Physics of Interferometric Gravitational Wave Detectors

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California Institute of Technology, USA

International Conference on Gravitation and Cosmology
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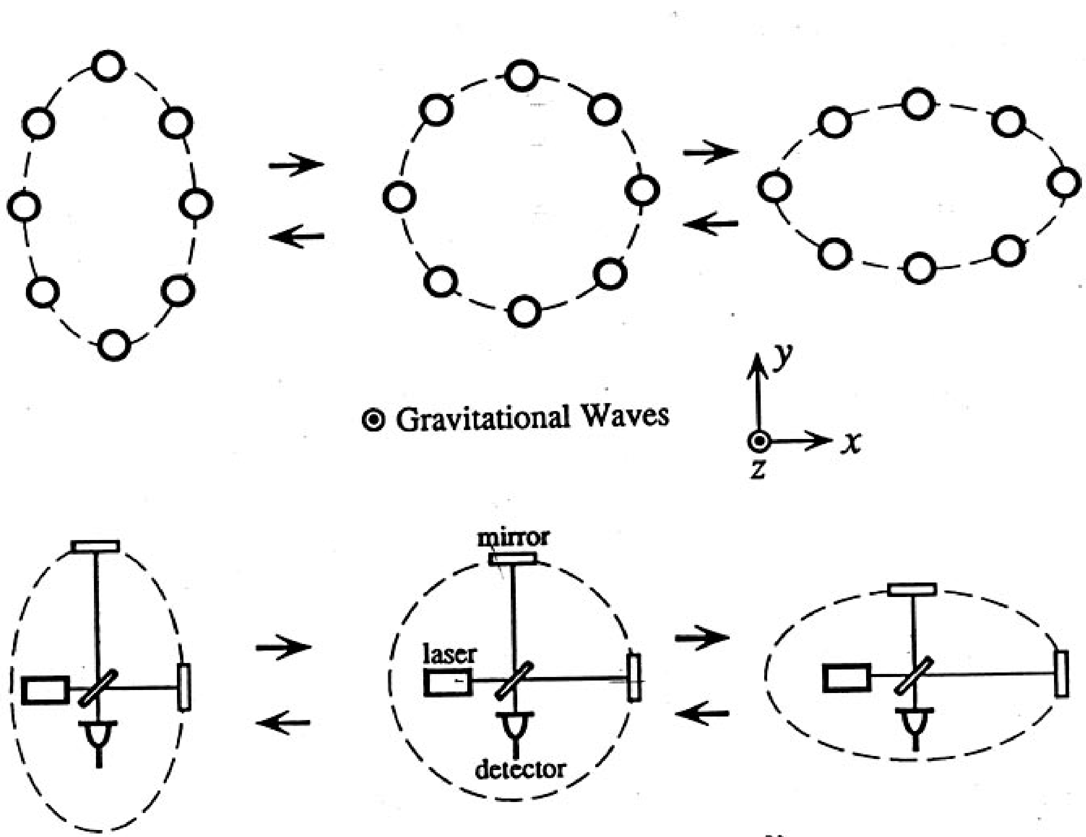
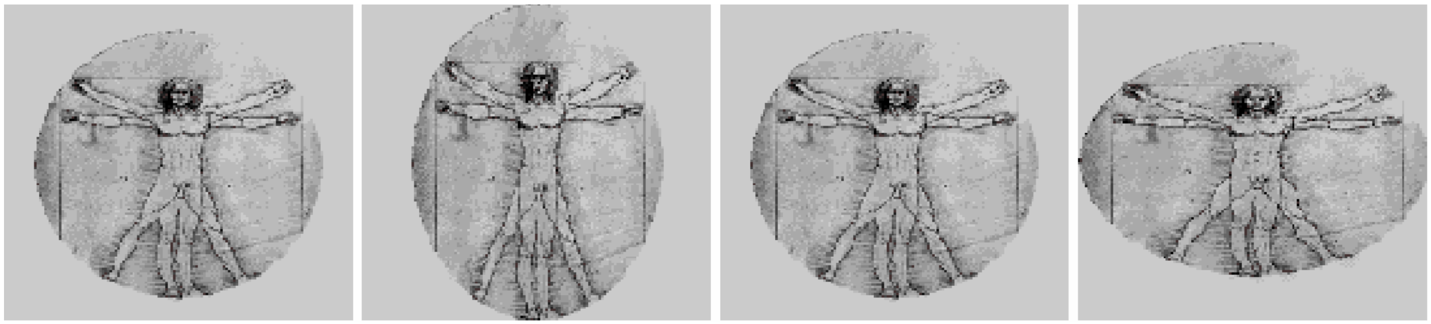
Interferometer as Gravitational wave detector



Gravitational Waves

the effect



Leonardo da Vinci's Vitruvian man

stretch and squash in perpendicular directions at the frequency of the gravitational waves

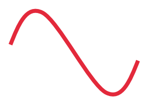
The effect is greatly exaggerated!!

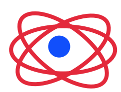
If the man were 4.5 light years high, he would grow by only a 'hairs width' LIGO (4 km), stretch (squash) = 10^{-18} m will be detected at frequencies of 10 Hz to 10^4 Hz. It can detect waves from a distance of 60 10^6 light years

How Small is 10^{-18} Meter?

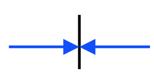
One meter

÷10,000  *Human hair, about 100 microns*

÷100  *Wavelength of light, about 1 micron*

÷10,000  *Atomic diameter, 10^{-10} meter*

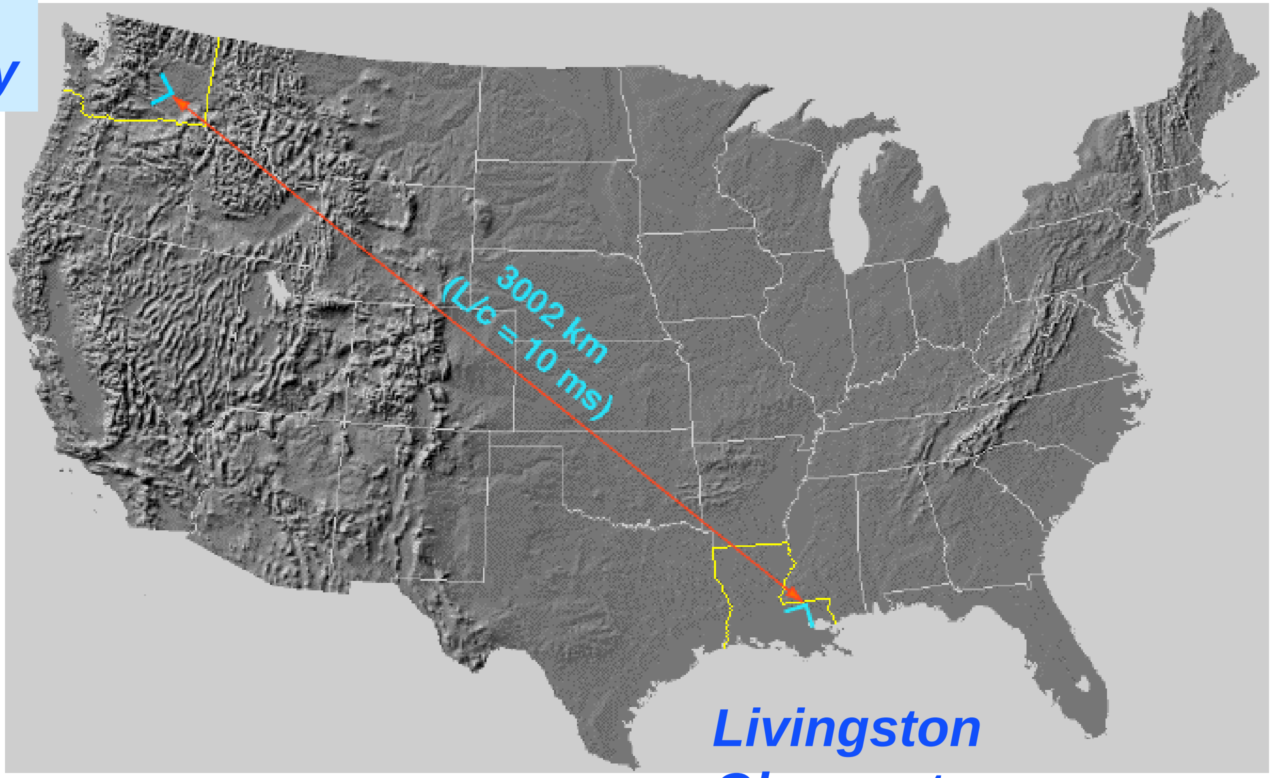
÷100,000  *Nuclear diameter, 10^{-15} meter*

÷1,000  *LIGO sensitivity, 10^{-18} meter*

The Laboratory Sites

Laser Interferometer Gravitational-wave Observatory (LIGO)

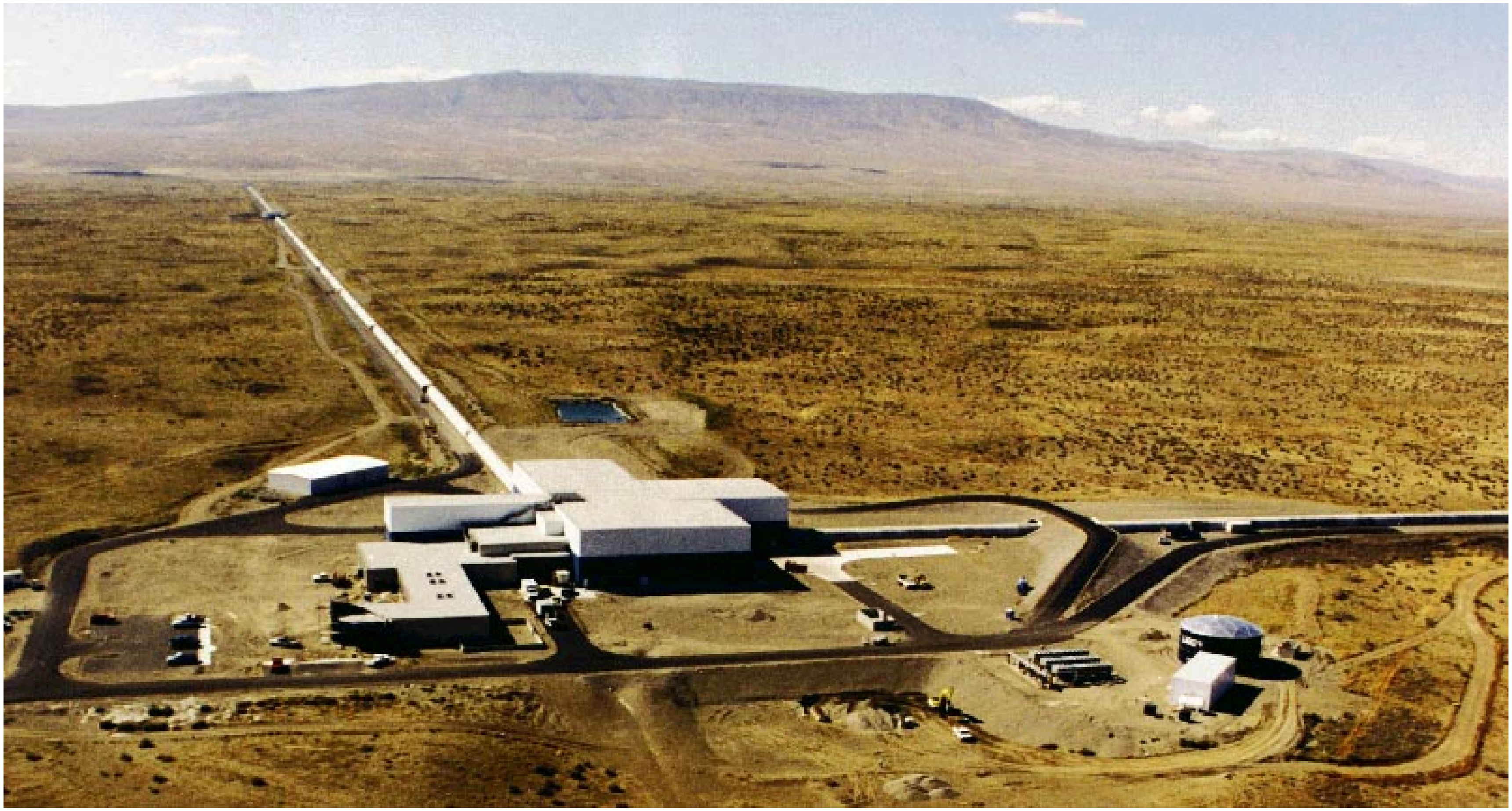
Hanford Observatory



Livingston Observatory

LIGO
Livingston Observatory

LIGO
Hanford Observatory



Outline of Talk

- Introduction to LIGO I configuration
- Length Control & Lock acquisition
- Misalignment & Wave-Front sensor
- Thermal Lensing & compensation
- Advanced LIGO features

LIGO Interferometer Optical Scheme

•Michelson interferometer with Fabry-Perot arm cavities

•Arm cavity storage time

$$\tau \sim 1/2\pi f_{GW}$$

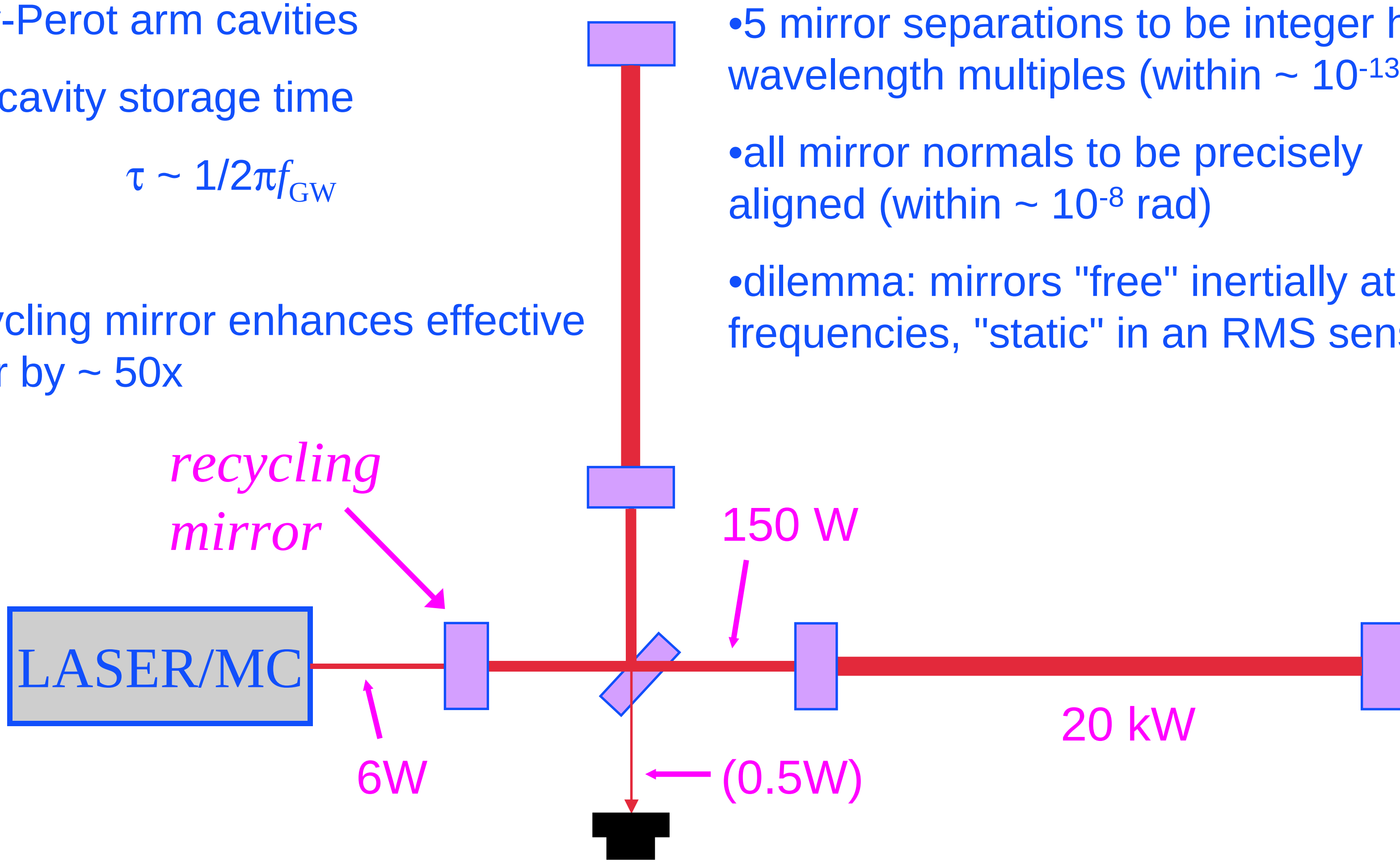
•Recycling mirror enhances effective power by ~ 50x

