Constraining the time-variation of the gravitational constant using gravitational-wave observations of binary neutron stars

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Motivation

- Dirac was the first to conjecture the possibility of the variation of the fundamental *constants* of nature.

- Alternative theories, especially scalar-tensor theories like Brans-Dicke, predict a time-varying gravitational constant *G*.

- There are bounds on the time variation of *G* from several observational channels. The strongest comes from Lunar Laser Ranging ($\sim 10^{-13}\,\mathrm{yr}^{-1}$). All bounds though come from low redshift or very high redshift (CMB).

- Can we add gravitational waves to the list?

Motivation

- There are some (very very weak!) constraints on the time variation of G from GW150914 and GW151226 [Yunes+ 2016].

- Gravitational waves carry an imprint of the *G* at the time of merger.

- Hence, if the inferred masses using the current value G_o fall outside the range of *allowed* neutron star masses by theory, it would be an indication that the value G_s at merger would be different than it is now.

Example Theory Constraints			
Repr. Parameters	GW150914	GW151226	Current Bounds
$\sqrt{\lvert\alpha_{\rm EdGB}\rvert}$ [km]	—	—	10^7 [56], 2 [57–59]
$\lvert\dot{\phi}\rvert$ [1/sec]	—	—	10^{-6} [60]
$\sqrt{\lvert\alpha_{\rm dCS}\rvert}$ [km]	—	—	10^8 [61, 62]
(c_+, c_-)	$(0.9, 2.1)$	$(0.8, 1.1)$	$(0.03, 0.003)$ [63, 64]
$(\beta_{\rm KG}, \lambda_{\rm KG})$	$(0.42, -)$	$(0.40, -)$	$(0.005, 0.1)$ [63, 64]
ℓ [μm]	5.4×10^{10}	2.0×10^9	10–10^3 [65–69]
$\lvert\dot{G}\rvert$ [10^{-12}/yr]	5.4×10^{18}	1.7×10^{17}	0.1–1 [70–74]
m_g [eV]	10^{-22} [19]	10^{-22} [5]	10^{-29}–10^{-18} [75–79]
E_*^{-1} [eV^{-1}] (time)	5.8×10^{-27}	3.3×10^{-26}	—
E_*^{-1} [eV^{-1}] (space)	1.0×10^{-26}	5.7×10^{-26}	3.9×10^{-53} [80]
$\eta_{\rm dsrt}/L_{\rm Pl} > 0$	1.3×10^{22}	3.8×10^{22}	—
$\eta_{\rm dsrt}/L_{\rm Pl} < 0$			2.1×10^{-7} [80]
$\alpha_{\rm edt}/L_{\rm Pl}^2 > 0$	5.5×10^{62}	2.5×10^{63}	2.7×10^2 [80]
$\alpha_{\rm edt}/L_{\rm Pl}^2 < 0$			—

Yunes+ 2016

Neutron Stars

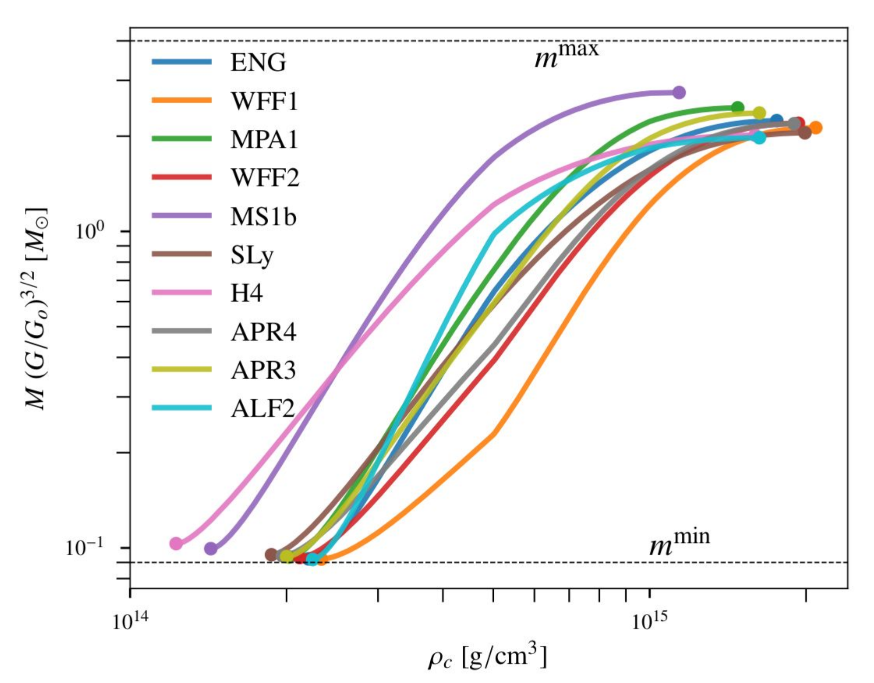
- The mass of a spherically symmetric star is determined by the TOV equation.

