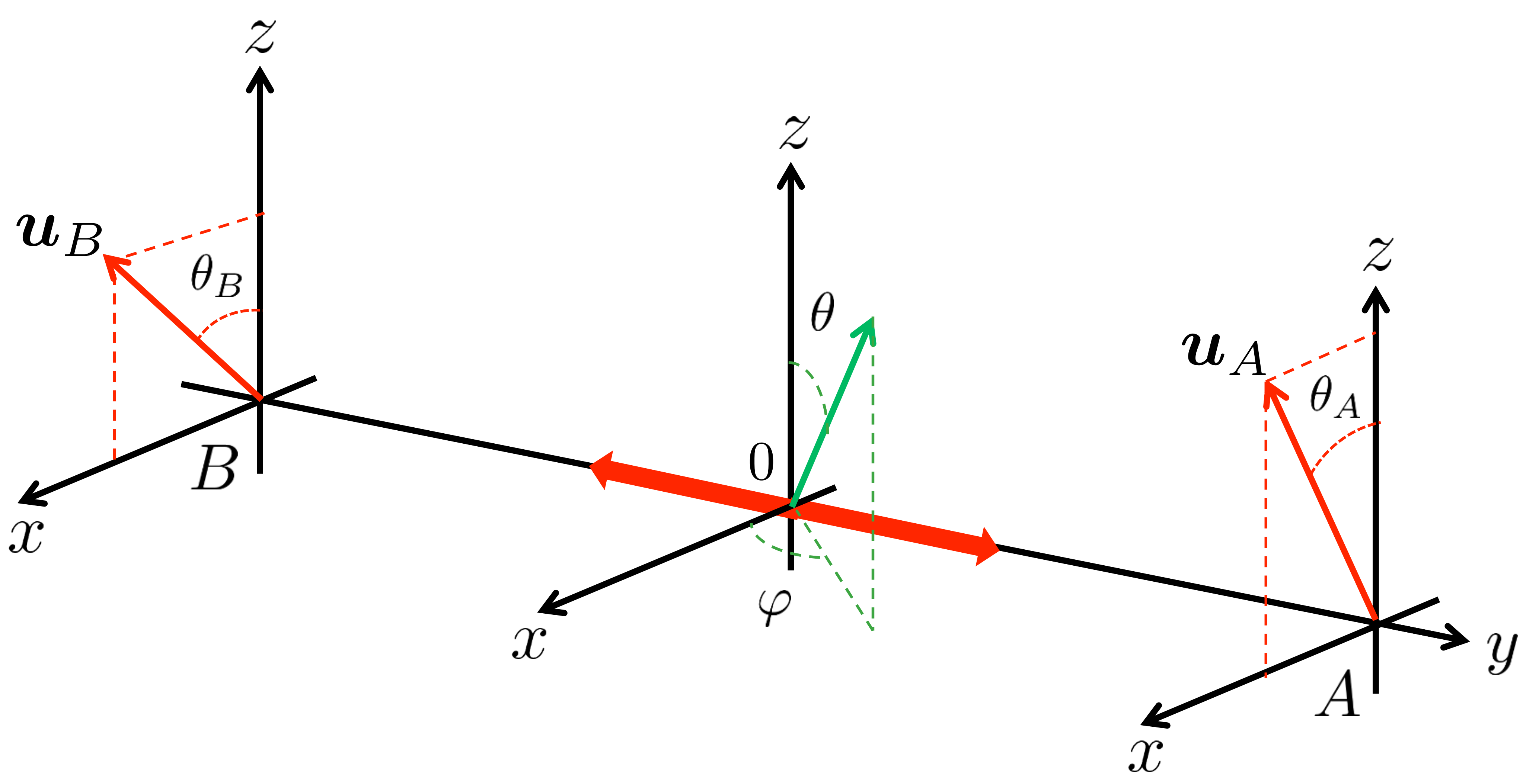
True for inflation, but also for most alternatives (such as contracting cosmologies)

- Strong statement (extraordinary statement requires extraordinary evidence)
- The consequences that can be inferred from this idea are consistent with observations
- This gives an indirect confirmation that cosmological structures have an quantum-mechanical origin

Any direct evidence?

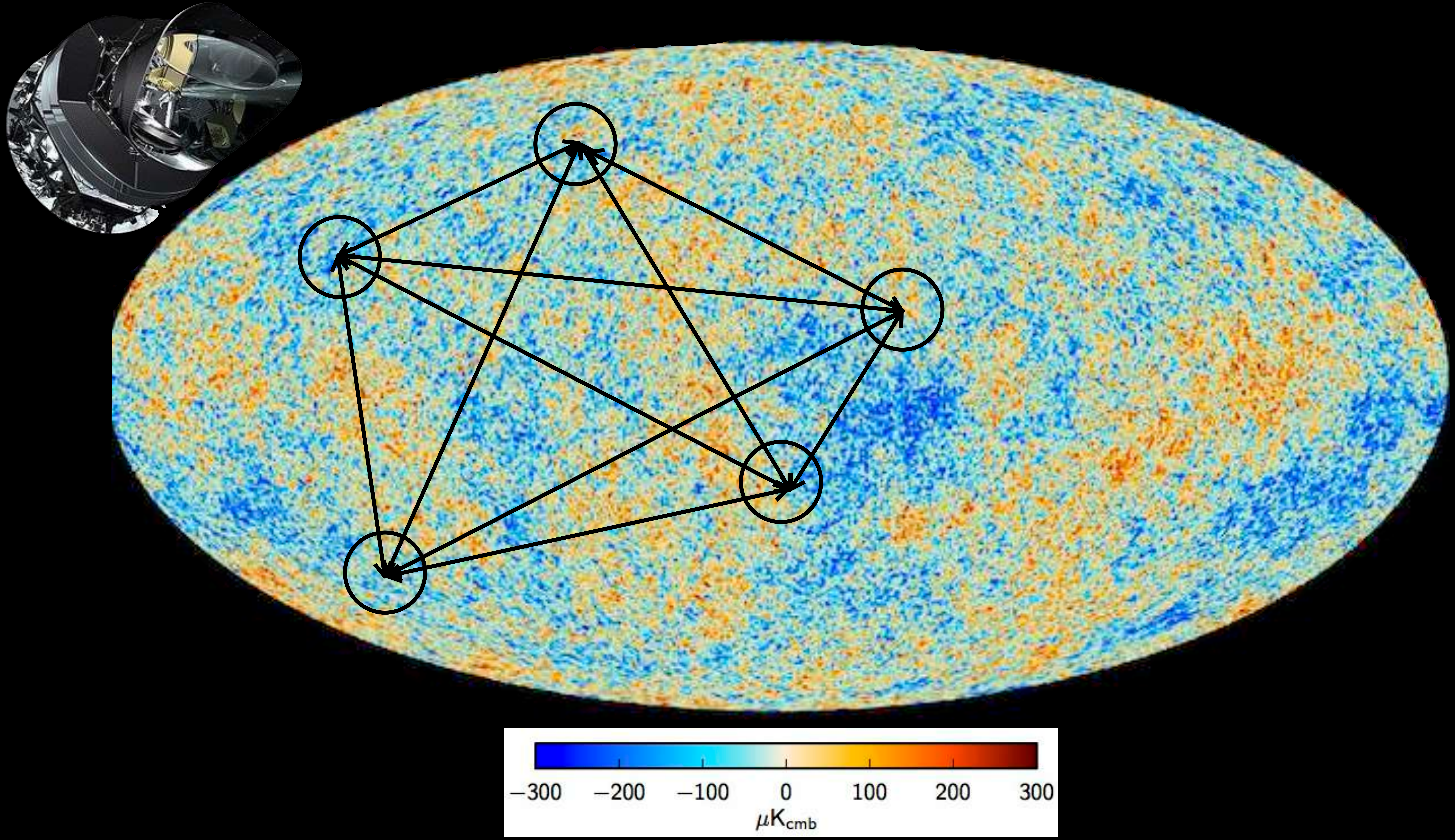
Role of the decaying mode: Lesgourgues, Kiefer, Polarski, Starobinsky (role of decaying mode)
Bell inequalities: Campo, Parentani // Maldacena // Martin, Vennin // Kanno, Shock, Soda // Choudhury, Panda, Singh
Entanglement entropy, Quantum discord: Lim // Martin, Vennin // Hollowood, Mc Donald // Espinosa, Garcia-Bellido
Higher-order statistics: Martin, Vennin // Green, Porto

The quantum and the classical worlds differ by the nature of the correlations they allow for



Are there some quantum correlations in the primordial density field?

Can we detect them?

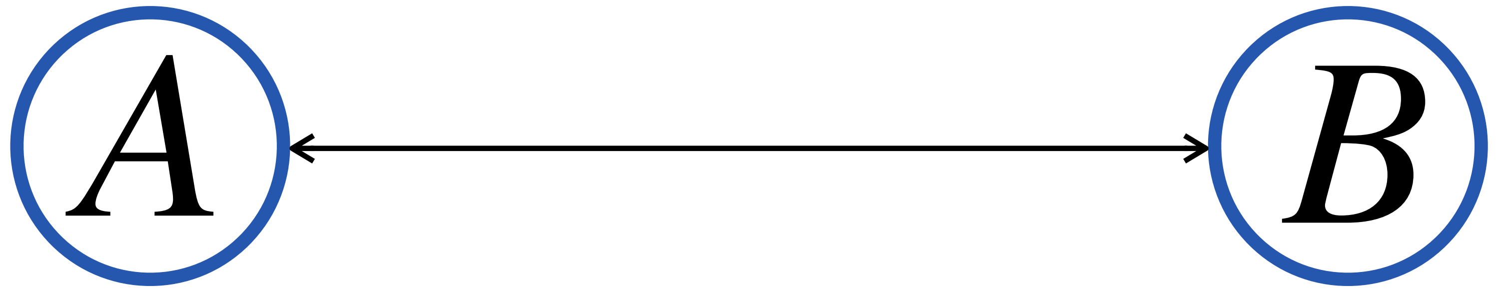

−300 −200 −100 0 100 200 300
μK_cmb

Quantum discord

Henderson and Vedral 2001; Ollivier and Zurek 2001

How to characterise the presence of quantum correlations?

Subsystem *A:* configurations $\{a_i\}$



Subsystem *B:* configurations $\{b_j\}$

CLASSICAL LEVEL: *p*

$$p(a_i) = \sum_j p(a_i, b_j)$$

Von-Neumann entropy: $S(A) = -\sum_i p(a_i) \ln p(a_i) > 0$

$S(A) = 0$ ⟶ The system *A* is entirely determined

$S(A, B) = S(B) + S(B|A)$ (Bayes theorem)

