Introduction to gravitational wave astronomy

Chandra Kant Mishra
IIT Madras

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Outline

- Gravitational Waves in theories of gravity and its properties
- GW detection principle — Freely falling frames, Tidal forces
- Detection methods— Chirps, Matched filtering

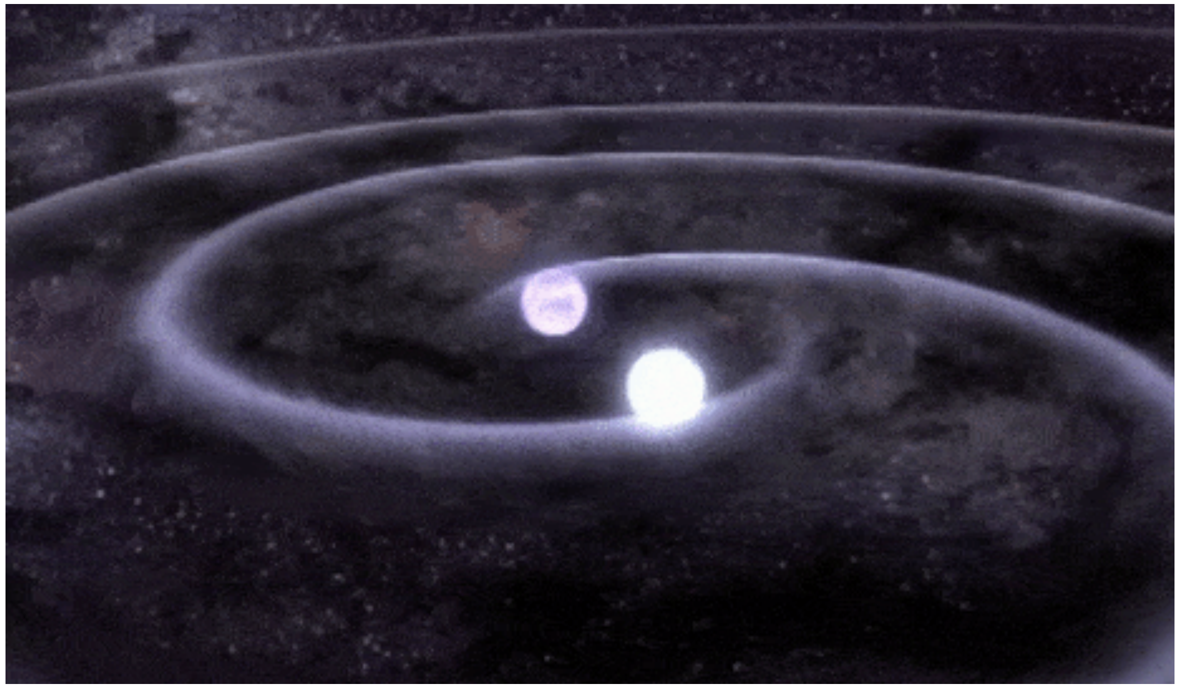
Newtonian Gravity

Universal Law of Gravitation

$$\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}}$$

Gravitational Waves are simply …

- Propagating gravitational fields.. (similar to EM waves which are propagating EM fields)
- Produced by acceleration of masses (EM waves produced due to accelerating charges)
- Transverse in nature (so are EM waves)
- Has two states of polarisations (similar to EM waves)
- Interacts very weakly with intervening matter (unlike EM waves)



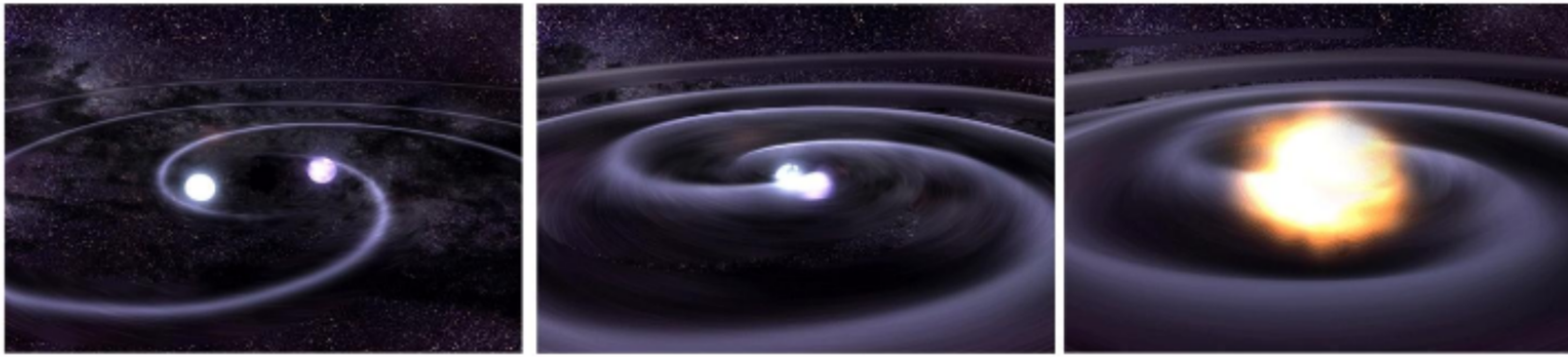
GWs are quadrupolar, EM waves are dipolar

Slide : **K G Arun**

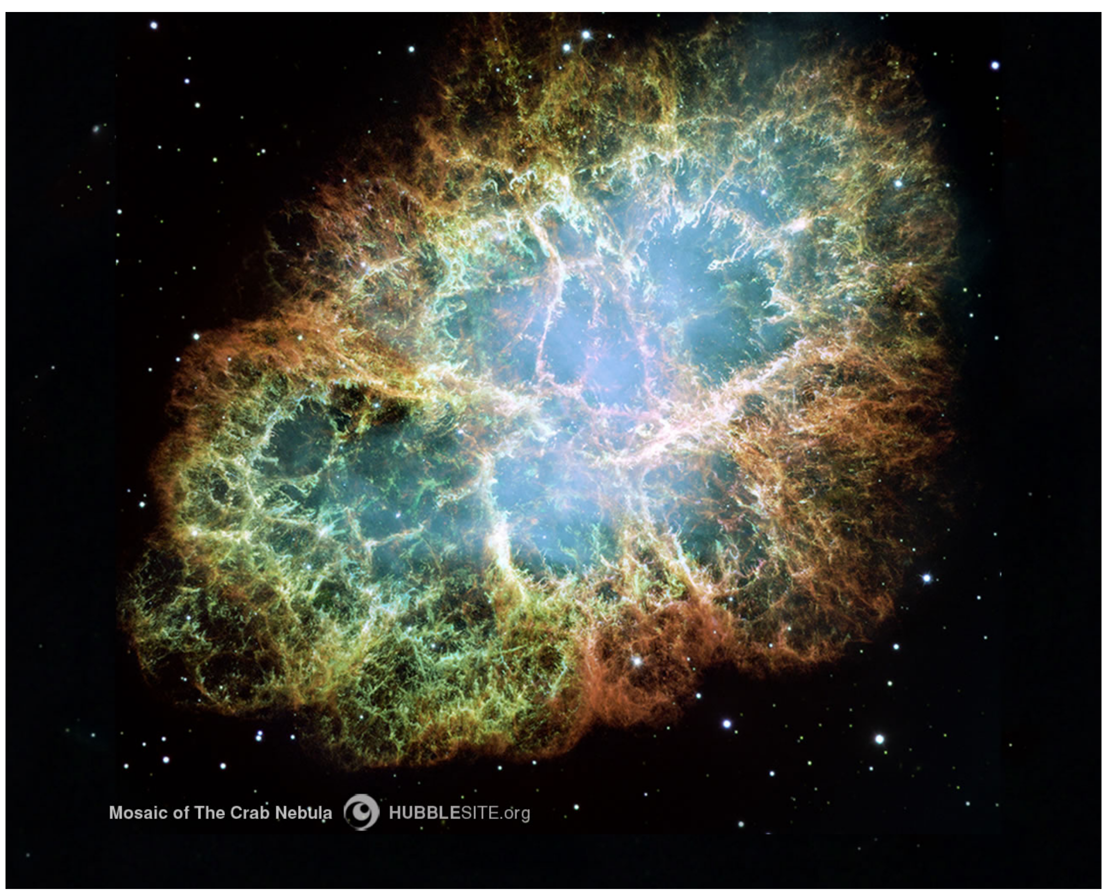
Generation of GWs

- Conservation of “mass” — No monopole radiation
- And the dipole radiation is forbidden due to “conservation of linear and angular momentum”.
- However, all (time-dependent) non-spherical motions according to General Relativity should produce these waves.

