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POST-NEWTONIAN DYNAMICS (MOTION AND RADIATION) OF COMPACT BINARIES

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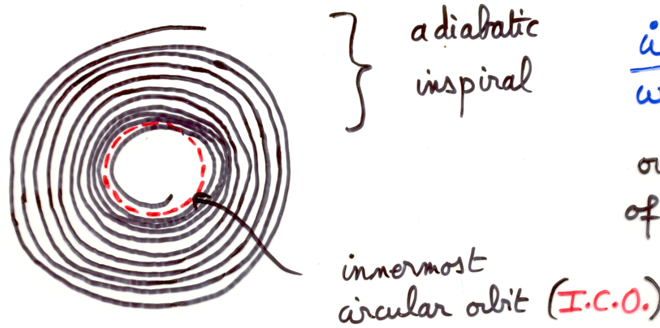
- POST-NEWTONIAN SOLUTION OF THE FIELD EQUATION
- CURRENT EXPRESSIONS NEEDED FOR THE TEMPLATES OF INSPIRALLING COMPACT BINARIES
- PROBLEM OF SELF-FIELD REGULARIZATION AT THE 3PN ORDER
- CONVERGENCE OF THE POST-NEWTONIAN EXPANSION

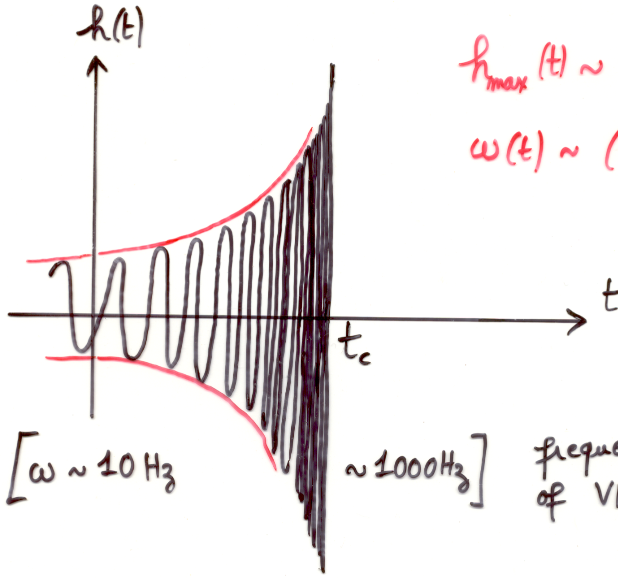
COMPACT BINARY INSPIRAL

Two compact objects describe an inspiralling orbit because of the loss of energy by gravitational radiation



$$\frac{\dot{\omega}}{\omega^2} = \mathcal{O}\left(\frac{1}{c^5}\right) \ll 1$$

order of magnitude of radiation reaction



$h_{max}(t) \sim (t_c - t)^{-1/4}$

$\omega(t) \sim (t_c - t)^{-3/8}$

using quadrupole formalism

$[\omega \sim 10\,\mathrm{Hz}$ $\sim 1000\,\mathrm{Hz}]$ frequency bandwidth of VIRGO

<u>Theoretical problem:</u> to compute the frequency and phase evolution (i.e. $\omega(t)$ and $\phi(t) = \int \omega(t)\,dt$) up to high post-Newtonian order say $3PN = \mathcal{O}\left(\frac{1}{c^6}\right)$

OUTLINE OF THE METHOD

- FIND THE SOLUTION OF EINSTEIN'S FIELD EQUATION

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu} \Rightarrow \nabla_\nu T^{\mu\nu} = 0$$

eq. of motion of matter source

(in a suitable coordinate system), AND VALID

  - FOR GENERAL <u>COMPACT-SUPPORT</u> AND <u>REGULAR</u> i.e. C^∞ MATTER TENSOR (describing some isolated hydrodynamical fluid)

  - IN THE FORM OF A FORMAL POST-NEWTONIAN EXPANSION (limited say to 3PN order)