Price to pay:

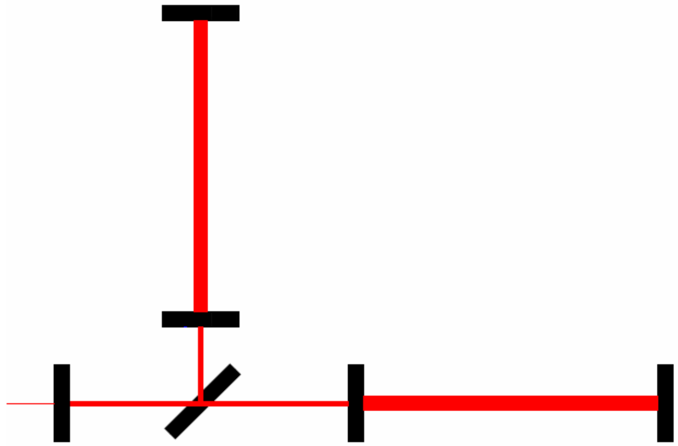
•5 mirror separations to be integer half-wavelength multiples (within ~ 10^{-13} m)

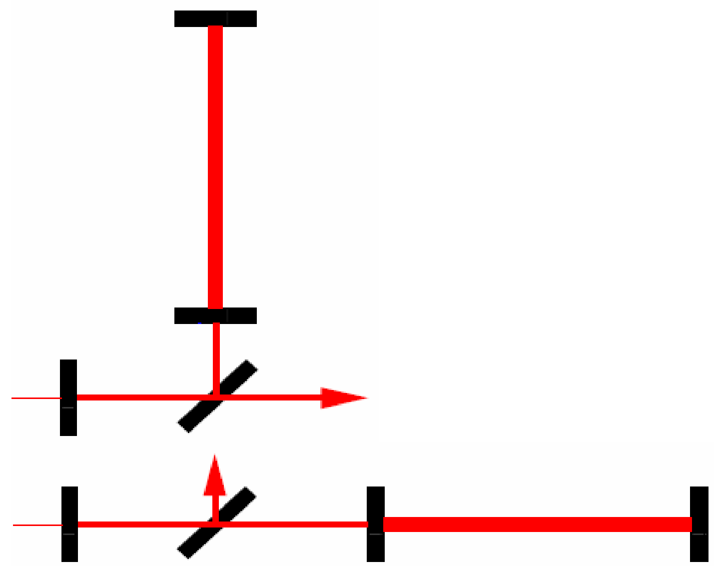
•all mirror normals to be precisely aligned (within ~ 10^{-8} rad)

•dilemma: mirrors "free" inertially at GW frequencies, "static" in an RMS sense



LIGO: A complex system





The system is

-Dynamic

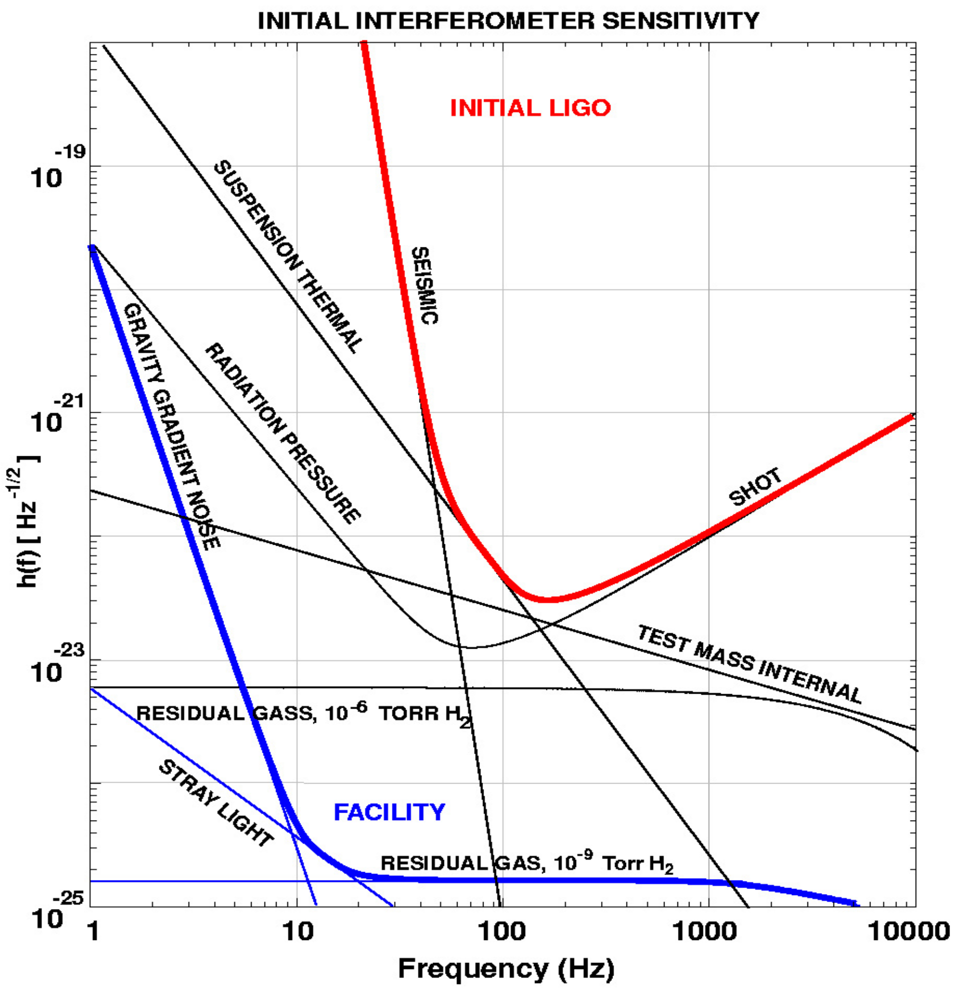
-Coupled

-Nonlinear

LIGO configuration:

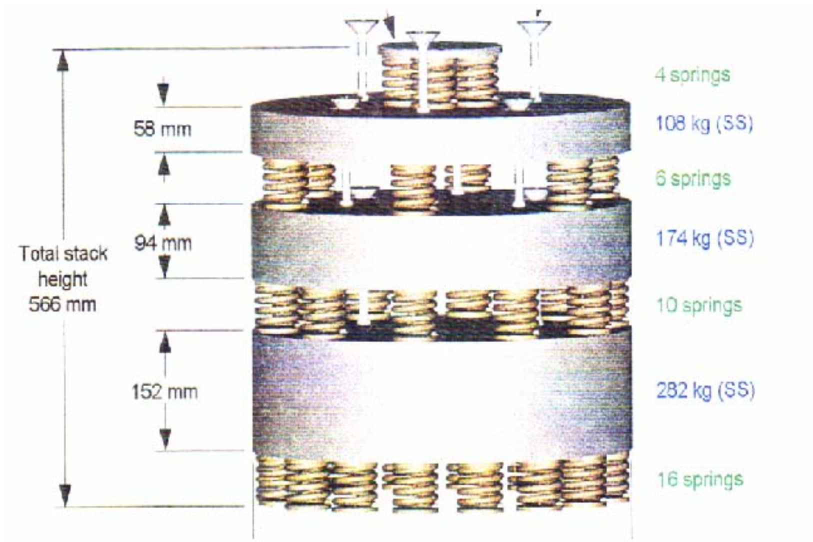
Equivalent to 2 coupled cavities which themselves are coupled together

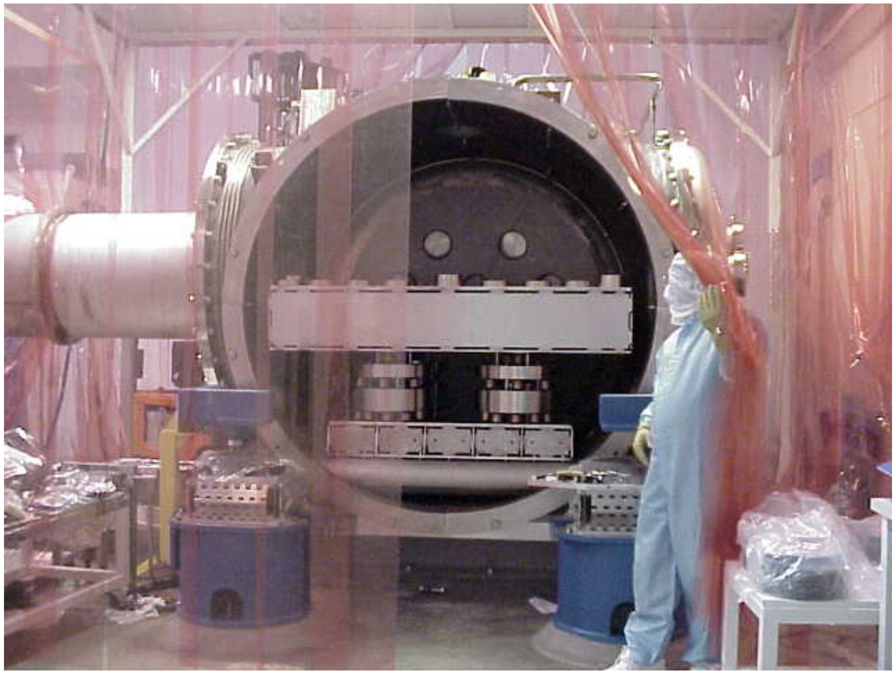
Initial LIGO Sensitivity Goal

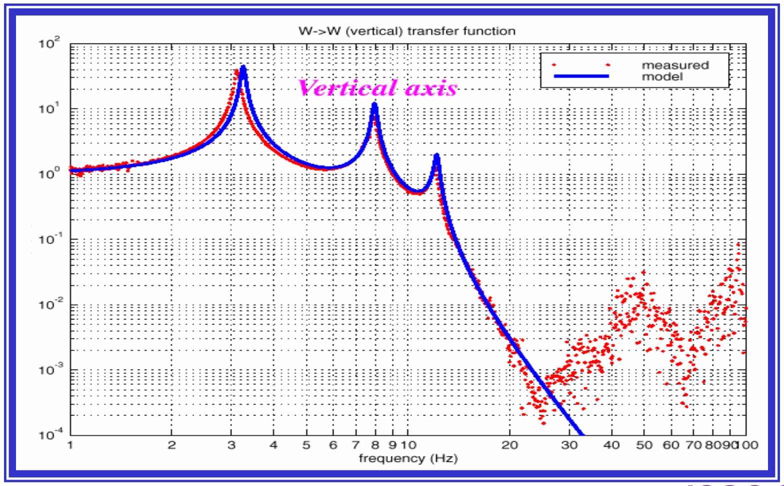


- Strain sensitivity $< 3 \times 10^{-23}$ $1/\mathrm{Hz}^{1/2}$ at 200 Hz
- Displacement Noise
  - » Seismic motion
  - » Thermal Noise
  - » Radiation Pressure
- Sensing Noise
  - » Photon Shot Noise
  - » Residual Gas

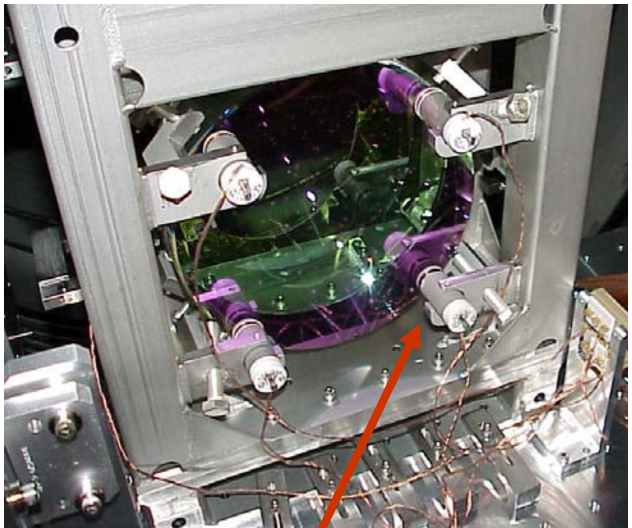
Seismic isolation stacks

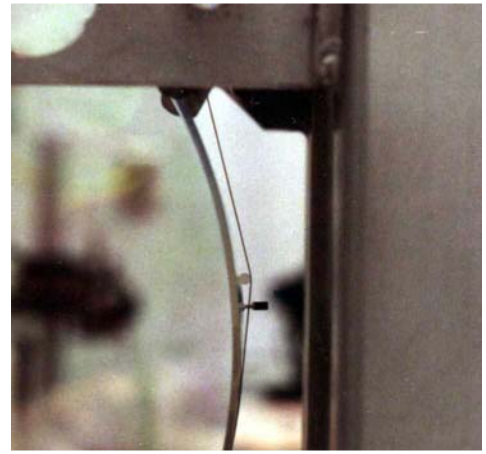






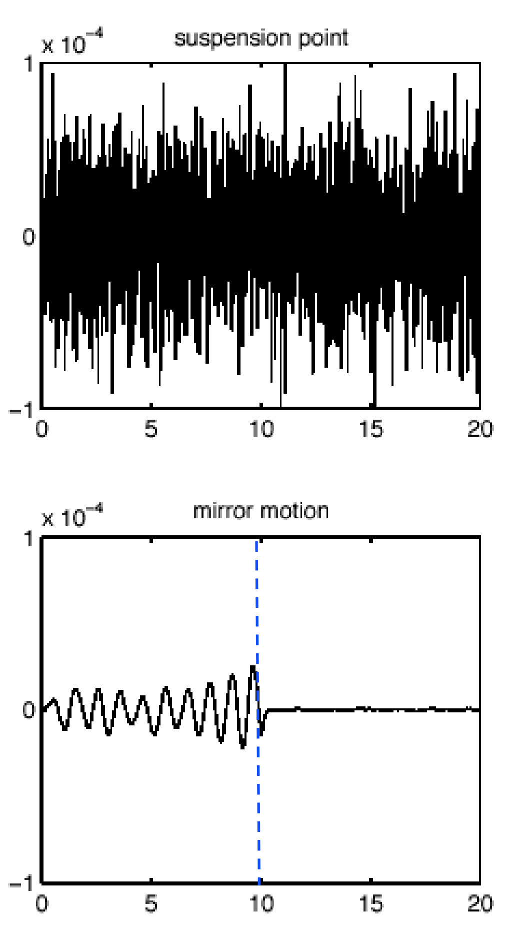
Suspended Core Optics



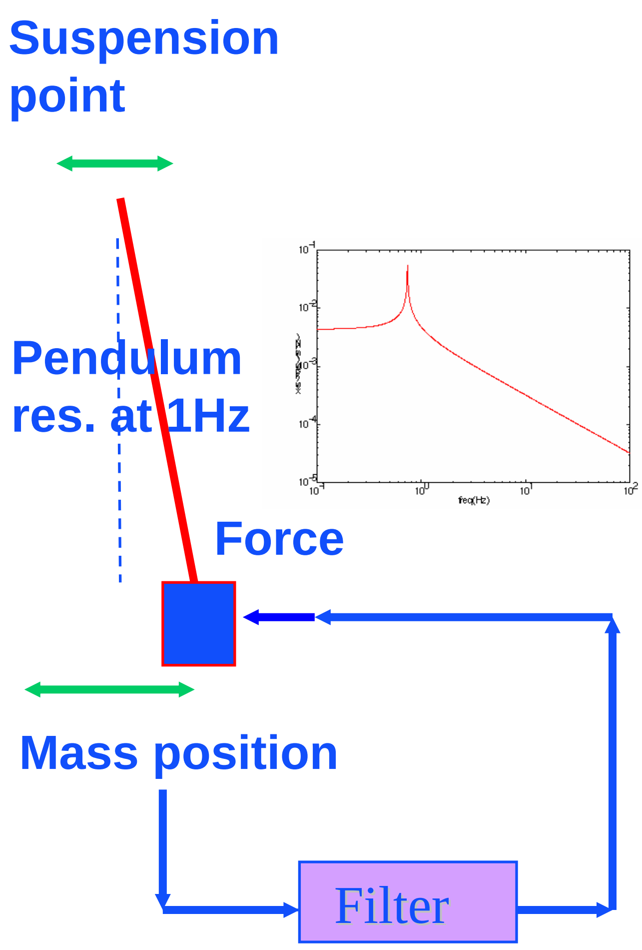


- Optics suspended as simple pendulums
- Local sensors/actuators for damping and control
- Coils push/pull on tiny magnets glued to optics
- Earthquake stops to prevent break-off of tiny magnets