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r), \quad \frac{dP(r)}{dr} = \frac{-G\, m(r)\, \rho(r)}{r^2} C(r),$$

C(r) is the relativistic correction.

- A dimensional analysis of the above reveals that the mass of the equilibrium configuration scales as $G^{-3/2}$. Can be verified by numerics.

- There exists a minimum (maximum) mass limit below (above) which a neutron star will get gravitationally unbound (collapse).



(Binary) Neutron Stars

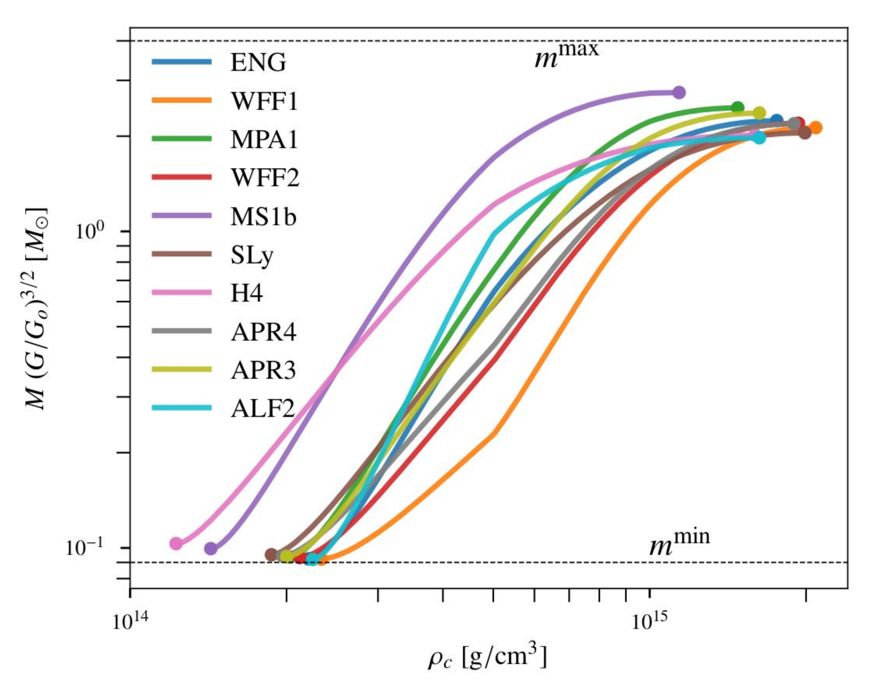
- We entertain the possibility that the value of G during the merger G_s could be different from its current value G_o

- The maximum and minimum masses between these two epochs scale as,

$$m_s^{\text{min, max}} = m^{\text{min, max}} (G_s/G_o)^{-3/2}$$

  The precise values of the masses will depend on the equation of state.

- We make the conservative choices $m^{max} = 4\ M_{sun}$ and $m^{min} = 0.1\ M_{sun}$.



(Binary) Neutron Stars

- The phase matching condition gives us

$$\left(\frac{\pi G_o M_o f}{c^3}\right)^{1/3} = \left(\frac{\pi G_s M_s f}{c^3}\right)^{1/3}, \quad \Rightarrow \quad M_o = \frac{G_s}{G_o} M_s \implies m_o = \frac{G_s}{G_o} m_s,$$

where m_o and m_s are the values of the masses at the current epoch and merger epoch respectively

- Invoking that m_s should lie between the maximum and minimum allowed NS masses at the epoch of merger, we get

$$m^{\min}(G_s/G_o)^{-1/2} \leq m_o \leq m^{\max}(G_s/G_o)^{-1/2}$$

- Caveat: we assume that the redshift of the source is known either from an independent electromagnetic observation, or if the event is nearby.