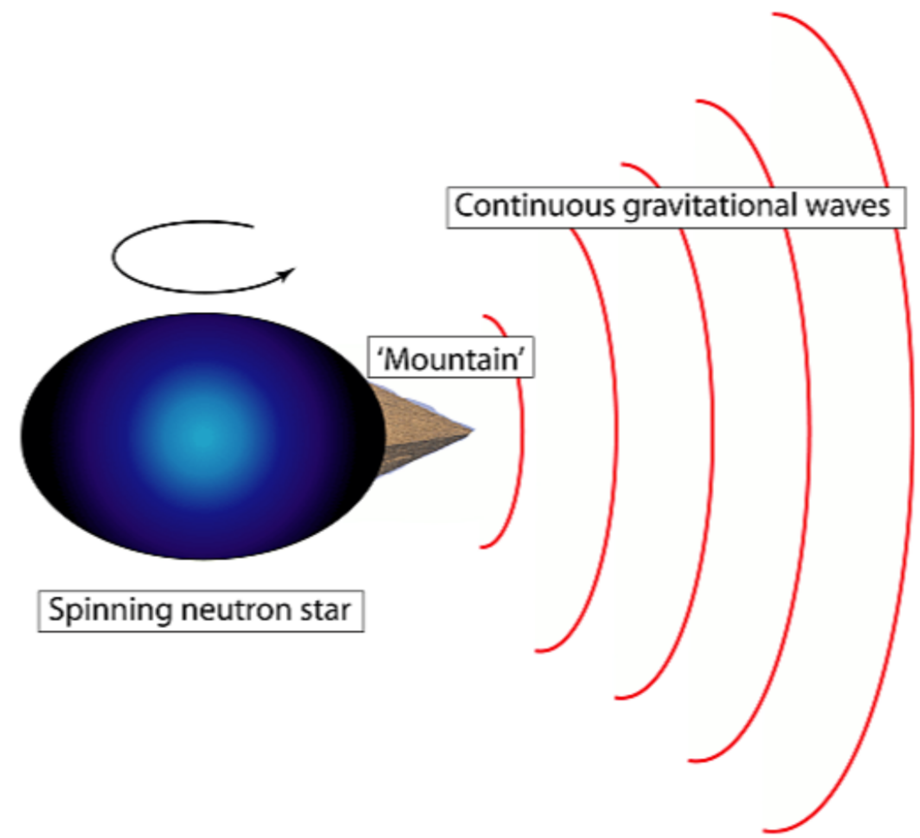
Time varying fields



source:ligo.org



Source: HUBBLESITE.org

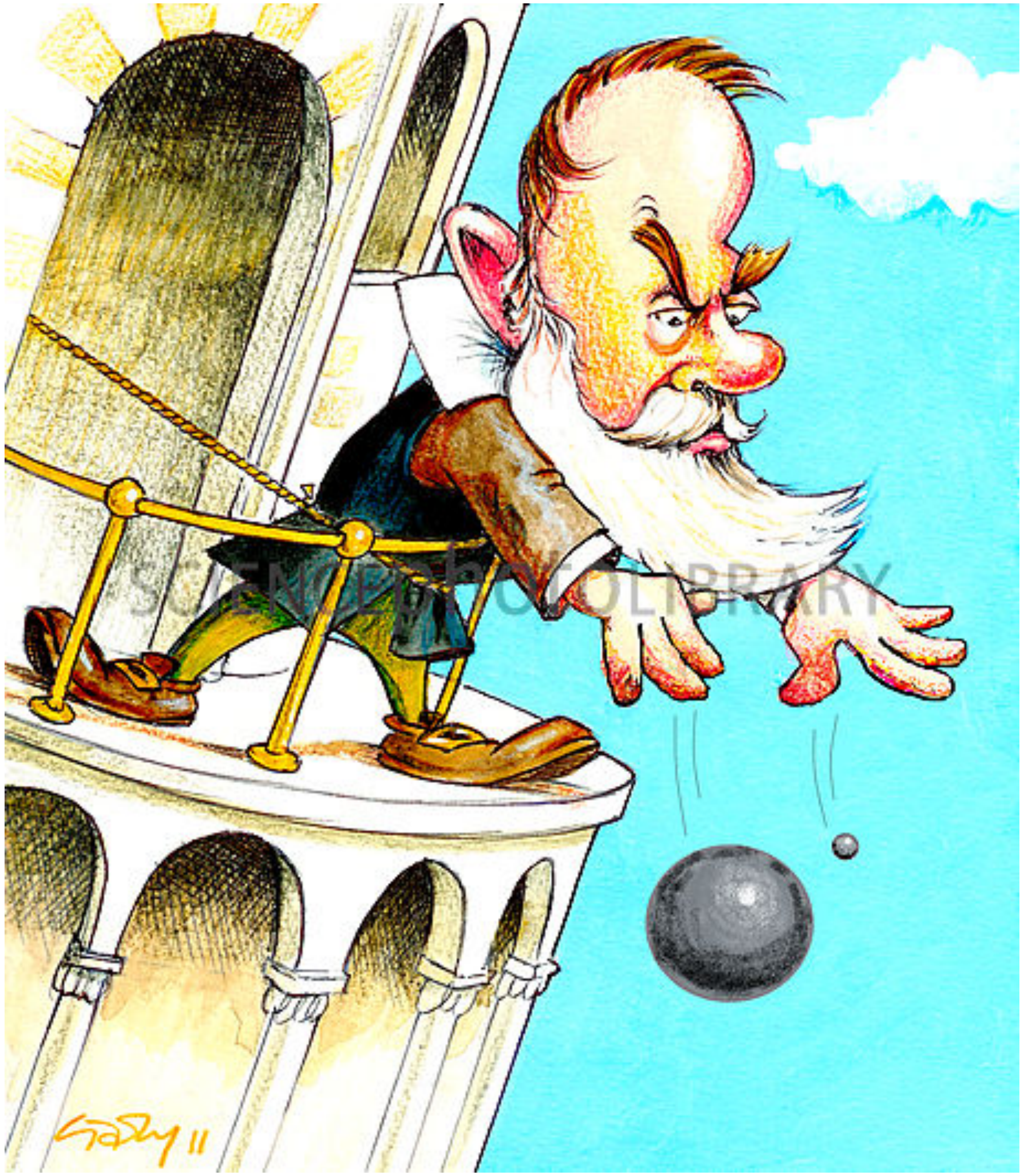


Credit: ANU Centre for Gravitational Physics

“Apples” down the Pisa tower

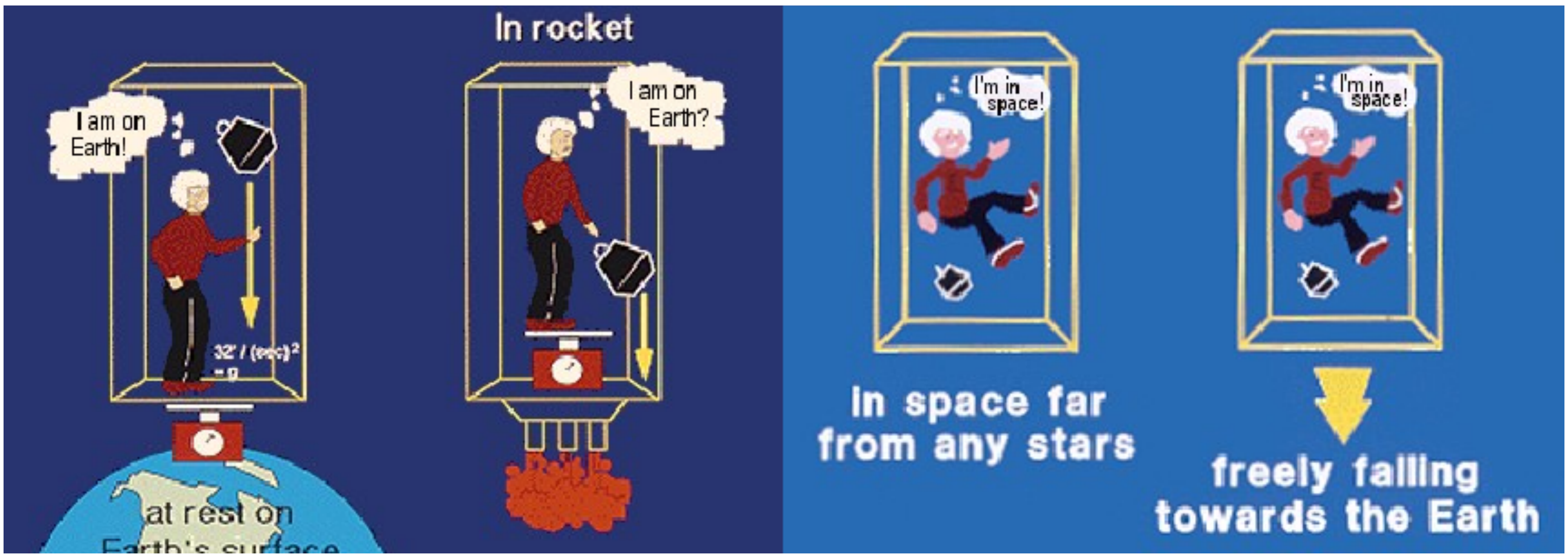
Gravitational acceleration

$$\mathbf{a} = \mathbf{F}/m = -\frac{GM}{r^2}\hat{\mathbf{r}}$$



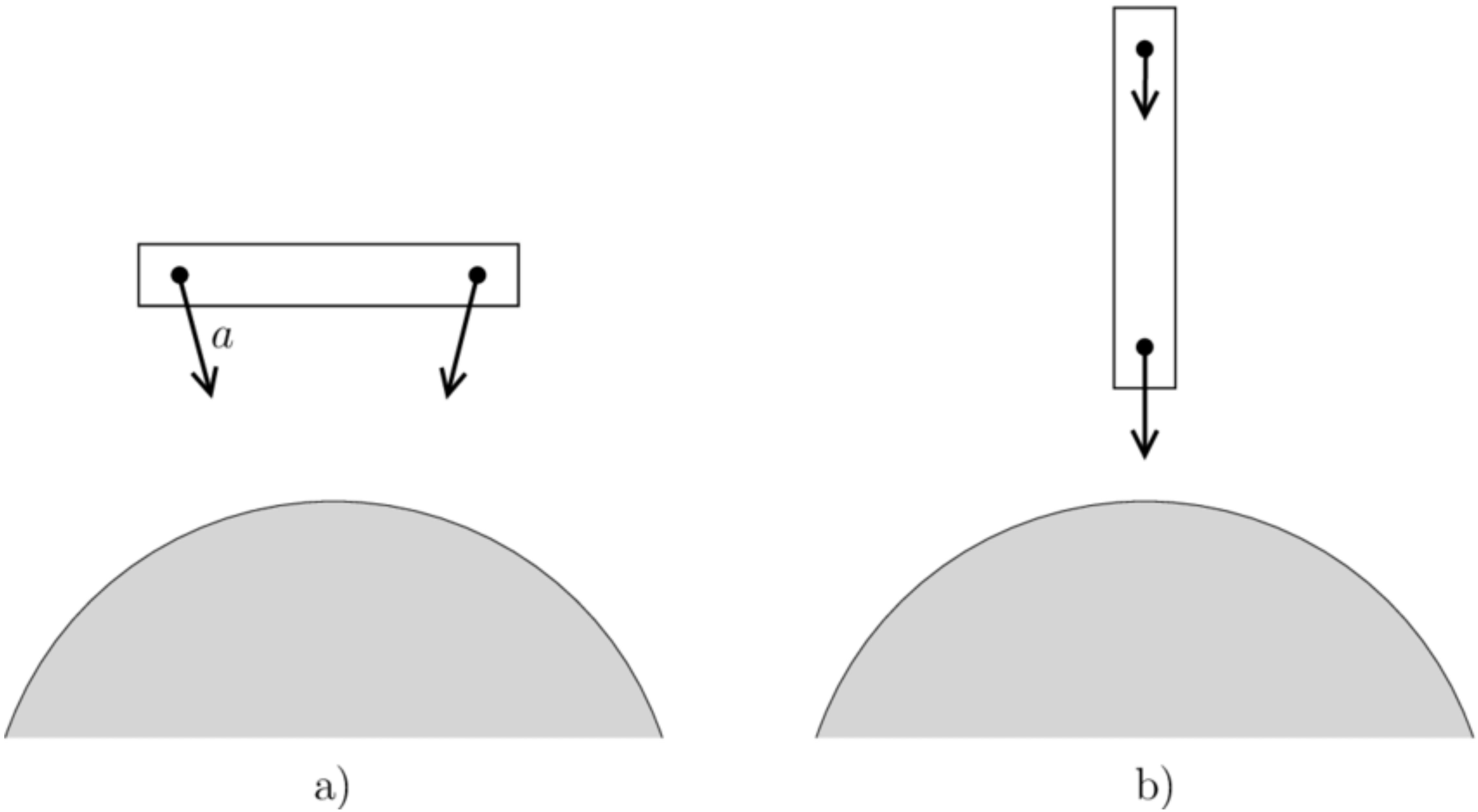
Credit: GARY BROWN/SCIENCE PHOTO LIBRARY