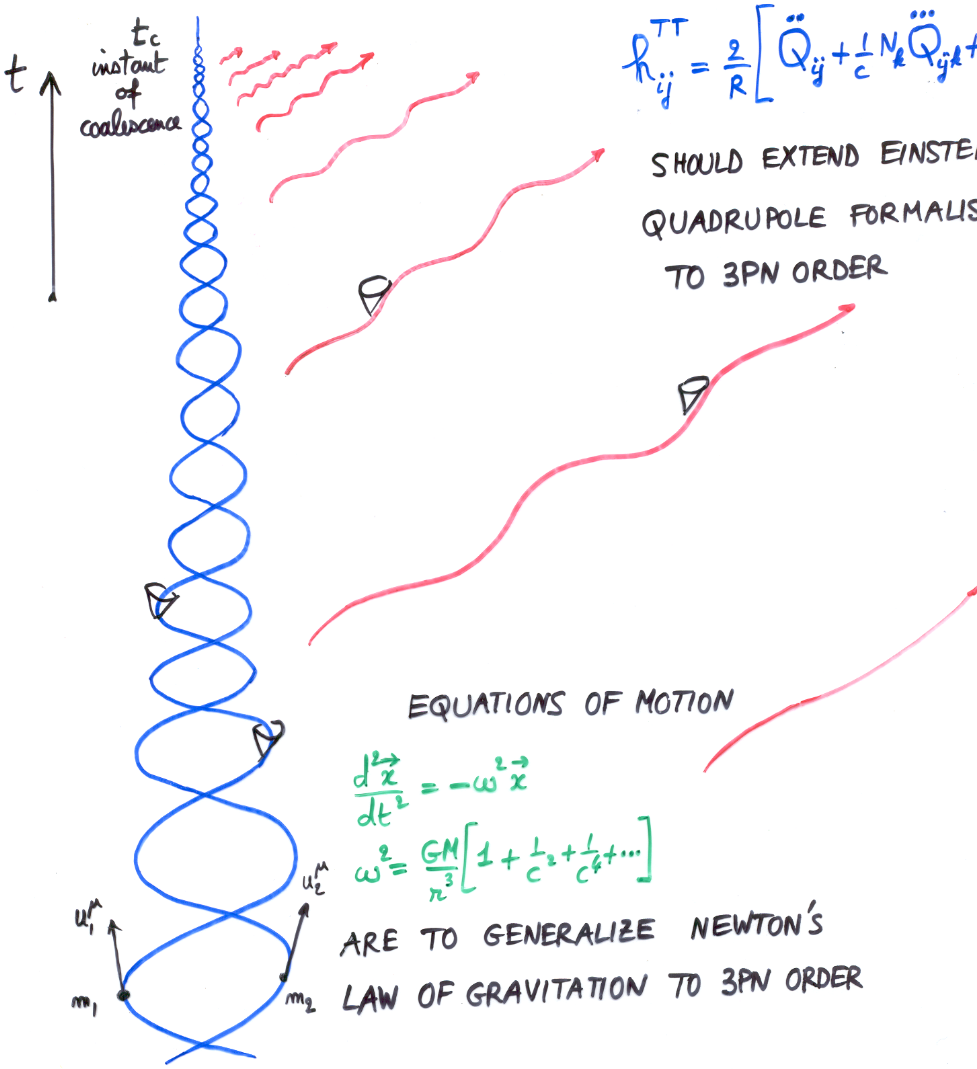
- APPLY THE LATTER SOLUTION TO A BINARY SYSTEM OF <u>POINT-PARTICLES</u>

DIRAC DELTA-FUNCTION

$$T^{\mu\nu} = \sum_{A=1,2} \frac{m_A v_A^\mu v_A^\nu}{\sqrt{-(g_{\rho\sigma})_A v_A^\rho v_A^\sigma}} \frac{\delta(\vec{x}-\vec{y}_A(t))}{\sqrt{-g}}$$

- SUPLEMENT THE CALCULATION WITH AN ASSUMPTION OF <u>SELF-FIELD REGULARIZATION</u> (to remove infinite self-field of point particles)

PROBLEMS OF DYNAMICS AND WAVE EMISSION



RADIATION FIELD

$$h_{ij}^{TT} = \frac{2}{R}\left[\ddot{Q}_{ij} + \frac{1}{c} N_k \dddot{Q}_{ijk} + \cdots\right]^{TT}$$

SHOULD EXTEND EINSTEIN'S QUADRUPOLE FORMALISM TO 3PN ORDER

EQUATIONS OF MOTION

$$\frac{d^2\vec{x}}{dt^2} = -\omega^2 \vec{x}$$

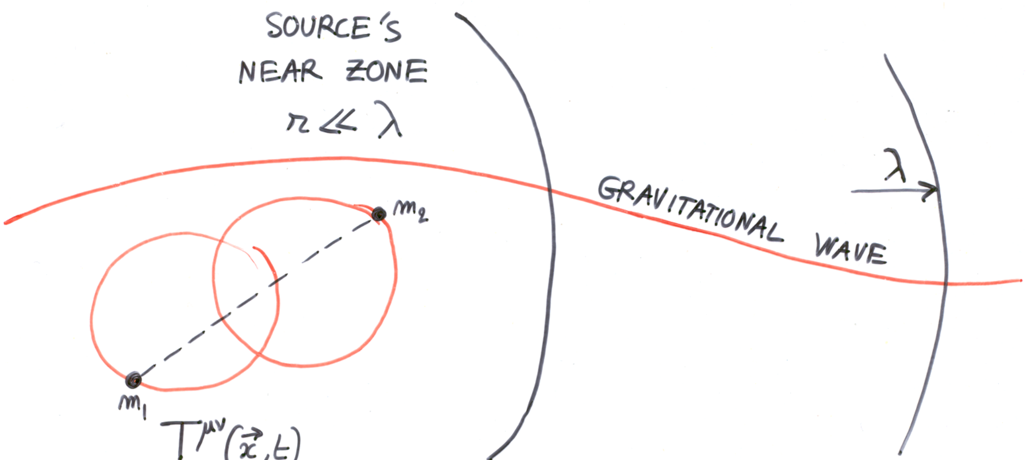
$$\omega^2 = \frac{GM}{r^3}\left[1 + \frac{1}{c^2} + \frac{1}{c^4} + \cdots\right]$$

ARE TO GENERALIZE NEWTON'S LAW OF GRAVITATION TO 3PN ORDER

ENERGY BALANCE ARGUMENT $\quad \frac{dE}{dt} = -F$

PHASING FORMULA $\quad \phi = \int \omega\, dt = -\int \frac{\omega\, dE}{F}$

POST-NEWTONIAN EXPANSION OF NEAR-ZONE FIELD



$T^{\mu\nu}(\vec{x},t)$

such that

$$\varepsilon = \mathrm{Max}\left\{ \frac{T^{0i}}{T^{00}}, \left(\frac{T^{ij}}{T^{00}}\right)^{1/2}, \left(\frac{U}{c^2}\right)^{1/2} \right\} \ll 1$$

(in practice $\varepsilon \lesssim 0.5$)

THE NEAR-ZONE COVERS ENTIRELY THE SOURCE

$$\varepsilon \sim \frac{v}{c} \sim \frac{a}{\lambda} \ll 1$$

FORMAL POST-NEWTONIAN EXPANSION $\left(h^{\alpha\beta} \equiv \sqrt{-g}\, g^{\alpha\beta} - \eta^{\alpha\beta}\right)$

$$\boxed{\overline{h}^{\alpha\beta}(\vec{x},t,c) = \sum_{n=2}^{+\infty} \frac{1}{c^n}\, \overline{h}_n^{\alpha\beta}(\vec{x},t,\ln c)}$$