QUANTUM LEVEL: ρ

$$\rho_A = \mathrm{Tr}_B(\rho_{A,B})$$

$$S(A) = -\mathrm{Tr}(\rho_A \ln \rho_A) > 0$$

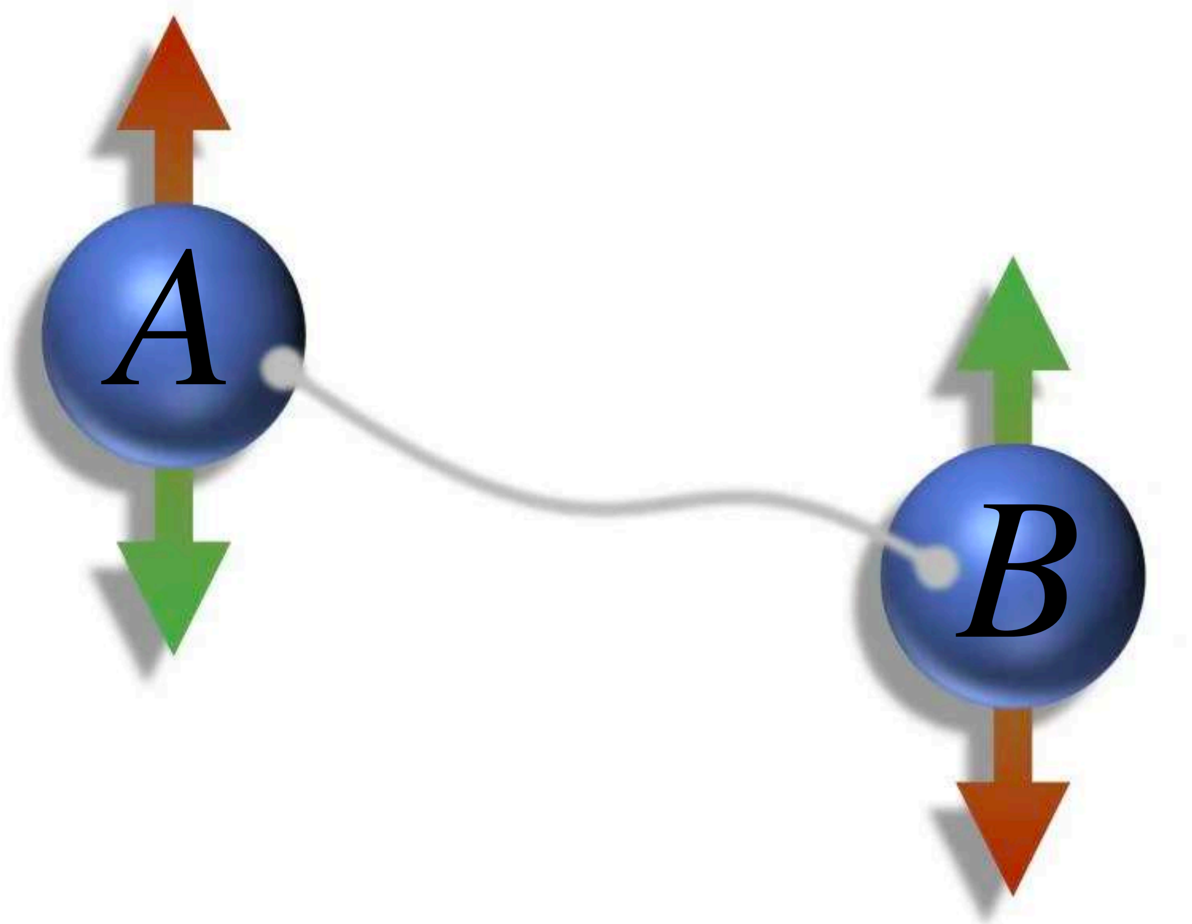
$S(A) = 0$ ⟶ The system *A* is in a pure state

$$S(A, B) \neq S(B) + S(B|A)$$

Quantum discord

Henderson and Vedral 2001; Ollivier and Zurek 2001

Example: Entangled State



$$|\psi\rangle = \frac{|\uparrow_A, \uparrow_B\rangle + |\downarrow_A, \downarrow_B\rangle}{\sqrt{2}}$$

$S(A, B) = 0$ (pure state)

$$\rho_B = \mathrm{Tr}_A(\rho_{A,B}) = \frac{1}{2}|\uparrow_B\rangle\langle\uparrow_B| + \frac{1}{2}|\downarrow_B\rangle\langle\downarrow_B| \longrightarrow S(B) = \ln(2)$$

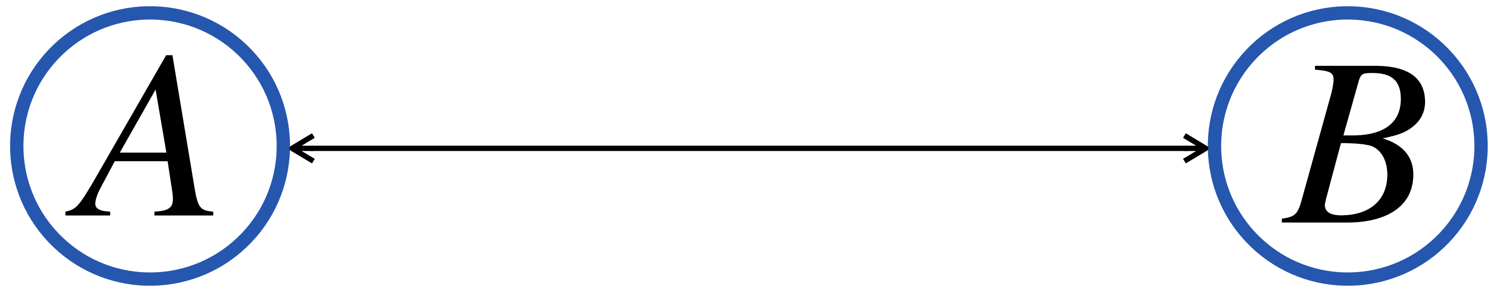
$$\underbrace{S(A, B)}_{0} \neq \underbrace{S(B)}_{\ln(2)} + \underbrace{S(B|A)}_{>0}$$

Quantum discord

Henderson and Vedral 2001; Ollivier and Zurek 2001

How to characterise the presence of quantum correlations?

Subsystem *A:* configurations $\{a_i\}$



Subsystem *B:* configurations $\{b_j\}$

CLASSICAL LEVEL: *p*	QUANTUM LEVEL: ρ
$p(a_i) = \sum_j p(a_i, b_j)$	$\rho_A = \mathrm{Tr}_B(\rho_{A,B})$
Von-Neumann entropy: $S(A) = -\sum_i p(a_i) \ln p(a_i) > 0$	$S(A) = -\mathrm{Tr}(\rho_A \ln \rho_A) > 0$
$S(A) = 0$ ⟶ The system *A* is entirely determined	$S(A) = 0$ ⟶ The system *A* is in a pure state
$S(A, B) = S(B) + S(B \mid A)$ (Bayes theorem)	$S(A, B) \neq S(B) + S(B \mid A)$

$$\mathscr{D}(A, B) = \min_{\{\hat{\Pi}_j\}} [S_{\hat{\Pi}_j}(A \mid B) + S(B) - S(A, B)]$$

Cosmological perturbations

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(t, \boldsymbol{x})$$

$$\phi = \bar{\phi}(t) + \widehat{\delta\phi}(t, \boldsymbol{x})$$

Scalar perturbations are described by a single combination of metric and field fluctuations that directly determines CMB temperature anisotropies →

$$\hat{\zeta}(t, \boldsymbol{x})$$

Expansion of Einstein-Hilbert + scalar field action at second order: independent parametric oscillators, one for each $\boldsymbol{k} \in \mathbb{R}^{3+}$

Interaction term between the quantum fluctuations and the classical background

Free term

$$\hat{H} = \int \mathrm{d}^3\boldsymbol{k} \left[\frac{k}{2}\left(\hat{c}_{\boldsymbol{k}}\hat{c}^{\dagger}_{\boldsymbol{k}} + \hat{c}_{-\boldsymbol{k}}\hat{c}^{\dagger}_{-\boldsymbol{k}}\right) - \frac{i}{2}\frac{\left(a\sqrt{\epsilon_1}\right)'}{a\sqrt{\epsilon_1}}\left(\hat{c}_{\boldsymbol{k}}\hat{c}_{-\boldsymbol{k}} - \hat{c}^{\dagger}_{-\boldsymbol{k}}\hat{c}^{\dagger}_{\boldsymbol{k}}\right)\right]$$

Creation / annihilation of pairs of particles

Pump field: time-dependent coupling constant
Depends only on the scale factor and its derivative
Vanishes if *a* is constant