Suspended mass with local control

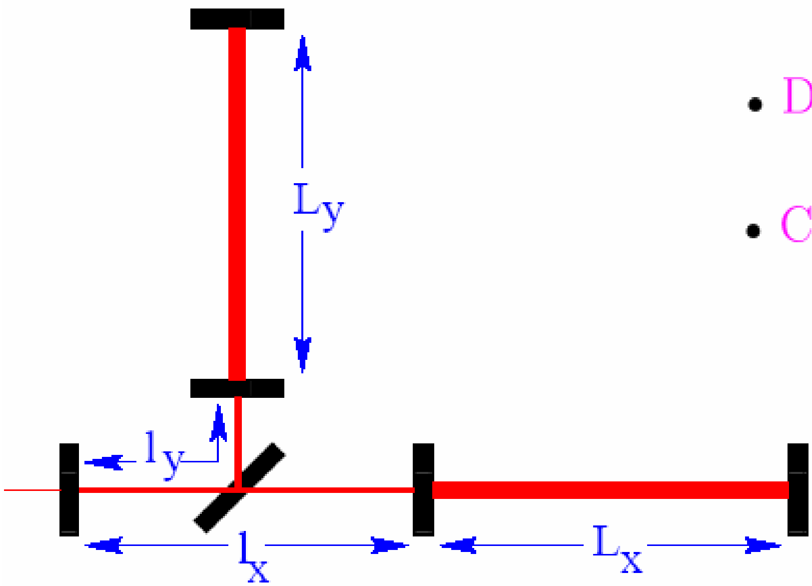


Control on at 10



Interferometer Length Sensing and Control

- Global Length Sensing and Control
  - » Reflection locking technique length and frequency sensing
  - » Control 4 longitudinal degrees of freedom and laser frequency
  - » Requirements:
    - – Differential arm length $<10^{-13}$ m rms
    - – Frequency noise $< 3\times10^{-7}$ Hz/Hz$^{1/2}$ at 100 Hz
    - – Controller noise for diff. arm length $<10^{-20}$ m/ Hz$^{1/2}$ at 150 Hz



- Differential Modes: $(L_x - L_y)/2, \quad (\ell_x - \ell_y)/2$
- Common Modes: $(L_x + L_y)/2, \quad (\ell_x + \ell_y)/2$

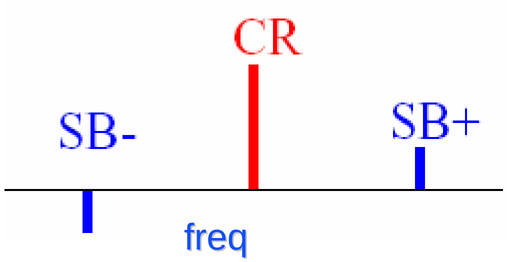
Phase modulation

- Phase Modulation

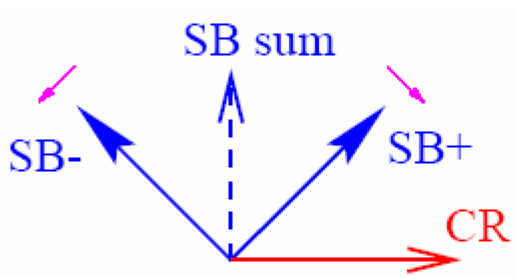
$$\exp\left(i\omega_c t\right) \times \exp\left[i\varepsilon_m \sin(\omega_m t)\right]$$

- Series Expansion

$$e^{i\omega_c t} \times \left[J_0\left(\varepsilon_m\right) + J_1\left(\varepsilon_m\right) e^{i\omega_m t} + J_{-1}\left(\varepsilon_m\right) e^{-i\omega_m t} + \ldots\right]$$



$$\simeq e^{i\omega_c t} \times \left[1 + \frac{\varepsilon_m}{2} e^{+i\,\omega_m t} - \frac{\varepsilon_m}{2} e^{-i\,\omega_m t}\right]$$



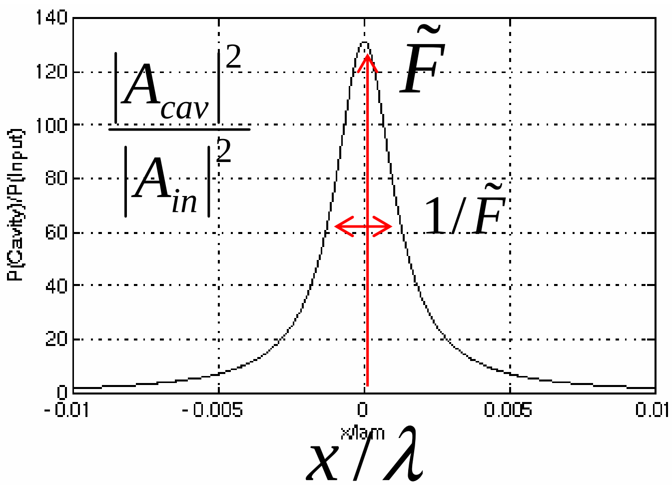
Phasor diagram

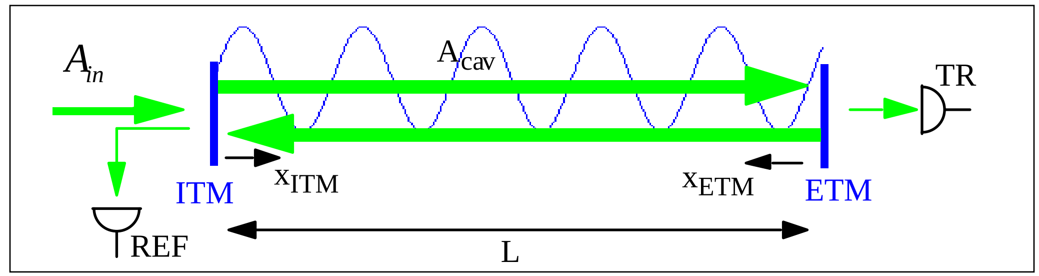
$$\simeq e^{i\omega_c t} \times \left[1 + i\varepsilon_m \sin\left(\omega_m t\right)\right]$$

Fabry-Perot Cavity dynamics

microscopic length offset: $x = n\lambda - L$

phase: $\phi = 4\pi x / \lambda$





$$A_{cav} = A_{in} \frac{t_{ITM}}{1 - r_{ITM} r_{ETM} e^{i\phi}}$$

$$\tilde{F} = \left(\frac{t_{ITM}}{1 - r_{ITM} r_{ETM}} \right)^2 \approx \frac{4}{t_{ITM}^{\ 2}}$$

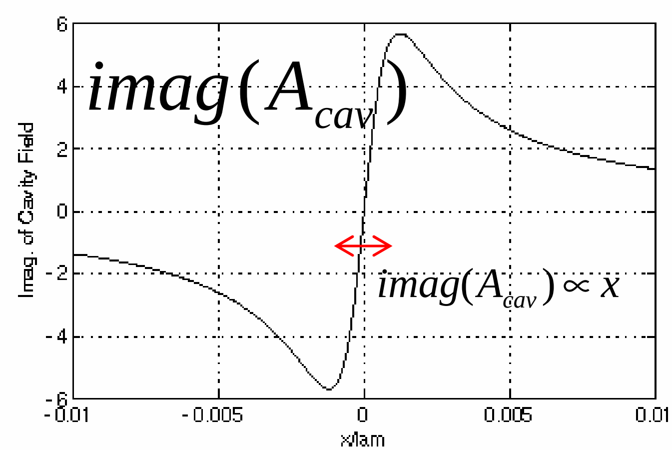
$$\approx A_{in} \sqrt{\tilde{F}} \left(1 + i2\pi \cdot \frac{x}{\lambda / \tilde{F}}\right)$$

$x << \lambda$

$t_{ITM}^{\ 2} = 0.03$

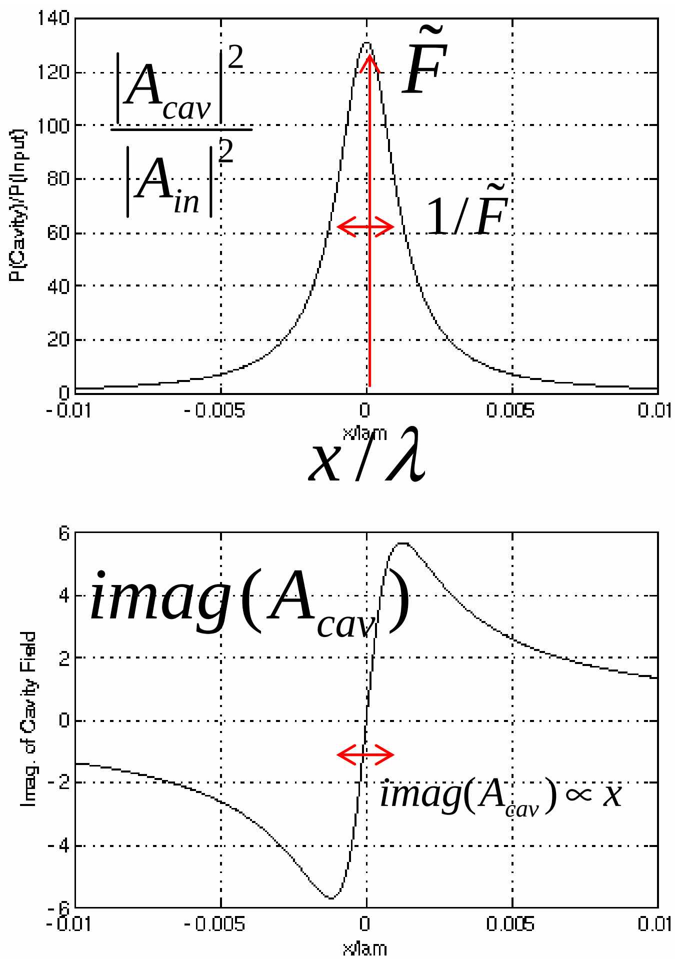
$t_{ETM}^{\ 2} = 10e^{-6}$

$\tilde{F} = 130$



$$A_{ref} = r_{ITM} - t_{ITM} r_{ETM} e^{i\phi} A_{cav}$$

Signal from Power? -No



$$A_{cav} = A_{in} \frac{t_{ITM}}{1 - r_{ITM} r_{ETM} e^{i\phi}}$$

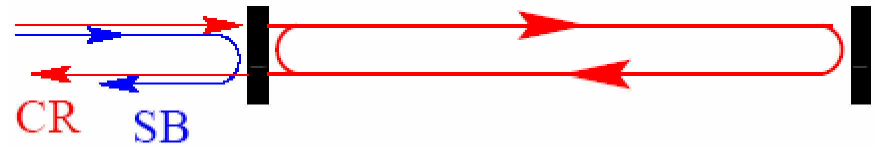
$$\approx A_{in} \sqrt{\tilde{F}} (1 + i2\pi \cdot \frac{x}{\lambda/\tilde{F}})$$

$$P_{cav} = A_{in}^2 F \times \left[1 - 4\pi^2 \left(\frac{x}{\lambda/F} \right)^2 \right]$$

$$\lambda/F \simeq 10^{-8} meter$$

Pound-Drever Signal

Power: $|CR + SB^{+} + SB^{-}|^{2}$



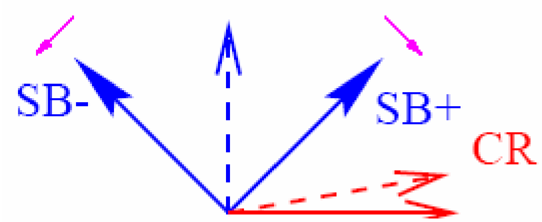
Signal (beating between CR and SBs):

$$2 \times Real\left(CR^{*} \times \left[SB^{+} + SB^{-}\right]\right)$$

$$= 2 \times Real\left(CR^{*} \times i\varepsilon_{m} \sin\left(\omega_{m} t\right)\right)$$

When CR has got a phase, ϕ due to length change:

$$\propto Real\left(\left(1 + i\phi\right) \times i\varepsilon_{m} \sin\left(\omega_{m} t\right)\right)$$



Phasor diagram

$$\propto \phi.\varepsilon_{m} \sin\left(\omega_{m} t\right)$$

Demodulate at Radio Frequency

signal: $\propto \phi.\varepsilon_m \sin\left(\omega_m t\right)$

Demodulated signal:

$$\phi.\varepsilon_m \sin^2\left(\omega_m t\right) = \frac{1}{2}\phi.\varepsilon_m \left[1 - \cos(2\omega_m t)\right]$$

Pass it thru low-pass filter:

$$\propto \phi.\varepsilon_m$$

Interferometer Operating Condition

Carrier resonant in all cavities

Ideal: No light in anti-sym port (Dark)

Differential length change ->

CR leaks thru anti-sym port

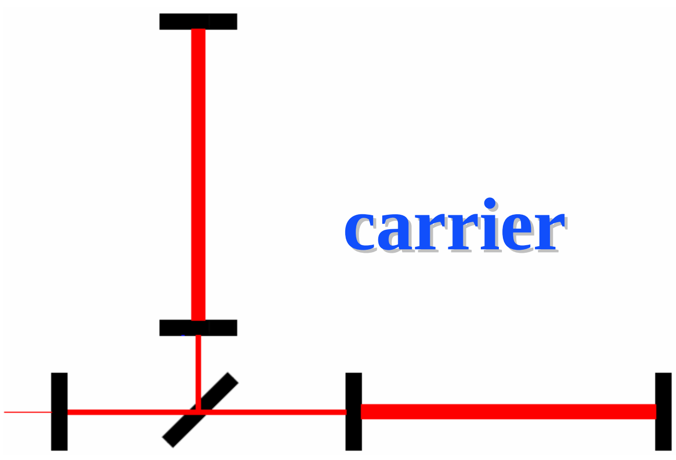
Beats with SB present there -> Signal

SBs resonant only in recycling cavity

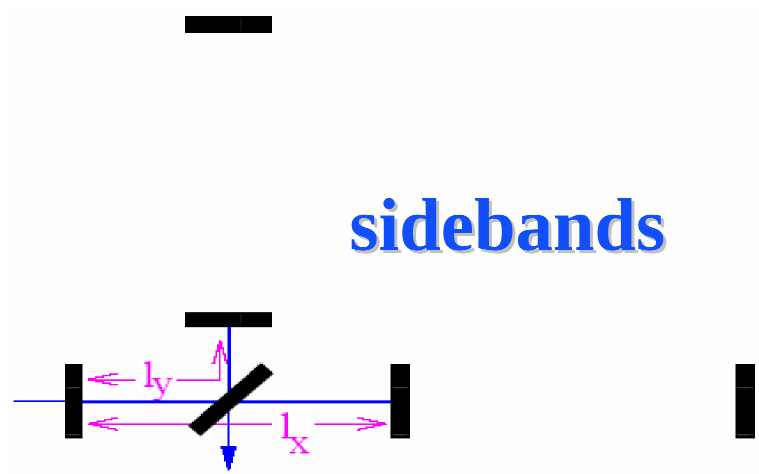
Difference in rec cav lengths allows SBs to leak thru anti-sym port $P_{SB}(dark) = [\omega(\ell_x - \ell_y)/c]^2$