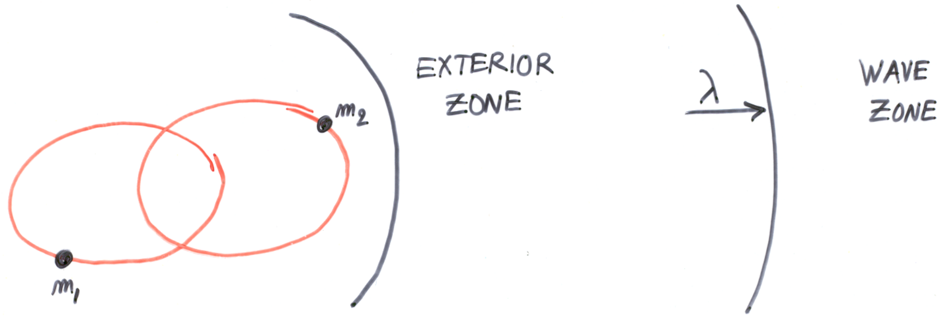
THIS IS A NEAR-ZONE EXPANSION (Fock 1959)

TYPICALLY $\left|\overline{h}_n^{\alpha\beta}\right| \longrightarrow +\infty$ WHEN $|\vec{x}| \longrightarrow +\infty$

(POST-MINKOWSKIAN) MULTIPOLAR FIELD IN SOURCE'S EXTERIOR

$$\boxed{\mathcal{M}(h^{\alpha\beta}) = G\, h^{\alpha\beta}_{(1)}[I_L, J_L] + G^2 h^{\alpha\beta}_{(2)}[I_L, J_L] + \dots}$$

(Blanchet and Damour 1986)



SOURCE IS DESCRIBED BY SOURCE MULTIPOLE MOMENTS

$I_L(t) \quad J_L(t)$

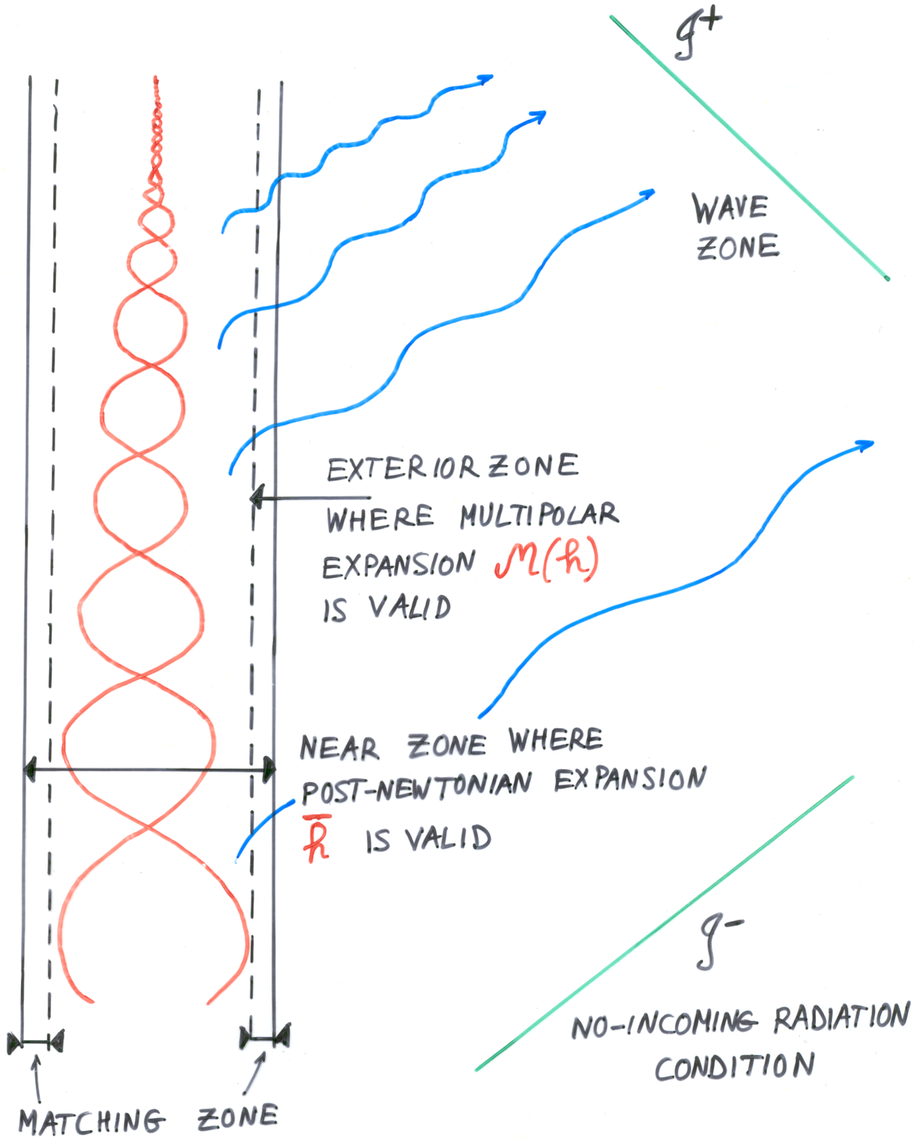
RADIATION FIELD IS DESCRIBED BY RADIATIVE MOMENTS

$U_L(t) \quad V_L(t)$

TWO PROBLEMS

- RELATING I_L, J_L TO THE STRESS-ENERGY TENSOR OF THE SOURCE $T^{\mu\nu}$
- FINDING THE RADIATIVE MOMENTS U_L, V_L IN TERMS OF THE SOURCE ONES I_L, J_L