$$\epsilon_1 = -\dot{H}/H^2$$

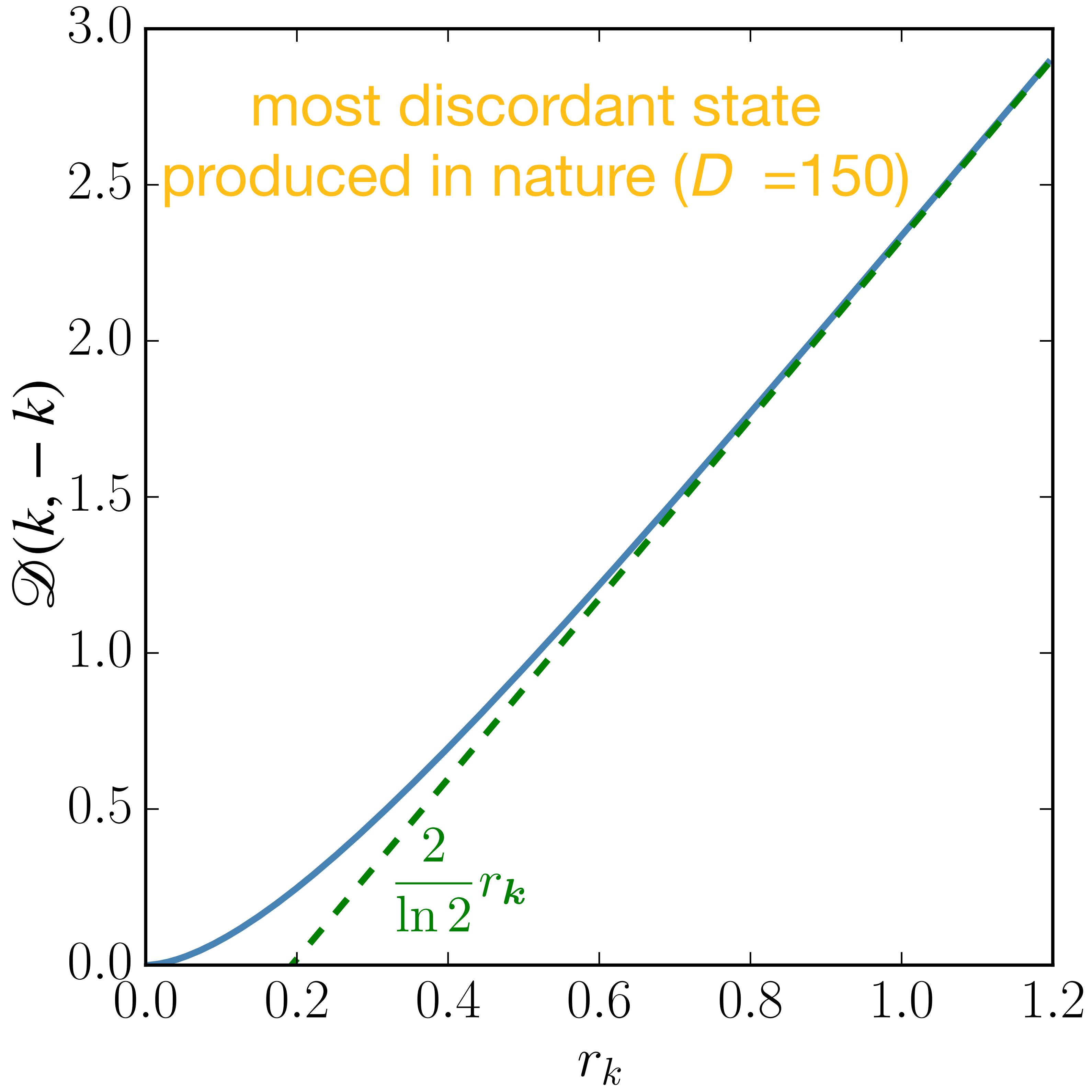
Cosmological perturbations

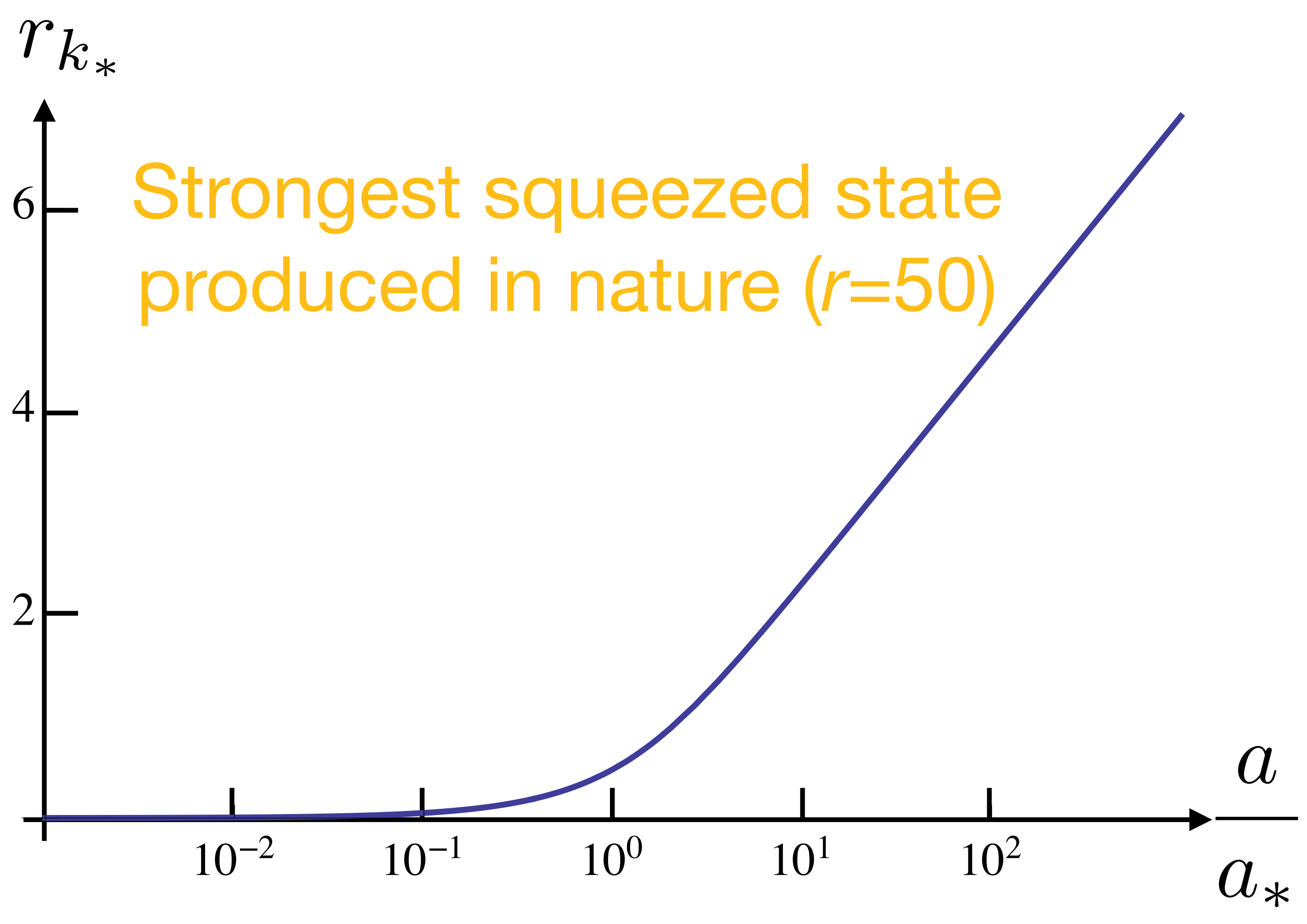
Two-mode squeezed sate $|\Psi_{\mathrm{CMB}}\rangle = \bigotimes_{\boldsymbol{k}\in\mathbb{R}^{3+}} |\Psi_{\boldsymbol{k}}\rangle$ with $|\Psi_{\boldsymbol{k}}\rangle = \frac{1}{\cosh r_k}\sum_{n=0}^{\infty} e^{2in\varphi_k}(-1)^n \tanh^n r_k |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$

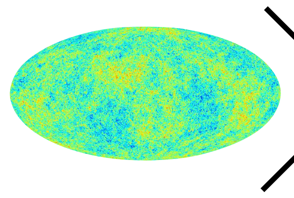
Entangled state
(correlations between modes ***k*** and -***k***)

Martin, Vennin 2015

$$\mathscr{D}(k,-k) = \cosh^2 r_k \log_2(\cosh^2 r_k) - \sinh^2 r_k \log_2(\sinh^2 r_k)$$

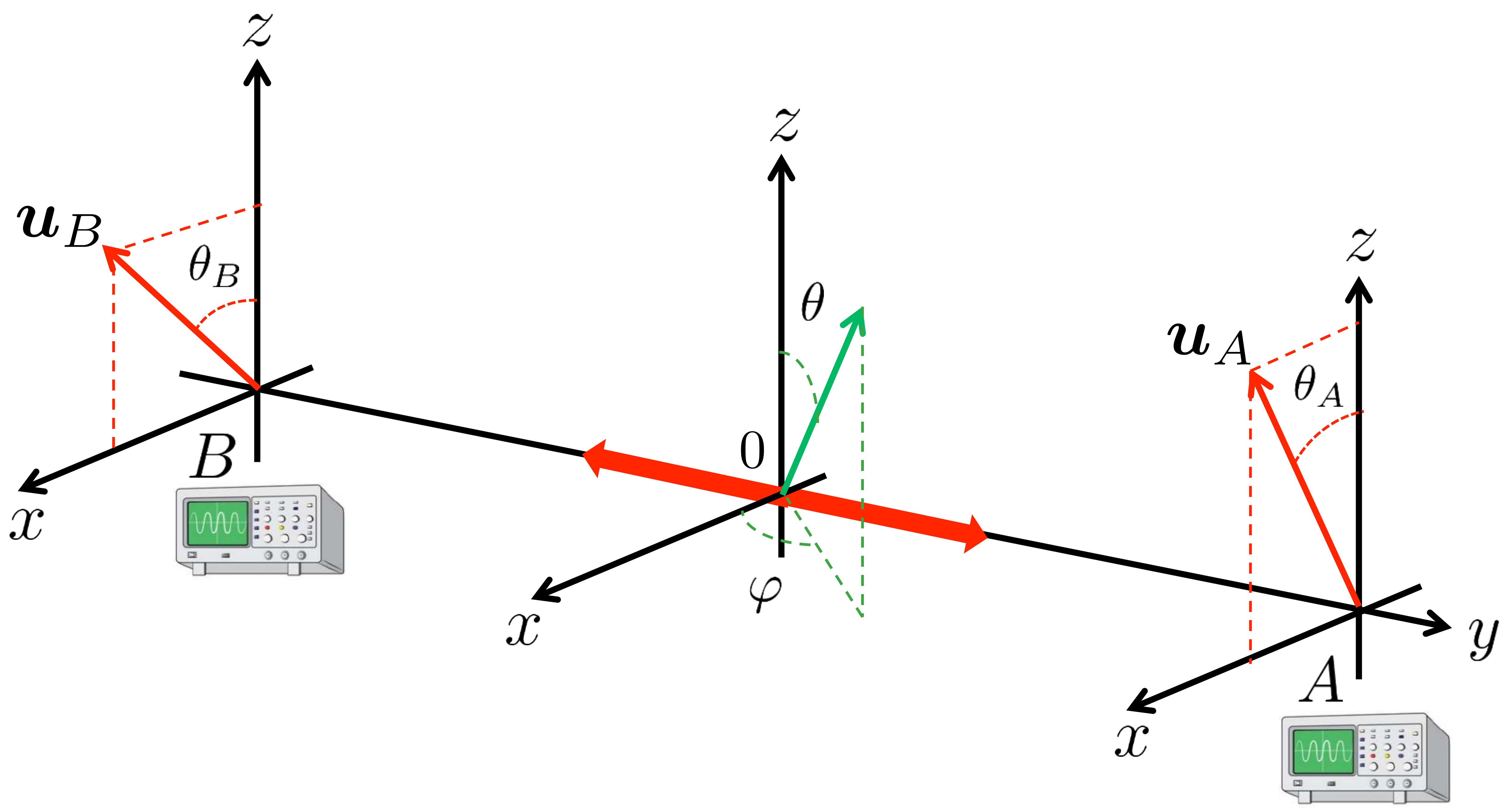




|  $\rangle$ = highly-non classical state? ⟶ Can we violate Bell's inequalities with the CMB?

Bell inequalities

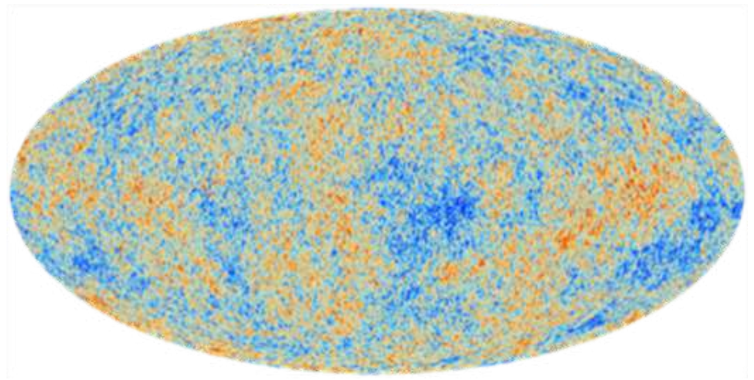
$$\hat{B} = \left(\boldsymbol{u}_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}_B \cdot \hat{\boldsymbol{S}}_B\right) + \left(\boldsymbol{u}_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}'_B \cdot \hat{\boldsymbol{S}}_B\right) + \left(\boldsymbol{u}'_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}_B \cdot \hat{\boldsymbol{S}}_B\right) - \left(\boldsymbol{u}'_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}'_B \cdot \hat{\boldsymbol{S}}_B\right)$$



Classically: $\left\langle \hat{B} \right\rangle < 2$

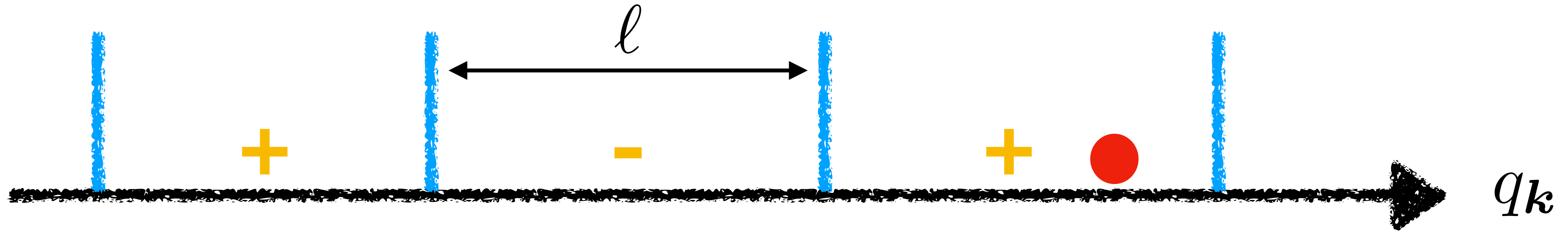
- Bipartite system: *k* and -*k*
- Entangled system: two-mode squeezed state
- improper, spin-like operators (Revzen 2006)

Bell inequalities



continuous variable

$$\hat{q}_{\boldsymbol{k}} = \frac{\hat{c}_{\boldsymbol{k}} + \hat{c}_{\boldsymbol{k}}^\dagger}{\sqrt{2k}} = \hat{q}_{\boldsymbol{k}}^\dagger$$



continuous spectrum

- Divide the real axis into intervals $[n\ell, (n+1)\ell]$
- Perform a measurement of $q_{\boldsymbol{k}}$
- Return $S_z(\ell) = (-1)^n$

Larsson 2004

$$\hat{S}_z(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{n\ell}^{(n+1)\ell} \mathrm{d}q_{\boldsymbol{k}} \, |q_{\boldsymbol{k}}\rangle \langle q_{\boldsymbol{k}}| \longrightarrow \hat{S}_z^2(\ell) = 1$$

$$\hat{S}_+(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{2n\ell}^{(2n+1)\ell} \mathrm{d}q_{\boldsymbol{k}} \, |q_{\boldsymbol{k}}\rangle \langle q_{\boldsymbol{k}} + \ell|$$