SB efficiency 100% when

$P_{SB}(dark) = P_{SB}(input)$



Dark port



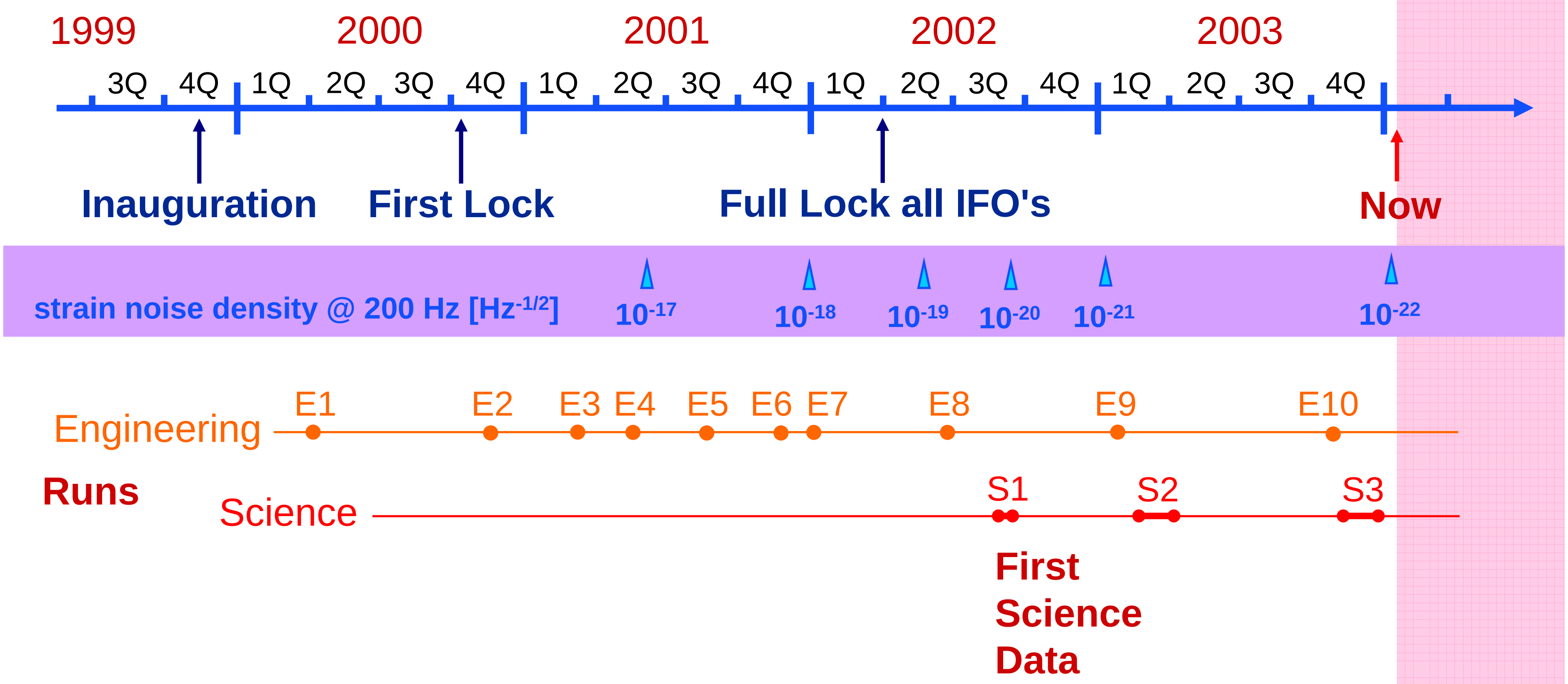
LIGO End to End (E2E)Simulation

Motivation for time-domain simulation:

- Assist detector design, commissioning, and data analysis
- To understand a complex system
  - » complex hardware : pre-stabilized laser, input optics, core optics, seismic isolation system on moving ground, suspension, sensors and actuators
  - » field : non-Gaussian field propagation through non perfect mirrors and lenses, misalignment and modal mismatch ..
  - » feedback loops : length and alignment controls, feedback to laser
  - » non-linearity : cavity dynamics to actuators
  - » coupling : alignment and longitudinal, frequency and motion, between arms
  - » noise : mechanical, thermal, sensor, field-induced, laser, etc : amplitude and frequency : creation, coupling and propagation
  - » wide dynamic range : $10^{-6} \sim 10^{-20}$ m

Current Team @ Caltech: B. Bhawal, M.Evans,V.Sannibale,H. Yamamoto

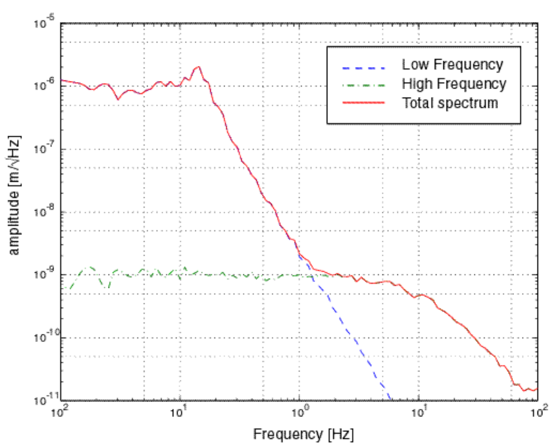
Time Line

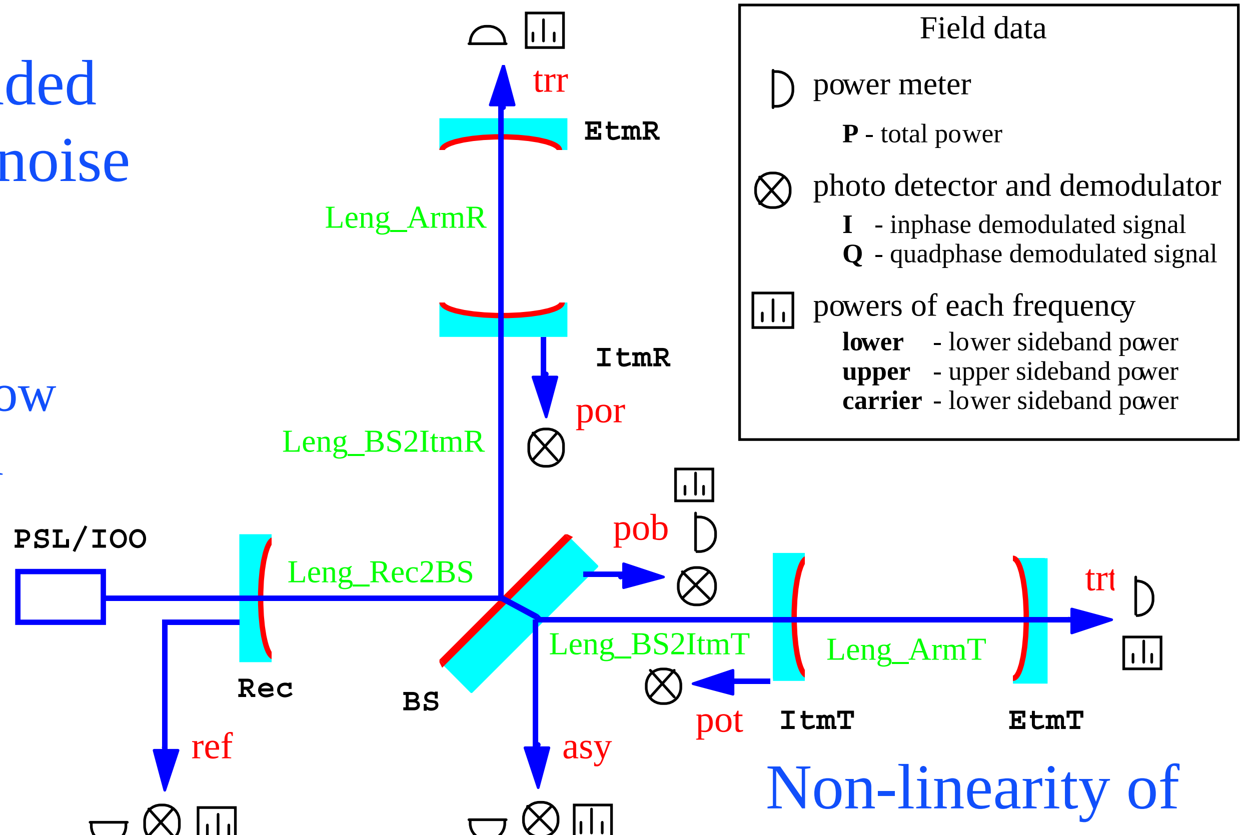

1999
2000
2001
2002
2003
3Q 4Q 1Q 2Q 3Q 4Q 1Q 2Q 3Q 4Q 1Q 2Q 3Q 4Q 1Q 2Q 3Q 4Q
Inauguration
First Lock
Full Lock all IFO's
Now
strain noise density @ 200 Hz [Hz$^{-1/2}$]
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
Engineering
E1 E2 E3 E4 E5 E6 E7 E8 E9 E10
Runs
Science
S1 S2 S3
First Science Data

Hanford 2k simulation setup

6 independent suspended mirrors with seismic noise

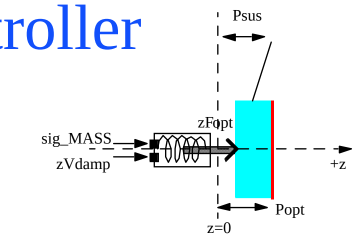
corner station :
strong correlation in the low frequency seismic motion





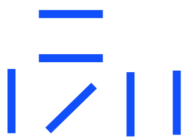
Han2k optics setup

Non-linearity of controller

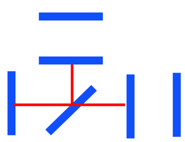


ICGC-2004, Kochi, India

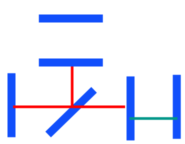
Multi step locking



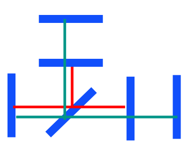
State 1 : Nothing is controlled. This is the starting point for lock acquisition.



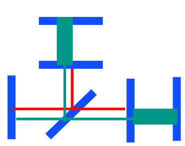
State 2 : The power recycling cavity is held on a carrier anti-resonance. In this state the sidebands resonate in the recycling cavity.



State 3 : One of the ETMs is controlled and the carrier resonates in the controlled arm.



State 4 : The remaining ETM is controlled and the carrier resonates in both arms and the recycling cavity.

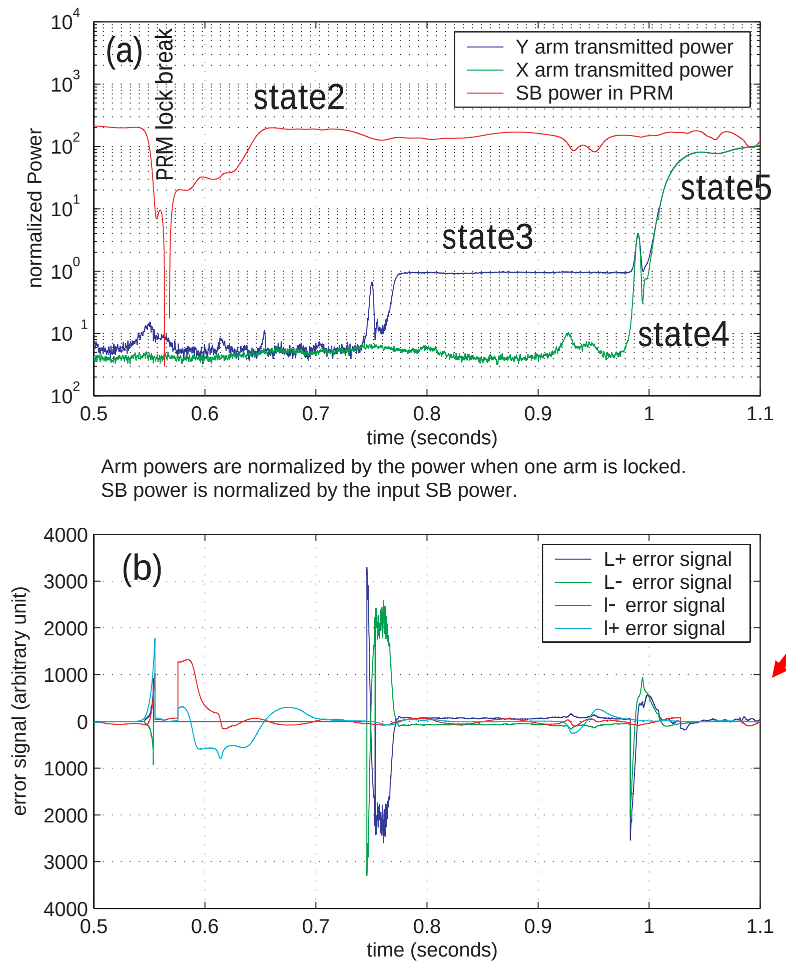


State 5 : The power in the IFO has stabilized at its operating level. This is the ending point for lock acquisition.