MATCHING TO A POST-NEWTONIAN SOURCE



MATCHING EQUATION $\boxed{\mathcal{M}(\bar{h}) = \overline{\mathcal{M}(h)}}$

GENERAL SOLUTION OF THE MATCHING EQUATION

(Blanchet 1995; 1998
Poujade and Blanchet 2002)

$$\mathcal{M}(h) = \sum_{\ell=0}^{+\infty} \partial_L \left(\frac{1}{r} \mathcal{F}_L \left(t - \frac{r}{c}\right) \right)$$ ← "LINEARIZED" MULTIPOLE EXPANSION

$$+ \underset{B=0}{\mathrm{FP}} \; \Box^{-1}_{\mathrm{Ret}} \left[r^B \mathcal{M}(\Lambda) \right]$$ ← DUE TO NON-LINEARITIES IN THE FIELD

THE SOURCE MULTIPOLE MOMENTS I_L, J_L ARE DEDUCED FROM

$$\boxed{\mathcal{F}_L(t) = \underset{B=0}{\mathrm{FP}} \int d^3x \, |\vec{x}|^B \, x_L \, \overline{\tau}(\vec{x}, t)}$$

post-Newtonian expansion of pseudo-tensor

WHERE

$$\tau^{\mu\nu} = T^{\mu\nu} + \frac{c^4}{16\pi G} \Lambda^{\mu\nu}(h, \partial h, \partial^2 h)$$

STRESS-ENERGY PSEUDO-TENSOR OF MATTER AND GRAVITATIONAL FIELDS

It can be shown that the formalism is <u>equivalent</u> to the one de Will and Wiseman (1996).

POST-NEWTONIAN WAVE FLUX

It is given in terms of the radiative moments

$$F = \frac{G}{c} \sum_{\ell=2}^{+\infty} \left\{ \frac{1}{c^{2\ell}} \left(\dot{U}_L \right)^2 + \frac{1}{c^{2\ell+2}} \left(\dot{V}_L \right)^2 \right\}$$

The radiative moments are given in terms of source moments

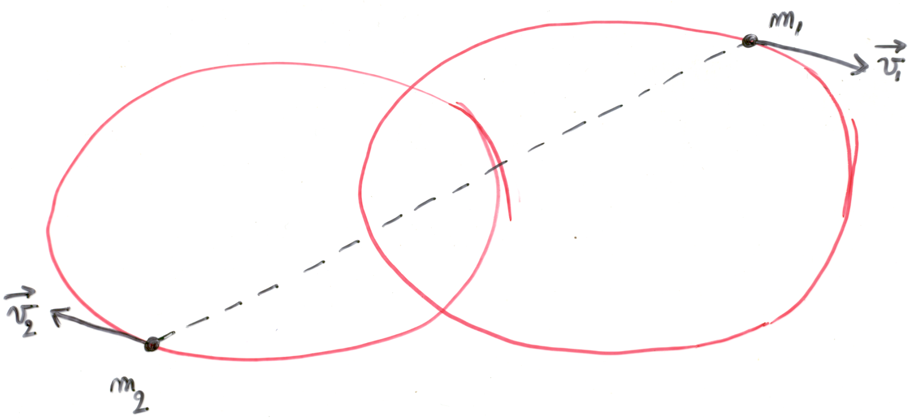
$$U_{ij}(t) = \ddot{I}_{ij}(t) + \underbrace{\frac{2GM}{c^3} \int_{-\infty}^{t} dt' \, \ddddot{I}_{ij}(t') \ln\left(\frac{t-t'}{2b}\right)}_{\text{tail integral}}$$

$$+ \frac{G}{c^5} \left\{ \underbrace{- \frac{2}{7} \int_{-\infty}^{t} dt' \, \dddot{I}_{k<i}(t') \, \dddot{I}_{j>k}(t')}_{\text{non-linear memory integral}} + \cdots \right\}$$

$$+ \underbrace{\frac{2G^2M^2}{c^6} \int_{-\infty}^{t} dt' \, \overset{(5)}{I}_{ij}(t') \left\{ \ln^2\left(\frac{t-t'}{2b}\right) + \ln\left(\frac{t-t'}{2b}\right) \right\}}_{\text{tail-of-tail integral}}$$

$$+ \mathcal{O}\left(\frac{1}{c^7}\right)$$

POST-NEWTONIAN EQUATIONS OF MOTION



The equations are expressed in Newtonian-like form, by means of the coordinate positions and velocities of particles.

$$\frac{dv_1^i}{dt} = A_N^i + \frac{1}{c^2} A_{1PN}^i + \frac{1}{c^4} A_{2PN}^i + \frac{1}{c^5} A_{2.5PN}^i + \frac{1}{c^6} A_{3PN}^i + \mathcal{O}\left(\frac{1}{c^7}\right)$$

The equations must

- possess the correct perturbative limit when $\nu = \frac{\mu}{M} \to 0$ (geodesics of Schwarzschild)
- stay invariant under a global Lorentz transformation
- be derivable from a Lagrangian when the radiation-reaction term is neglected