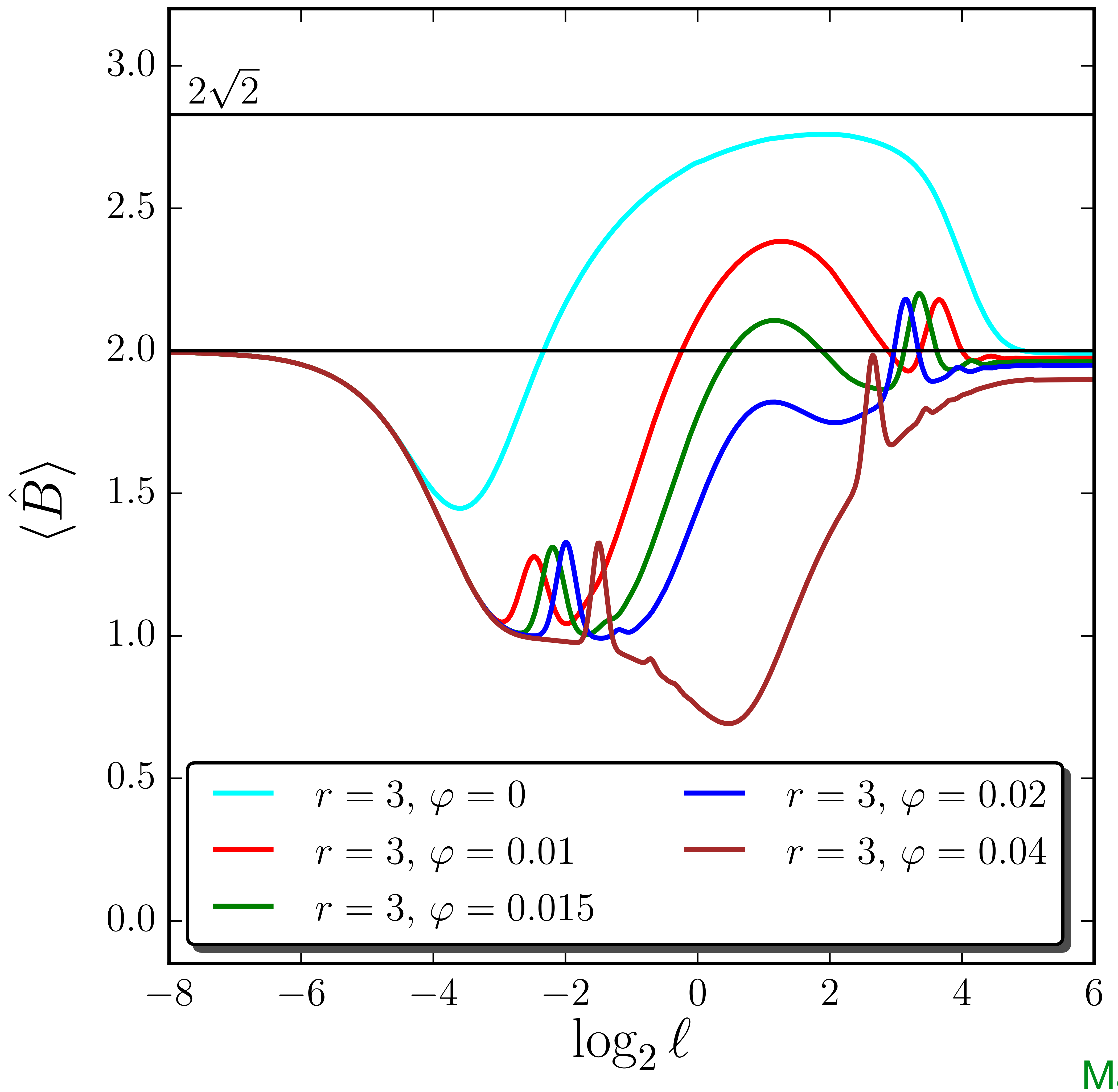
$$\longrightarrow \hat{S}_x(\ell) = \hat{S}_+(\ell) + \hat{S}_+^\dagger(\ell)$$

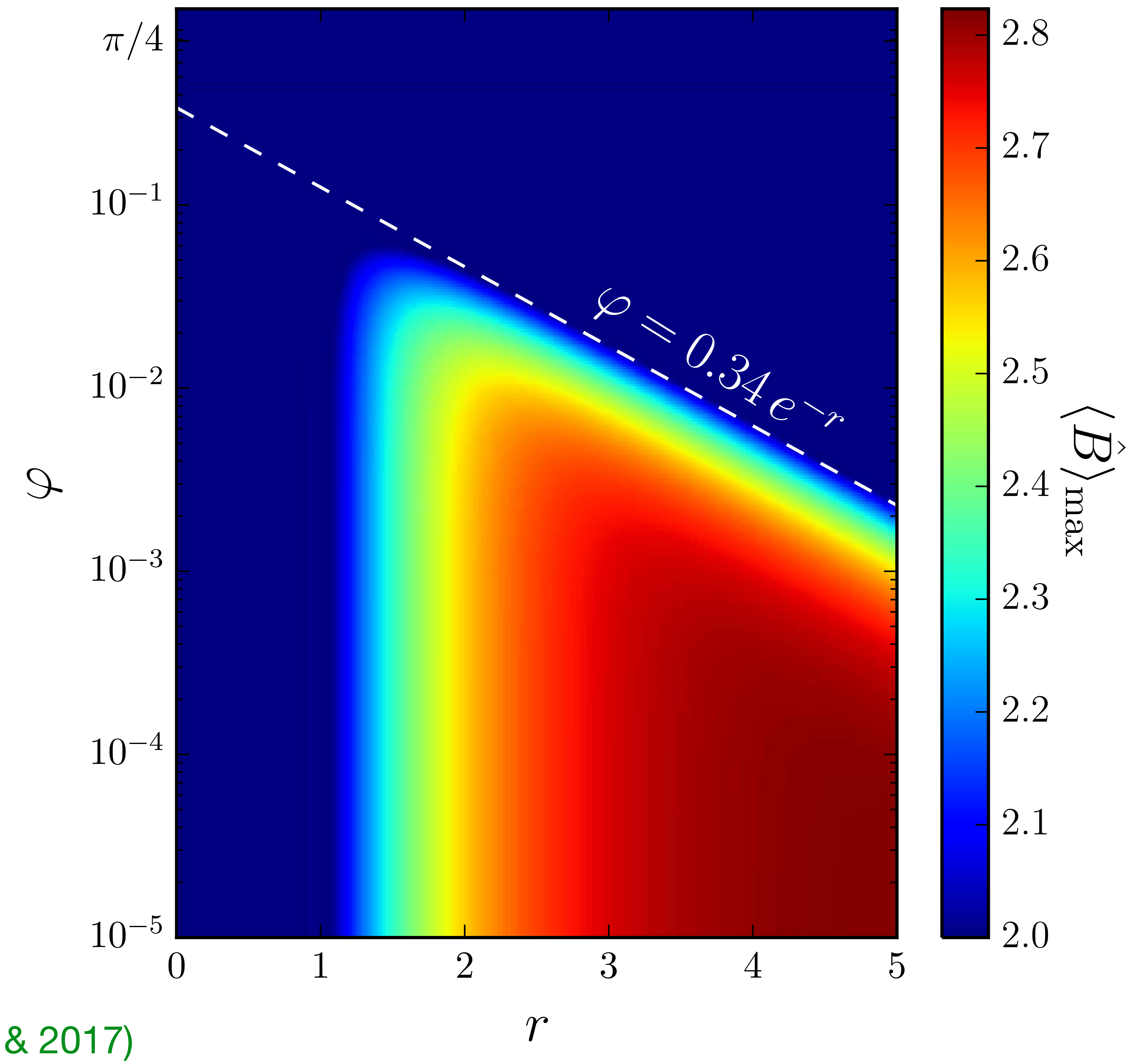
$$\longrightarrow \hat{S}_y(\ell) = -i\left[\hat{S}_+(\ell) - \hat{S}_+^\dagger(\ell)\right]$$

$$\longrightarrow \left[\hat{S}_i(\ell), \hat{S}_j(\ell)\right] = 2i\epsilon_{ijk}\hat{S}_k(\ell)$$ obey spin algebra

Bell inequalities in the CMB

classically: $\left\langle \hat{B}\left(\ell\right) \right\rangle < 2$



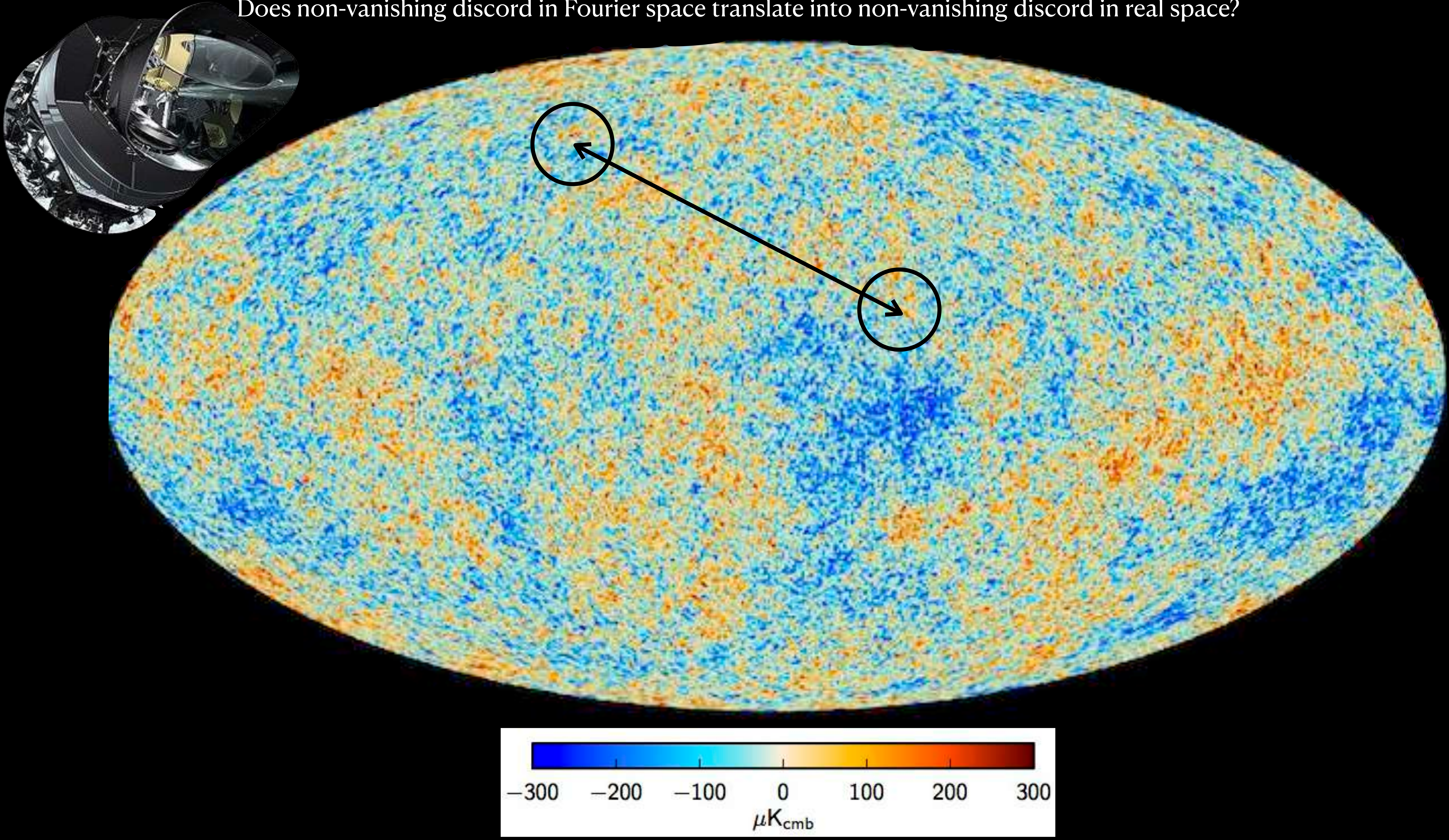


Martin, Vennin (2016 & 2017)

Wait a minute … do we measure correlations in Fourier space?