Lock acquisition

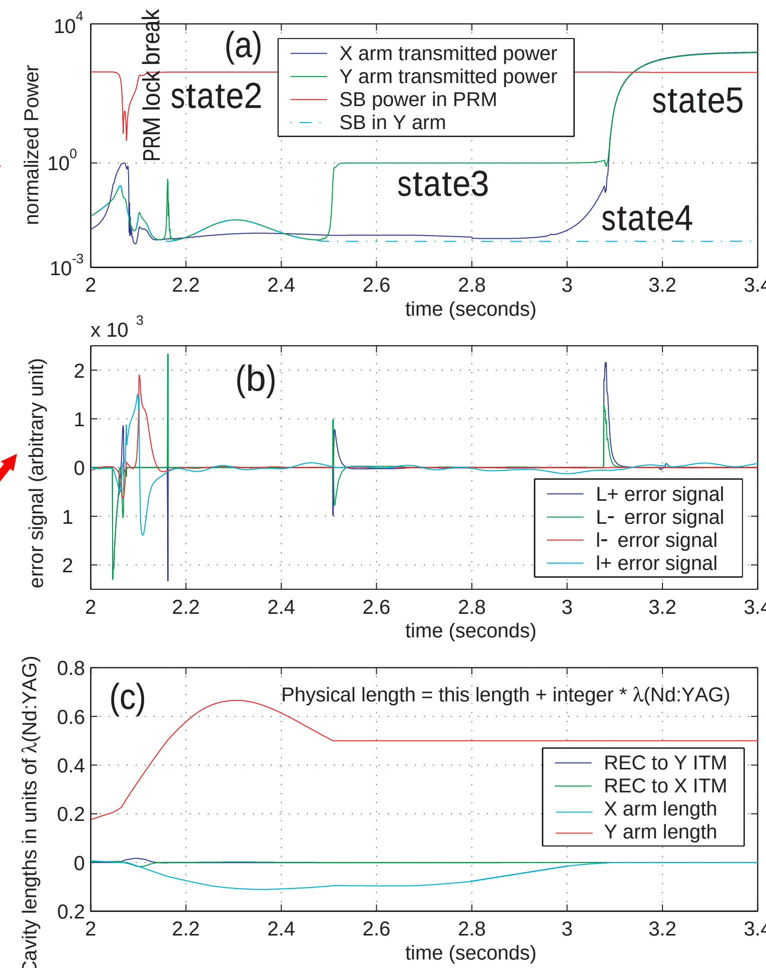
real and simulated

Figure 1. LHO 2k IFO data



observable

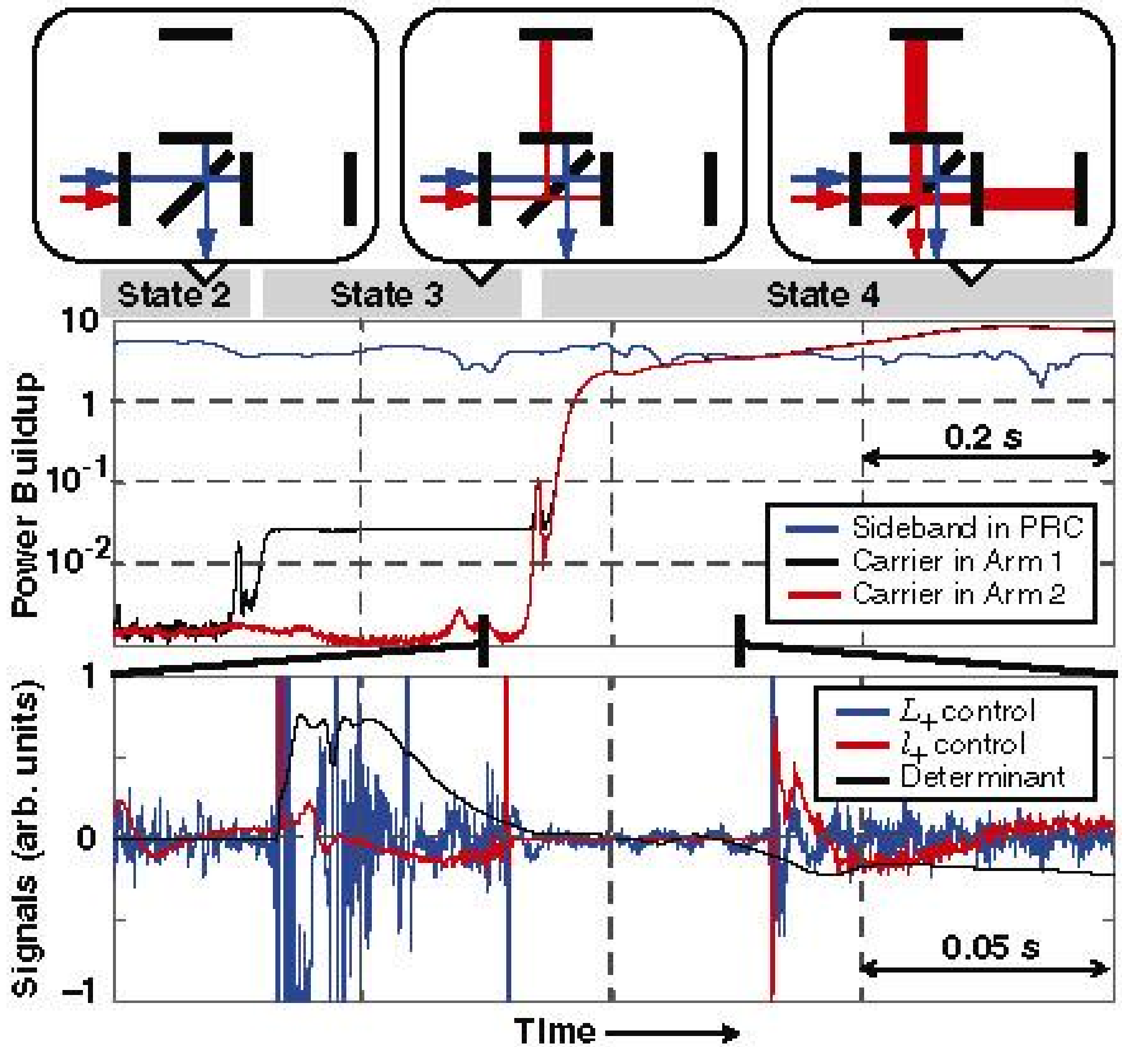
Figure 2. Simulated signal



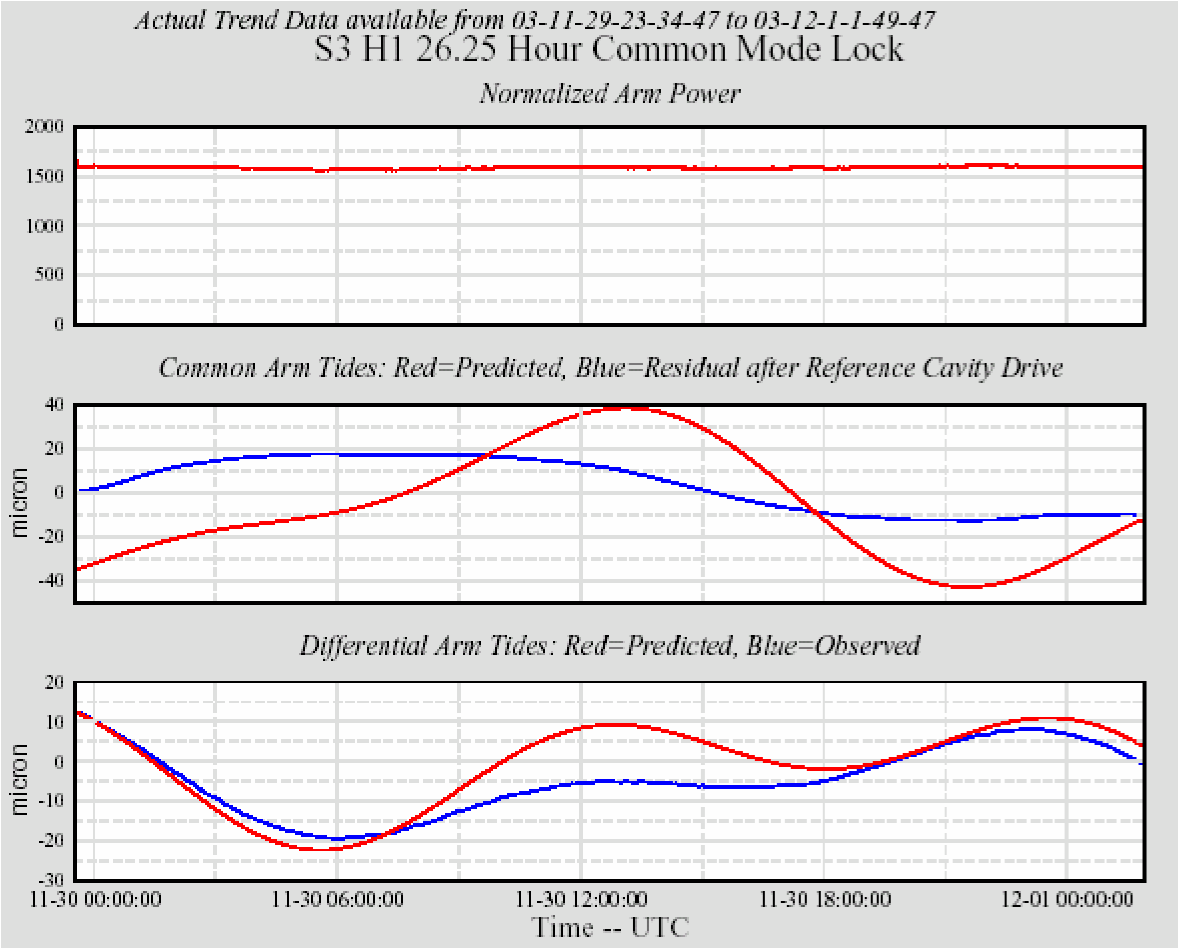
Not experimentally observable

Guided Lock Acquisition

- Fast sensors monitor circulating powers, RF sidebands in cavities
- Sequencing code digitally switches feedback state at proper transition times
- Loop gains are actively scaled (*every sample*) to match instantaneous carrier & sideband buildups
- Designed by Matt Evans (PhD thesis)



Tidal Compensation during locked state



Daily strains

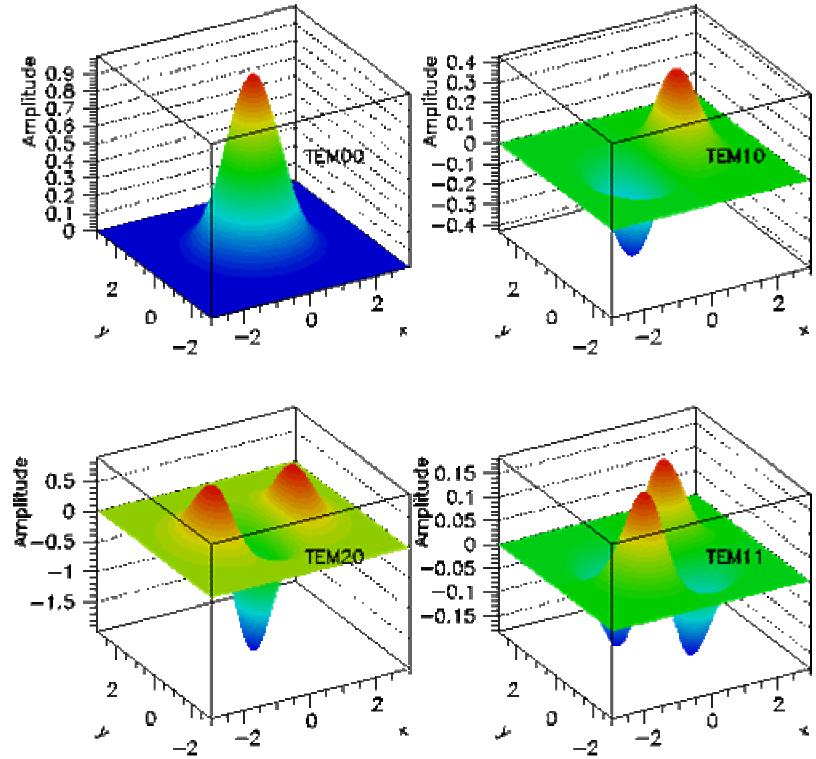
~3e-8

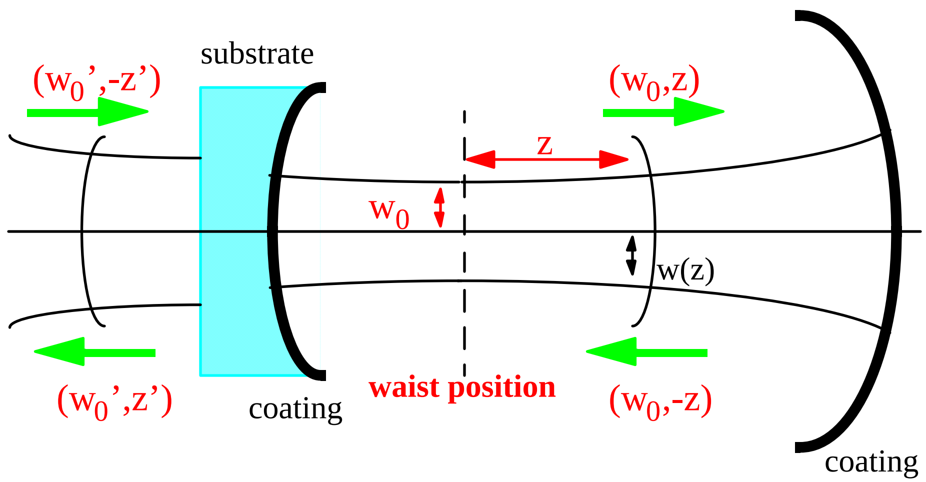
For 4 Km

~120micrometer

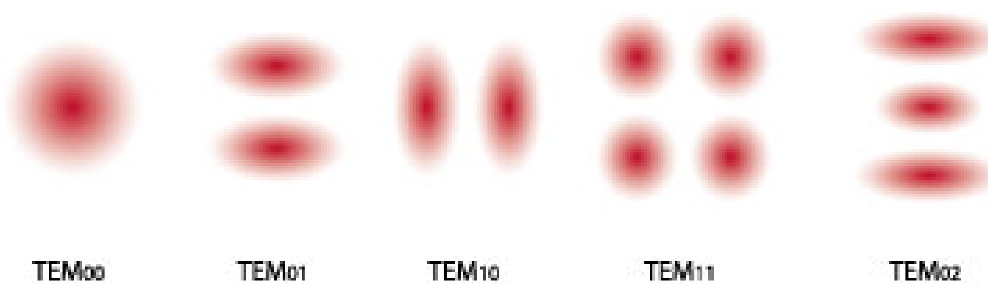
Misalignments: Cavity & its eigenmodes

amplitude



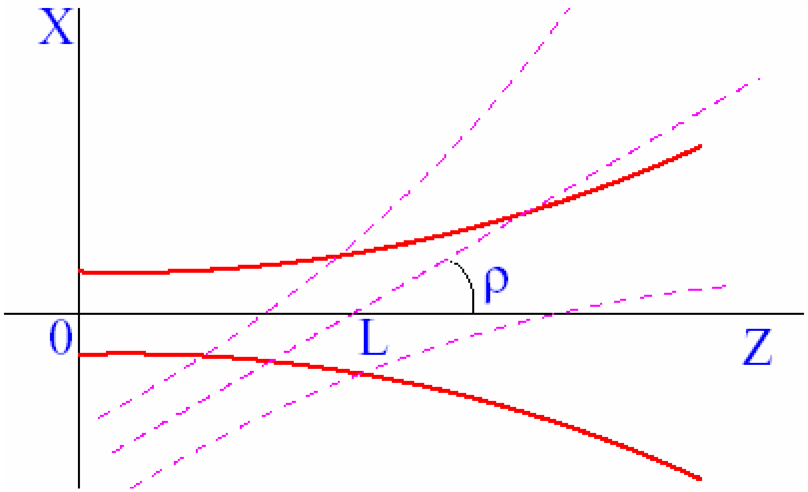


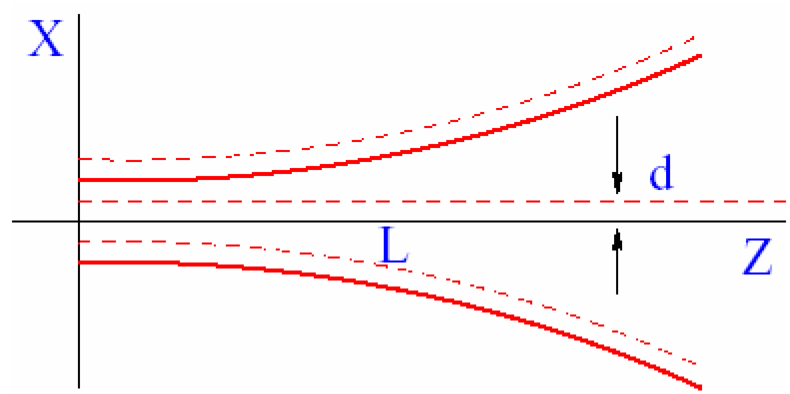
power



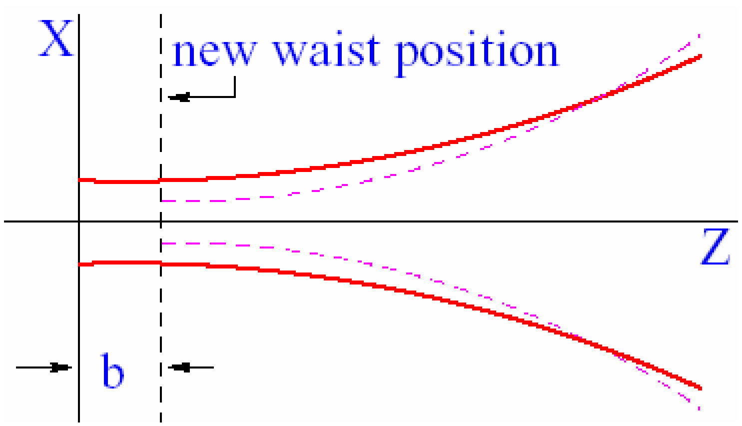
Perturbations on Beam

- Initial Beam: AU_o with k (mode no.), w (waist)





- Rotation(ρ): $A\left[U_0 + i\rho\frac{kw}{\sqrt{2\pi}}U_1\right]$
- Lateral displacement (d): $A\left[U_0 + \left(\sqrt{\frac{2}{\pi}}\right)\frac{d}{w}U_1\right]$



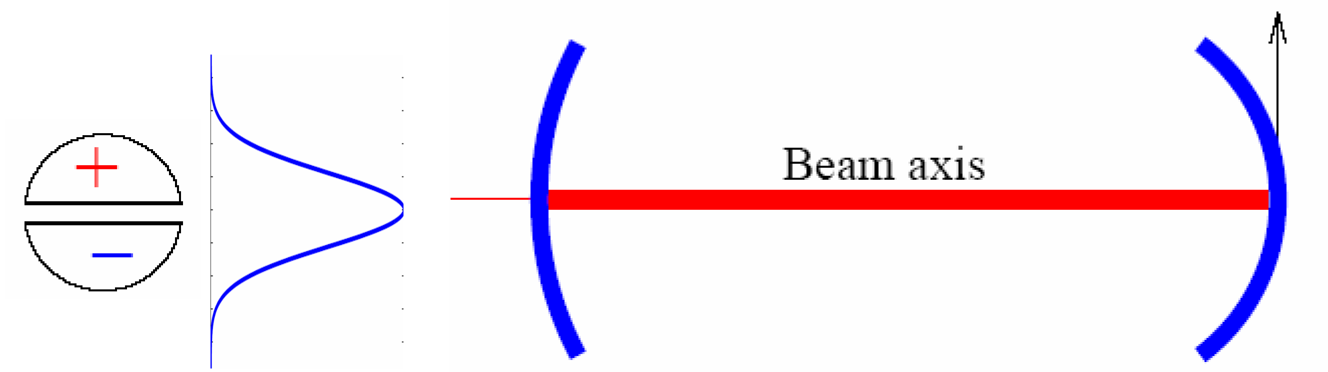
- Waist-position mismatch (b):
  $A\left[U_0 + i\frac{b}{2kw^2}\{U_0 + U_2\}\right]$
- Waist-size mismatch (s) : $A\left[U_0 + \frac{s}{2w}U_2\right]$