Equivalence principle

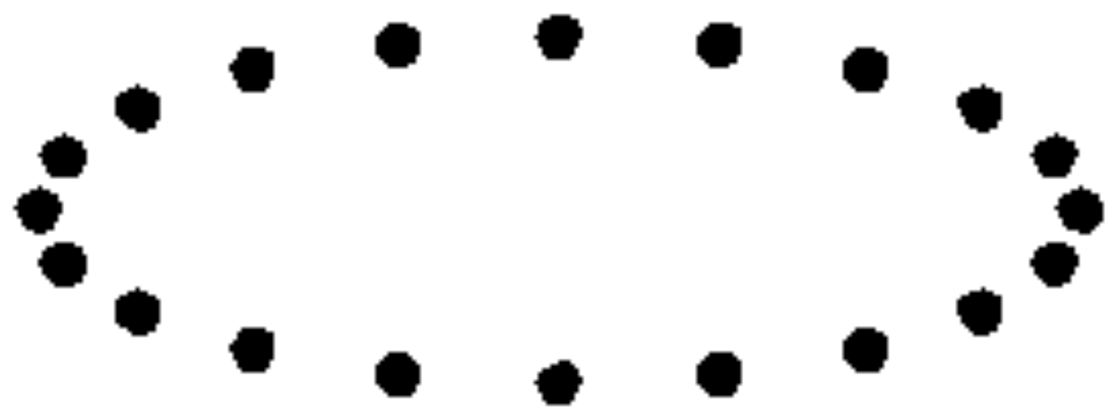


Source: Time Travel Research Center

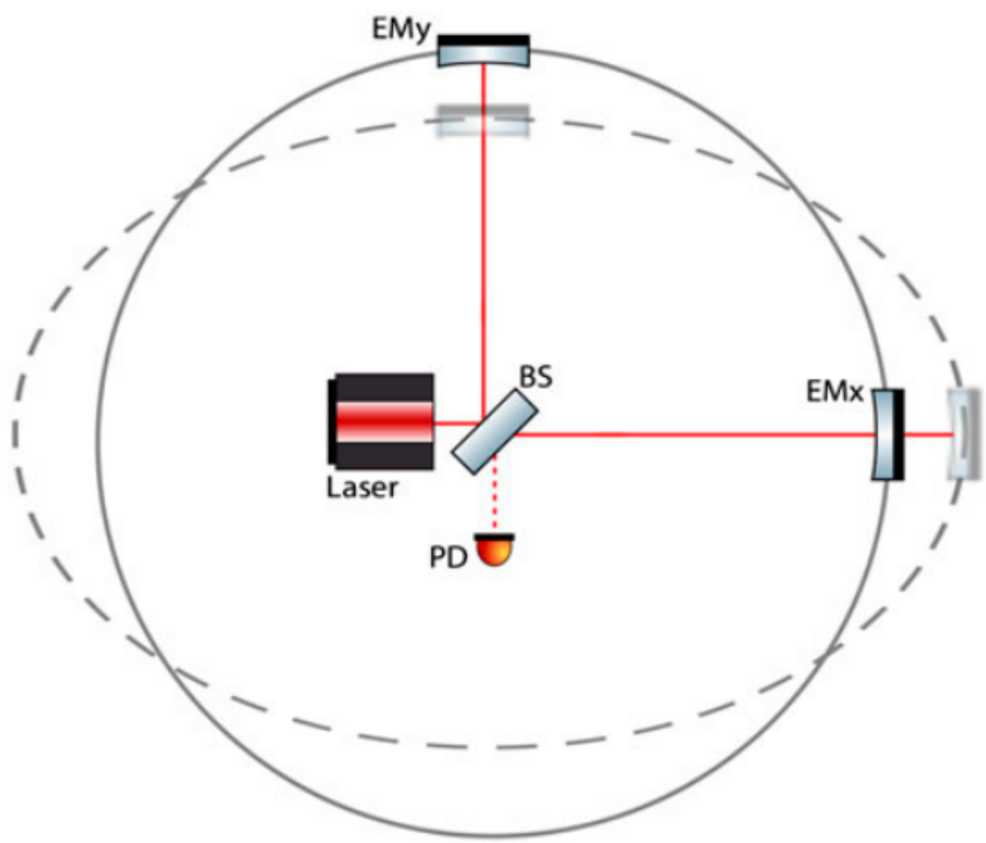
Tidal forces



Detection principle

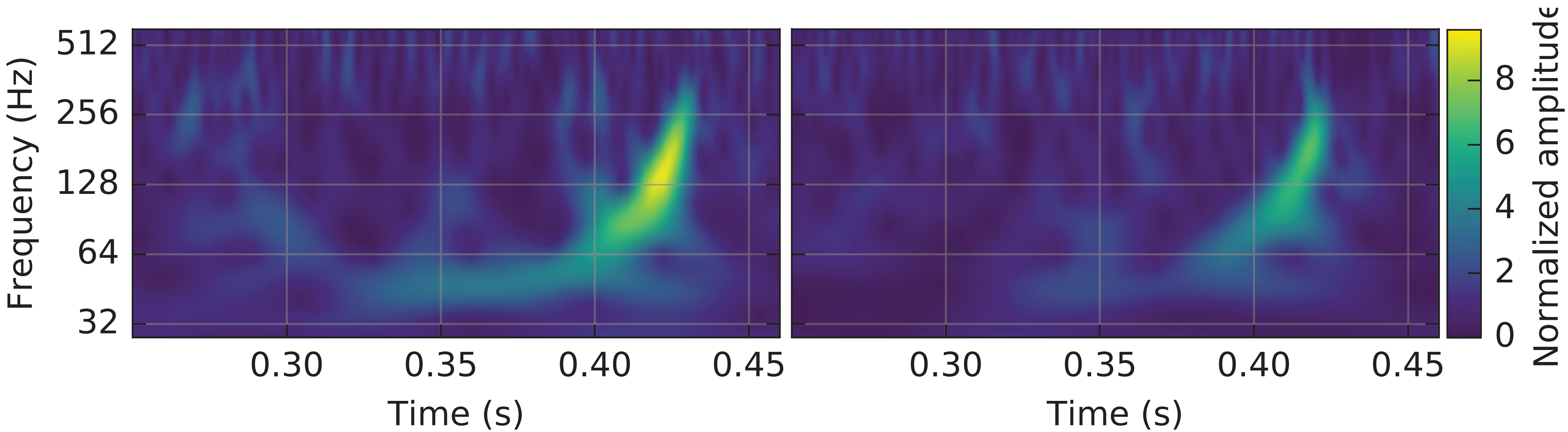


Effect of Gravitational Waves



$$h = \frac{2\Delta l}{l}$$