Does it make sense to test for locality in Fourier space?

Does non-vanishing discord in Fourier space translate into non-vanishing discord in real space?



Real-space entanglement of quantum fields

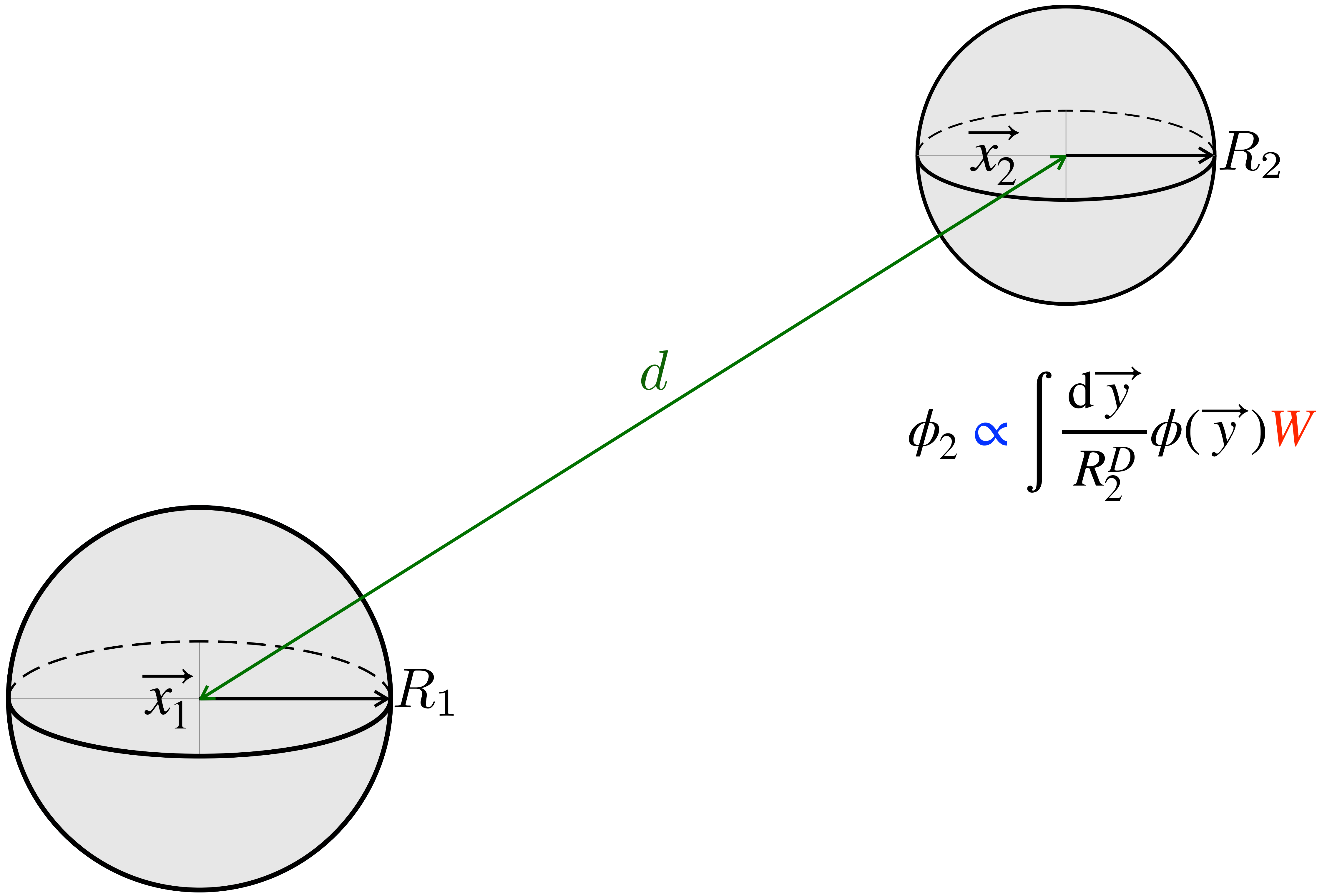
Previous work: Casini, Huerta (2009) // Datta (2009) // Shiba (2012) (for mutual information only, using numerical lattice simulations)

New approach (Martin, Vennin 2021) :

$$[\phi_i, \pi_j] = i\delta_{i,j}$$

The fields need to be rescaled

The support of the window function needs to be compact



$$\phi_2 \propto \int \frac{\mathrm{d}\vec{y}}{R_2^D}\phi(\vec{y})W\left(\frac{|\vec{y}-\vec{x}_2|}{R_2}\right)$$

$$\phi_1 \propto \int \frac{\mathrm{d}\vec{y}}{R_1^D}\phi(\vec{y})W\left(\frac{|\vec{y}-\vec{x}_1|}{R_1}\right)$$

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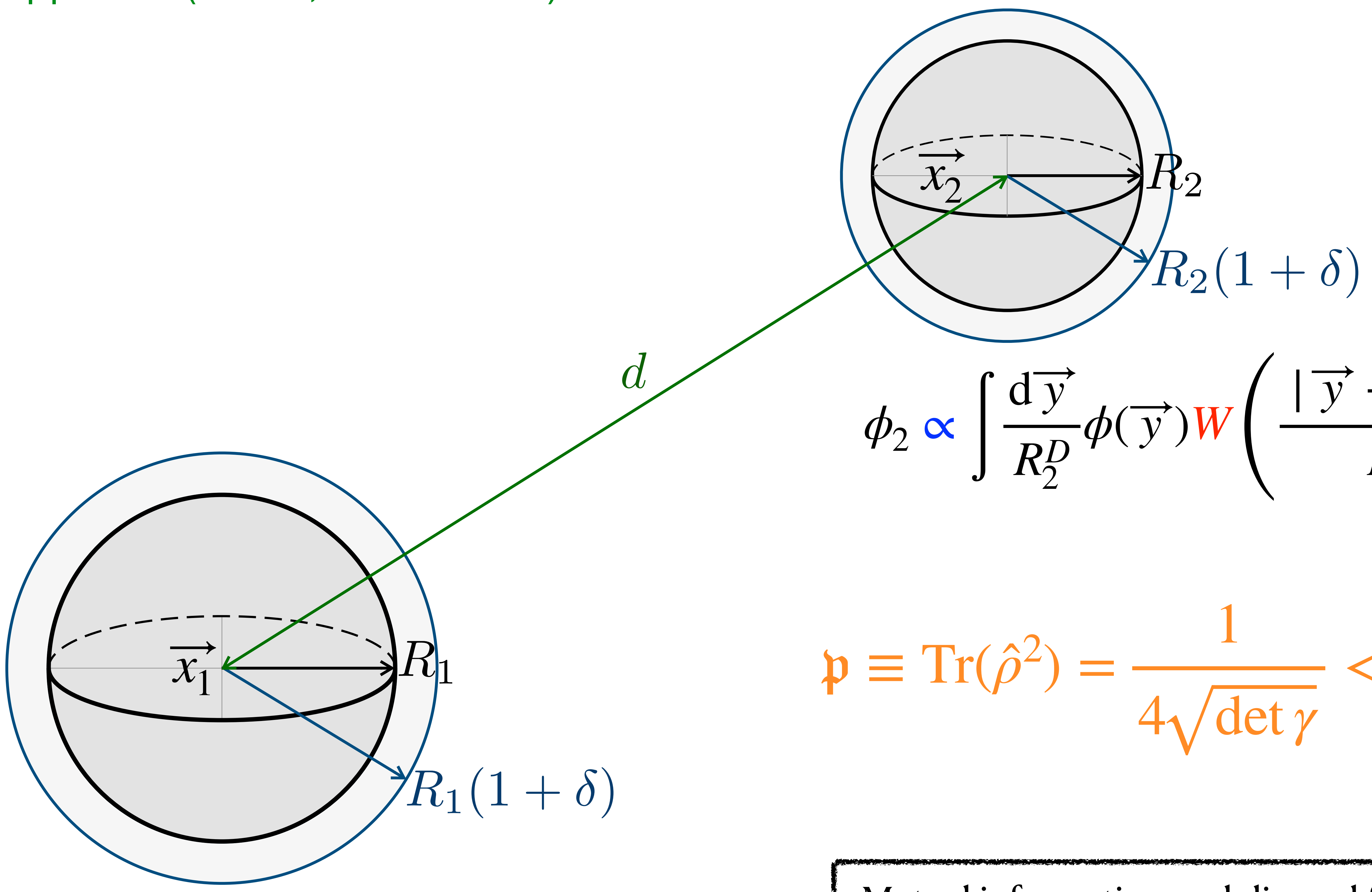


$$\phi_2 \propto \int \frac{\mathrm{d}\vec{y}}{R_2^D}\phi(\vec{y})W\left(\frac{|\vec{y}-\vec{x}_2|}{R_2}\right)$$

$$\phi_1 \propto \int \frac{\mathrm{d}\vec{y}}{R_1^D}\phi(\vec{y})W\left(\frac{|\vec{y}-\vec{x}_1|}{R_1}\right)$$

$$\mathfrak{p} \equiv \mathrm{Tr}(\hat{\rho}^2) = \frac{1}{4\sqrt{\det\gamma}} < 1$$

Mutual information and discord for bipartite Gaussian systems entirely given by the symplectic spectrum of the (sub-blocks of the) covariance matrix [Adesso & Data 2010]

$$[\phi_i, \pi_j] = i\delta_{i,j}$$

The fields need to be rescaled

The support of the window function needs to be compact

Bipartite, "particle-like" system

For Gaussian quantum fields, this is a Gaussian bipartite system with correlation matrix

$$\gamma \propto \int \mathrm{d}\ln \widetilde{W}^2(kR) \begin{pmatrix} \mathcal{P}_{\phi\phi}(k) & \mathcal{P}_{\phi\pi}(k) & \mathcal{P}_{\phi\phi}(k)\,\mathrm{sinc}\,(kd) & \mathcal{P}_{\phi\pi}(k)\,\mathrm{sinc}\,(kd) \\ - & \mathcal{P}_{\pi\pi}(k) & \mathcal{P}_{\phi\phi}(k)\,\mathrm{sinc}\,(kd) & \mathcal{P}_{\pi\pi}(k)\,\mathrm{sinc}\,(kd) \\ - & - & \mathcal{P}_{\phi\phi}(k) & \mathcal{P}_{\phi\pi}(k) \\ - & - & - & \mathcal{P}_{\pi\pi}(k) \end{pmatrix}$$