$$
\begin{aligned}
A^i_{3\mathrm{PN}} \;=\; & \Big[\frac{Gm_2}{r_{12}^2}\Big(\frac{35}{16}(n_{12}v_2)^6 - \frac{15}{8}(n_{12}v_2)^4 v_1^2 + \frac{15}{2}(n_{12}v_2)^4(v_1v_2) + 3(n_{12}v_2)^2(v_1v_2)^2 \\
- \; & \frac{15}{2}(n_{12}v_2)^4 v_2^2 + \frac{3}{2}(n_{12}v_2)^2 v_1^2 v_2^2 - 12(n_{12}v_2)^2(v_1v_2)v_2^2 - 2(v_1v_2)^2 v_2^2 \\
+ \; & \frac{15}{2}(n_{12}v_2)^2 v_2^4 + 4(v_1v_2)v_2^4 - 2v_2^6\Big) + \frac{G^2m_1m_2}{r_{12}^3}\Big(-\frac{171}{8}(n_{12}v_1)^4 \\
+ \; & \frac{171}{2}(n_{12}v_1)^3(n_{12}v_2) - \frac{723}{4}(n_{12}v_1)^2(n_{12}v_2)^2 + \frac{383}{2}(n_{12}v_1)(n_{12}v_2)^3 \\
- \; & \frac{455}{8}(n_{12}v_2)^4 + \frac{229}{4}(n_{12}v_1)^2 v_1^2 - \frac{205}{2}(n_{12}v_1)(n_{12}v_2)v_1^2 + \frac{191}{4}(n_{12}v_2)^2 v_1^2 - \frac{91}{8}v_1^4 \\
- \; & \frac{229}{2}(n_{12}v_1)^2(v_1v_2) + 244(n_{12}v_1)(n_{12}v_2)(v_1v_2) - \frac{225}{2}(n_{12}v_2)^2(v_1v_2) \\
+ \; & \frac{91}{2}v_1^2(v_1v_2) - \frac{177}{4}(v_1v_2)^2 + \frac{229}{4}(n_{12}v_1)^2 v_2^2 - \frac{283}{2}(n_{12}v_1)(n_{12}v_2)v_2^2 \\
+ \; & \frac{259}{4}(n_{12}v_2)^2 v_2^2 - \frac{91}{4}v_1^2 v_2^2 + 43(v_1v_2)v_2^2 - \frac{81}{8}v_2^4\Big) + \frac{G^2m_2^2}{r_{12}^3}\Big(-6(n_{12}v_1)^2(n_{12}v_2)^2 \\
+ \; & 12(n_{12}v_1)(n_{12}v_2)^3 + 6(n_{12}v_2)^4 + 4(n_{12}v_1)(n_{12}v_2)(v_1v_2) + 12(n_{12}v_2)^2(v_1v_2) \\
+ \; & 4(v_1v_2)^2 - 4(n_{12}v_1)(n_{12}v_2)v_2^2 - 12(n_{12}v_2)^2 v_2^2 - 8(v_1v_2)v_2^2 + 4v_2^4\Big) \\
+ \; & \frac{G^3m_2^3}{r_{12}^4}\Big(-(n_{12}v_1)^2 + 2(n_{12}v_1)(n_{12}v_2) + \frac{43}{2}(n_{12}v_2)^2 + 18(v_1v_2) - 9v_2^2\Big) \\
+ \; & \frac{G^3m_1m_2^2}{r_{12}^4}\Big(\frac{415}{8}(n_{12}v_1)^2 - \frac{375}{4}(n_{12}v_1)(n_{12}v_2) + \frac{1113}{8}(n_{12}v_2)^2 \\
- \; & \frac{615}{64}(n_{12}v_{12})^2\pi^2 + \frac{123}{64}v_{12}^2\pi^2 + 18v_1^2 + 33(v_1v_2) - \frac{33}{2}v_2^2\Big) \\
+ \; & \frac{G^3m_1^2m_2}{r_{12}^4}\Big(-\frac{45887}{168}(n_{12}v_1)^2 + \frac{24025}{42}(n_{12}v_1)(n_{12}v_2) - \frac{10469}{42}(n_{12}v_2)^2 \\
+ \; & \frac{48197}{840}v_1^2 - \frac{36227}{420}(v_1v_2) + \frac{36227}{840}v_2^2 + 110(n_{12}v_{12})^2\ln\left(\frac{r_{12}}{r'_1}\right) - 22v_{12}^2\ln\left(\frac{r_{12}}{r'_1}\right)\Big) \\
+ \; & \frac{G^4m_1^3m_2}{r_{12}^5}\Big(-\frac{3187}{1260} + \frac{44}{3}\ln\left(\frac{r_{12}}{r'_1}\right)\Big) + \frac{G^4m_1^2m_2^2}{r_{12}^5}\Big(\frac{34763}{210} - \frac{44}{3}\lambda - \frac{41}{16}\pi^2\Big) \\
+ \; & \frac{G^4m_1m_2^3}{r_{12}^5}\Big(\frac{10478}{63} - \frac{44}{3}\lambda - \frac{41}{16}\pi^2 - \frac{44}{3}\ln\left(\frac{r_{12}}{r'_2}\right)\Big) + 16\frac{G^4m_2^4}{r_{12}^5}\Big]n^i_{12} \\
+ \; & \Big[\frac{Gm_2}{r_{12}^2}\Big(\frac{15}{2}(n_{12}v_1)(n_{12}v_2)^4 - \frac{45}{8}(n_{12}v_2)^5 - \frac{3}{2}(n_{12}v_2)^3 v_1^2 + 6(n_{12}v_1)(n_{12}v_2)^2(v_1v_2) \\
- \; & 6(n_{12}v_2)^3(v_1v_2) - 2(n_{12}v_2)(v_1v_2)^2 - 12(n_{12}v_1)(n_{12}v_2)^2 v_2^2 + 12(n_{12}v_2)^3 v_2^2 \\
+ \; & (n_{12}v_2)v_1^2 v_2^2 - 4(n_{12}v_1)(v_1v_2)v_2^2 + 8(n_{12}v_2)(v_1v_2)v_2^2 + 4(n_{12}v_1)v_2^4 - 7(n_{12}v_2)v_2^4\Big) \\
+ \; & \frac{G^2m_2^2}{r_{12}^3}\Big(-2(n_{12}v_1)^2(n_{12}v_2) + 8(n_{12}v_1)(n_{12}v_2)^2 + 2(n_{12}v_2)^3 \\
+ \; & 2(n_{12}v_1)(v_1v_2) + 4(n_{12}v_2)(v_1v_2) - 2(n_{12}v_1)v_2^2 - 4(n_{12}v_2)v_2^2\Big) \\
+ \; & \frac{G^2m_1m_2}{r_{12}^3}\Big(-\frac{243}{4}(n_{12}v_1)^3 + \frac{565}{4}(n_{12}v_1)^2(n_{12}v_2) - \frac{269}{4}(n_{12}v_1)(n_{12}v_2)^2 \\
- \; & \frac{95}{12}(n_{12}v_2)^3 + \frac{207}{8}(n_{12}v_1)v_1^2 - \frac{137}{8}(n_{12}v_2)v_1^2 - 36(n_{12}v_1)(v_1v_2) \\
+ \; & \frac{27}{4}(n_{12}v_2)(v_1v_2) + \frac{81}{8}(n_{12}v_1)v_2^2 + \frac{83}{8}(n_{12}v_2)v_2^2\Big) + \frac{G^3m_2^3}{r_{12}^4}\Big(4(n_{12}v_1) + 5(n_{12}v_2)\Big) \\
+ \; & \frac{G^3m_1m_2^2}{r_{12}^4}\Big(-\frac{307}{8}(n_{12}v_1) + \frac{479}{8}(n_{12}v_2) + \frac{123}{32}(n_{12}v_{12})\pi^2\Big) \\
+ \; & \frac{G^3m_1^2m_2}{r_{12}^4}\Big(\frac{31397}{420}(n_{12}v_1) - \frac{36227}{420}(n_{12}v_2) - 44(n_{12}v_{12})\ln\left(\frac{r_{12}}{r'_1}\right)\Big)\Big]v^i_{12}
\end{aligned}
$$