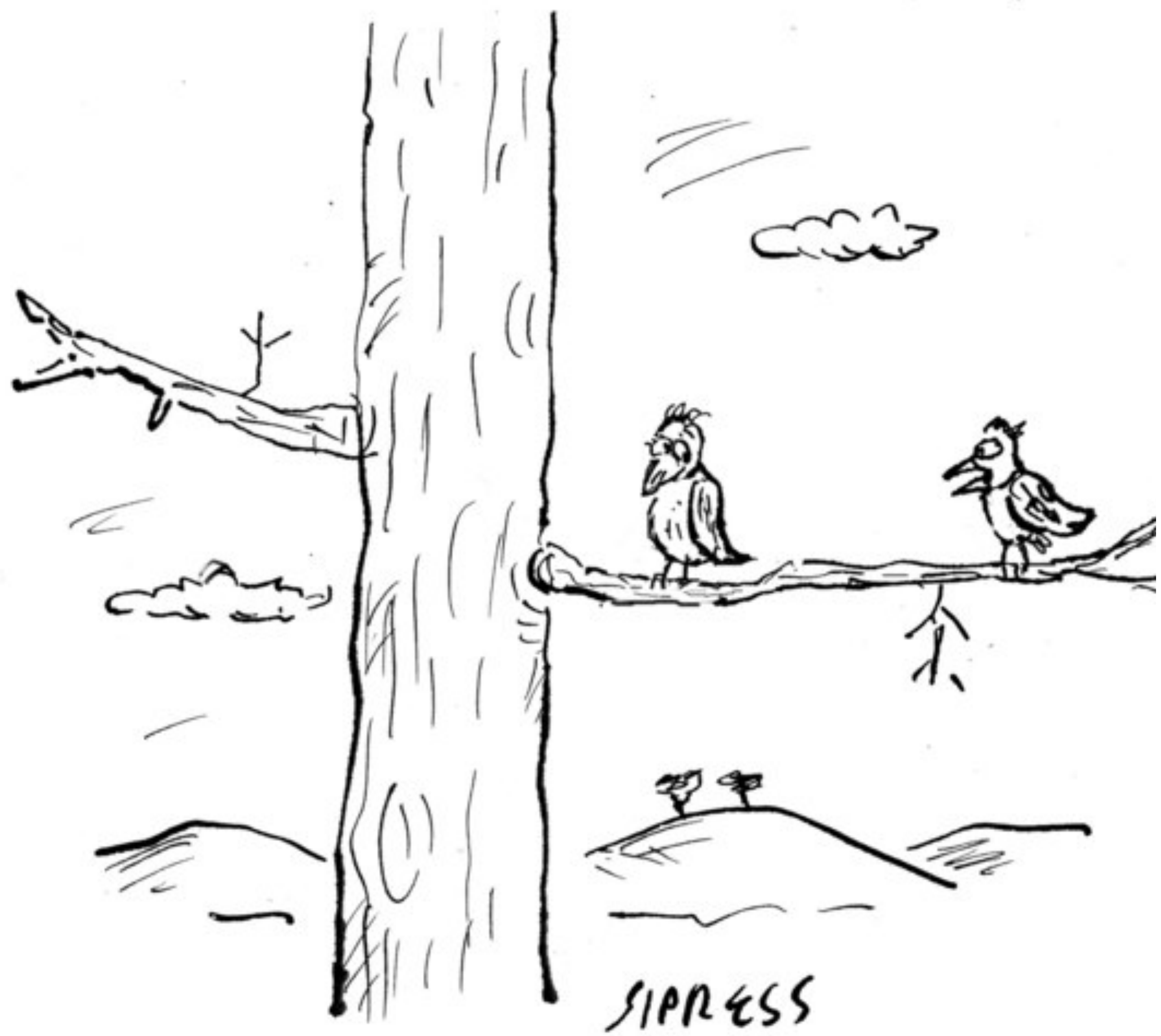
Chirps



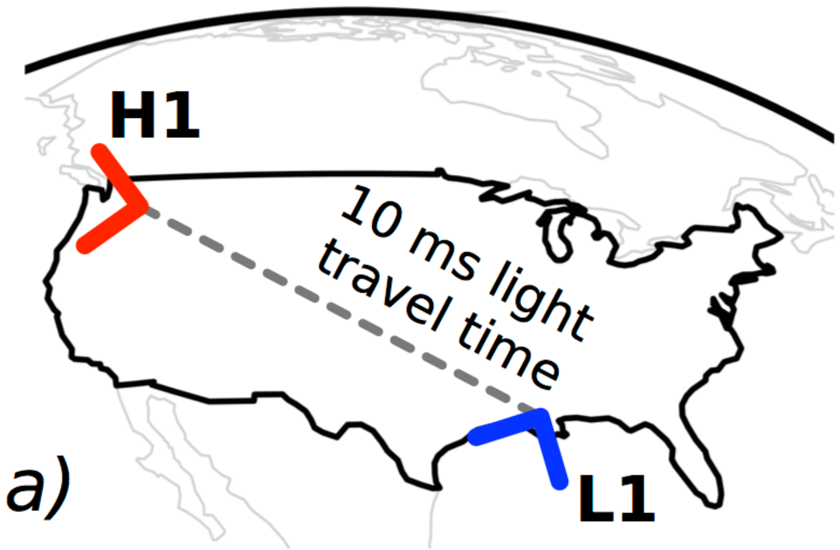
Video Credit: UffcomINFN

DAILY CARTOON: FRIDAY, FEBRUARY 12TH

By David Sipress February 12, 2016

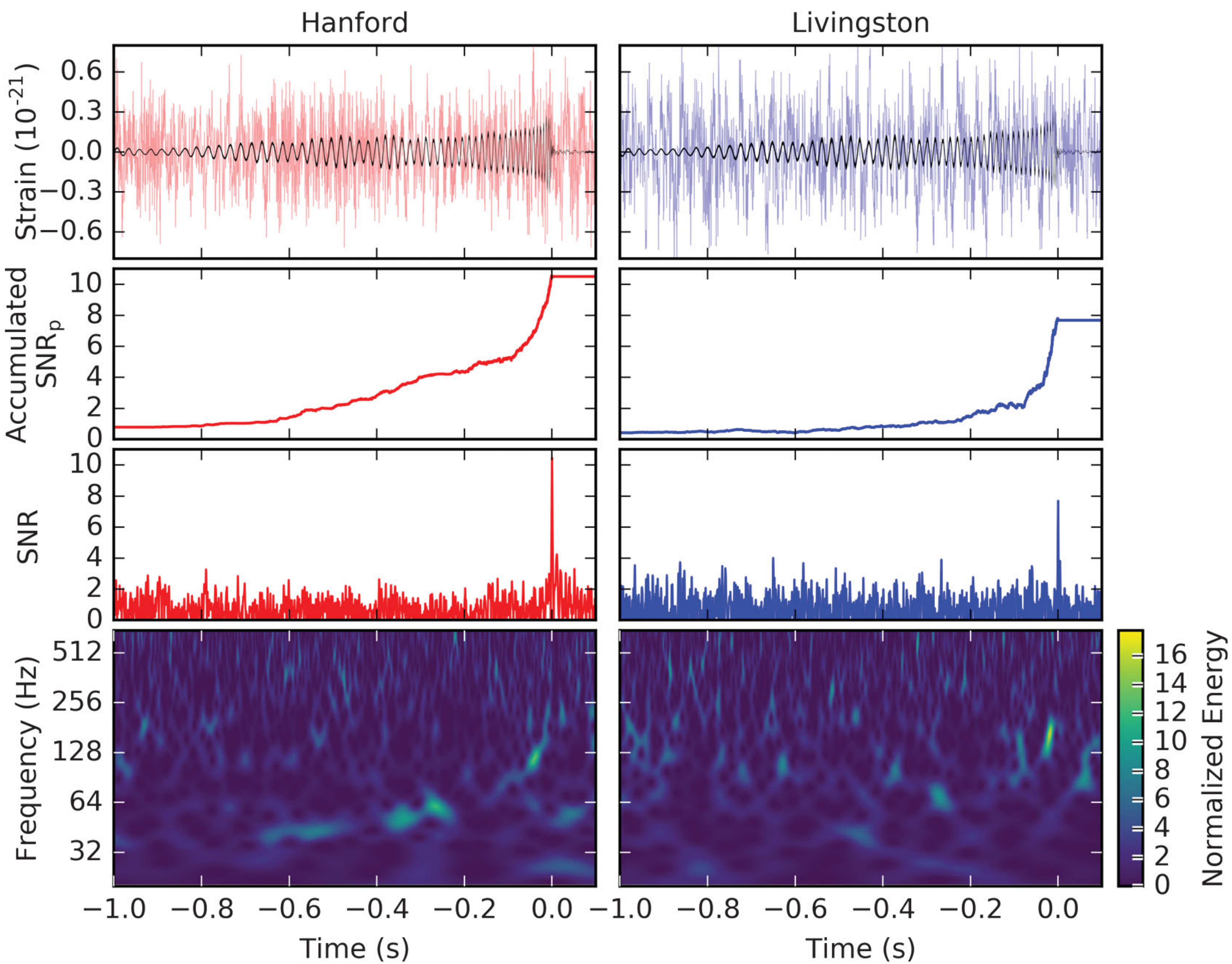


"Was that you I heard just now, or was it two black holes colliding?"