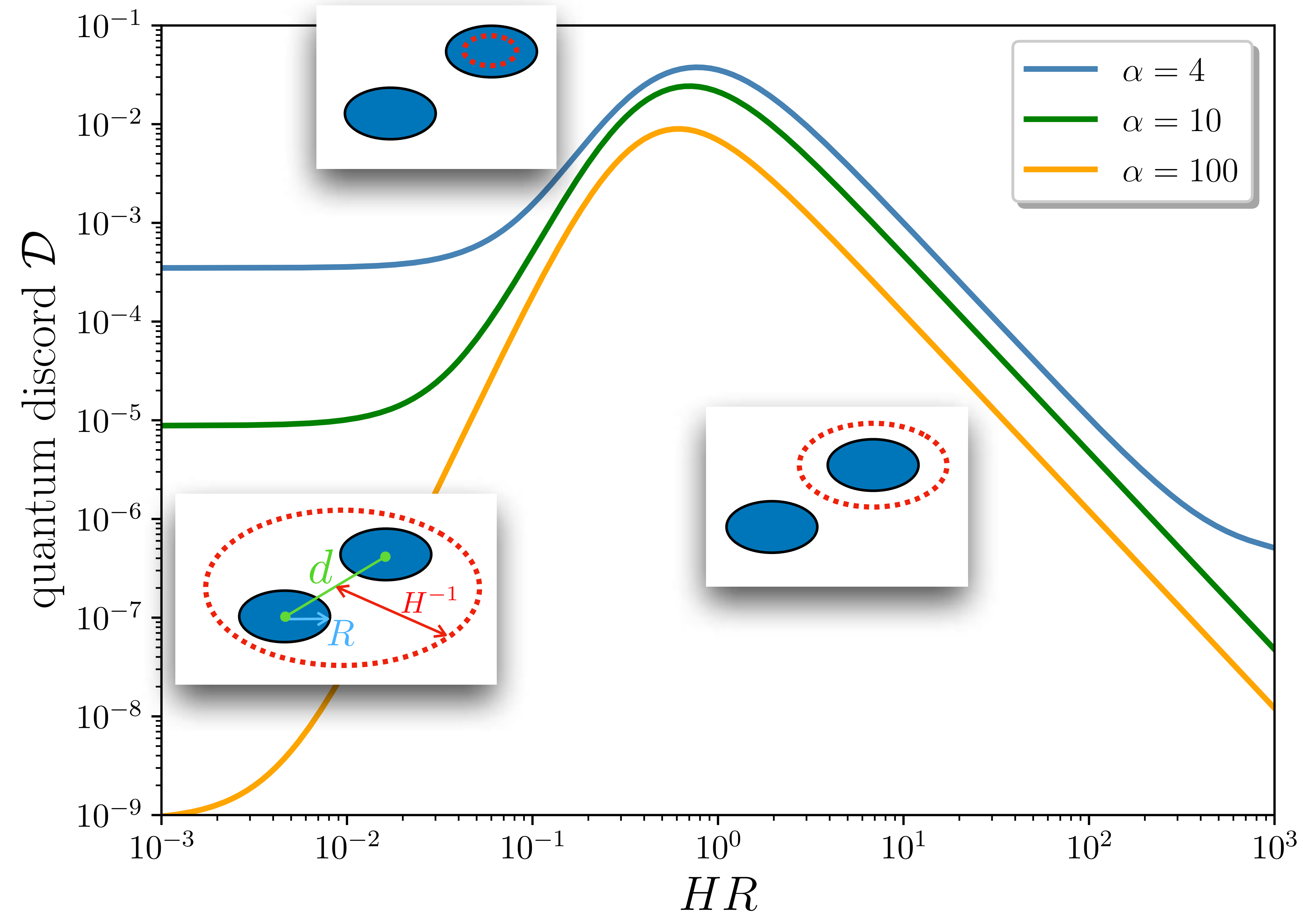
Does not describe a pure state

Real-space entanglement of quantum fields

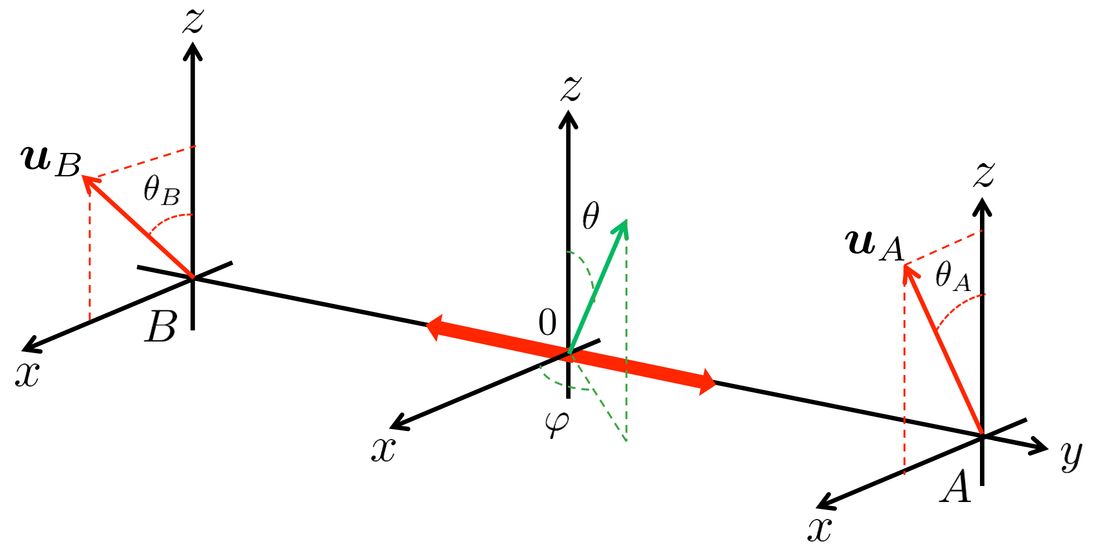
Cosmological perturbations

Martin, Vennin (2021)

$\alpha = d/R$



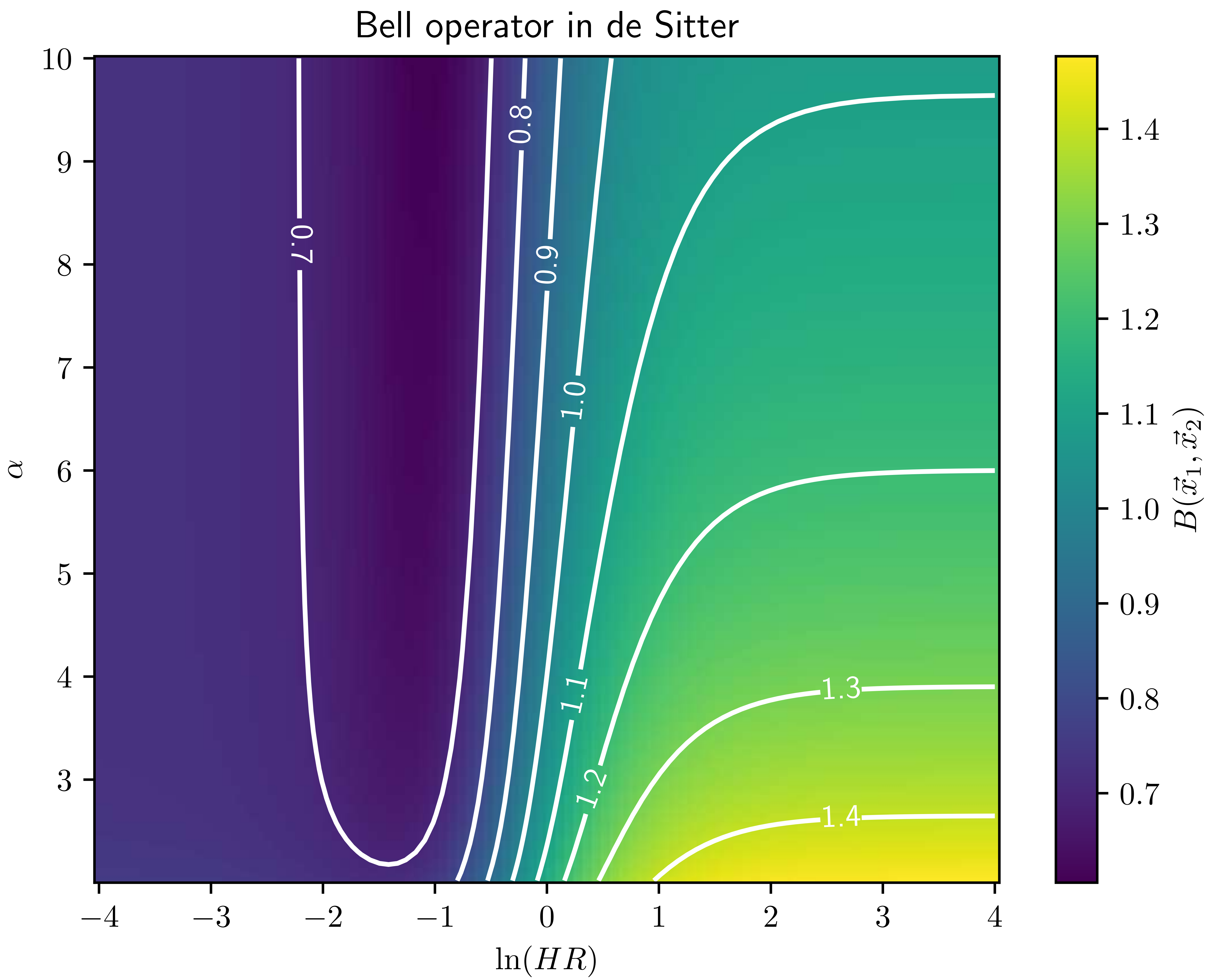
- Flat space results recovered at sub-Hubble scales
- Enhancement at large distance compared to flat space
- Smaller mutual information and discord than in Fourier space (effect of self-decoherence)
- Best place to look for quantum effects: Hubble scale at the end of inflation (compromise between correlations vs "self decoherence", or between particle creation vs decaying mode)