self-field regularization ambiguity

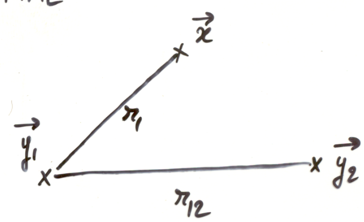
r'_1 and r'_2 are gauge constants

(Blanchet and Faye 2000)

PROBLEM OF SELF-FIELD REGULARIZATION

CONSIDER THE NEWTONIAN POTENTIAL

$$U(\vec{x}) = \frac{Gm_1}{r_1} + \frac{Gm_2}{r_2}$$



- HOW TO COMPUTE THE VALUE OF A SINGULAR FUNCTION AT A SINGULAR POINT ?

$$U(\vec{y}_1) \quad ? \qquad\qquad U(\vec{y}_1) = \frac{Gm_2}{r_{12}} \quad ?$$

- WHAT IS THE MEANING OF

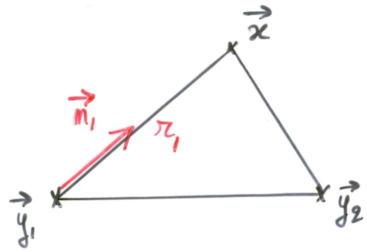
$$U(\vec{x})\,\delta(\vec{x}-\vec{y}_1) \quad ? \qquad \left(\text{ill-defined object in distribution theory}\right)$$

- HOW TO PERFORM PARTIAL DERIVATIVES ?

$$\partial_i \partial_j U = \left(\partial_i \partial_j U\right)_{\text{ordinary}} + \frac{4\pi}{3}\delta_{ij}\left[Gm_1\,\delta(\vec{x}-\vec{y}_1) + 1 \leftrightarrow 2\right] \quad ?$$

HADAMARD SELF-FIELD REGULARIZATION

(Hadamard 1932; Schwartz 1978)
see also review by Sellier 1993



Let $F(\vec{x})$ be a function singular at source points $\vec{y}_1$ and $\vec{y}_2$.

When $r_1 \to 0$
$$F(\vec{x}) = \sum_{p_0 \leq p \leq N} r_1^p \, \underset{1}{f_p}(\vec{n}_1) + o(r_1^N)$$

- Hadamard's partie finie of F at 1:

$$(F)_1 \equiv \int \frac{d\Omega_1}{4\pi} \, \underset{1}{f_0}(\vec{n}_1)$$

- Hadamard's partie finie of $\int d^3x \, F$:

$$\underset{s_1 s_2}{\mathrm{Pf}} \int d^3x \, F \equiv \lim_{s \to 0} \left\{ \int_{\mathbb{R}^3 - B_1(s) \cup B_2(s)} d^3x \, F - \sum_{p < -3} \frac{s^{p+3}}{p+3} \left(\frac{F}{r_1^p} \right)_1 - \ln\left(\frac{s}{s_1}\right) (r_1^3 F)_1 \right\} + 1 \leftrightarrow 2$$

- "Non_distributivity" of Hadamard's regularization

$$(FG)_1 \neq (F)_1 (G)_1$$

Jaranowski and Schäfer (1998) noticed that this is a source of "ambiguity" for certain integrals occuring at 3PN order.

A NEW HADAMARD-TYPE REGULARIZATION

(Blanchet and Faye 2000; 2001)

- Let $\mathcal{F}$ be the space of singular functions such as F. If $F \in \mathcal{F}$ we introduce the <u>pseudo-function</u> $\mathrm{Pf}\, F$

$$\forall G \in \mathcal{F}, \qquad \langle \mathrm{Pf}\, F, G \rangle = \mathrm{Pf} \int d^3x\, FG$$

When restricted to smooth functions (with compact support), $\mathrm{Pf}\, F$ is a distribution in Schwartz's sense.

- The pseudo-function $\mathrm{Pf}\, \delta_1$ is defined by

$$\forall F \in \mathcal{F}, \qquad \langle \mathrm{Pf}\, \delta_1, F \rangle = (F)_1$$

- Derivatives of pseudo-functions obey the rule of "integration by parts"

$$\forall F, G \in \mathcal{F}, \quad \langle \partial_i (\mathrm{Pf}\, F), G \rangle = - \langle \partial_i (\mathrm{Pf}\, G), F \rangle$$

Using this regularization all integrals at 3PN order are computed "unambiguously". But the formalism introduces some arbitrary constants.