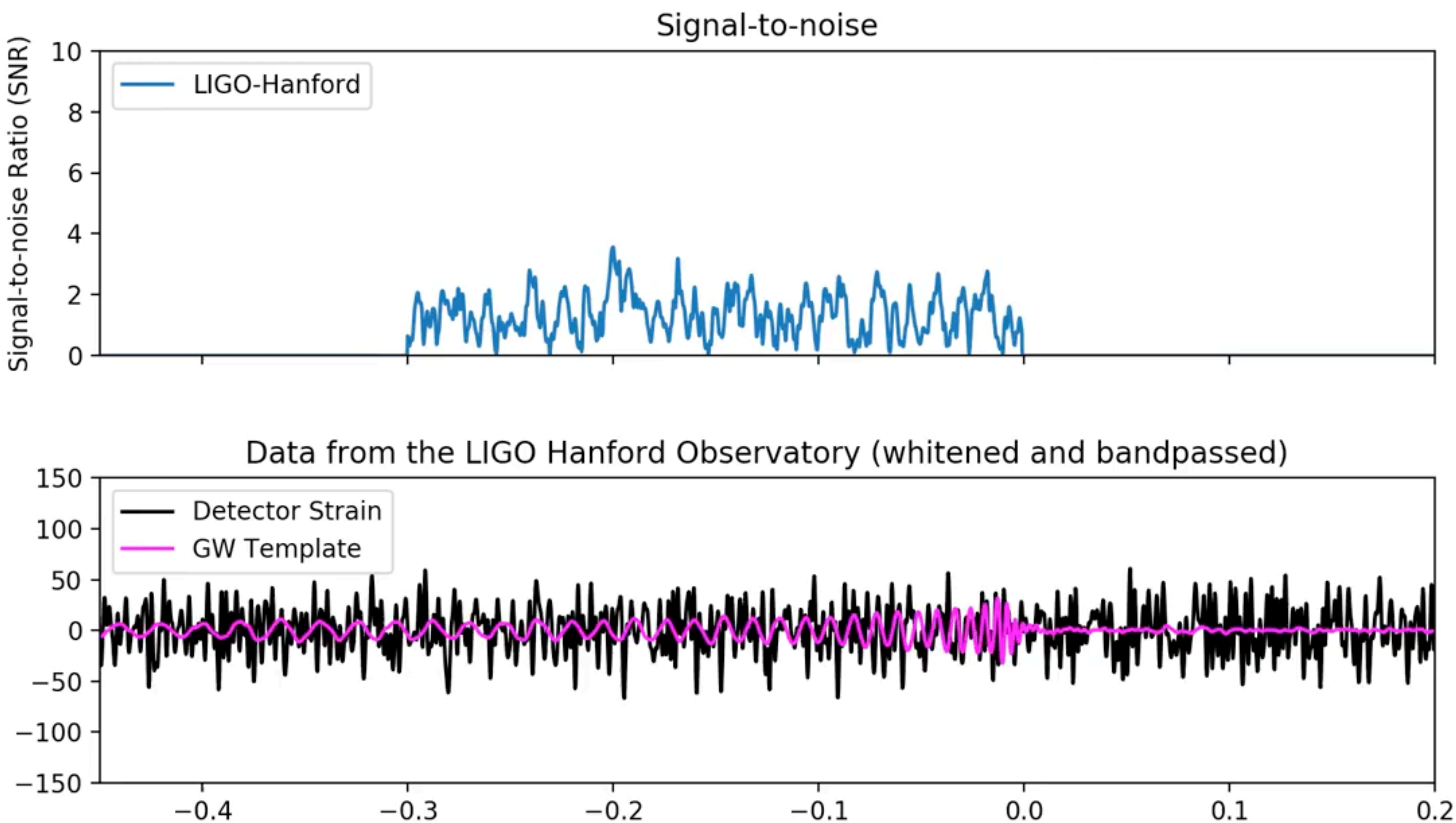
The LIGO Twins

Image Credit: LIGO Caltech

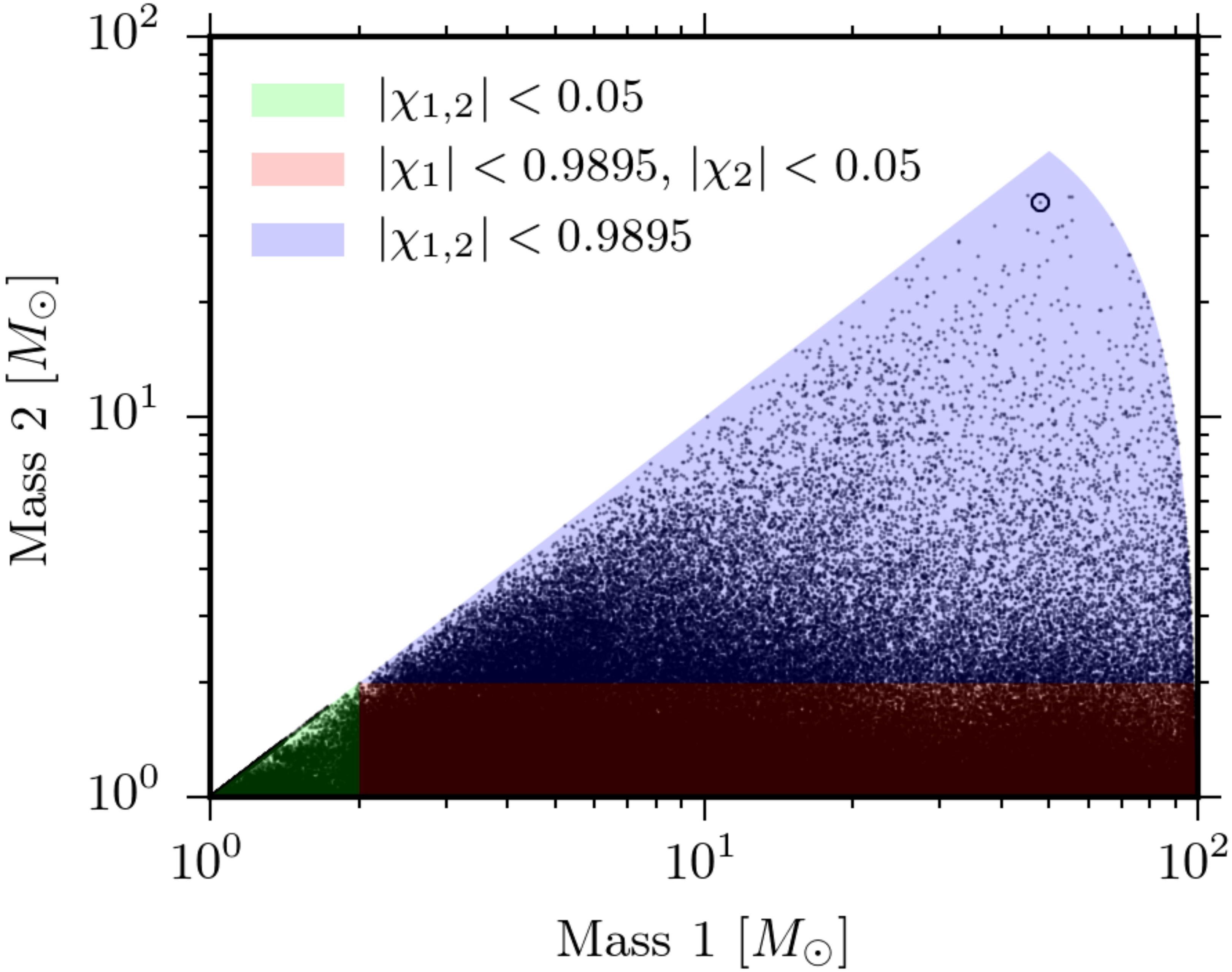


Hanford
Livingston
Strain (10^{-21})
0.6
0.3
0.0
−0.3
−0.6
Accumulated SNR_p
10
8
6
4
2
0
SNR
Frequency (Hz)
512
256
128
64
32
Normalized Energy
16
14
12
10
8
6
4
2
0
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
Time (s)

Matched filtering



Template Bank



250,000 waveforms

[LVC, PRD 96, 122003 (2016)]

BBH Waveforms

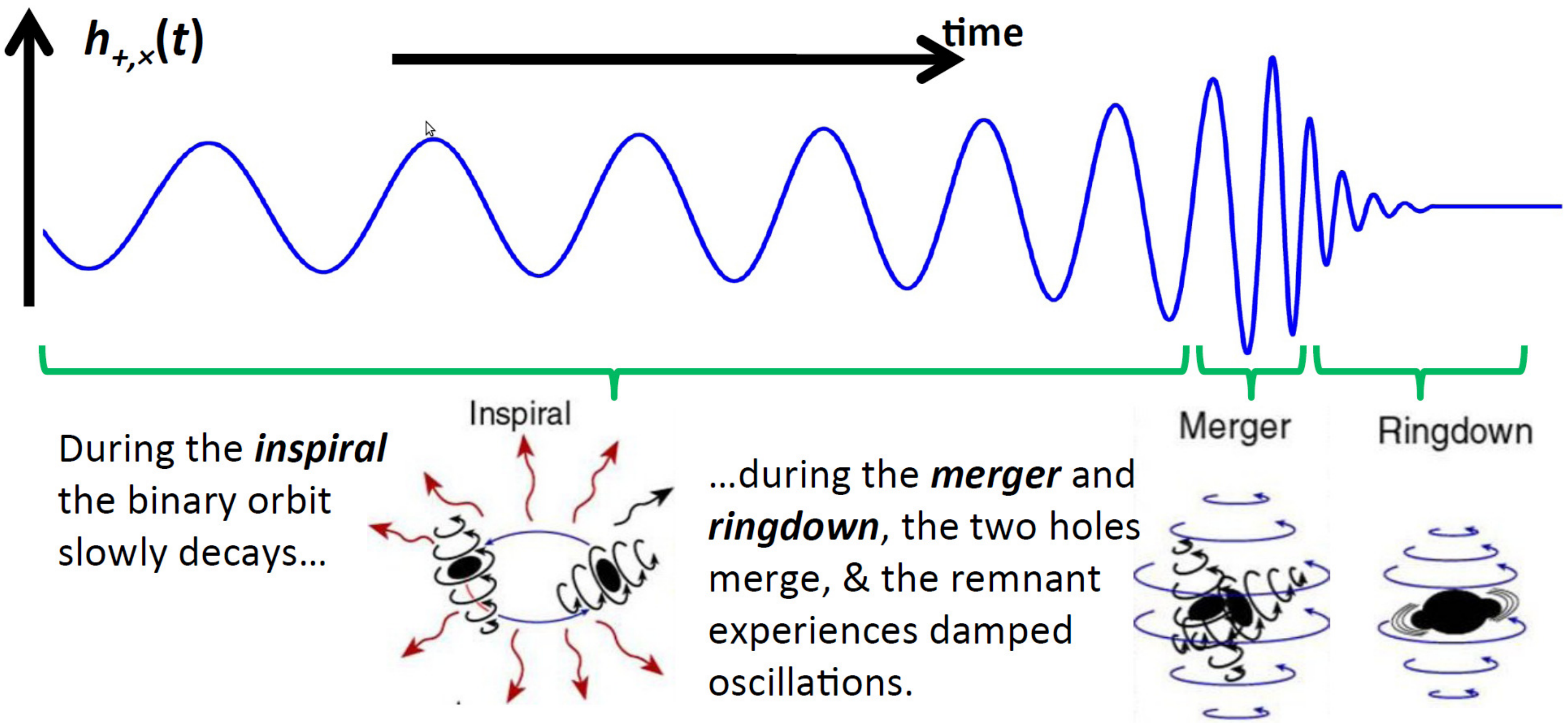
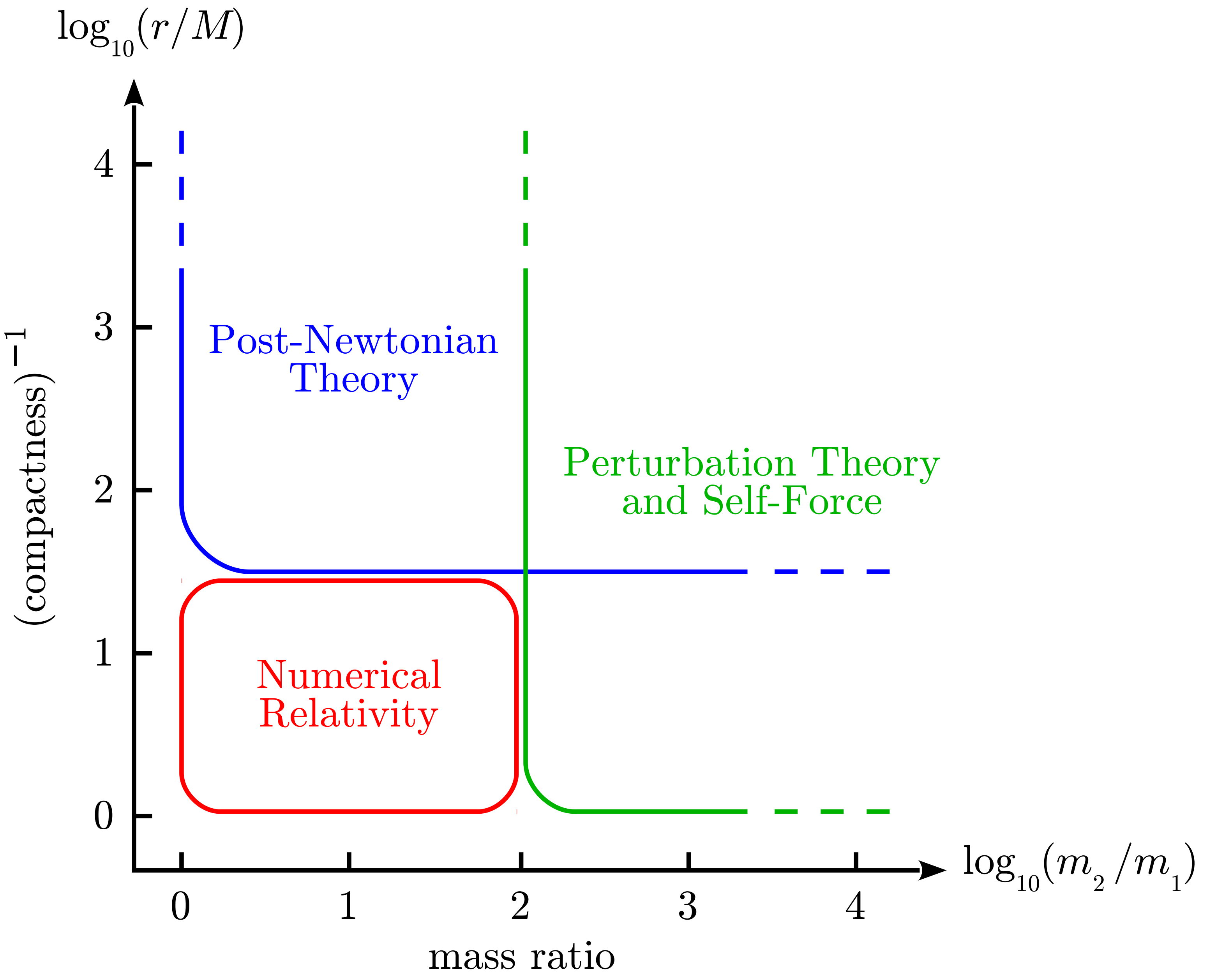