Results

- Using GW170817 (with EM counterpart), the value of *G* at the merger epoch is constrained to

$$4 \times 10^{-3} G_o < G_s < 9 G_o$$

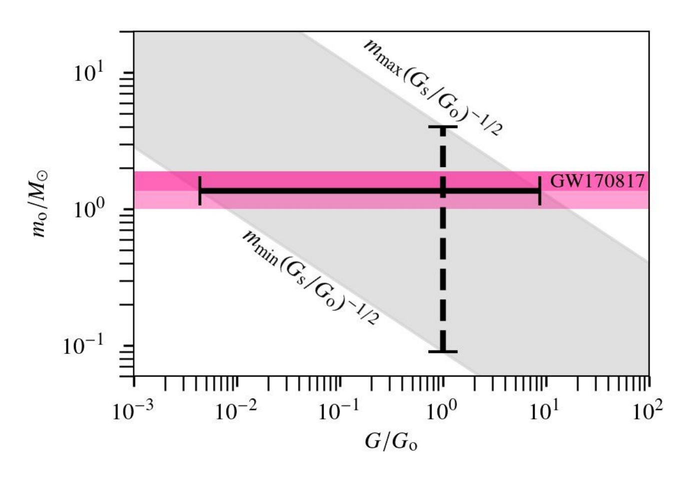
and its average rate of change is constrained to

$$-7 \times 10^{-9}\text{yr}^{-1} < \dot{G}/G_\text{o} < 5 \times 10^{-8}\text{yr}^{-1}$$

- Using GW190425 (assuming low-redshift event), the constraint is,

$$-4 \times 10^{-9}\text{yr}^{-1} < \dot{G}/G_\text{o} < 2 \times 10^{-8}\text{yr}^{-1}$$

- These bounds assume pretty conservative values of the min/max masses. Bounds go as $(1 / \; m^{min/max})^2$.



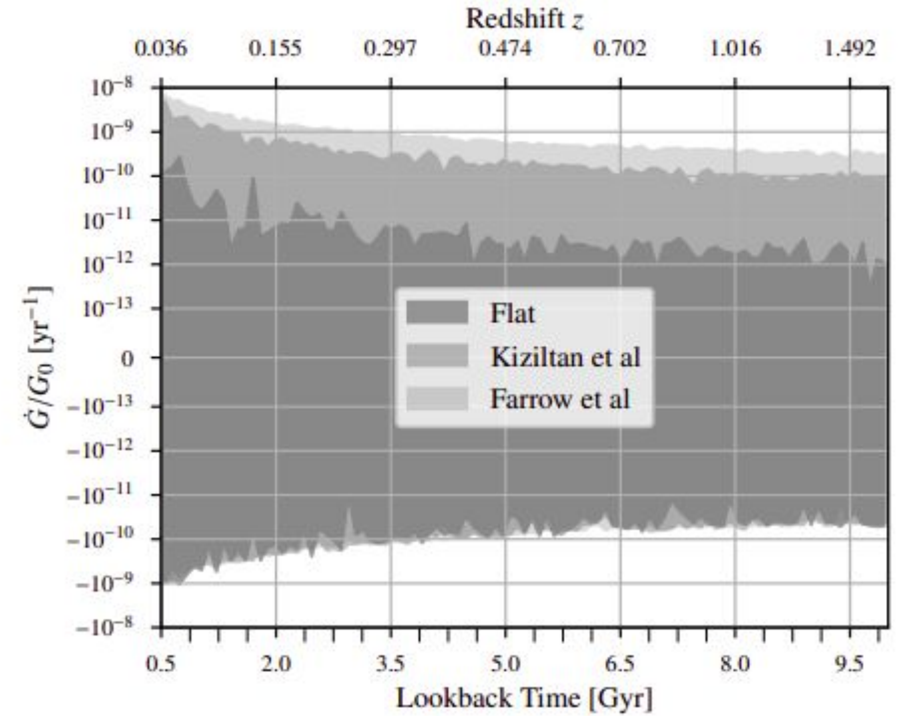
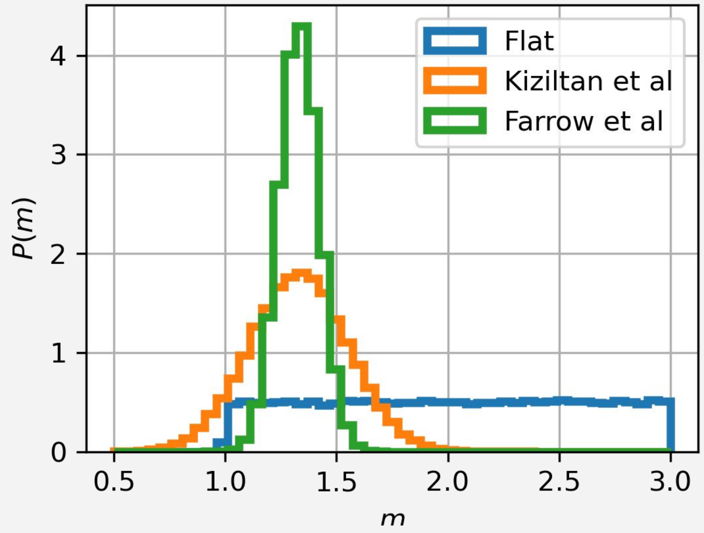