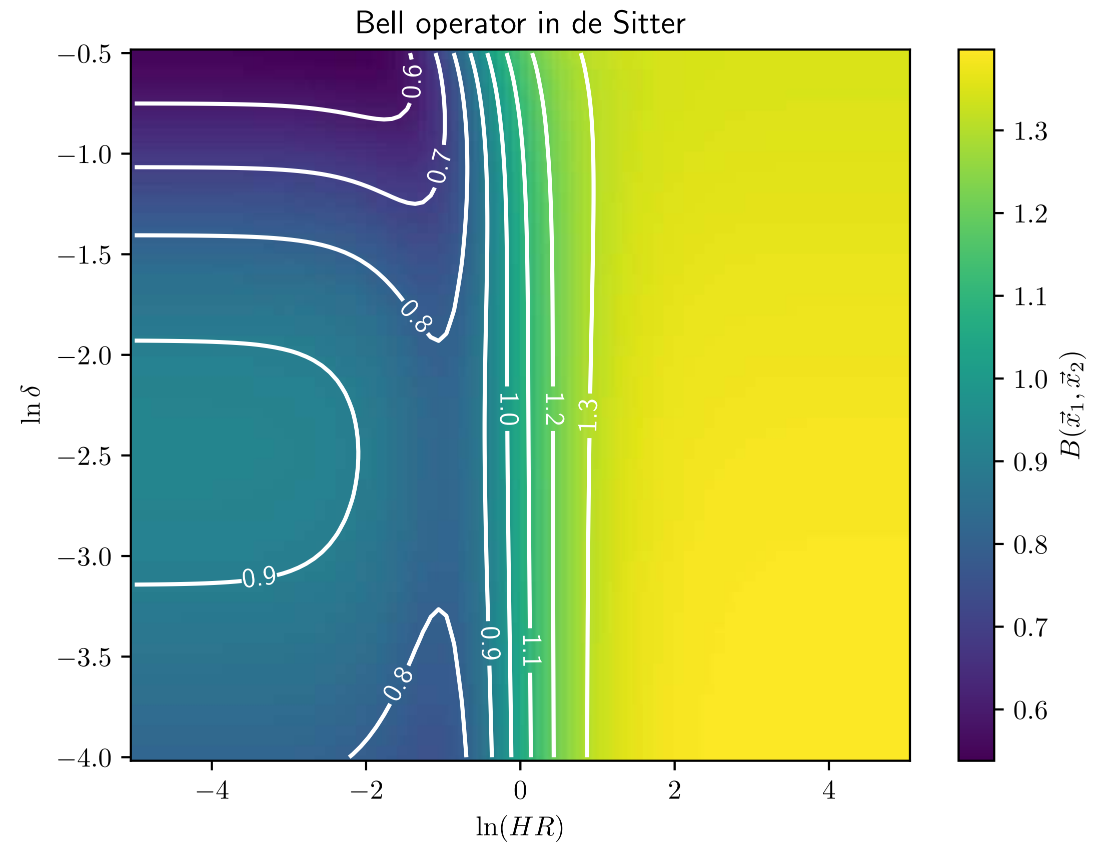


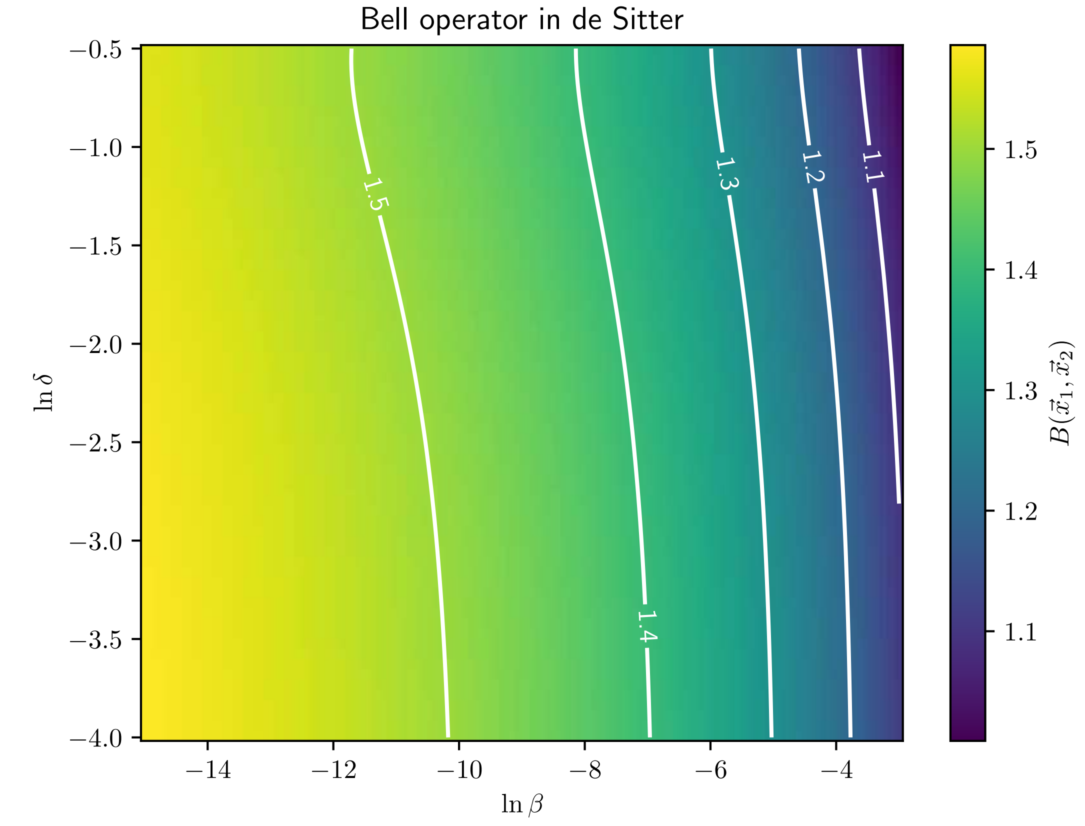
Real-space Bell inequalities

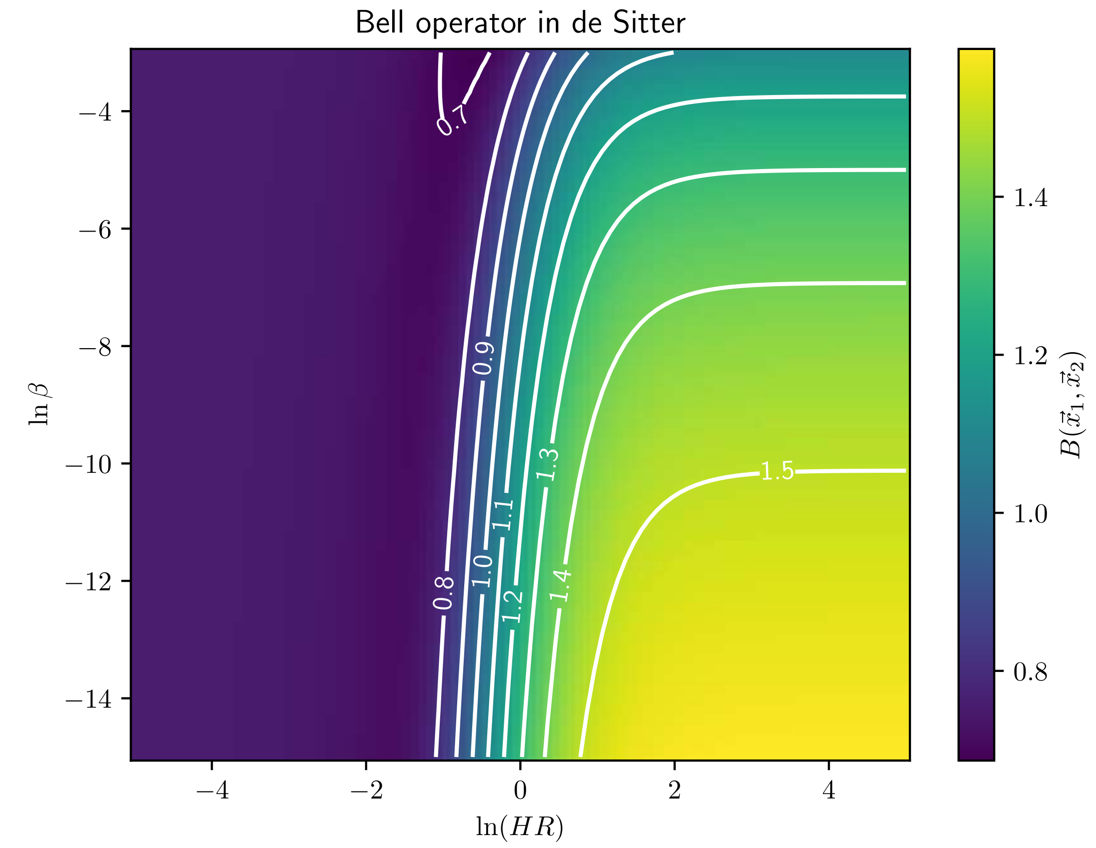
Cosmological perturbations

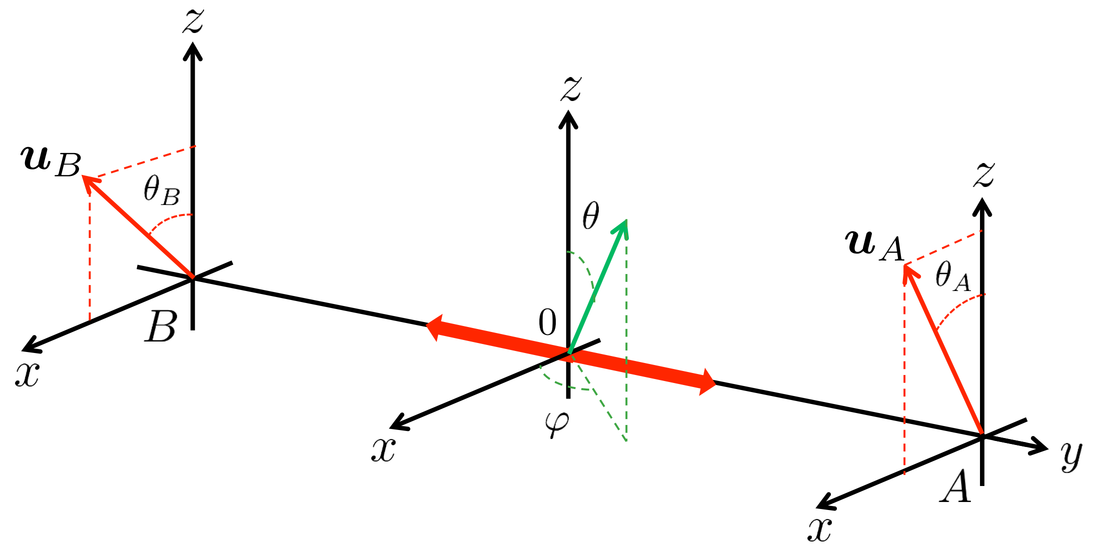
Espinosa-Portales, Vennin (to appear soon)







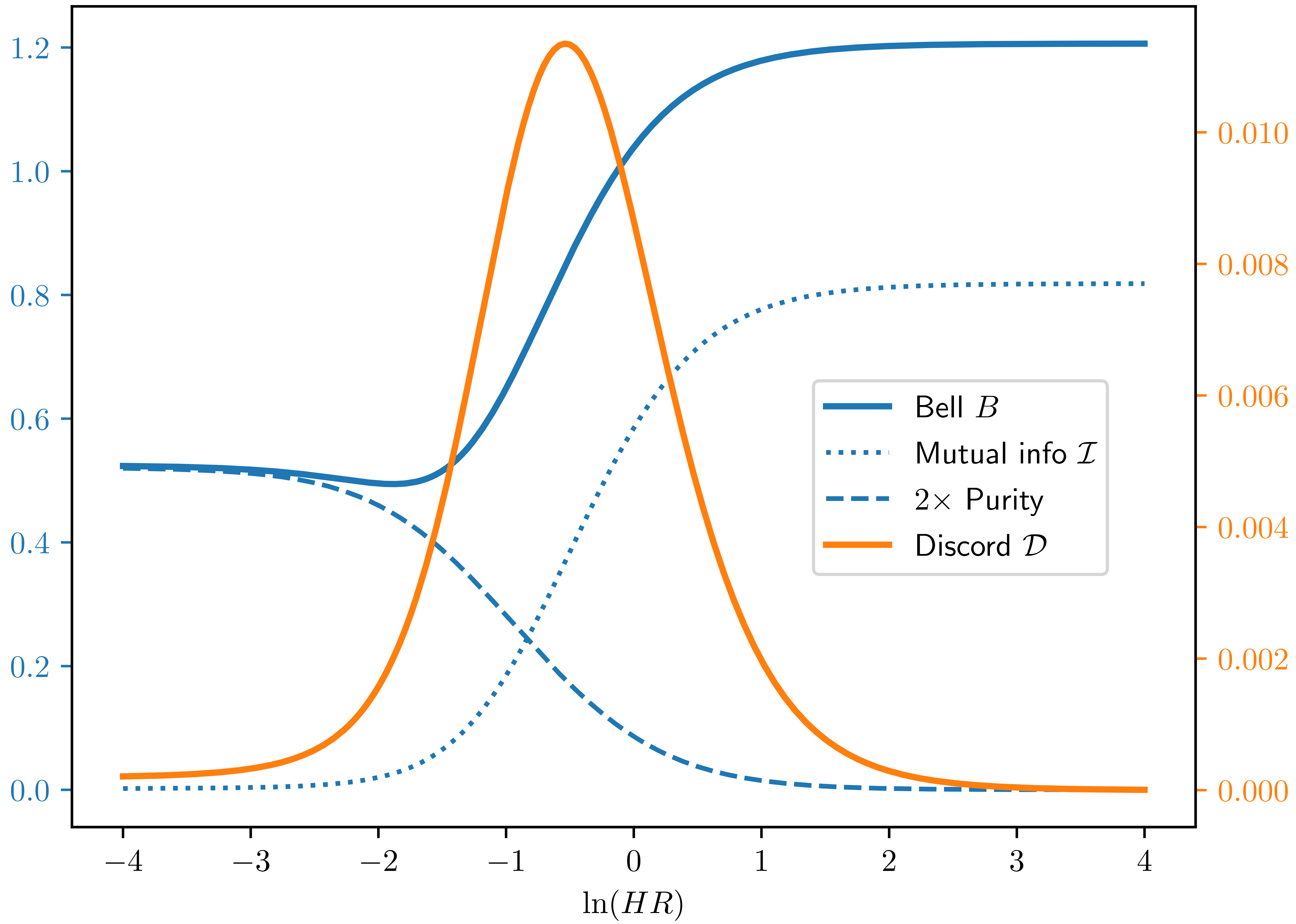




Real-space Bell inequalities

Cosmological perturbations

Espinosa-Portales, Vennin (to appear 2022)