Image Credit: Kip Thorne

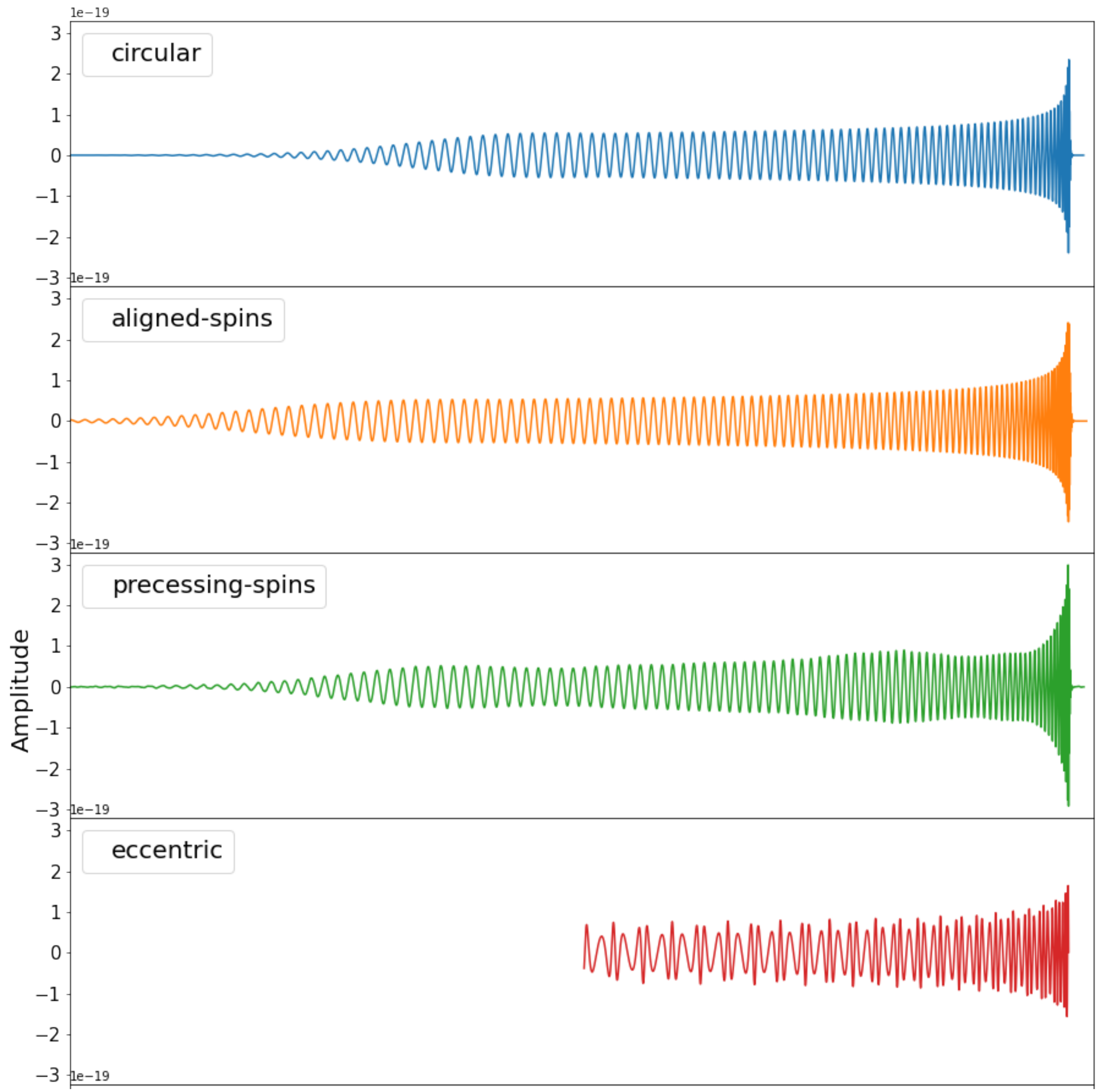
Approximation schemes



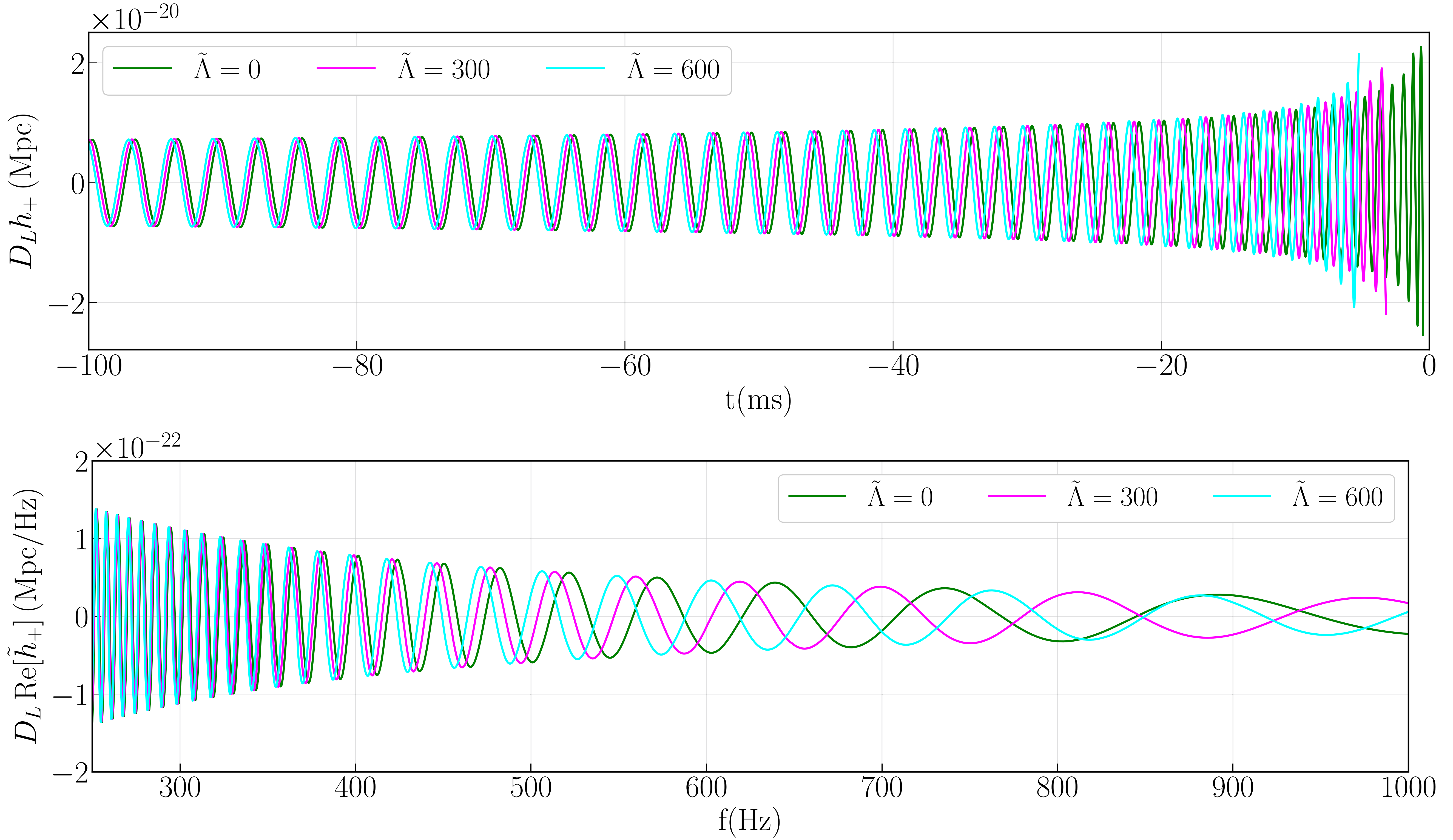
Extracting source properties

- In general a compact binary is fully characterised by a set of 17 parameters comprising of
  - Component Masses (2)
  - Component spins (6)
  - Binary's distance (1)
  - Binary's location (2)
  - Binary's orientation (2)
  - Orbital eccentricity (1) — if binary is not circular
  - Effect of matter — tidal effects