Alignment Sensing and Control

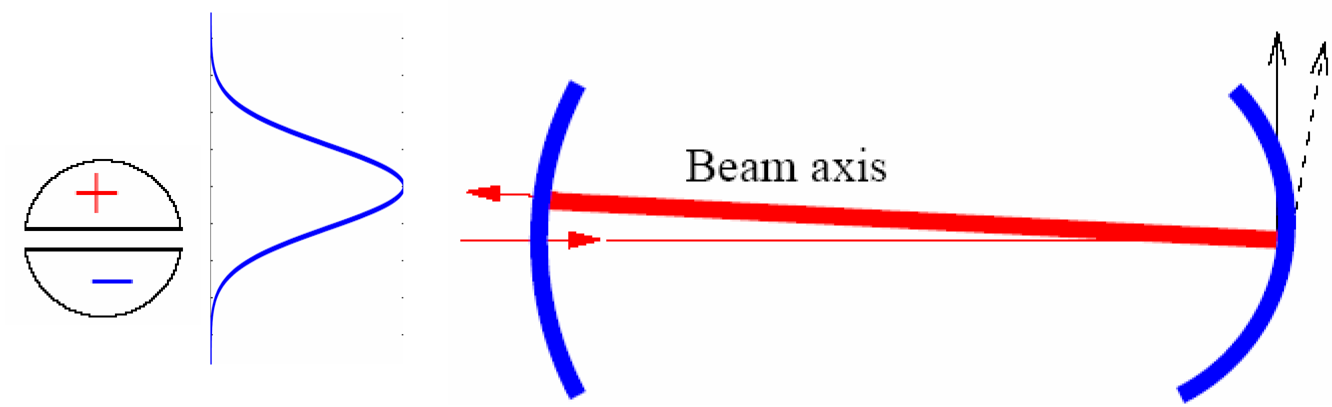
- Alignment sensing and control
  - » Digital Control of 12 mirror angles and the input beam direction
  - » Requirement: angular fluctuations <10^{-8} rad rms
  - » Wavefront sensors (split photodetectors)

Misalignment signal

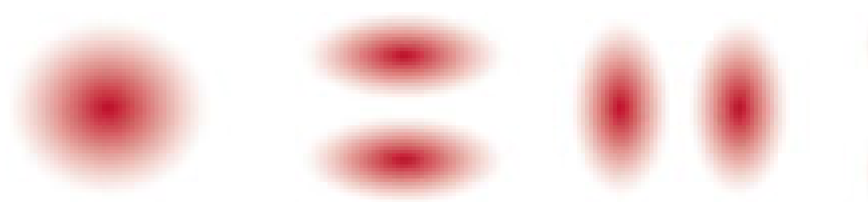
Signal 0 <=



Signal non0 <=



Higher Order Modes due to misalignment



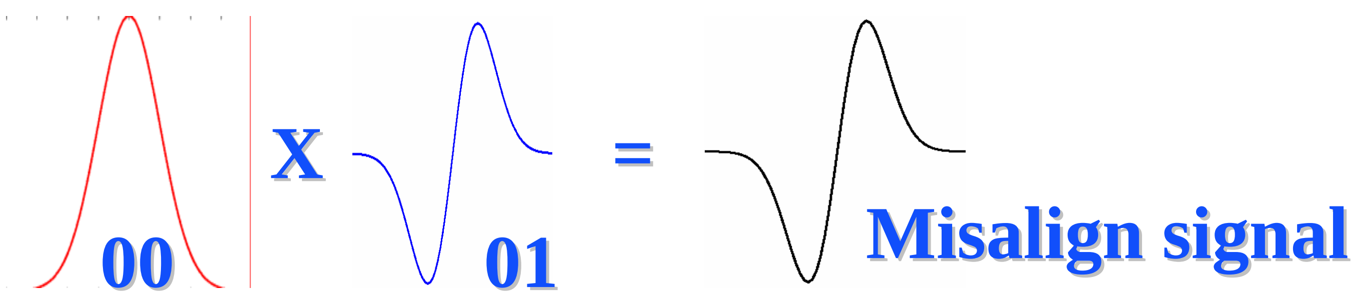
TEM00 TEM01 TEM10

power

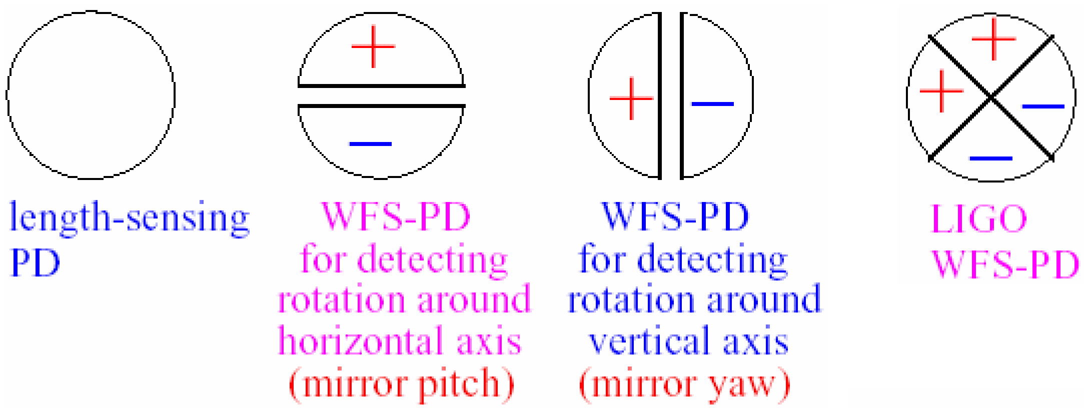
Signal in Radio frequency

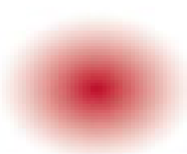
- Signal- 1st Even term: $\propto CR_{00} \times SB_{00}^{*}$
- Signal- 1st Odd term: $\propto CR_{00} \times SB_{01}^{*} + CR_{01} \times SB_{00}^{*}$

  Or, $\propto CR_{00} \times SB_{10}^{*} + CR_{10} \times SB_{00}^{*}$

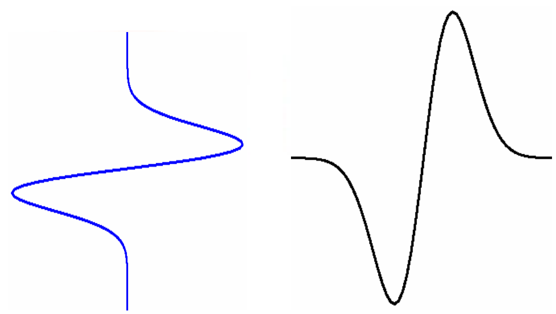


Wave Front Sensor & Misalignment





$CR_{00} \times SB_{00}$



$CR_{00} \times SB_{10} + CR_{10} \times SB_{00}$

$CR_{00} \times SB_{01} + CR_{01} \times SB_{00}$

Wave Front Sensors in Hanford 4KM IFO

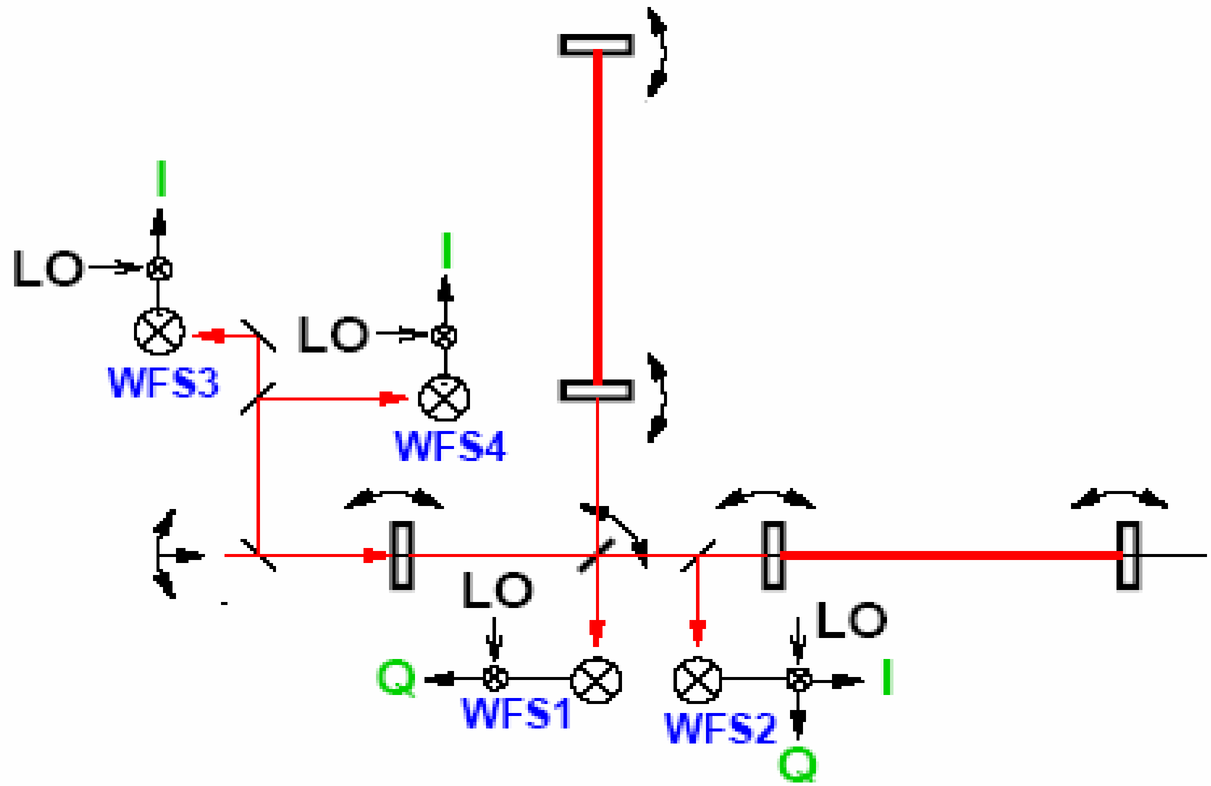
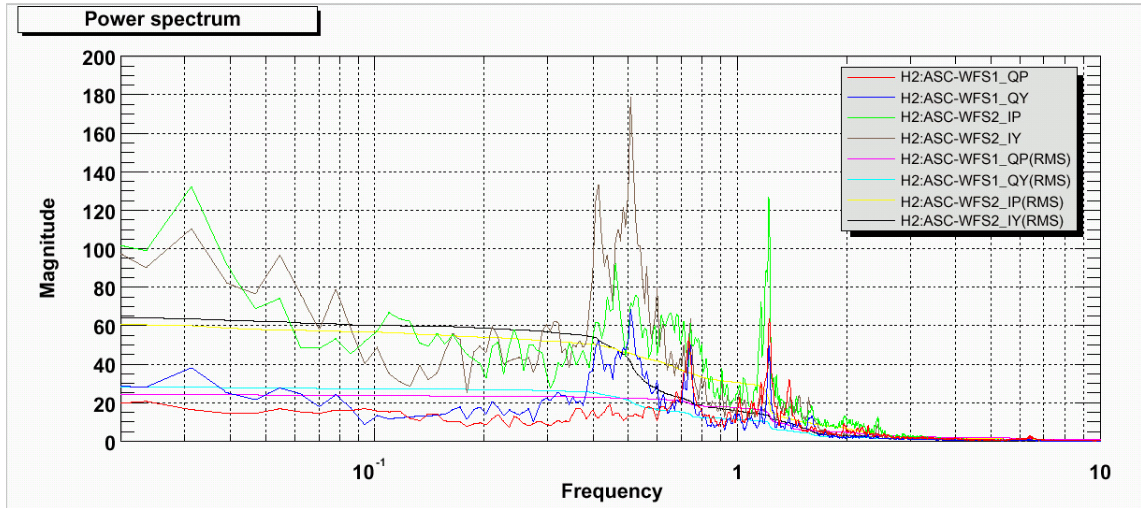


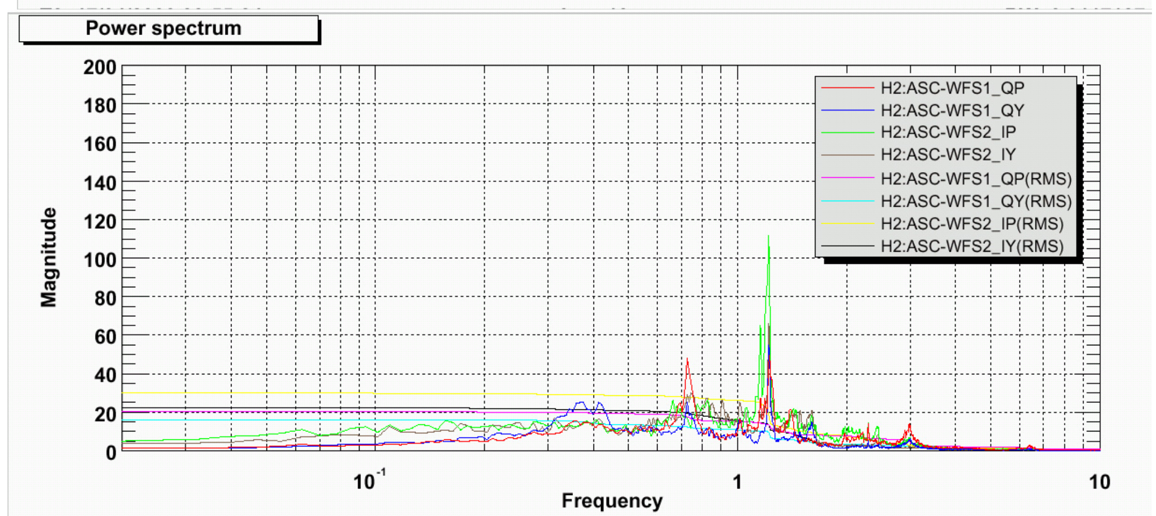
Figure 1: The location of the different WFSs in H_1 during S3.