Need to access the decaying mode

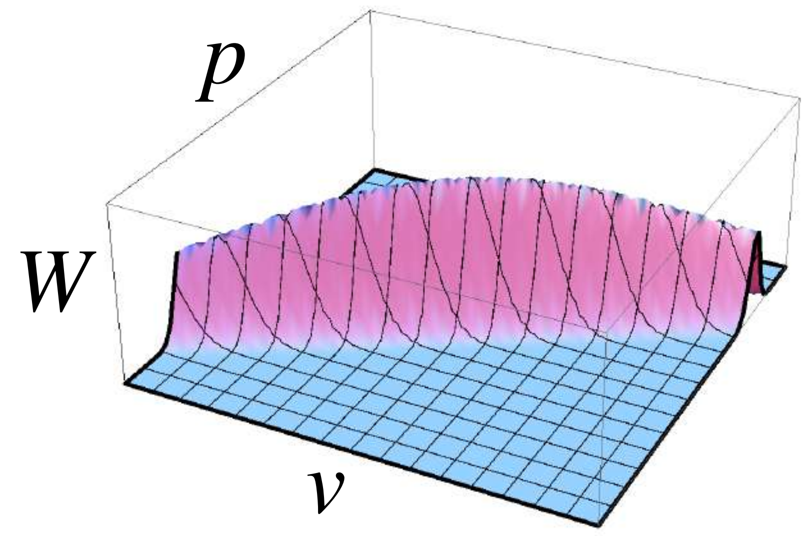
How to measure $\hat{S}_+(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{2n\ell}^{(2n+1)\ell} \mathrm{d}q_{\boldsymbol{k}} \, |q_{\boldsymbol{k}}\rangle \langle q_{\boldsymbol{k}} + \ell|$?

requires to access phase information

conjugated momentum $\pi_{\boldsymbol{k}}$

decaying mode

$\zeta'_{\boldsymbol{k}} \sim e^{-r_{\boldsymbol{k}}} \zeta_{\boldsymbol{k}}$



$\tilde{A} \rightarrow A$ in the large squeezing limit

Example: $O = v_{\boldsymbol{k}} v_{\boldsymbol{k}}^\dagger p_{\boldsymbol{k}} p_{\boldsymbol{k}}^\dagger + p_{\boldsymbol{k}} p_{\boldsymbol{k}}^\dagger v_{\boldsymbol{k}} v_{\boldsymbol{k}}^\dagger$

$\tilde{O} = O - 1/4$ with $\langle \hat{O} \rangle = e^{2\left(N - N_{\text{Hubble crossing}}\right)}$

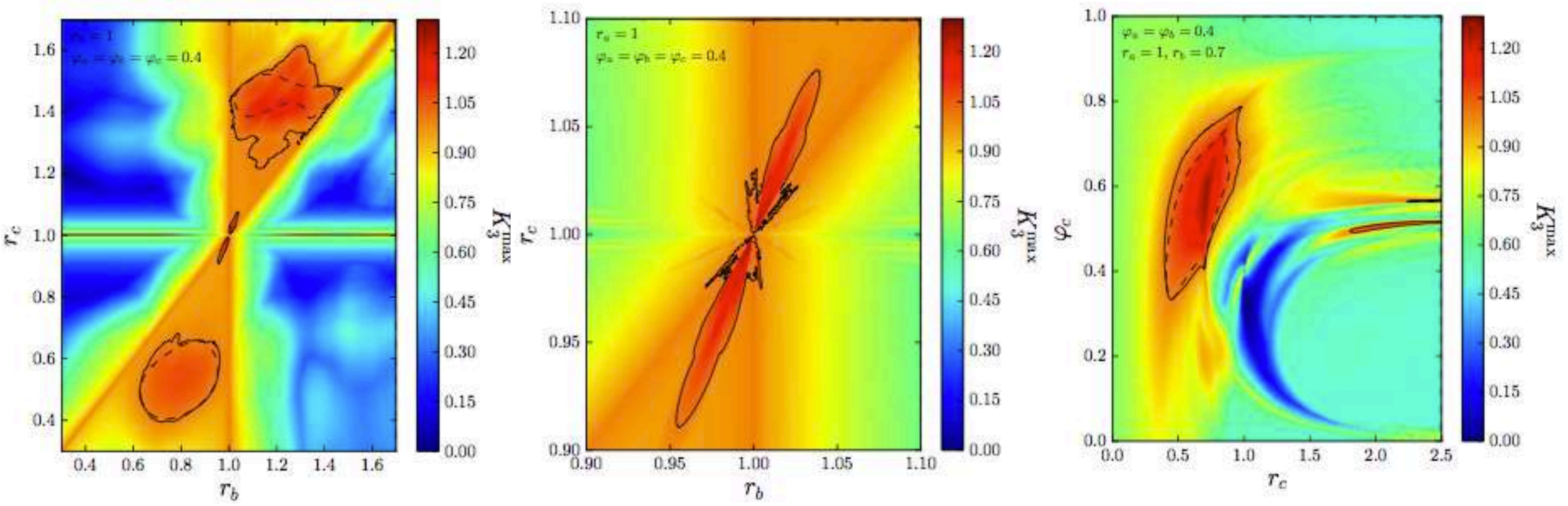
Can we detect quantum correlations using "position" measurements only?

$\widetilde{\zeta_{\boldsymbol{k}}} = \zeta_{\boldsymbol{k}}$ and $\widetilde{f}(\zeta_{\boldsymbol{k}}) = f(\zeta_{\boldsymbol{k}})$ so according to Revzen's theorem: not with Bell inequalities!

Generalised Bell inequalities in the CMB

	Type of inequality	Assumptions	Requires bipartite system	involves single spin measurement only
$\langle \hat{S}_1^a(t)\hat{S}_2^b(t)\rangle$	Spatial Bell	realism and locality	yes	no
$\langle \hat{S}_1^a(t)\hat{S}_1^b(t')\rangle$	Temporal Bell	realism and non-invasiveness	no	no
$\langle \hat{S}_1^a(t)\hat{S}_1^a(t')\hat{S}_1^a(t'')\rangle$	Legget-Garg (≥3 measurement times)	realism and non-invasiveness	no	yes
$\langle \hat{S}_1^a(t)\hat{S}_2^a(t')\rangle$	Bipartite temporal Bell	realism and locality	yes	yes

J. Martin, V.V. (2016) for LGI;



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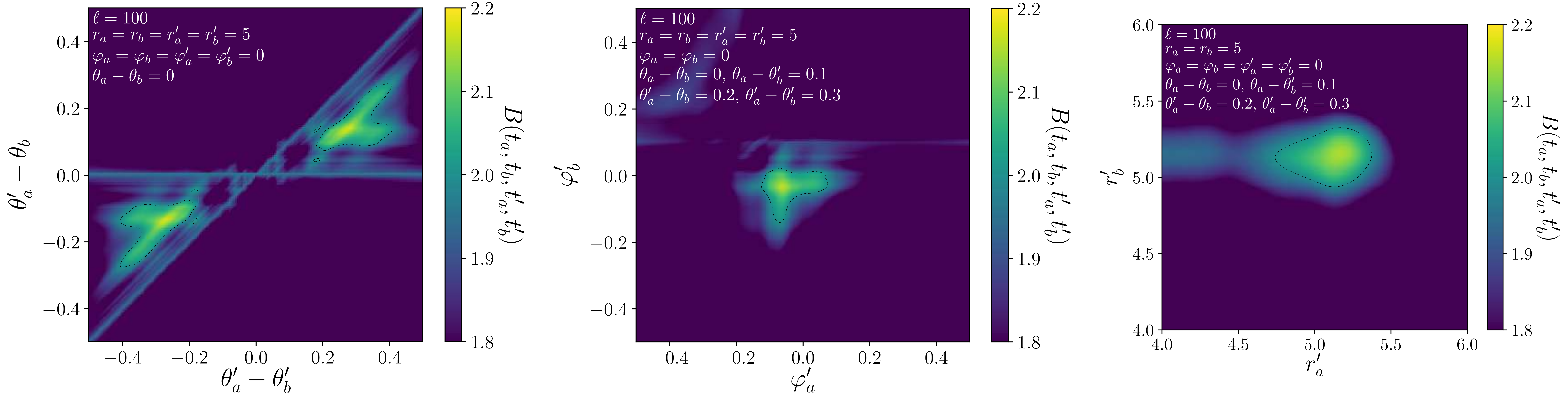
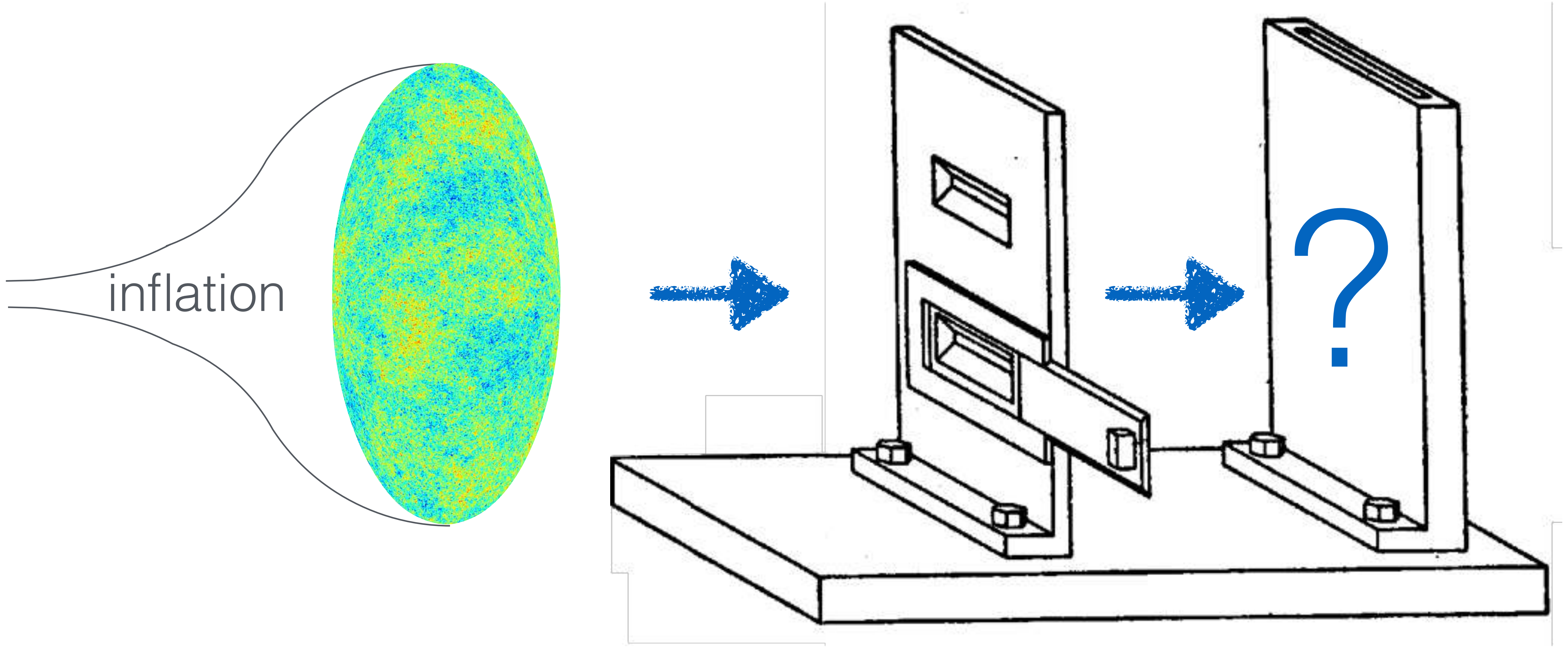
K. Ando, V.V. (2020) for BTBI



But requires to measure zeta at different times … cross correlate measurement at different redshifts?

Conclusions

- **Cosmological perturbations** are placed in a two-mode highly **squeezed state** in the very early Universe
- Such a state has a large **quantum discord in Fourier space**, denoting the presence of **large quantum correlations** between particles created with opposite wave momenta
- In **real space**, quantum discord is much more suppressed, and we de not report violations of Bell inequalities.
- Even if we found successful Bell operators, would it require to “measure” somehow the (exponentially suppressed, at least in the standard setup) **decaying mode ?**
- What about **non-Gaussianities?**
- What about **decoherence?**



Thank you for your attention!