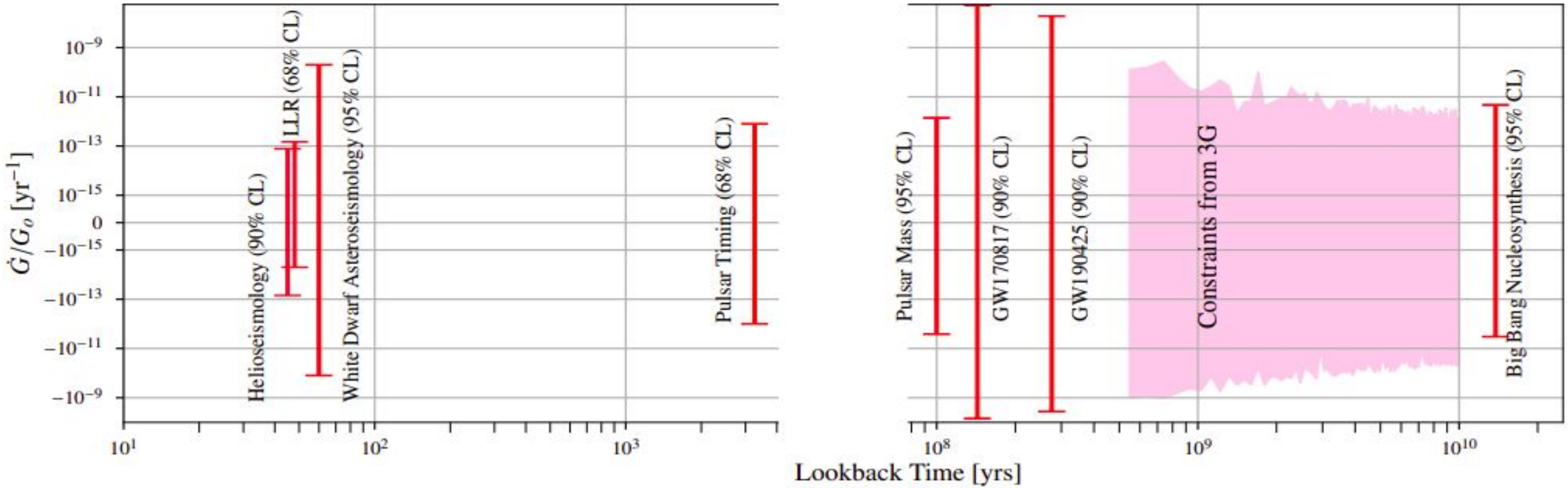
FIG. 4: Expected constraints on $\dot{G}/G_o$ from 10 year observations of third-generation GW detectors. We assume three different mass distributions of neutron stars, and that ~ 1% of the mergers will have a detectable electromagnetic counterpart from which the cosmological redshift can be estimated.

Results

- What does the future look like?
  - Simulate a population of BNSs in the next generation of detectors, based on three different mass distributions [Kiziltan+ 2013, Farrow+ 2019]



Results



$\dot{G}/G_o$ [yr^{-1}]
10^{-9}
10^{-11}
10^{-13}
10^{-15}
0
-10^{-15}
-10^{-13}
-10^{-11}
-10^{-9}
Helioseismology (90% CL)
LLR (68% CL)
White Dwarf Asteroseismology (95% CL)
Pulsar Timing (68% CL)
Pulsar Mass (95% CL)
GW170817 (90% CL)
GW190425 (90% CL)
Constraints from 3G
Big Bang Nucleosynthesis (95% CL)
10^1
10^2
10^3
10^8
10^9
10^{10}
Lookback Time [yrs]

Summary

- We outlined a method to constrain the time variation of the gravitational constant using gravitational-wave observations from binary neutron stars.

- These constraints are fourteen orders of magnitude better than any other constraints from gravitational waves, and are comparable to some other non-GW constraints.

- These constraints will improve by a couple of orders of magnitude using future detectors, and will probe an epoch inaccessible to any other observational probe.

Future Work

- Include change in redshift evolution due to varying G, which will be important for high-z 3G BNS detections.
- Combine information from multiple events for stronger constraints, possibly using a fully bayesian approach.
- Build a map of G across cosmic time and hence place constraints on models of GR/cosmology with time-varying G.

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