Activation of Wavefront sensors

Alignment fluctuations before engaging wavefront sensors



After engaging wavefront sensors

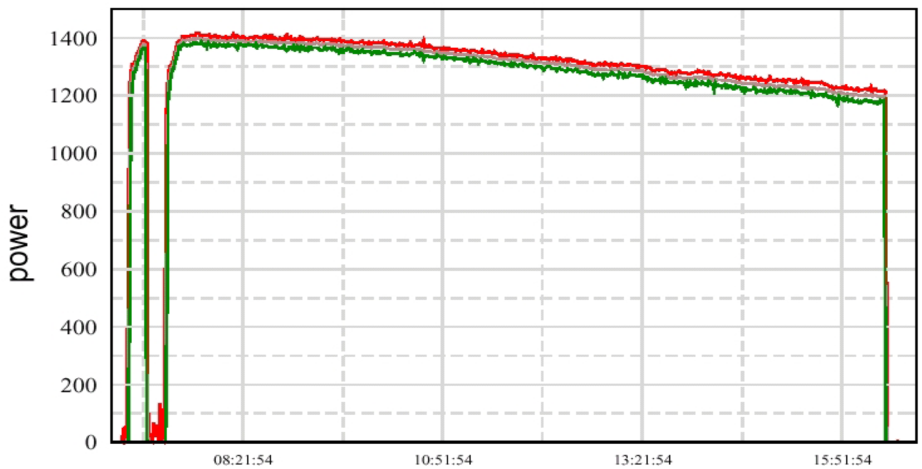


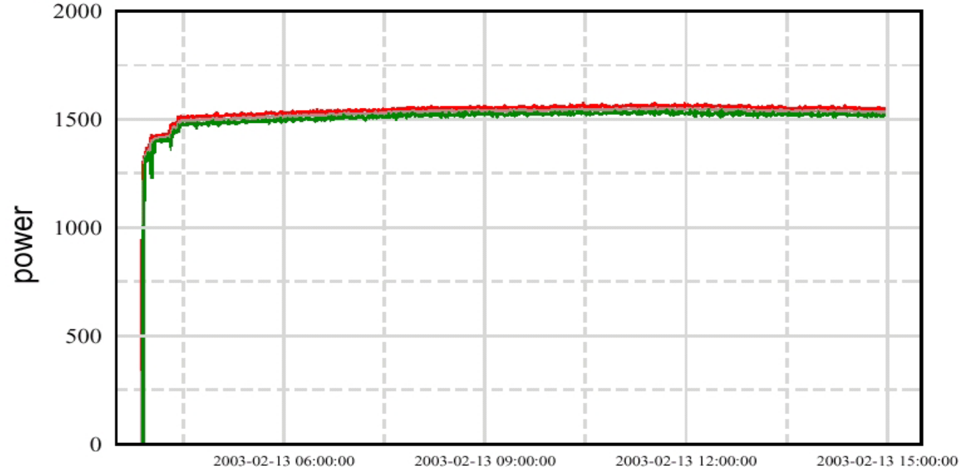
WFS Alignment System

WFS OFF

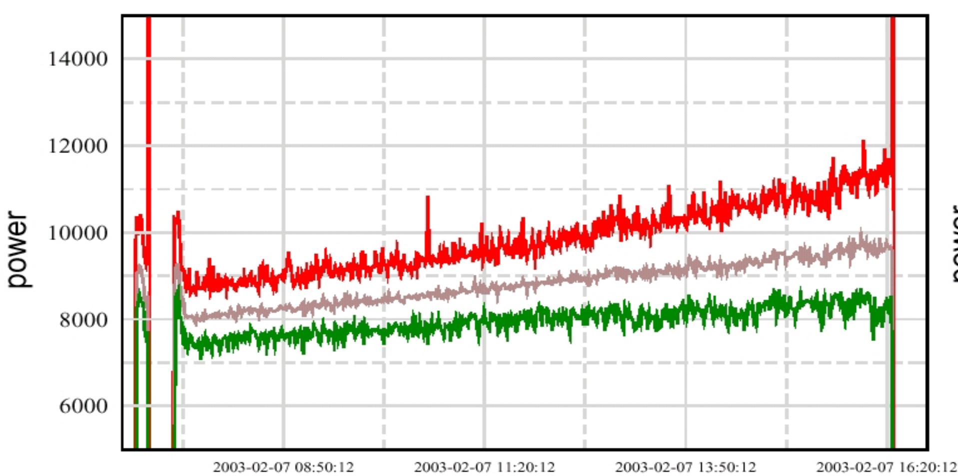
WFS ON

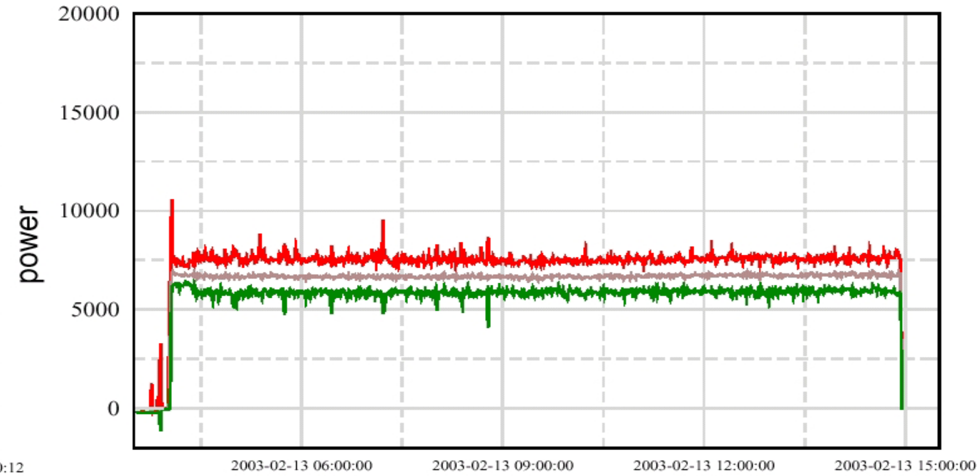
Arm Cavity Power



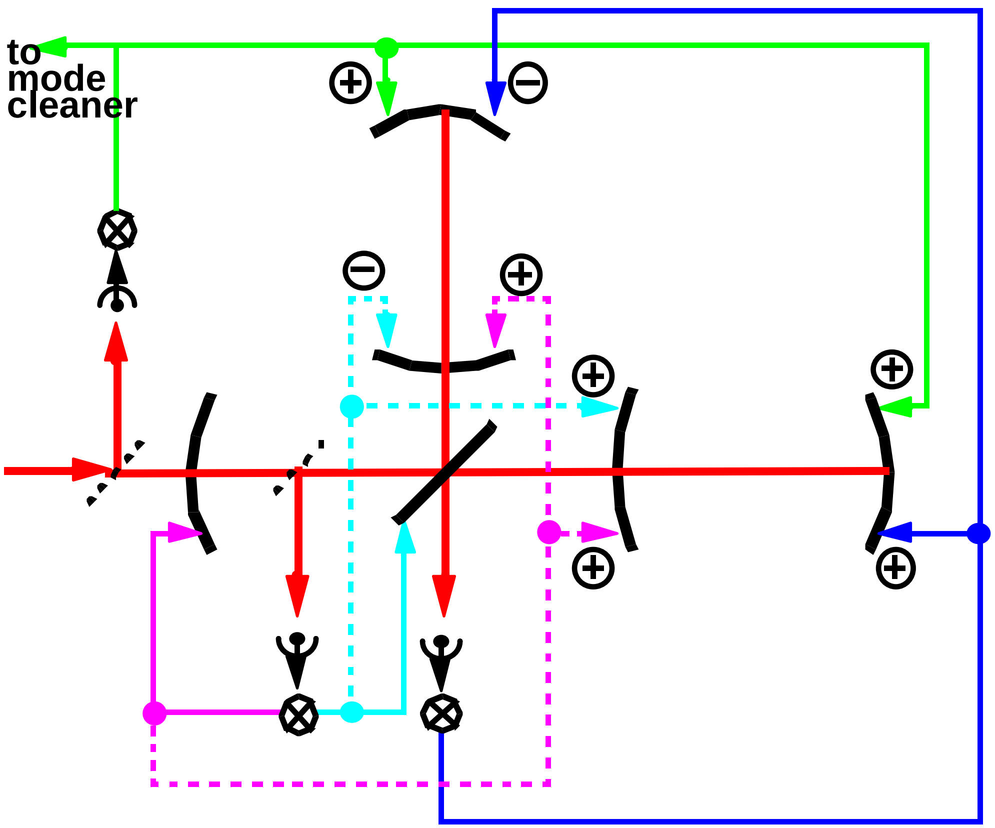


Anti-symmetric Port Power





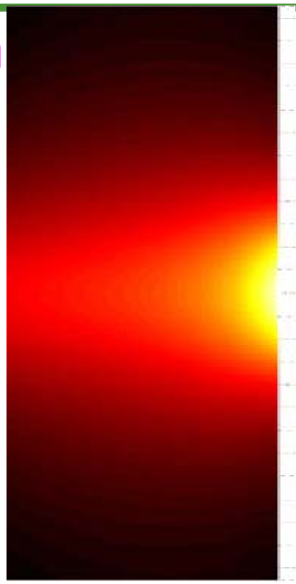
Feedback Control Systems



example: cavity length sensing & control topology

- Array of sensors detects mirror separations, angles
- Signal processing derives stabilizing forces for each mirror, filters noise
- 5 main length loops shown; total ~ 25 degrees of freedom
- Operating points held to about 0.001 Å, .01 µrad RMS
- Typ. loop bandwidths from ~ few Hz (angles) to > 10 kHz (laser wavelength)

Thermal lensing



- Lens Definition: varying optical path length along trans direction
- Any concave LIGO mirror is a lens ….
- … BUT Temperature gradient inside substrate
  - -> gradient of refractive index across transverse direction
  - -> Change in Optical path length across trans. direction
  - -> Change in transmitted beam wavefront

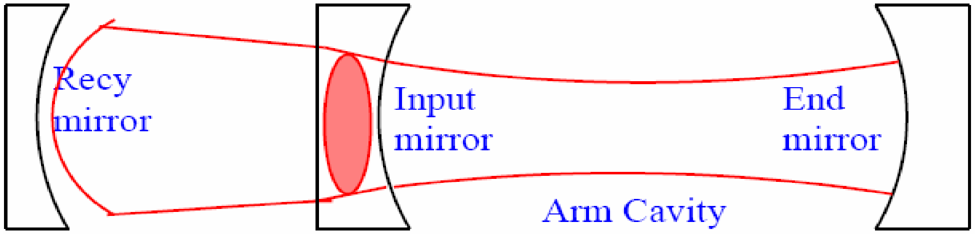
…that's the effect of Thermal lensing

- Substrate heat absorption in Input mirrror:

-> InputPower X RecyclingGain X SubstrateAbsorpCoeff

- Surface heat absorption in Input mirrror:

-> InputPower X RecyclingGain X ArmGain X SurfaceAbsorpCoeff



Mode-mismatch

Thermal Lensing in Input mirror

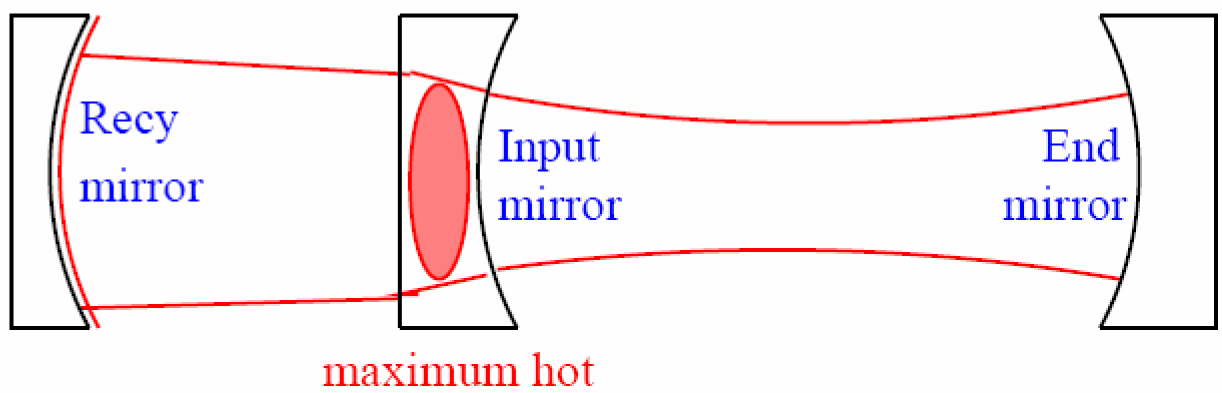
» Carrier modes mainly determined by arms (negligible effect of thermal lensing on carrier in LIGO I)

» Sidebands coupled into recycling cavity

» Recycling cavity is **nearly degenerate** [small cavity (~9 m) with large mirror curvature (~15 km)

» Original "point design" depends on specific, balanced **thermal lensing:**

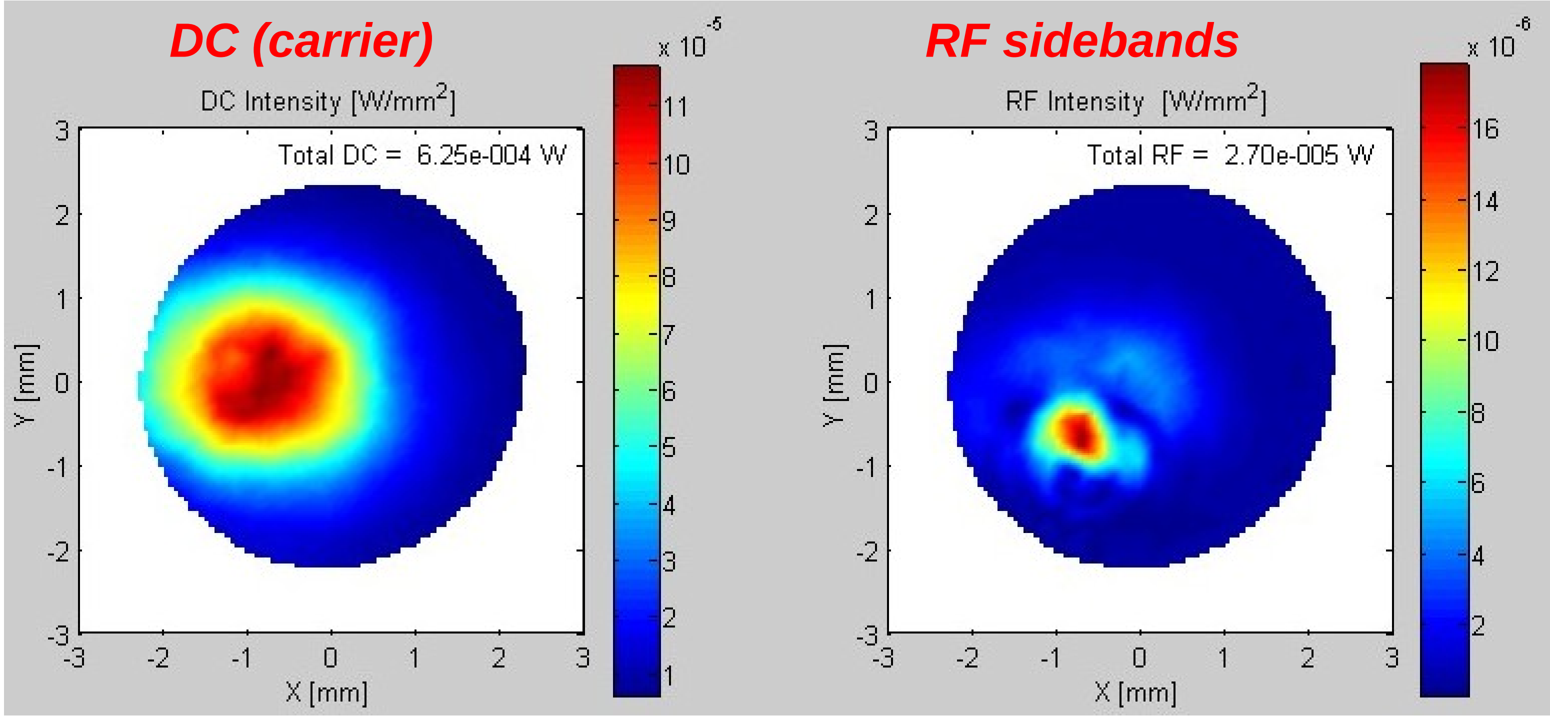
Beam wavefront matches with Recycling mirror ROC when maximum hot (Problem: what happens if “maximum hot” state is different from prediction?)



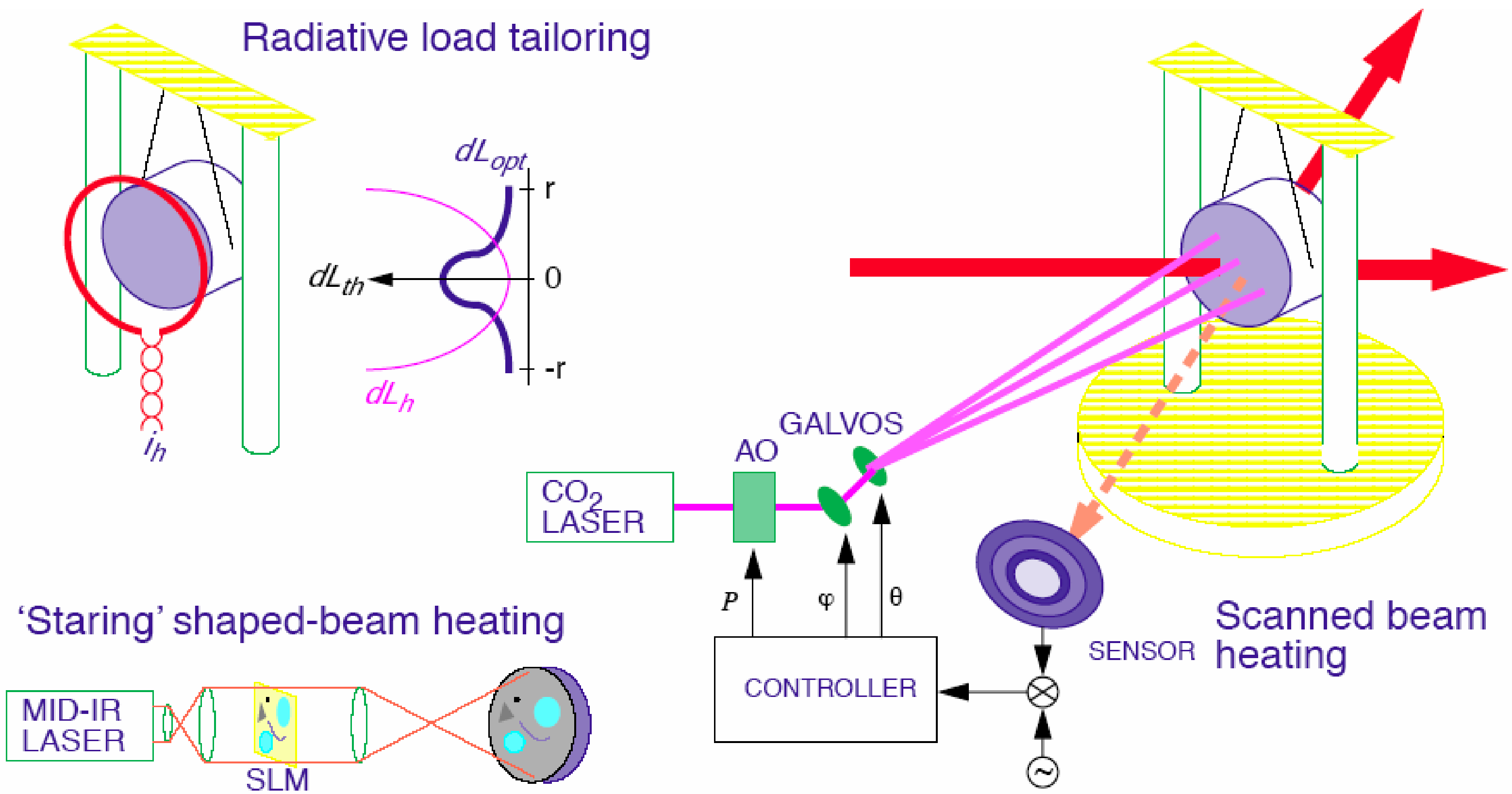
Bad mode-overlap for SBs

- RF sideband efficiency found to be very low
  - » H1 efficiency: ~6% (anti-symmetric port relative to input)
  - » incorrect/insufficient ITM thermal lens makes the recycling cavity an unstable resonator

⇒Bad mode overlap!