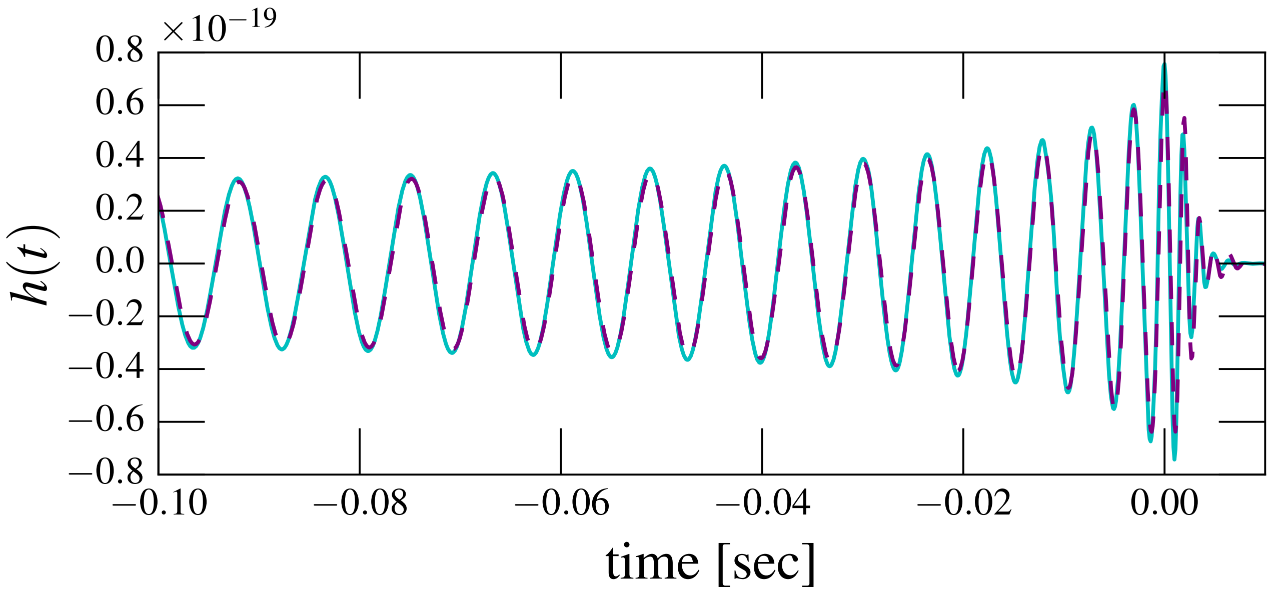
Approximation schemes

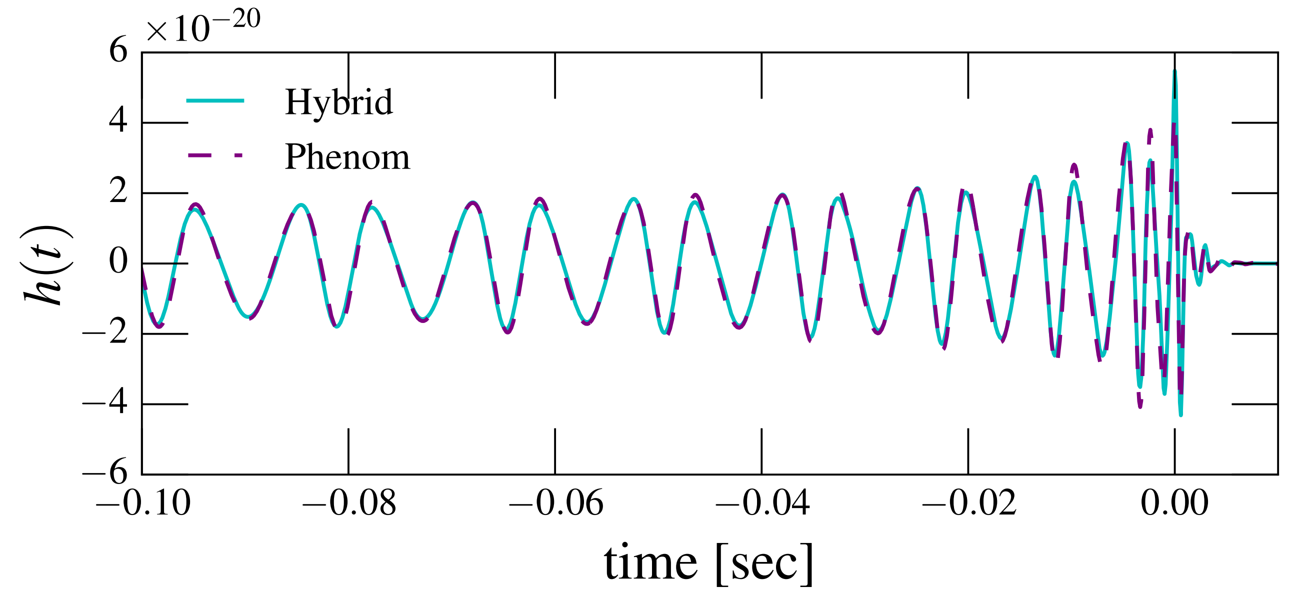


Matter effects



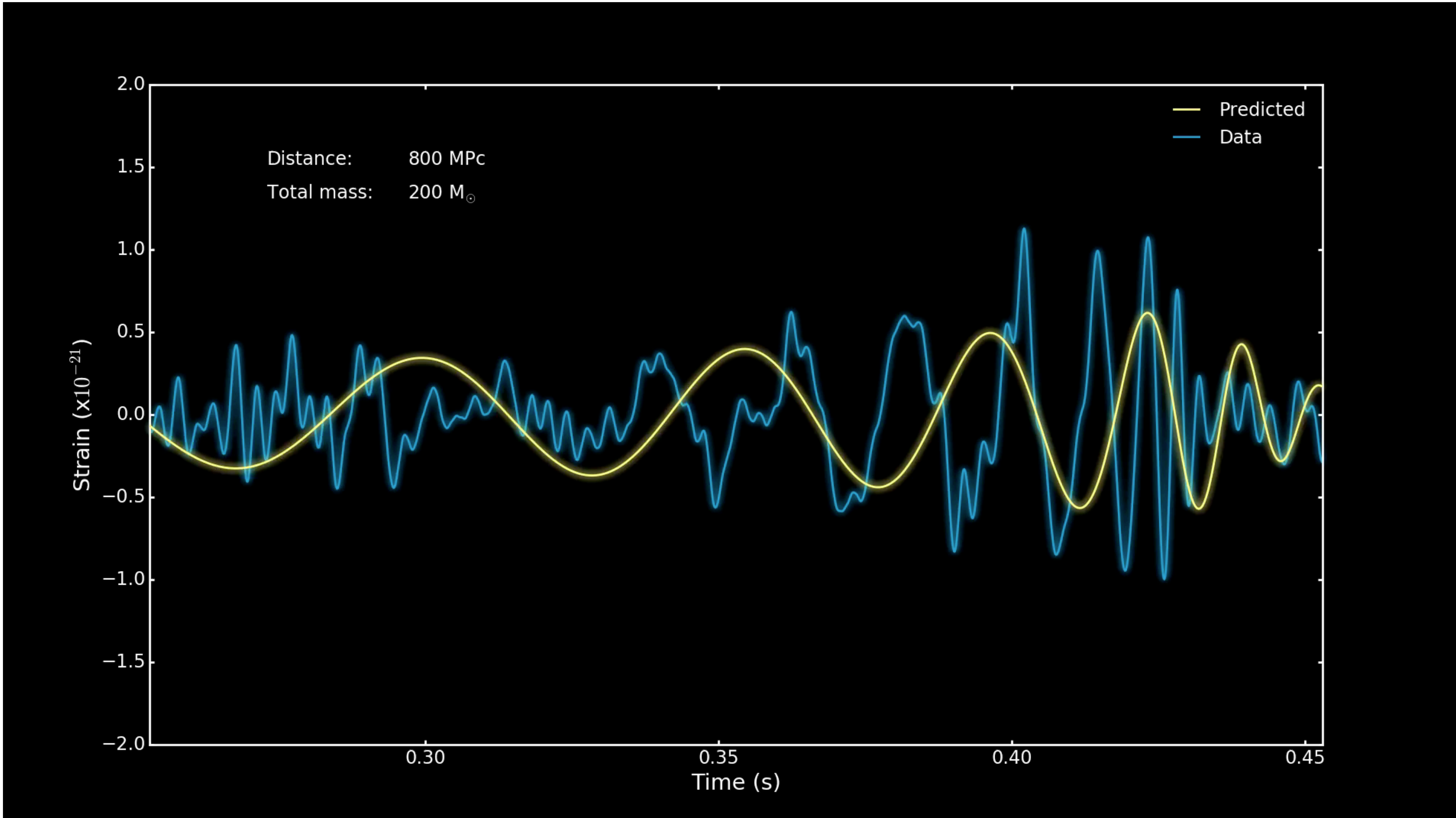
Higher order Modes





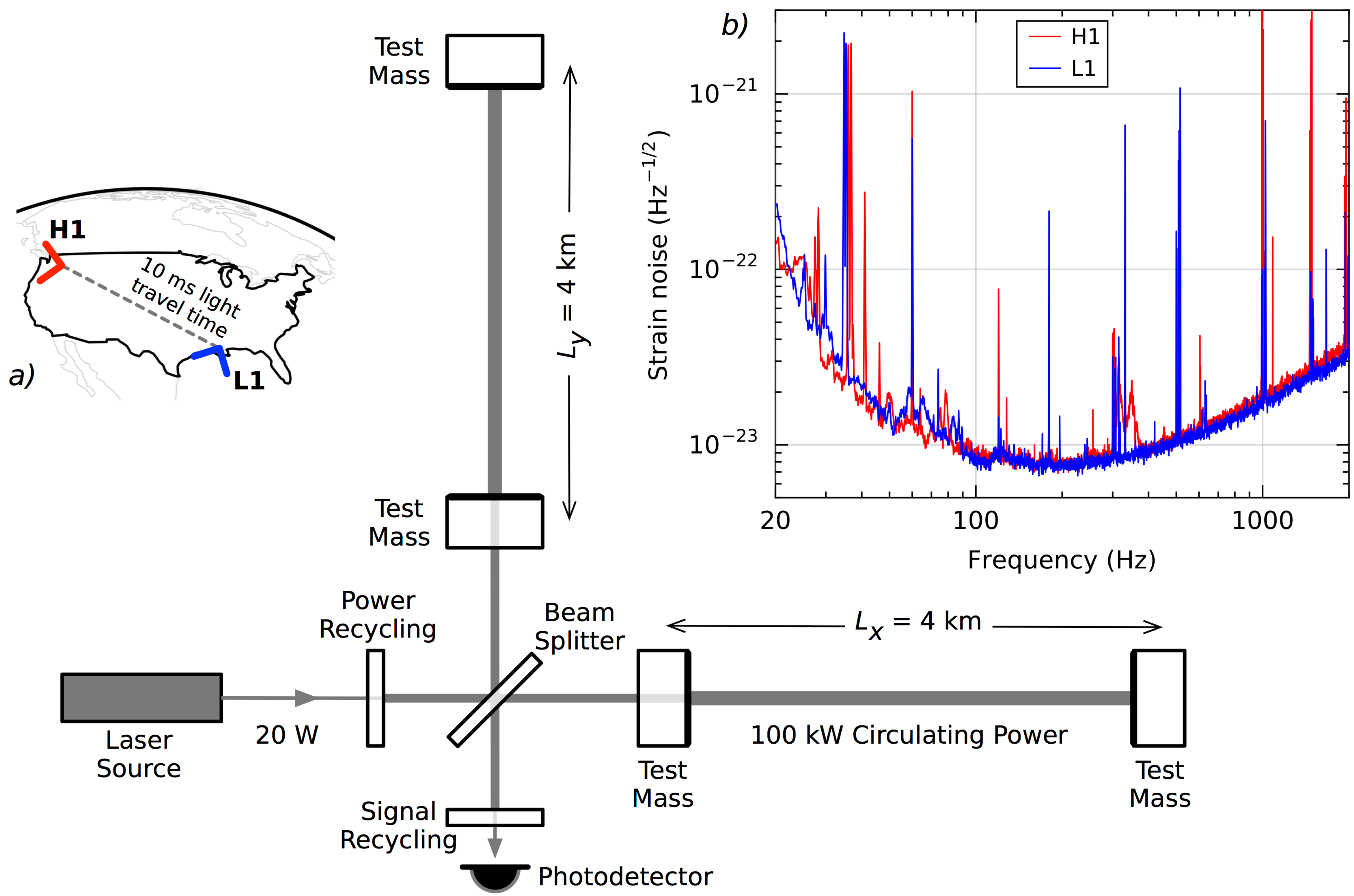
[Mehta et al., PRD 96 124010 (2022)]