REGULARIZATION AMBIGUITIES AT 3PN ORDER

- Jaranowski and Schäfer (1998; 1999) introduce <u>two</u> constants unknown in the 3PN ADM-Hamiltonian

  $\omega_{kinetic} \qquad \omega_{static}$

- Blanchet and Faye (2000ab) find <u>one</u> constant λ in the 3PN equations of motion. Comparison with the ADM-Hamiltonian shows

$$\boxed{\begin{aligned} \omega_{kinetic} &= \frac{41}{24} \\ \omega_{static} &= -\frac{11}{3}\lambda - \frac{1987}{840} \end{aligned}}$$

- Damour, Jaranowski and Schäfer (2000) recover the value of $\omega_{kinetic}$ by imposing the Lorentz-invariance of the ADM-Hamiltonian.

- Blanchet, Iyer and Joguet (2002) find <u>three</u> constants ξ, κ, ζ in the 3PN mass quadrupole but only one combination is relevant to circular binaries

$$\boxed{\theta = \xi + 2\kappa + \zeta}$$

WHAT IS KNOWN ABOUT λ, θ

- $\omega_{static} \approx -9$ guessed by Damour et al (2001) in order that post-Newtonian resummation techniques (Padé approximants and Effective-one-body approach) agree at 3PN order

- $$\boxed{\omega_{static} = 0 \iff \lambda = -\frac{1987}{3080}}$$

  computed in GR by

dimensional regularization (instead of Hadamard's)
- in the ADM-Hamiltonian formalism (Damour, Jaranowski, Schäfer 2001)
- in the equations of motion in harmonic coordinates (Blanchet, Damour, Esposito-Farèse 2003)

- surface-integral approach applicable to compact binaries with strong internal gravity (Itoh and Futamase 2003, Itoh 2003)

- Nothing is known about the value of θ

DIMENSIONAL REGULARIZATION

- A beautiful technique for fixing the incompleteness/ambiguity of Hadamard regularization.

- Cannot be used alone (at 3PN order) but necessitates beforehand the computation of the finite part when

$$\varepsilon \equiv d-3 \longrightarrow 0$$

which is nothing but the Hadamard <u>partie finie</u>.

- For instance, in the case of a Poisson integral

$$P(\vec{x}) = \Delta^{-1} F(\vec{x})$$

one computes the <u>difference</u> (at point $\vec{x}=\vec{y}_1$)

$$\boxed{\mathcal{D}P(\vec{y}_1) \equiv \underbrace{P^{(3+\varepsilon)}(\vec{y}_1)}_{\substack{\text{value of potential} \\ \text{in } d=3+\varepsilon}} - \underbrace{(P)_1}_{\substack{\text{partie finie in} \\ \text{Hadamard's sense}}}}$$

This yields simple poles (at 3PN) and crucial terms linked with the finite part

$$\mathcal{D}P(\vec{y}_1) = \frac{a_{-1}}{\varepsilon} + a_0 + \mathcal{O}(\varepsilon)$$

DIMREG APPLIED TO THE EQUATIONS OF MOTION

- Blanchet and Faye (2000ab) obtain the acceleration of particle 1 as

$$\vec{a}_1^{(BF)}(\lambda) = \underbrace{\vec{a}_1^{(\text{pure Had})}(\lambda)}_{\substack{\text{due to standard Hadamard}\\ \text{regularization, where } \lambda \text{ comes}\\ \text{from a certain logarithmic ratio}}} + \underbrace{\vec{a}_1^{(\text{non distr})} + \vec{a}_1^{(\text{Lorentz})}}_{\substack{\text{corrections due to improved}\\ \text{version of Hadamard}\\ \text{regularization}}}$$

- Applying DimReg we compute the difference

$$\mathcal{D}\vec{a}_1 \equiv \vec{a}_1^{(\text{DimReg})} - \vec{a}_1^{(\text{pure Had})}$$

and require

$$\boxed{\vec{a}_1^{(\text{DimReg})} = \vec{a}_1^{(BF)} + \underbrace{\delta_\xi \vec{a}_1}_{\substack{\text{change of gauge}\\ \text{in } d=3+\varepsilon \text{ dimensions}}} + \mathcal{O}(\varepsilon)}$$

- This yields an equation for λ

$$\mathcal{D}\vec{a}_1(\lambda) = \vec{a}_1^{(\text{non distr})} + \vec{a}_1^{(\text{Lorentz})} + \underbrace{\delta_\xi \vec{a}_1}_{\substack{\text{contains all the}\\ \text{poles} \sim 1/\varepsilon}}$$

which is solved uniquely by

$$\boxed{\lambda = -\frac{1987}{3080}}$$

(Blanchet, Damour and Esposito-Farèse 2003, in perfect agreement with Damour, Jaranowski and Schäfer 2001).
and Itoh and Futamase 2003

CENTER-OF-MASS ENERGY AT 3PN ORDER

POST-NEWTONIAN PARAMETER $x \equiv \left(\frac{GM\omega}{c^3}\right)^{2/3}$

SYMMETRIC MASS RATIO $\nu \equiv \mu/M$ such that $0 < \nu \leq \frac{1}{4}$

$$\boxed{\begin{aligned} E = -\frac{\mu x}{2}\Big\{ & 1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right)x^2 \\ & + \left(-\frac{675}{64} + \left[\frac{209323}{4032} - \frac{205}{96}\pi^2 - \frac{110}{9}\lambda\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right)x^3 \\ & + \mathcal{O}(x^4)\Big\} \end{aligned}}$$

MINIMUM OF $E(x)$ DEFINES THE I.C.O.