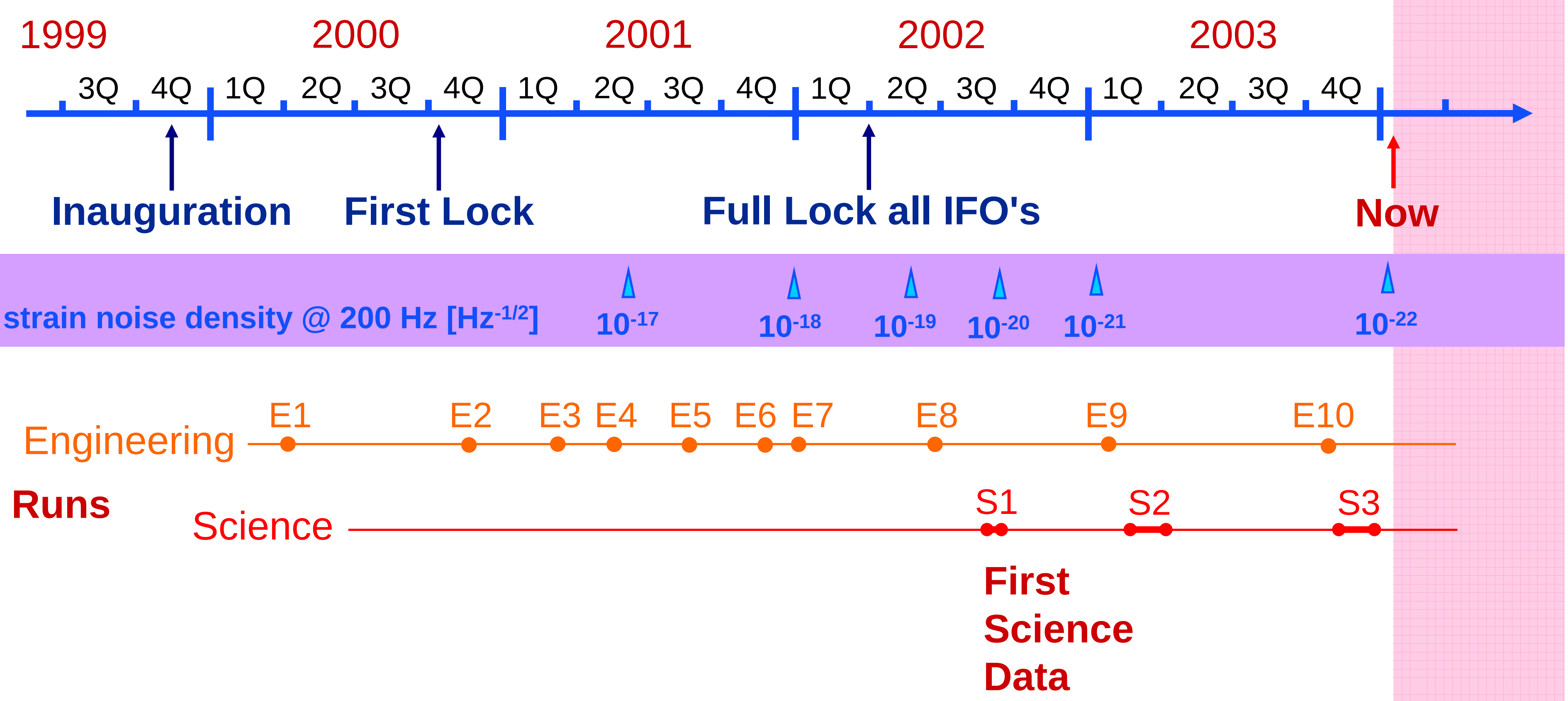


Thermal Compensation to the Rescue



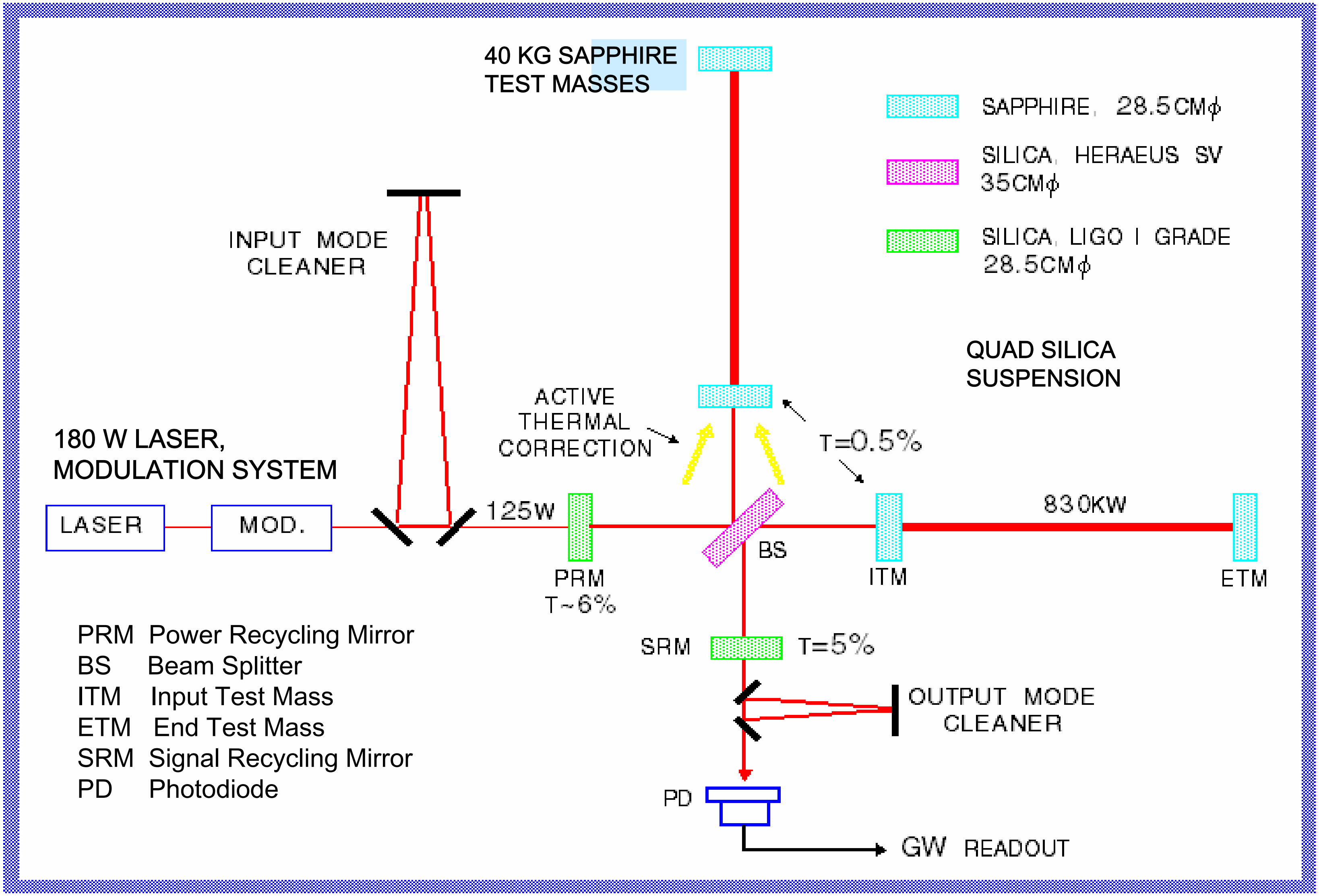
- 10W CW TEM00 CO2 Laser (10.6mm)

Time Line



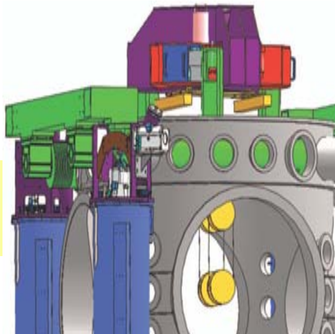
1999
2000
2001
2002
2003
3Q 4Q 1Q 2Q 3Q 4Q 1Q 2Q 3Q 4Q 1Q 2Q 3Q 4Q 1Q 2Q 3Q 4Q
Inauguration
First Lock
Full Lock all IFO's
Now
strain noise density @ 200 Hz [Hz$^{-1/2}$]
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
Engineering
E1 E2 E3 E4 E5 E6 E7 E8 E9 E10
Runs
Science
S1 S2 S3
First Science Data

Advanced LIGO: Dual Recycling

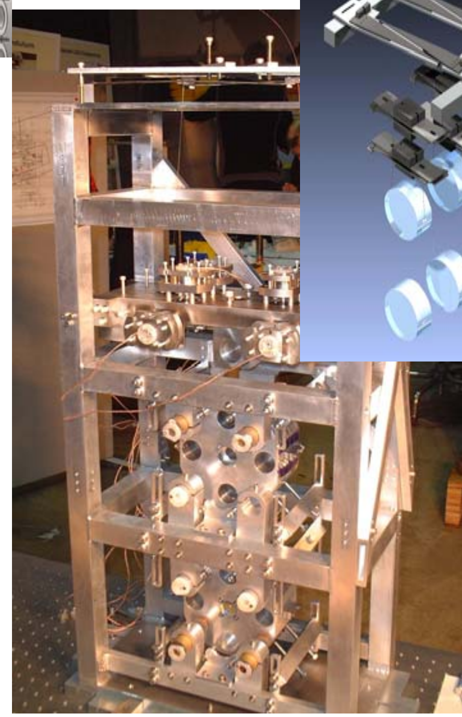


Advanced LIGO

Active Seismic

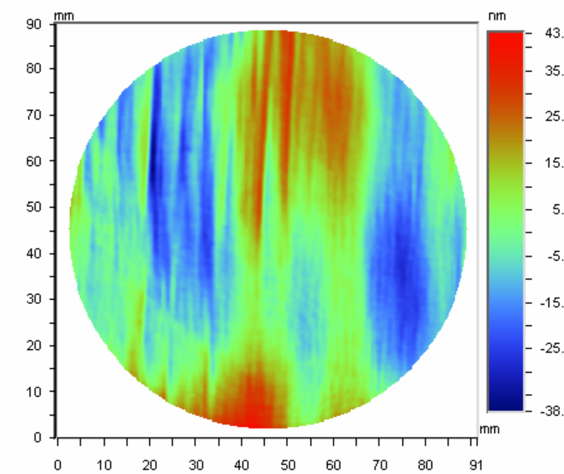


Multiple Suspensions



Silica Suspension

Sapphire Optics



Date: 10/25/2001
Time: 13:59:18
Wavelength: 1.064 um
Pupil: 100.0 %
PV: 81.6271 nm
RMS: 13.2016 nm

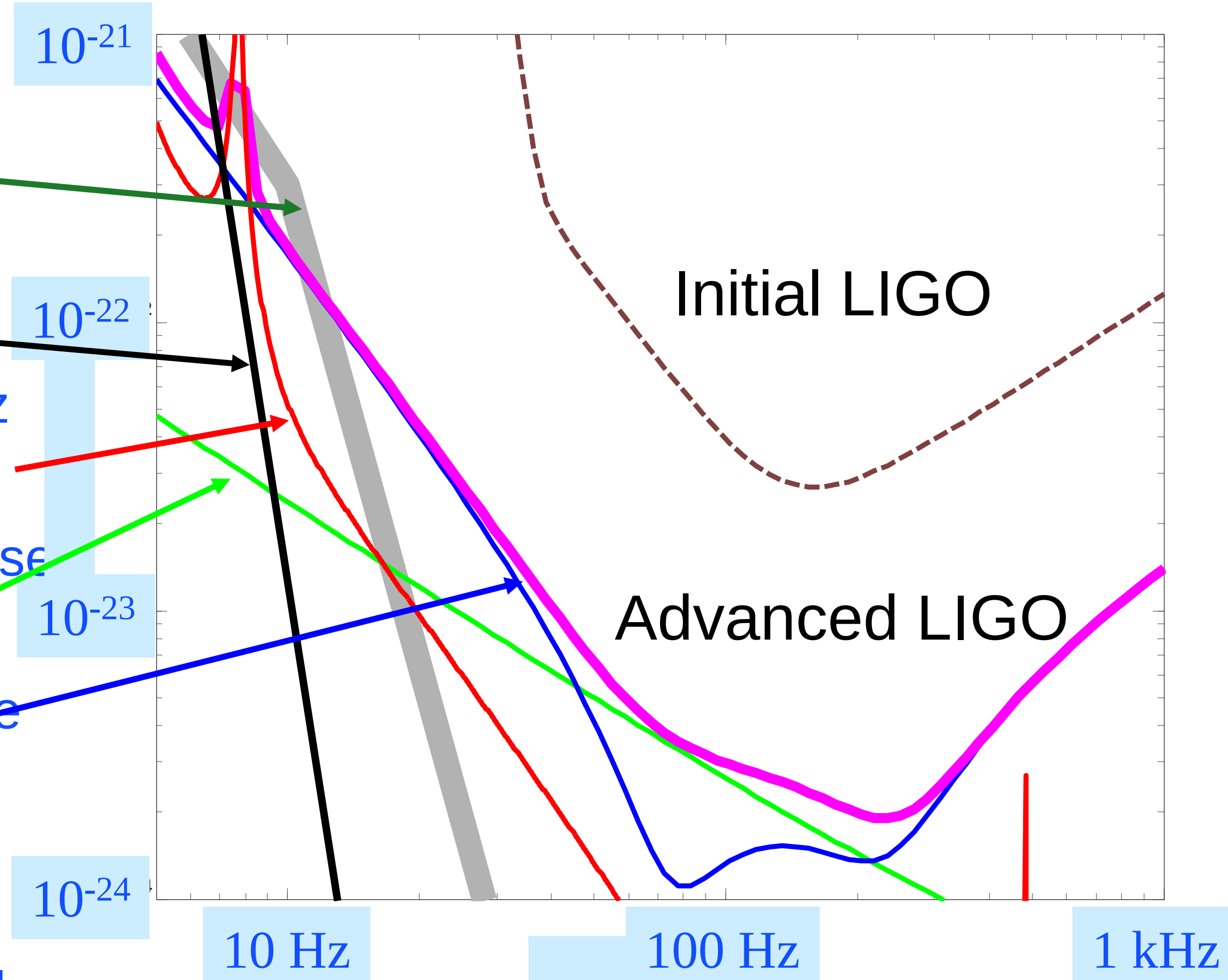
X Center: 172.00
Y Center: 145.00
Radius: 163.00 pix
Terms: None
Filters: None
Masks:

Higher Power Laser

Dual recycling

Anatomy of the projected Advanced LIGO detector performance

- Newtonian background, estimate for LIGO sites
- Seismic ‘cutoff’ at 10 Hz
- Suspension thermal noise
- Test mass thermal noise
- Unified quantum noise dominates at most frequencies for full power, broadband tuning



How much of sky can we see?

Improve amplitude sensitivity by a factor of 10x, and…

⇒ Number of sources goes up 1000x!