Parameter Estimation



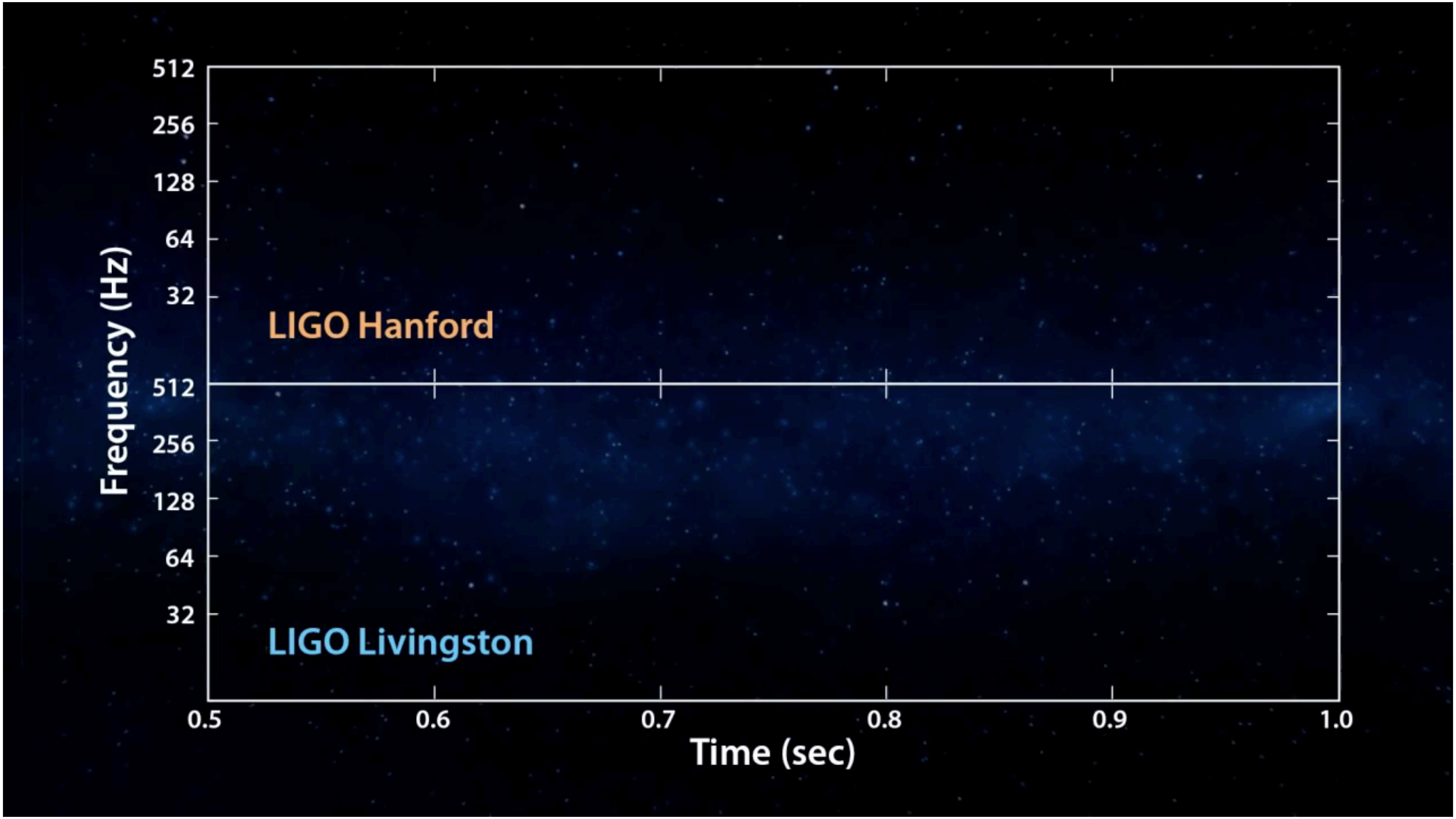
additional slides

LIGO Schematic Diagram



LIGO-Virgo Collaboration, PRL 116, 061102 (2016)

Chirps



Video Credit: UffcomINFN

Linearised Gravity

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

$$+$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \rightarrow \quad \boxed{\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}}$$

$$\nabla^2 - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} = \Box^2 \quad \& \quad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h$$

$$\boxed{\bar{h}_{\mu\nu}(t, \bar{x}) = \frac{4G}{c^4} \int \frac{T_{\mu\nu}(t - |\bar{x} - \bar{x}'|/c, \bar{x}')}{|\bar{x} - \bar{x}'|} d^3 x'}$$

Static weak fields

$$\bar{h}_{\mu\nu}(t,\bar{x}) = \frac{4G}{c^4}\int \frac{T_{\mu\nu}(t-|\bar{x}-\bar{x}'|/c,\bar{x}')}{|\bar{x}-\bar{x}'|}d^3x'$$

$$+$$

$$T^{00} = \rho c^2 \quad \longrightarrow \quad \bar{h}^{00} = \frac{4G}{c^4}\int \frac{\rho(\bar{x}')}{|\bar{x}-\bar{x}'|}d^3x'$$

$$\bar{h}^{00} = -\frac{4\Phi(\bar{x})}{c^2} \quad \text{OR} \quad \boxed{h^{00} = -\frac{2\Phi(\bar{x})}{c^2}}$$

$$\bar{\Phi} = -G\int \frac{\rho(\bar{x}')}{|\bar{x}-\bar{x}'|}d^3x'$$

Quadrupole approximation

EE in Vacuum

$$\Box h_{\mu\nu} = 0$$

Fourier Mode in z-dir

$$h_{\mu\nu} = A_{\mu\nu} e^{\mathrm{i}k(t-z)}$$

Non-zero components

$$A_{11} = -A_{22} = A_+ \text{ and } A_{12} = A_{21} = A_\times$$

$$A_{\mu\nu} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & A_+ & A_\times & 0 \\ 0 & A_\times & -A_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Effect on matter

Geodesic Deviation Equation

$$\frac{D^2(\delta x^\lambda)}{D\tau^2} = R^\lambda_{\mu\nu\kappa}\left(\frac{dx^\mu}{d\tau}\right)\left(\frac{dx^\nu}{d\tau}\right)\delta x^\kappa$$

For weak fields

$$\frac{d^2}{dt^2}(\delta x^i) = -R^i_{0j0}\delta x^j \quad \longrightarrow \quad \frac{d^2}{dt^2}(\delta x_i) = \frac{1}{2}\frac{\partial^2 h_{ij}}{\partial t^2}\delta x^j$$

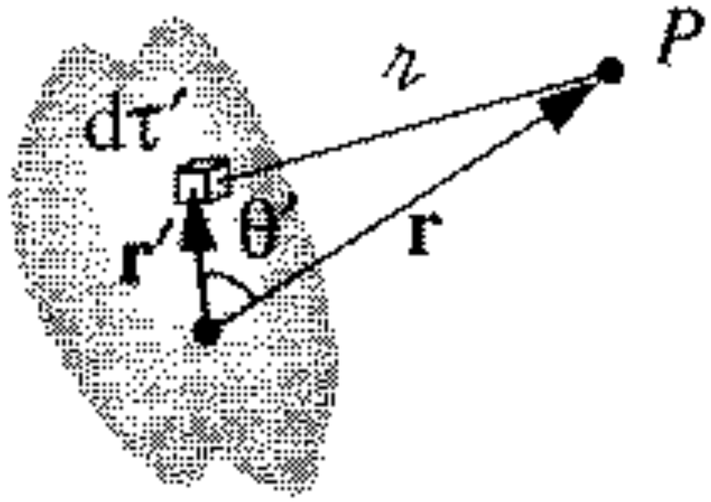
Solution

$$\delta x_i = \delta x_{i,0} + \frac{1}{2}h_{ij}\delta x^j$$

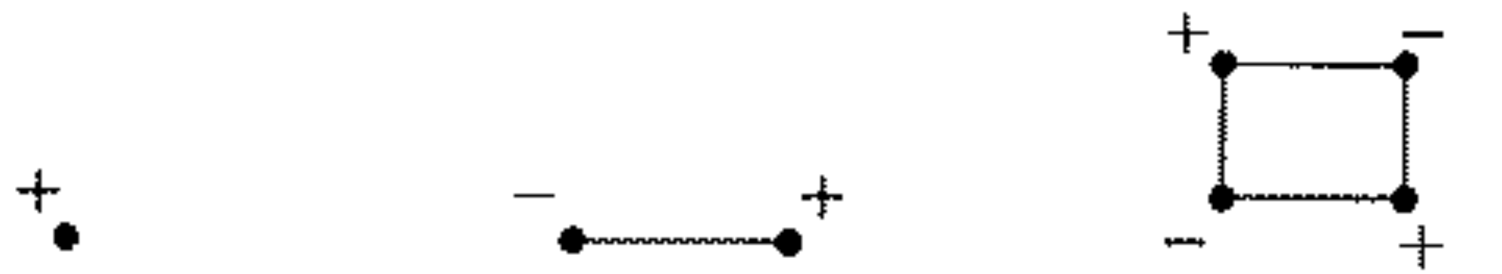
It is easy to see for the polarisation mode with amplitude (A_+) we get

$$\begin{aligned}\delta x_1 &= \delta x_{1,0} + \frac{1}{2}A_+ e^{ikt}\delta x^j \\ \delta x_2 &= \delta x_{2,0} - \frac{1}{2}A_+ e^{ikt}\delta x^j\end{aligned}$$

Multipole Expansion



$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}}\rho(\mathbf{r}')\,d\tau'.$$



Monopole
($V \sim 1/r$)

Dipole
($V \sim 1/r^2$)

Quadrupole
($V \sim 1/r^3$)

$$\begin{aligned} V(\mathbf{r}) \;&=\; \frac{1}{4\pi\epsilon_0}\left[\frac{1}{r}\int \rho(\mathbf{r}')\,d\tau' + \frac{1}{r^2}\int r'\cos\theta'\rho(\mathbf{r}')\,d\tau' \right. \\ &\left. + \frac{1}{r^3}\int (r')^2\left(\frac{3}{2}\cos^2\theta' - \frac{1}{2}\right)\rho(\mathbf{r}')\,d\tau' + \ldots\right] \end{aligned}$$

Monopole and dipole radiation

Monopole

$$M = \int \rho dx^3$$

Dipole

$$d_i = \int \rho x_i d^3 x,$$

$$\dot{d}_i = \int \rho v_i d^3 x$$

Quadrupole approximation

Quadrupole

$$Q_{jk} = \int \rho x_j x_k d^3x.$$

Quadrupole Formula

$$h_{jk} = \frac{2G}{rc^4} \frac{d^2 Q_{jk}}{dt^2}$$