AT THE I.C.O. WE HAVE $v_{ICO} \sim 0.5c$

HENCE $x_{ICO} \sim 0.2$

RELATIVE CONTRIBUTION OF 3PN TERM IS

$$\boxed{3PN = (\text{coefficient of order } 1)\, x^3 \sim 0.01}$$

WHICH GIVES AN ESTIMATE OF ACCURACY OF 3PN APPROXIMATION.

GRAVITATIONAL-WAVE FLUX TO 3.5PN ORDER

We pose $x \equiv \left(\frac{GM\omega}{c^3}\right)^{2/3}$ and $\nu \equiv \frac{\mu}{M}$ such that $0 < \nu \leq \frac{1}{4}$

$$\mathcal{F} = \frac{32c^5}{5G} x^5 \nu^2 \Big\{ 1 + \overbrace{\left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x}^{1\,\text{PN}}$$

Wagoner and Will 1976; Blanchet and Schäfer 1989

$$+ \overbrace{4\pi x^{3/2}}^{1.5\,\text{PN}}$$

Poisson 1993; Wiseman 1993; Blanchet and Schäfer 1993

$$+ \overbrace{\left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)x^2}^{2\,\text{PN}}$$

Blanchet, Damour, Iyer, Will and Wiseman 1995

TWO INDEPENDENT CALCULATIONS

$$+ \overbrace{\left(-\frac{8191}{672} - \frac{535}{24}\nu\right)\pi x^{5/2}}^{2.5\,\text{PN}}$$

$$+ \overbrace{\left(\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16x)\right.}^{3\,\text{PN}}$$

$$\left. + \left[-\frac{11497453}{272160} + \frac{41\pi^2}{48} + \frac{176}{9}\lambda - \frac{88}{3}\theta\right]\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right)x^3$$

$$+ \overbrace{\left(-\frac{16285}{504} + \frac{176419}{1512}\nu + \frac{19897}{378}\nu^2\right)\pi x^{7/2}}^{3.5\,\text{PN}}$$

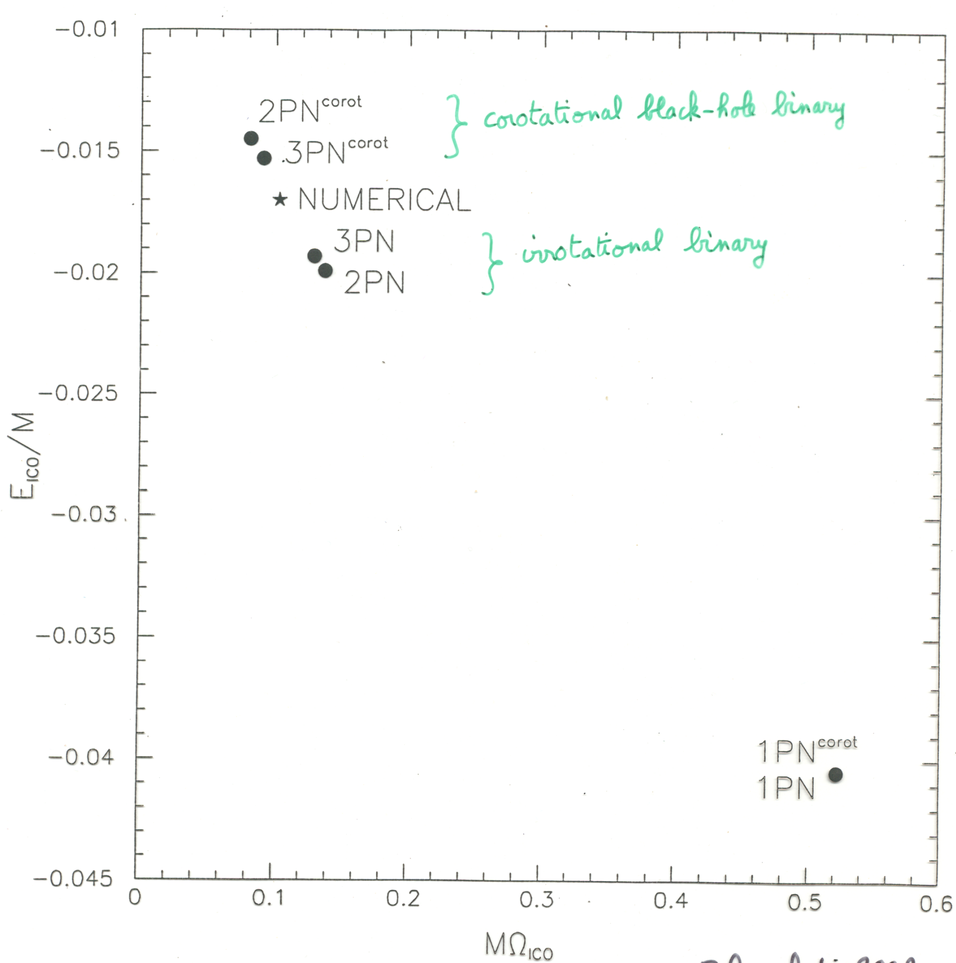
$$+ \mathcal{O}(x^4) \Big\}$$

Blanchet 1996; 1998; Blanchet, Iyer and Joguet 2001

The test-mass limit ($\nu \to 0$) is in perfect agreement with black-hole perturbations (Poisson 1993; Tagoshi and Sasaki 1994; Sasaki 1994; Tanaka, Tagoshi and Sasaki 1996)

I.C.O. = INNERMOST CIRCULAR ORBIT

= minimum of binary's energy function $E(\omega)$ for circular orbit



Blanchet 2002

NUMERICAL POINT : Gourgoulhon, Grandclément, Bonazzola (2002) obtained with helical Killing vector (HKV) and conformal flatness.